Investigating Dynamics of Machine Tool Spindles under Operational Conditions

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Abstract. Chatter is one of the major problems in machining and can be avoided by stability diagrams which are generated using frequency response functions (FRF) at the tool tip. During cutting operations, discrepancies between the stability diagrams obtained by using FRFs measured at the idle state and the actual stability of the process are frequently observed. These deviations can be attributed to the changes of machine dynamics under cutting conditions. In this paper, the effects of the cutting process on the spindle dynamics are investigated both experimentally and analytically. The variations in the spindle dynamics are attributed to the changes in the bearing parameters. FRFs under cutting conditions are obtained through the input-output relations of the cutting forces and the vibration response which are measured simultaneously. Experimentally and analytically obtained FRFs are then used in the identification of the bearing parameters under cutting conditions. Thus, bearing properties obtained at idle and cutting conditions are compared and variations in their values are obtained.

Introduction

Chatter continues to be a major problem causing poor surface finish and low material removal rate for machining applications. In order to avoid chatter, mechanisms of the dynamic cutting process have been examined in detail for decades [1-5], and stability lobe diagrams, which provide stable regions in terms of the depth of cut and spindle speed combinations, have been developed [3-6]. For generation of stability diagrams, frequency response functions (FRF) at the tool tip are needed. In general, tool point FRF is obtained experimentally using impact testing and modal analysis while the machine center is idle. However during cutting operations, discrepancies between the stability diagrams and actual stability of the process are frequently observed. One major contributor to these deviations is the changes of machine dynamics under cutting conditions. Spindle is one of the most dynamically flexible parts of high speed machining centers. Dynamic properties of high speed spindles may vary due to gyroscopic moments, thermal expansions and centrifugal forces. These effects appear during the cutting operation and cause significant changes both on the spindle shaft dynamics and bearing characteristics [7-8]. In order to identify the effects of the gyroscopic moments and centrifugal forces, Xiong et al. [9] used Finite element modeling (FEM) and also Movahhedy and Mosaddegh [10] used FEM in order to predict the chatter in high speed milling process including gyroscopic effects. In addition to the gyroscopic effects, various factors affecting the spindle dynamics during cutting are studied by the researchers [11-12]. Also, bearing characteristics under high rotational speeds affect the spindle FRF [13-14]. During the cutting process, centrifugal forces and gyroscopic forces act on the balls of the bearings and centrifugal forces press the balls toward the outer race. This effect causes to change in the contact angles and kinematics of the balls as well as redistributing the contact loads in the bearing which leads to decrease in the stiffness of the bearings [13]. On the contrary, damping of the ball bearings increases under the rotational effects [14]. Thermal expansions may also cause variations in the contact conditions, and thus affect dynamic properties of the bearings. Since bearing properties mainly affect the spindle modes of the machine center FRF [15], both changes in the spindle and bearing dynamics should be considered for the accurate prediction of the chatter during cutting operation.

In addition to the modeling, there have been several experimental studies on the spindle dynamics under cutting conditions. For that purpose, non-contact sensors such as Laser Doppler Vibrometers (LDV) can be used in measurement of rotating structures [16-21]. Also Tatar and Gren [22] used LDV to measure the response of machining centers. Similarly, Zaghbani and Songmene [23] used operational modal analysis in the determination of the dynamics of the milling machine during cutting. However, in these studies, FRFs could not be obtained due to the harmonic content of the cutting forces in milling operations. In order to overcome the harmonic content problem of the cutting forces, Opitz and Weck [24] proposed a spectral measurement method using a workpiece which has a random surface profile. Based on the work of Opitz and Weck [24], Minis et al. [25] also used spectral measurement techniques in measuring dynamics of a lathe.

In this paper, dynamics of a milling machine spindle during operation is investigated. The observed variations are attributed to the changes in the bearing parameters which are also identified under operating conditions. For the identification of the bearing parameters, system FRFs are determined experimentally during cutting operation. In order to determine the spindle FRF, instead of exciting the system by an external exciter such as an impact hammer, actual cutting forces are measured by a dynamometer and taken as the excitation source of the system. In addition to the measured cutting forces, response of the milling machine to these forces is measured by laser vibrometer, and spindle dynamics is determined by using the cutting force - response relation with two different approaches. First, it is assumed that the system dynamics is not affected by relatively small spindle rotational speed variations and system FRF is obtained for the standard cutting operation. In addition to this approach, cutting operation is performed on a specially designed workpiece proposed by the Opitz and Weck [24], and system FRF is obtained for each spindle speed uniquely. Finally, using the analytical model of the spindle – holder – tool assembly, and experimentally obtained FRFs, bearing properties are identified for static and rotating cases. During the identification of the bearing properties, it is assumed that the changes in the spindle dynamics are due to the changes in bearing parameters under operating conditions.

Spectral Density Measurements

In dynamic systems, system FRF can be obtained by the input-output relationship. In literature there exist different types of estimators which are suitable for different types of systems and conditions [26-28]. In real life applications, determination of the system FRF is highly dependent on the accurate estimation of the noise exist in the measurement of the input and output signals. For instance, for the system given in Fig. 1, $x(t)$ and $y(t)$ are the input and output signals, respectively. $x_m(t)$ and $y_m(t)$ are the measured input and output signals of the system, respectively, and $n_x(t)$ and $n_y(t)$ represent the noises in the measured signals.

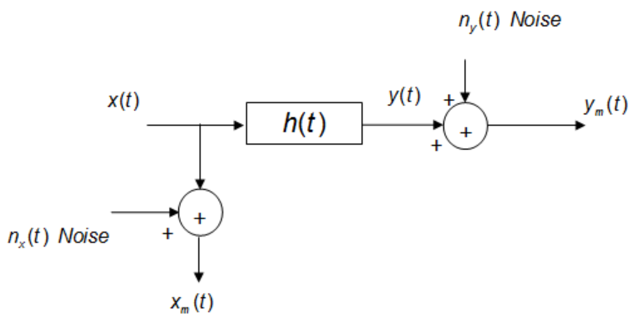

Figure 1: The effects of the measurement noise on both input and output.

In order to determine the system FRF, noise levels in the input and output signals should be examined carefully. When there exist noises both in the input and the output signal measurements as shown in Fig. 1, system FRF can be determined by the estimator H3 defined as follows [28] :

$$H_3(f)=\frac{S_{y_m y_m}(f)-\kappa(f)S_{x_m x_m}(f)}{2S_{y_m x_m}(f)}+\frac{\sqrt{\left[S_{x_m x_m}(f)\kappa(f)-S_{y_m y_m}(f)\right]^2+4\left|S_{x_m y_m}(f)\right|^2\kappa(f)}}{2S_{y_m x_m}(f)}. \quad (1)$$

where $\kappa(f)$ is the ratio of the spectra of the measurement noises in the input and output signals.

$$\kappa(f)=\frac{n_x(f)}{n_y(f)}. \quad (2)$$

In real applications, it is difficult to determine the noise ratio of the input and output signals. In such cases, it can be assumed that the ratio of the spectra of the measurement noises is unity ($\kappa(f)=1$) [28].

In system identification, in addition to the correct estimator choice, another important criterion is the consistency of the input and output signals. In cases where there are additional inputs to the system that cannot be estimated or when there is a nonlinear relation between the input and output signals, applied FRF estimators will not give correct results. At that point, coherence function between the input and output signals can provide valuable information about the accuracy of the identification process. For a linear time invariant system coherence function between input and output signals can be defined as follows:

$$\gamma_{xy}^2=\frac{\left|S_{xy}(f)\right|^2}{S_{xx}(f)S_{yy}(f)}. \quad (3)$$

where $S_{xy}(f)$ is the cross power spectrum between input and output signal. Similarly, S_{xx} and S_{yy} are power spectral density functions of the input and output signals, respectively. Generally, coherence functions greater that 0.75 can be taken as acceptable in the system identification processes.

Determination of the System FRF During Cutting

Standard Cutting. In order to determine the system dynamics during machining, the response of the system to cutting forces can be used. For this purpose, cutting forces can be measured and used as an input to the system and the response of the system to them can be measured using a laser sensor, and finally the system FRF can be obtained by using the input-output relation between cutting forces and vibration measurements.

Since spindle part of the milling machine used in the experiments (a 5-axis high speed DMG Evo 50 machining center) is placed inside a casing, response measurements are taken from the rotating holder part. First, the milling machine is excited at the tool tip by an instrumented impact hammer and the response of the system is measured at the tool holder with a laser vibrometer for the idle state of the spindle. Obtained cross FRF (G_{12}) is given in Fig. 2. Here, subscript 1 represents the response point which is on the holder, and subscript 2 represents the excitation point which is the tool tip.

In order to obtain tool point – holder cross FRF during machining operation, cutting operation was performed using a 25 mm diameter end mill with 50 mm overhang length and 1-tooth on an aluminum alloy in down milling mode using 1 mm axial depth of cut, 12.5 mm radial immersion and 2025 rpm spindle speed.

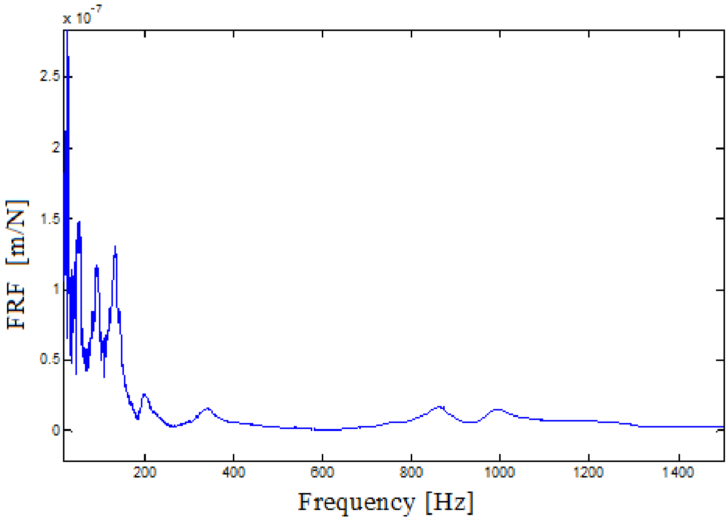

Figure 2: Tool point–holder cross FRF (G_{12}).

During the cutting operation, cutting forces are measured with a Kistler table type dynamometer which is directly attached to the workpiece and the response of the system is measured at the holder by using a laser vibrometer. Frequency spectrum of the measured cutting force in the x-direction and frequency spectrum of the laser vibrometer measurement are given in Fig. 3 and Fig. 4, respectively.

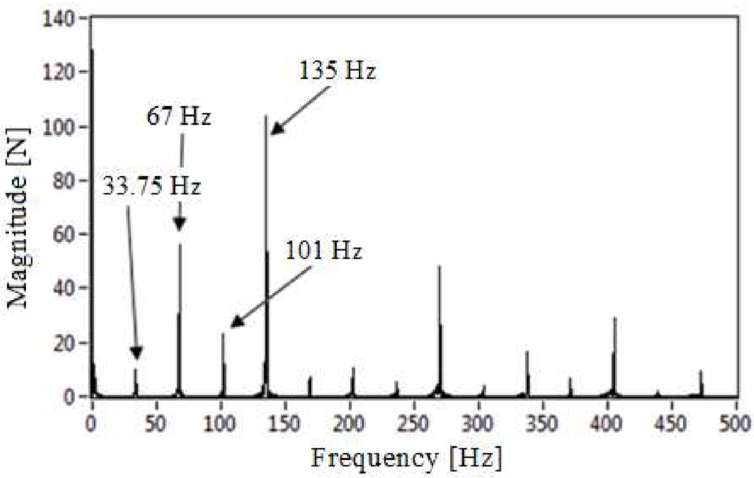

Figure 3: Spectrum of the cutting force in the x-direction with respect to the spindle axis.

As seen from Fig. 3 and Fig. 4, main problem in this approach is the harmonic characteristics of the cutting forces. Since the cutting forces excite the system mainly at the tooth passing frequency and at its higher harmonics, system responds to the cutting force excitation at the same frequencies. For example, for the cutting operation with 1 tooth and 2025 rpm spindle speed, tooth passing frequency is 33.75 Hz and its higher harmonics are 67 Hz, 101 Hz and 134 Hz. Therefore, it is not possible to obtain FRF of the system in a certain frequency band. Instead, specific points of the FRF for the given cutting operation are obtained. In order to overcome this problem and obtain FRF for each frequency in the interested band, it is assumed that the system dynamics is not affected significantly from the relatively small spindle speed variations. Based on this assumption, the system is excited around each mode using the pre-determined spindle speeds. For example, the required spindle speeds in order to excite the system around 134 Hz are given in Table 1 for the tool having 4 and 1 cutting edges.

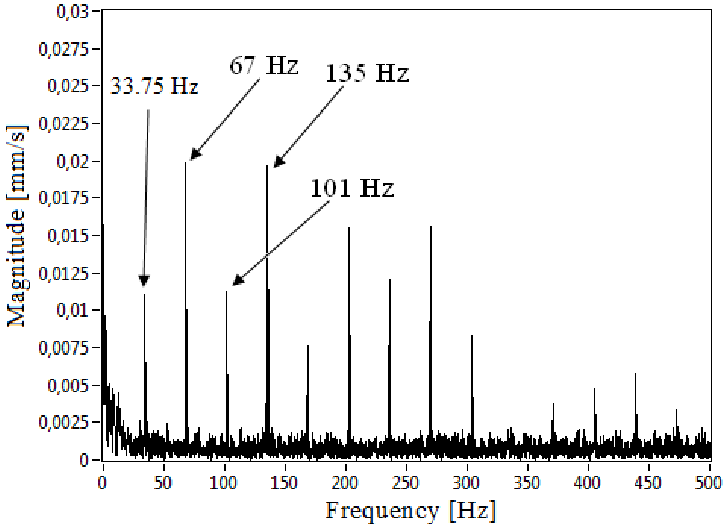

Figure 4: Frequency spectrum of the laser vibrometer measurement.

Table 1: Spindle speeds and corresponding tooth passing frequencies for the tool with 4 and 1 cutting edge.

Tooth passing frequency [Hz]	Spindle speed with 4 cutting edge [rpm]	Spindle speed with 1 cutting edge [rpm]
80	1200	4800
90	1350	5400
100	1500	6000
110	1650	6600
120	1800	7200
130	1950	7800
140	2100	8400
150	2250	9000

As seen from Table 1, system can be excited at the same frequency with higher spindle speeds by changing the number of cutting edges of the tool. To be able to excite the spindle effectively, relatively large radial and axial depth of cuts are chosen.

In order to determine the tool point – holder cross FRF, cutting tests were performed on a 5 – axis machining center using spindle speeds given in Table 1 where a 25 mm diameter end mill with 50 mm overhang was attached to the holder. Cutting was performed using 1 mm axial depth of cut, 30% radial immersion and 2025 rpm spindle speed, where feed was chosen as 0.25 mm/revolution. During the cutting operation, cutting forces were measured with a Kistler table type dynamometer which is directly attached to the workpiece and the response of the system was measured by a laser vibrometer at the holder. Measurement points on the milling machine are shown in Fig. 5 where point 1 and 2 represent the response and force measurement points, respectively.

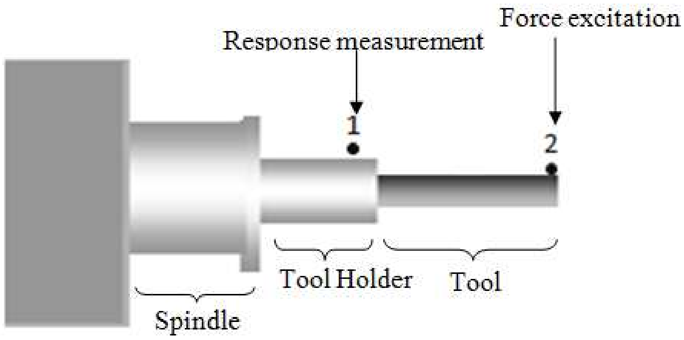

Figure 5: Measurement locations for the cutting forces and response on the machining center.

Assuming that the system dynamics is not affected significantly by relatively small spindle speed variations, as mentioned earlier, FRFs given in Fig. 6 can be taken as tool point – holder cross FRFs of the system.

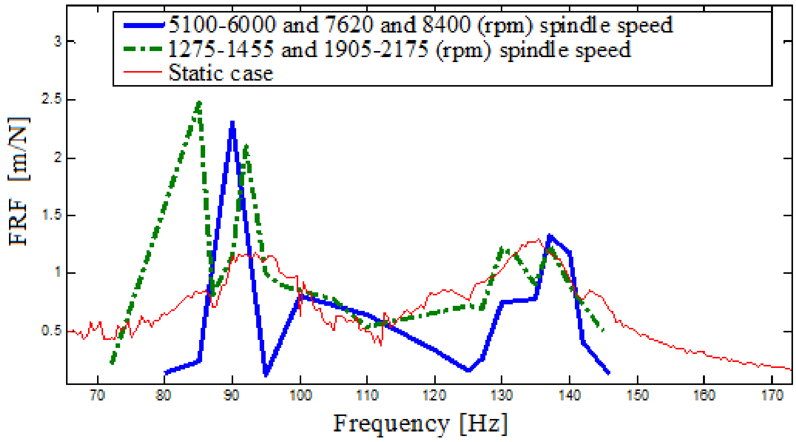

Figure 6: Tool point – holder cross FRFs for static case and during cutting operation.

As can be seen from Fig. 6, during cutting significant changes in the FRFs are observed. This is more pronounced at the first spindle mode located at 90 Hz, which is shifted to the lower frequencies during cutting operation. For the second mode located at 135 Hz, no significant change is observed compared to the first mode. But it should be kept in mind that FRFs given in Fig. 6 are obtained by using the assumption that system dynamics is not affected from relatively small spindle speed variations.

Cutting with specially designed workpiece. Rather than exciting the system with harmonic cutting forces as shown in Fig. 3 and Fig. 4, exciting the system with random forces can provide much more valuable information for the determination of the spindle FRF. In order to excite the system with random cutting forces, specially designed workpiece can be used [23]. For that purpose, a workpiece having randomly distributed channels with random thicknesses is prepared as shown in Fig. 7.

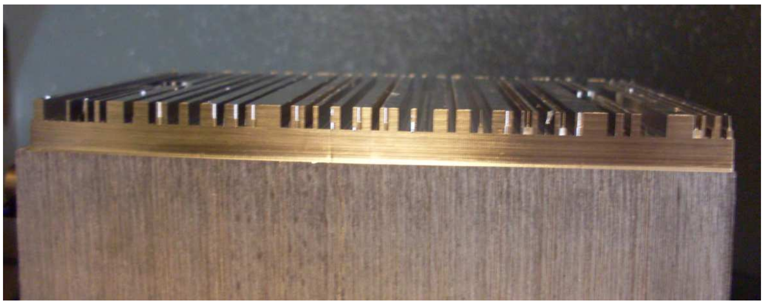

Figure 7: Workpiece with random surface profile.

Cutting operation is performed on the same 5-axis machining center where 25 mm diameter end mill with 50 mm overhang length is attached to the holder. Cutting was performed with 1 mm axial depth of cut, 30% radial depth of cut and 500 rpm spindle speed. During the machining operation, cutting forces were measured with a dynamometer which is directly connected to the workpiece and response of the system is measured at the tool holder of the system with a laser vibrometer.

During the determination of the tool point – holder cross FRF, H3 estimator is used due to the feedback characterisctics of the cutting operation. Coherence function between the cutting force in the x direction with respect to the spindle axis and vibrometer measurement is shown in Fig. 8. As seen from Fig. 8, for low frequecies around the interested spindle modes, which are located between 70 Hz and 140 Hz, coherence function is around 0.6-0.8 which decreases with frequency. Especially for frequencies larger than 800 Hz, coherence is close to 0. This high frequency behaviour of the identfication process is an expected result due to the limited measurement capacity of the dynamometer.

The tool point – spindle cross FRF obtained by using H3 estimator is given in Fig. 9. As it can be seen from this figure as well, similar changes are observed in the spindle FRF for the three different spindle speeds considered. For the spindlle mode located at 100 Hz, there is decrease in the stiffness. For the second mode located in the 140-160 Hz frequency band, in addition to the decrease in the stiffnesss, damping values are increased. These results are consistent with the expected bearing stiffness and damping changes under rotating conditions as stated in [11-12] since bearing properties affect mainly the spindle modes [13]. As seen from Fig. 7 and Fig. 9, in both approaches significant changes are observed in the spindle FRFs.

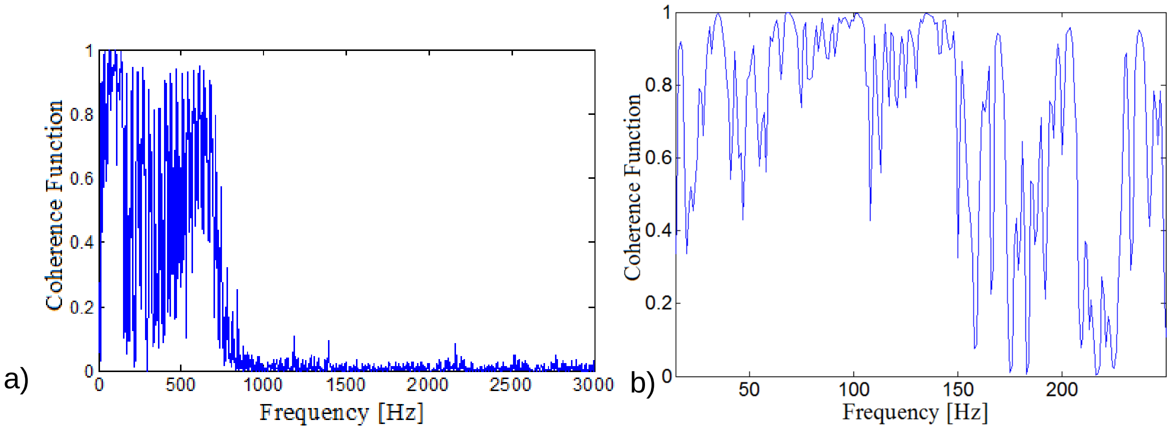

Figure 8: Coherence function between the cutting force in the x direction with respect to the spindle axis and vibrometer measurement.

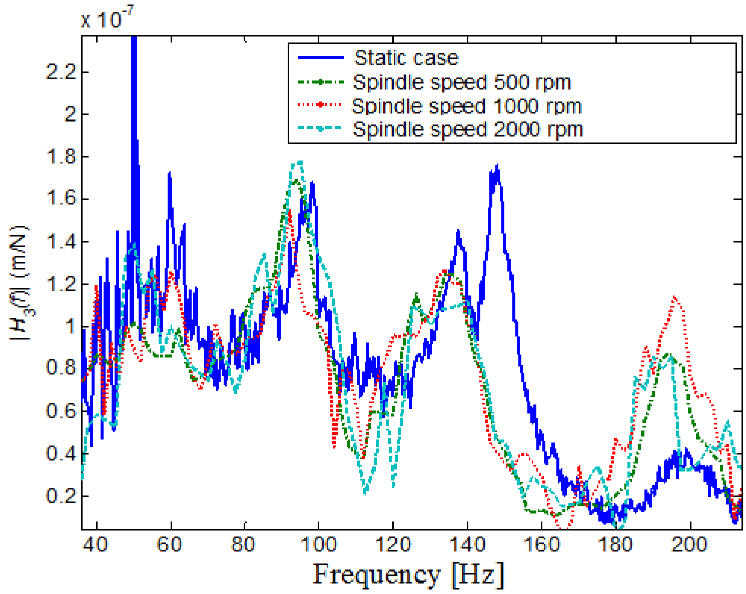

Figure 9: Tool point – holder cross FRFs for static case and during cutting operation at spindle speeds 500 rpm, 1000 rpm and 2000 rpm.

Identification of Bearing Parameters

In this study, variations in spindle modes are assumed to be due to changes in bearing parameters under operating conditions. In order to identify the variations of the bearing stiffness properties, system subassemblies (spindle, holder and tool) are modelled analytically by the model proposed by Erturk et. al. [29]. However in this study, being different from the method proposed in [29], subassembly FRFs are coupled using impedance coupling method [30], since the receptance coupling gives only the end point FRFs. Thus, by using the impedance coupling method, FRFs at the connection points are also kept. Moreover, it is possible to obtain cross FRF (G12) of the spindle – holder – tool assembly, where - similar to the experimental case - subscript 1 represents the response point (which is the holder) and subscript 2 represents the excitation point (which is the tool tip). For the milling machine, subassemblies and coupling connection points used in the analytical model are shown in Fig. 10. In Fig. 10, point C and C' represent the rigid impedance coupling point which is also the response measurement point during the experiments.

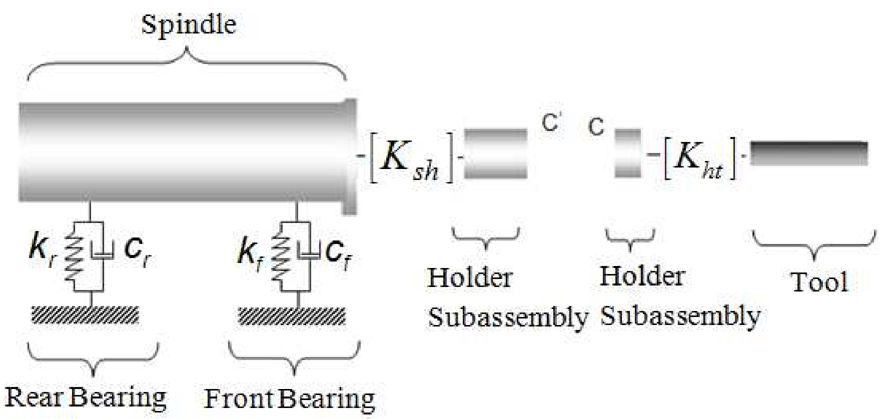

Figure 10: System subassemblies and coupling procedure.

In order to obtain cross FRF at the response measurement point of the experiments, holder is divided into two subassemblies as shown in Fig. 10. One of the holder subassembly is coupled with the spindle by the contact stiffness and damping parameters at the spindle – holder connection and the second holder subassembly is coupled with the tool by the contact stiffness and damping parameters at the holder – tool connection. Thus, spindle – holder and holder – tool subassemblies are obtained. Bearing dynamics are included into the system dynamics by using the structural modification method suggested by Özgüven [31].

In Fig. 10, k_f and c_f are the linear displacement – to – force stiffness and damping values of the front bearing, respectively; k_r and c_r are linear displacement – to – force stiffness and damping values of the rear bearing, respectively. $[K_{sh}]$ and $[K_{ht}]$ represent the contact parameters at the spindle – holder and holder – tool connections, respectively. These contact parameter matrices contain translational and rotational stiffness and damping values, which are given as follows:

$$[K_{sh}] = \begin{bmatrix} k_y^{sh} + i\omega c_y^{sh} & 0 \\ 0 & k_\theta^{sh} + i\omega c_\theta^{sh} \end{bmatrix} \tag{4}$$

$$[K_{ht}] = \begin{bmatrix} k_y^{ht} + i\omega c_y^{ht} & 0 \\ 0 & k_\theta^{ht} + i\omega c_\theta^{ht} \end{bmatrix} \tag{5}$$

where k_y is the linear displacement – to – force stiffness, k_θ is the angular displacement – to – moment stiffness, c_y is the linear displacement – to – force damping, c_θ is the angular displacement – to – moment damping, ω is the excitation frequency and i is the imaginary number. Also superscripts ht and sh represent holder – tool and spindle – holder connections, respectively.

Finally, spindle – holder and holder – tool subassemblies are coupled rigidly by the impedance coupling method as shown in Fig. 11 and cross FRF of the spindle – holder – tool assembly is obtained for the idle state of the milling machine.

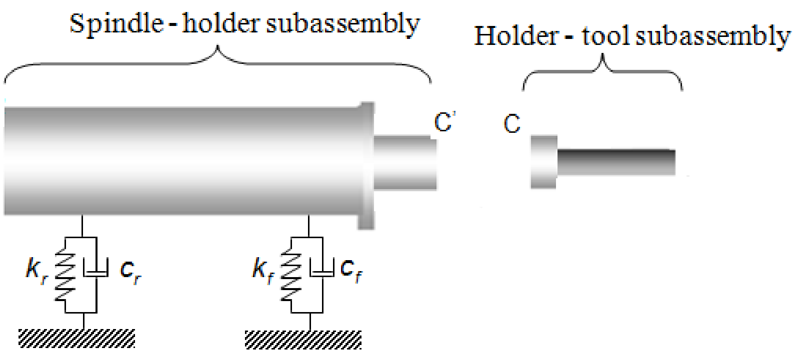

Figure 11: Rigid impedance coupling of the spindle - holder and holder – tool subassemblies.

In Fig. 11, point C and C' represent the rigid impedance coupling point which is also the response measurement point during the experiments.

In order to identify the bearing properties, cross FRF of the system is obtained by impact testing at the idle state of the machine. Also spindle – holder – tool assembly is modeled analytically. Using both experimentally obtained FRF and analytical model prediction, bearing properties are identified by manually tuning the translational stiffness and damping values. Experimentally obtained FRF and analytical model prediction are shown in Fig. 12. Identified bearing properties are given in Table 2.

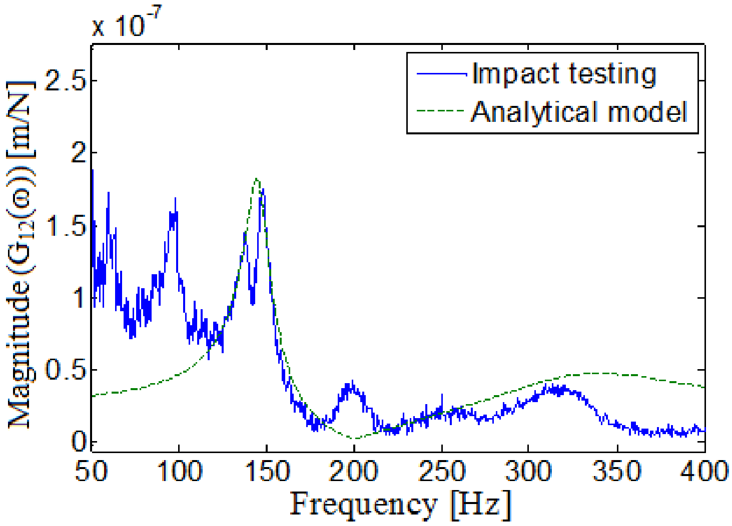

Figure 12: Analytically and experimentally obtained cross FRF for the idle state of the spindle.

Table 2: Front and rear bearing properties.

	Front Bearing	Rear Bearing
Translational Stiffness [N/m]	6.25×106	2.25×107
Translational Damping [N.s/m]	30	1500

In the previous section it is observed that during cutting operation there exist variations is the system dynamics and these variations can be assumed to be due to changes in bearing parameters. Thus, by using experimentally obtained FRFs during cutting operation along with analytical model, bearing properties during operation can be identified and variations in the bearing properties during operation can be studied. For that purpose, similar to the static case, bearing properties are identified by manually tuning the bearings translational stiffness and damping values. Obtained bearing properties are given in Table 3. Analytically obtained FRF which is obtained by using the identified bearing properties is shown in Fig. 13 with the experimentally obtained FRF for 500 rpm spindle speed and also with system FRF for the idle state.

As seen from Fig. 9, since the same deviations are observed in the spindle dynamics for the analyzed rotational speed range, identified bearing properties for the 500 rpm, 1000 rpm and 2000 rpm are very close.

Table 3: Front and rear bearing properties.

	Front Bearing	Rear Bearing
Translational Stiffness [N/m]	5.25×106	2.0×107
Translational Damping [N.s/m]	150	1500

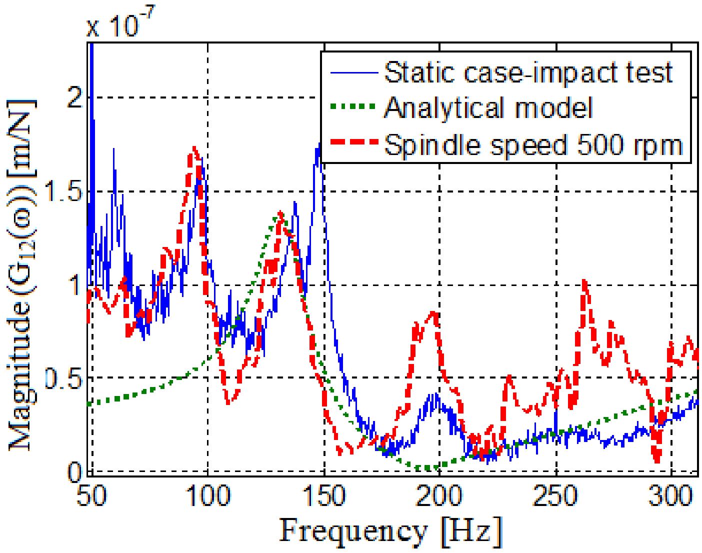

Figure 13: Analytically and experimentally obtained cross FRFs of the system.

Conclusion

In this paper, spindle dynamics during cutting operation is investigated and bearing parameters are identified under operating conditions. For the identification of the bearing properties, tool point FRF is determined experimentally and analytically. Finally, using both FRFs and by manually tuning bearing parameters, parameter identification is performed.

For the determination of the tool point FRF during cutting operation experimentally, cutting forces are used as an input to the system and the response of the system to the cutting forces is taken as the output of the system. Thus system dynamics is investigated with the examination of the input and output relation between cutting forces and system response. Main problem of the identification of the system FRF is the harmonic characteristics of the cutting forces. Since cutting forces excite the system at the tooth passing frequency and its higher harmonics, system responds at the same frequencies and it becomes impossible to obtain FRF for a certain frequency band. In order to overcome this problem, two different approaches are applied in this paper. First, it is assumed that the dynamics is not affected by the relatively small spindle rotational speed variations. Based on this assumption, in order to excite the system around each mode of the system, required spindle speeds are obtained and system FRF is determined. In addition to this approach, also spectral measurement techniques are applied to the cutting operation. In order to overcome the harmonic content problem of the cutting forces, specially designed workpiece which has randomly distributed channels on the upper surface is used during operation. In the second approach, system FRF is obtained for each spindle speed uniquely. In both approach, significant changes are observed. Finally, spindle – holder – tool assembly is modeled analytically by using Timoshenko beam theory and bearing properties are identified by using both experimentally and analytically obtained FRFs.

It should also be noted that in the analytical modeling of the spindle, holder and tool subassemblies, gyroscopic effects are not considered, and it is assumed that changes in the spindle modes are due to the changes in the bearing parameters.

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