Economics of Collective Refusals to Supply *

Izak Atiyas[†], Toker Doganoglu[‡], and Firat Inceoglu[§]

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Abstract

This paper examines situations where vertically integrated firms refuse to supply an input to an independent competitor in the downstream market. The treatment of such cases by competition or regulatory authorities is based on the assumption that such outcomes can only arise if there is collusion in the upstream markets. We argue that this is not always the case. In particular, we argue that proper antitrust or regulatory assessment of such cases requires analysis of the nature of competition, the shape and elasticity of the demand curve, the observability of upstream contracts, and even the number of potential downstream competitors.

Keywords: collective dominance, collective refusal to supply, tacit collusion, upstream competition.

JEL Classification: L13, L40, L41, L51.

Introduction

This paper examines situations where vertically integrated firms refuse to supply an input to an independent firm that competes with them in the downstream market. We will argue that the general treatment of such cases of collective refusal to supply by competition or regulatory authorities is based on the assumption that such market outcomes can only arise if the vertically integrated firms restrict competition in the upstream markets either through outright agreements or concerted practices or through market coordination. In competition law enforcement, collective refusal to deal implemented through agreements or concerted practices would be treated as per-se illegal. Such behavior is typically treated as a violation similar to other horizontal infringements (such as fixing prices, dividing markets etc.). Collective refusal to supply is also believed to arise through tacit collusion or market coordination when firms are collectively dominant. In the mobile telecommunications markets, identification of such collective dominance in access and call origination markets have led some regulatory authorities to impose remedies, namely access and roaming regulations, on the collectively dominant firms.

These approaches of competition or regulatory authorities are implicitly based on models where collective refusal to supply can only arise when the vertically integrated firms restrict competition in the upstream market through an agreement, concerted behavior or coordination.[1] In other words, in these models implicitly guiding competition law or regulatory enforcement, the assumption is that the best response of any firm to competitors who refuse to supply the input to downstream competitor would be to supply the input. Hence, collective refusal to supply would only arise (presumably) in a repeated game context.

We will show that this basic assumption does not always hold. Specifically, we investigate a number of examples where collective refusal to supply by vertically integrated firms to independent competitors in the downstream markets constitutes the Nash equilibrium of a number of stylized one-shot models where vertically integrated firms compete with each other in the upstream market and with each other and an independent firm in the downstream market. We note that such equilibria are taken to represent competitive cases not calling for ex-ante or ex-post intervention through competition law or sector specific regulation. An important implication of these observations is that the current treatment of collective refusals to deal in competition law and sector specific regulation is not consistent with the basic tenets that have guided either body

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[1] As discussed below, the term coordination is used to denote situations where competitors are not engaged in outright agreement or concerted practice but may still reduce the extent of competition by coordinating their actions. For example, in the theory of tacit collusion, which has actually inspired much of antitrust and regulatory enforcement in this area, such coordination may be implemented through simple trigger strategies.

of law. In particular, we will argue that proper antitrust or regulatory assessment of situations where vertically integrated firms refuse to supply downstream competitors (i.e. whether an observation of collective refusal to supply should raise any competition law or regulatory concern) requires more detailed analysis of a number of issues including the nature of competition, the shape and elasticity of the demand curve, whether upstream contracts are observable, and even the number of potential competitors in the downstream market.

A number of recent papers have examined issues similar to ours. Dewenter and Haucap (2006) study a very similar situation but with no concern for role of contracts, nor upstream competition. Their work suggests that in a homogeneous good Cournot model collective refusal to supply can only arise as a collusive outcome. We show that this is due to their demand specification and that with different specification of demand collective refusal to supply can arise as the equilibrium of a simple one-shot game. A growing strand of literature focus on upstream competition. Ordover and Schaffer (2007) and Höffler and Schmidt (2008) study models where vertically integrated firms sell homogeneous/differentiated upstream input goods to their downstream rivals using linear whole sale prices and linear differentiated downstream demand. Neither of these papers asks whether firms not selling to their downstream rivals would be an equilibrium, which is our main concern. Bourreau et.al. (2010) study a more general model. In a specific setting, they suggest that equilibrium foreclosure occurs when products are close substitutes. We consider two part wholesale pricing in the same setup, and show that non-collusive equilibrium foreclosure is less likely to occur.[TO BE EXTENDED]

The paper is organized as follows. In section 1, we briefly summarize several examples from Europe where collective refusal to supply by some upstream firms is used as grounds for either introducing ex-ante regulation, or for levying ex-post fines. In section 2, we present a reduced form model to guide our subsequent analysis of a variety of different stylized models. In section 3, focusing on homogeneous upstream and downstream markets, and assuming Cournot competition in the downstream market, we demonstrate how the assumptions on the demand curve and unobservability of upstream contracts can yield drastically different conclusions regarding the collusiveness of observations of collective refusals to supply. In section 4, by using a differentiated products model for downstream products, we demonstrate how substitutability between the the products, the complexity of feasible contracts as well as the number of potential downstream competitors may alter the nature of the assessment of an observation of collective refusal to supply. Section 5 concludes.

1   Background

This section discusses antitrust and regulatory policy towards instances of collective refusal to deal.

In the European Union, collective refusals to supply are most relevant for cases involving collective dominance. Collective dominance is said to exist when firms can coordinate their actions to increase profits without resorting to agreements or concerted practices. The doctrine of collective dominance is closely inspired by the economic theory of tacit collusion. In particular, since the *Airtours* decision of the Court of First Instance (CFI), the finding of collective dominance requires, among others, that the dominant firms be able to mutually observe each others' behavior and that there are incentives that prevent firms from deviating the common conduct because of threats of credible and effective punishment.

While the doctrine of collective dominance has not yet led to a case where firms have been found in breach of Article 102, it has played a significant role in merger review and in the mobile telecommunications industry where a number of regulators have imposed access or roaming obligations on the basis of a finding of collective dominance in the relevant markets.

The ComReg decision in Ireland to designate O_2 and Vodafone as jointly dominant in the wholesale mobile access and call origination and market is a good example of the degree to which the doctrine of collective dominance relies on the theory of tacit collusion, and also how retail prices and offering access to upstream elements are treated as if they have similar strategic properties. In that decision, the conditions for establishing the existence of joint dominance included "the ability to coordinate", and "enforceability of compliance" (Doyle, 2006, p. 152). The ability to coordinate was assessed along two dimensions: the first was whether the competitors could coordinate on prices. The second was whether the competitors could coordinate on denying independent entities access to upstream products, in particular, wholesale airtime. Hence, prices and access to upstream products were treated as having similar strategic properties. The assumption, as summarized by Doyle (.., p. 160) was that "if, for example, O_2 and Vodafone were not tacitly colluding, there would be a strong incentive to offer access to an entity capable of delivering higher profits to the upstream access provider." This is another way of saying that offering access when the competitor is not offering access is a best response in the stage game. A similar reasoning appears in the discussion of "enforceability of compliance". Building on the theory of tacit collusion with respect to prices, ComReg argued "the only deterrent or disciplinary mechanism required by Vodafone or O_2 to compel parallel behavior is the implicit threat of their reversion to the normal conditions of competition." (ComReg,

2004, p. 52). Regarding the specific dimension of providing access to wholesale products, ComReg stated: "If either did so, the other could retaliate in two ways – either by supplying similar services itself, or by cutting its retail prices. One or other, or both, responses could be rational, if the other firm undertook a course of action which reduced the retail price of services provided over its network. Overall it would appear that the potential reversion to the normal conditions of competition at the retail level constitutes a sufficiently serious disciplinary mechanism to perpetuate the consciously parallel market behavior being witnessed at present." (ibid). Implicit in this argument is the belief that offering access when the competitor is not offering access is a best response in the stage game, and hence the threat of retaliation is credible.

In the US, situations of joint refusal to supply are discussed under the topic of "group boycotts". The Federal Trade Commission (FTC) discusses group boycotts in the following manner: "Any company may, on its own, refuse to do business with another firm, but an agreement among competitors not to do business with targeted individuals or businesses may be an illegal boycott, especially if the group of competitors working together has market power."[2] However, the term group boycott seems to cover many different market structures and forms of behavior, including not only joint refusals to supply by upstream firms to downstream competitors, but also situations where a group of (presumably not integrated) firms may put pressure on upstream suppliers to refuse to supply to their competitors in the downstream market, or a where firms are refused acceptance to membership in trade organizations. [EXPAND GIVE REFERENCES.] As discussed by many legal scholars, this has generated quite a bit of confusion about the proper antitrust treatment of concerted refusals to deal (Glazer 2002, Robinson 2002). One of the important points of controversy is whether concerted or group refusals to deal should be treated as a *per-se* violation of antitrust law. In practice, it is safe to say that concerted refusal to deal by upstream firms to downstream purchasers would be considered to be *per-se* illegal. [CHECK, PROVIDE REFERENCE AND EXPAND ARGUMENT]

A case of collective refusal to supply in the cement industry investigated by the Turkish Competition Authority (TCA) is closely related to the issues examined in this paper. Cement industry typically contains two types of firms: vertically integrated producers who produce both clinker, which is used as an input in the production of cement, and cement itself. Hence these types of firms are potentially active in both in the upstream clinker market and downstream cement market. Producers that are not vertically integrated may purchase clinker from the upstream market, possibly from vertically integrated firms and grind them in their grinding

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[2]See http://www.ftc.gov/bc/antitrust/group_boycotts.shtm

last accessed March 11, 2011.

plants and use it to produce cement. The two types of firms typically compete in the down-stream cement market. In 2003, a cement producer in Turkey complained that two vertically integrated cement producers refused to supply it with clinker. The complainant and the vertically integrated firms were competitors in the downstream market. The TCA decided that the vertically integrated firms' behavior constituted a concerted action and fined them under Article 4 of the Law on the Protection of Competition (Competition Law, CL, for short), which is the analogue of Article 101 of the Treaty on the Functioning of the European Union. An interesting aspect of the Turkish CL is that it allows the TCA to reach a finding of abuse on the basis of a presumption of concerted behavior. This presumption can be established on circumstantial evidence, such as parallel behavior in the market (and the defendants can absolve themselves of responsibility by proving that their behavior had rational and economic reasons). In this case, the circumstantial evidence was the joint occurrence of refusal to supply. But this detail aside, the important point here is that in the TCA's approach, refusal to supply clinker was a result of concerted action: That is, absent concerted action between the clinker producers, each would have found it optimal to supply clinker to the downstream independent cement producer. The TCA treated the decision to supply or not as if it had similar strategtic characteristics with choosing the level of output and/or choosing the level of prices, where absent concerted behavior, the optimal way to respond to a competitor that raises its price or reduces the level of output (say, a competitor that keeps its output or price at levels that maximize joint static profits) would have been to increase output or reduce prices.

[WRITE THE ROAMING DECISION FOR MOBILES.]

In all of the cases described above, the authorities' approach to collective refusal to supply (presumably implemented through agreements, concerted actions or simply through market coordination that falls short of an agreement or concerted behavior[3]) is implicitly based on a model of oligopoly where each of the competing firms' best response is to supply when the rivals are refusing to supply. In that approach the market outcome of collective refusal to supply can only arise (presumably in a repeated interaction framework) if the competitors in the upstream market coordinate their behavior (for example through trigger-like strategies. In the next sections, we show that this is not necessarily the case.

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[3]In the sections below we will discuss situations where in the stage game the only Nash equilibrium entails both vertically integrated firms (VIFs) supplying to the independent downstream competitor and where the repeated versions of the game entail equilibria where both VIFs refuse to supply. Following the tradition in the literature, we will refer to the latter as "collusive" collective refusal to supply. While, strictly speaking, the theory of tacit collusion does not differentiate between agreements, concerted practices and pure coordination, it is generally presumed that tacit collusion entails one or some of such behavior.

2 A Reduced Form Model

Consider an industry with two vertically integrated firms (VIFs), $i = 1, 2$. The VIFs produce a homogenous input good that is needed for the production of a downstream product. There is a third downstream competitor, $i = 3$, that can compete with the VIFs provided that it can obtain the input from one or both of the upstream firms. It takes one unit of the upstream input to produce one unit of the downstream product.

A one time (one-shot) interaction between these parties is composed of three stages. The timing of these stages is as follows:

Stage 1: The VIF's offer upstream sales contracts to firm 3.

Stage 2: Firm 3 decides which, if any, offer to accept.

Stage 3: All firms that obtain the input compete in the downstream product market.

As stated in the introduction, we aim to investigate when and whether both VIFs refusing to sell the input to firm 3 can be an equilibrium of this game.[4] Let C denote the contract between one of the VIFs and the downstream competitor in case they agree on a deal. In order to determine the equilibrium strategies of the VIFs one needs to know:

1. The profits earned by the VIFs when they both refuse to sell to firm 3: $\hat{\Pi}_1$ and $\hat{\Pi}_2$

2. The profit of the VIF, say firm 2, that contracts with and sell to firm 3 with the optimal contract $\tilde{C}$ when the other VIF refuses to sell: $\tilde{\Pi}_2(\tilde{C})$

3. The profits earned by the VIFs when they compete at the downstream as well as the upstream market offering equilibrium contracts C_1^* and C_2^*: $\Pi_1^*(C_1^*)$ and $\Pi_2^*(C_2^*)$.

Suppose that the available contracts are sufficiently sophisticated. Then, if a vertical sale takes place, the parties will agree on a contract that maximizes their joint profits conditional on the set of feasible contracts, the equilibrium of the pricing game, and the alternative offers.

Next we will start by describing the possible equilibria of the game in a general formulation. This will allow us to identify the conditions under which a refusal to deal by both VIFs can be called collusive. Then we will solve the game for different modes of competition and market features and show that the possibility of a collusive outcome is highly dependent on these.

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[4]In this setup, a refusal to sell would simply correspond to a contract offer that included an infinitely high payment for the sale of the input.

First of all, note that whenever it is in the interest of one of the VIFs, say firm 2, not to sell the input when the other VIF also refuses to sell, that is, whenever

$$\tilde{\Pi}_2(\tilde{C}) \leq \hat{\Pi}_2, \qquad (1)$$

both firms not selling is a Nash equilibrium. Even though the outcome of this equilibrium involves both firm 1 and firm 2 refusing to sell to firm 3, there is nothing anticompetitive about it. Both firms compete in good faith and simply find it individually optimal to not sell the input to their downstream competitor. Obviously, there may be other equilibria where both VIFs want to sell the input even when condition (1) is satisfied. If one VIF decides sell to firm 3, then it could very well be optimal for the other VIF to supply the input, too. If, on the other hand, firm 2 prefers to sell to firm 3, that is if

$$\tilde{\Pi}_2(\tilde{C}) > \hat{\Pi}_2,$$

then both VIFs will compete in the upstream market by offering 'reasonable contracts'. In this case, the only Nash equilibrium of the game is one where both firms want to supply the input to their downstream competitor. Notice that in an equilibrium where both VIFs are willing to sell the input to the third firm they should obtain the same profit. This is true because otherwise one of the VIFs would have an incentive to adjust its offer and earn more. In other words, we must have[5]

$$\Pi_1^*(C_1^*) = \Pi_2^*(C_2^*).$$

So far, we have considered the situation in the framework of a one-shot game. Now suppose that the one-shot game we described above is played infinitely many times. In each period the VIFs make offers to firm 3, and then all active firms compete at the downstream level. In this setup, there will be a scope for collusion if the VIFs can earn more by coordinating their strategies and by collectively refusing to supply the input. That is, whenever

$$\Pi_i^*(C_i) < \hat{\Pi}_i \qquad i = 1, 2$$

firms may collude by, for example, adopting trigger strategies. In such an equilibrium of this repeated game, the VIFs will be able to sustain strategies that do not constitute a Nash equilibrium of the one-shot game due to fear of future punishments in case of deviation. It is for this reason that we take these strategies to yield collective refusal to supply outcome which we consider to be collusive. To reiterate, our definition of a collusive outcome in the remainder of the

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[5]In such a case, it is also reasonable to expect that both firms offer the same contract in equilibrium as well, however at this stage we will not impose this. Rather, we will show this to be the case in a series of stylized models.

paper is one which can not be supported as the equilibrium outcome of a static, one-shot game, but which can result when firms adopt history dependent strategies which involves punishments in case of deviation from the collusive outcome. On the other hand, if a collective refusal arises as an equilibrium outcome in a static, one-shot game, we take this to be a competitive outcome. It is clearly debateable whether such a distinction is appropriate. We leave this debate to future research, and proceed with these definitions.

In the following two sections we will look for the existence of such collusive outcomes under two different commonly used modes of competition.

3 Homogenous Goods Quantity Competition

In order to abstract from the effects of heterogeneity on foreclosure and entry, let us assume that all three firms can produce identical downstream products at zero cost beyond the acquisition cost of the upstream input. Furthermore, let the two VIFs have equal upstream production costs that consist of a constant marginal cost of c. Finally, we assume that competition between active firms in the downstream market takes place in quantities q_j, $j \in \{1, 2, 3\}$. Under these assumptions, when only the two VIFs are active in the market, they will earn equal Cournot duopoly profits, $\hat{\Pi}_1 = \hat{\Pi}_2 = \pi(2)$.

We limit the set of contracts that can be offered by a VIF to the downstream competitor to two-part tariffs of the form $F + wq$, where F is a fixed payment and w is the per unit wholesale price. Two-part tariffs are quite common in practice and provide a reasonable amount of contractual flexibility. We further assume that all contracts are publicly observable. This assumption allows us to compare our results to the results of related papers such as Dewenter and Haucap (2006), Höffler and Schmidt (2008) and Bourreau et. al. (2010). At the end of this subsection, however, we are going to consider unobservable contracts and show that then the implications may be radically different.

3.1 Linear Demand

We start by characterizing the equilibria of the one-shot game in a linear demand setting and then move on to more general demand formulations.[6] Essentially, our goal is to see if both VIFs

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[6]Dewenter and Haucap (2006) study basically the same issue with linear demands. However, they do not consider two part tariff contracts but assume contracts which only contain fixed payments and zero marginal costs in the upstream. They conclude, as our Lemma 1 below implies in a slightly more general setting, that with linear demands all observed collective refusals must be collusive.

not selling to the third firm can constitute a pair of Nash equilibrium strategies of the one-shot game.

Lemma 1 *Suppose that the market demand function is linear in price and upstream marginal costs are constant. When one of the VIFs refuses to sell the input to firm 3, the other VIF will always find it optimal to contract with firm 3. Thus, there exists no equilibrium where both VIFs refuse to sell to firm 3. The only Nash equilibrium is one where both firms offering the upstream product with a per-unit fee that is equal to marginal cost, $w = c$, and a fixed fee of zero. All firms earn three firm oligopoly profits in equilibrium.*

Proof. Straightforward. See the following text for a sketch.

When his rival refuses to sell, the VIF that contracts with firm 3 will set the per-unit fee equal to its marginal cost, $\tilde{w} = c$ and capture the downstream profits via setting the fixed fee, $\tilde{F}$, to three firm Cournot profit, $\pi(3)$. Clearly, this offer will be the best response of the contracting VIF if it leads to a higher profit. As it turns out, in a linear demand model we will always have that twice the three firm Cournot profits will be larger than duopoly profits,[7] i.e. $\tilde{\Pi}_2(\tilde{C}) = 2\pi(3) > \pi(2) = \hat{P}i_2$ and therefore a VIF always has unilateral incentives to supply firm 3. But in this case, the other VIF has an incentive to offer a contract with a slightly lower fixed fee. This undercutting continues until it no longer is profitable, namely until both VIFs offer a contract to the third firm that entails a per-unit fee that is equal to marginal cost, $w^* = c$, and no fixed fee, $F^* = 0$. In this case, all firms earn Cournot profits in a three firm oligopoly, i.e. $\Pi_1^*(C^*) = \Pi_1^*(C^*) = \Pi_3^* = \pi(3)$. The VIFs will compete in supplying the input to the third firm.

Let us now turn our attention on the incentives to collude. The only equilibrium of the one-shot game will involve a three firm symmetric oligopoly. Obviously, both VIFs would prefer an outcome where the third firm cannot obtain the input and remains out of the market. In a setup where the one-shot game is repeated infinitely many times, the VIFs can earn more by coordinating their strategies and by collectively refusing to supply the input by adopting trigger strategies, given that their discount rates are high enough. In other words, in a dynamic setup

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[7]This result is actually quite well-known in the horizontal mergers literature. The analogy is as follows: One can think of the contracting VIF as branching out a new division since it captures the whole profit of the third firm. Then, as long as downstream horizontal mergers are not profitable, both firms would find it profitable to branch out. Salant et. al. (1983) show that with a linear demand, constant marginal costs of production and Cournot competition every two firm merger is unprofitable if there are at least three firms in the market. In a related model with linear demands and constant marginal costs, Baye et. al. (1996) show that if firms can operate independent divisions by incurring a fixed cost, in equilibrium they will prefer operating multiple divisions.

the VIFs do have incentives to collude on not supplying the third firm. In a world described by linear demand and constant marginal costs, the striking implication for antitrust policy is then that every observed outcome where two rival VIFs do not supply the input to a downstream competitor is collusive and therefore anticompetitive. On the other hand, as we show in the following subsection, this result is highly model specific.

3.2 Cobb-Douglas Demand

We, next, repeat the same analysis with a demand function that can arise when consumers have Cobb-Douglas preferences. In this case, the demand is unit elastic at every level of aggregate output.

Lemma 2 *Let the inverse market demand function for the downstream product be $P(Q) = AQ^{-1}$, where $Q = q_1 + q_2 + q_3$. If one of the VIFs refuses to sell the input to firm 3, then not selling is also the best response of the other VIF. Accordingly, there is a Nash equilibrium of the one-shot game at which both firms refuse to supply input to firm 3.*

Proof. See Appendix.

The above lemma has a rather different implication in terms of antitrust policy. With the same setup but a different demand function, the behavior of the two firms can be completely different. We can no longer reach the conclusion that there is a concerted practice when neither firm supplies the input to a potential downstream competitor. Such a refusal to sell is the result of two firms competing in good faith. Our findings in Lemma 1 and in Lemma 2 suggest that the shape of the market demand should play a crucial role in deciding whether an observation of a collective refusal to sell by upstream firms is the result of collusive behavior.

3.3 Unobservable contracts

Our analysis above considered the case where any contract that is agreed upon becomes publicly known before firms compete in the downstream market. The nature of the strategic interaction would be considerably different if firms did not observe contracts as they competed in the downstream market. In our base setting, where we analyze only one of the VIFs making an upstream offer to firm 3, while its rival does not, the quantity choice of the non-contracting VIF could not depend on the unobserved wholesale price agreement reached by the other firms. In turn, the contracting firm cannot affect the output choice of his non-contracting rival by its choice of the wholesale price.

To highlight the role of contract non-observability let us return back to our simple setup with a linear downstream demand and constant marginal costs of producing the input. Consider once again the case where VIF 1 refuses to trade with firm 3, while VIF 2 offers a two part tariff wholesale contract. The sequence of events is as follows: VIF 2 makes a take-it-or-leave-it contract offer to firm 3. All firms observe whether there is an agreement or not, however the details of the contract are known only to VIF 2 and firm 3. Subsequently, all firms compete in the downstream market by selecting their production levels. We summarize our findings under unobservable contracts in the next proposition.

Proposition 1 *Suppose that the market demand function is linear in price and that upstream marginal costs are constant. Furthermore, suppose that the details of the contracts are known only to the contracting parties. Then, if one of the VIFs refuses to sell the input to firm 3, the other VIF will make a contract offer that implies an equilibrium quantity of zero by firm 3. That is, the VIF which makes the contract offer effectively refuses to sell as a best response. As a result, in equilibrium the downstream competitor cannot obtain the input and the two VIFs earn duopoly profits.*

Proof. See Appendix.

Contrasting our results with observable and unobservable contracts indicate another difficulty in assessing whether an observation of collective refusals to supply is collusive. When contracts are observable, the contracting VIF gains a first mover advantage. By its commitment on a wholesale price it can commit to a larger output collectively by itself and firm 3. This, in turn, reduces the output of the non-contracting VIF, thereby increasing the combined profits of the two contracting firms. In the earlier linear demand example, the contracting VIF is able implement the Stackelberg outcome (in terms of its profits) by means of signing an observable wholesale contract with firm 3. Since, both VIFs desire to obtain this Stackelberg advantage, they ultimately compete away any gains due to their ability to commit to a higher output by means of a wholesale contract. The only Nash equilibrium of the one-shot game involves both VIFs offering to supply the input to firm 3.

When contracts are unobservable, on the other hand, the contracting VIF cannot alter the output decision of its rival VIF by signing a wholesale contract. In this case, if the contracting VIF were to have two downstream plants, its best response would be to allocate a duopoly equilibrium quantity in anyway between these two plants. However, the contracting VIF cannot induce the duopoly output from itself and firm 3 in the downstream market with just controlling the per unit wholesale price unless it chooses a wholesale price which induces firm 3 to produce

zero units. Because of this, in equilibrium, both VIFs will not sell to firm 3 when they believe their rival also is not going to sell. In such a case, two VIFs competing in good faith can easily refuse sales to a third downstream rival, and there is nothing really collusive about it. This result stands in sharp contrast with our earlier findings with observable wholesale contracts, where the only possible way to sustain an outcome where both VIFs refuse sales were to engage in (tacitly) collusive behavior.

4 Differentiated Goods Price Competition

While with homogeneous goods, selling to a downstream competitor can be profitable only due the commitment role played by an observable wholesale contract, with differentiated goods in the downstream market, the trade-offs faced by the firms are more delicate. Introducing an additional variant will benefit consumers on the one hand, and increase downstream competition on the other hand. If the first effect dominates, firms can appropriate some of these consumer benefits as profits, while when the second effect is more important firms may end up earning less. In order to understand the mechanics of such tradeoffs, we now consider a differentiated products demand model. Assume that the downstream products of the the VIFs and firm 3 are perceived to be different by the consumer. The upstream product is still homogenous. Competition between active firms in the downstream market takes place in prices p_j, $j \in \{1, 2, 3\}$. The cost structure is the same as before and contracts between the VIFs and firm 3 take the form of two-part tariffs.

As we have seen in the homogenous goods case, the functional form of downstream demand plays an important role in the outcomes of the model. Acknowledging this fact, we are going to proceed with a very specific but commonly used demand model to highlight the role of the complexity of feasible contracts, the degree of substitution between different downstream products, and the number of potential downstream competitors play in determining whether foreclosure is collusive or not. We adopt a modified version of the Shubik and Levitan (1971) model of linear demand.

Let a representative consumer of the the downstream products have the following quasi-linear utility function:

$$U(q_1, q_2, q_3; \gamma) = \sum_{j=1}^{3} q_j - \frac{1}{2} \left(\sum_{j=1}^{3} q_j \right)^2 - \frac{3}{2(1+\gamma)} \left[\sum_{j=1}^{3} q_j^2 - \frac{\left(\sum_{j=1}^{3} q_j \right)^2}{n} \right]$$

It is straightforward to derive the corresponding demand functions. Demand for product j,

when all three firms are active, is given by

$$q_j(p_1, p_2, p_3) = \frac{1}{3}\left[1 - p_j + \beta\left(p_j - \frac{1}{3}(p_1 + p_2 + p_3)\right)\right].$$

As noted by Höffler and Schmidt (2008), when only the first two products are available, one cannot simply substitute $q_3 = 0$ in the above demand function. Instead, we need to derive the corresponding demand functions once again from the utility maximization problem of the representative consumer subject to the constraint that $q_3 = 0$. In this case, the demand functions for the products of the VIFs are

$$q_i(p_1, p_2) = \frac{1 + \beta}{3 + 2\beta}\left[1 - p_i - \frac{\beta}{3}(p_i - p_j)\right]$$

for $i \neq j \in \{1, 2\}$.

The only parameter of the demand functions, β, represents the substitutability of the downstream products. The degree of product differentiation decreases in β, and as $\beta \to \infty$ the products become perfect substitutes.

Let us revisit the problem of the supply of the upstream input to firm 3. First of all, when neither VIF supplies the input, they both earn

$$\Pi_i(p_1, p_2) = p_i q_i(p_1, p_2)$$

with $i \in \{1, 2\}$. The equilibrium of the price competition game is symmetric and yields the following equilibrium prices

$$\hat{p}_1 = \hat{p}_2 = \frac{3}{\beta + 6}$$

which in turn yield equilibrium profits of

$$\hat{\Pi}_1(\hat{p}_1, \hat{p}_2) = \hat{\Pi}_2(\hat{p}_1, \hat{p}_2) = \frac{3(1 + \beta)(3 + \beta)}{(\beta + 6)^2(2 + 3\beta)}.$$

The next step of the analysis considers whether a VIF would like to supply the input when its rival VIF refuses to sell it to firm 3. Again, since firm 3 has no other alternative to obtain the input (by assumption), we assume that the contracting VIF will design a take-it-or-leave-it two-part tariff offer, $wq + F$. The optimal contract will obviously depend on the nature of downstream competition. In the competition stage, firm 3 will have a marginal cost of obtaining the input which will be equal to the per-unit wholesale price it pays. Furthermore, the contracting VIF will take the effect of its downstream price on its upstream revenue into account.

Suppose that firm 1 refuses to sell the input to firm 3, and firm 2 makes the above mentioned two-part tariff offer. The profit function of firm 1 will simply be given by

$$\Pi_1(p_1, p_2, p_3) = p_1 q_1(p_1, p_2, p_3).$$

On the other hand, the variable profit function of firm 2 will consist of its own downstream as well as upstream profits:

$$\pi_2(p_1, p_2, p_3) = \underbrace{p_2 q_2(p_1, p_2, p_3)}_{\text{Downstream Profit}} + \underbrace{w q_3(p_1, p_2, p_3)}_{\text{Upstream Profit}}.$$

Finally, the variable profit function of firm 3 is given by

$$\pi_3(p_1, p_2, p_3) = (p_3 - w) q_3(p_1, p_2, p_3).$$

Incorporating the fixed fee paid by firm 3 to firm 2 in return for the sale of the input, we arrive at the profit functions of these two firms:

$$\Pi_2(p_1, p_2, p_3) = \pi_2(p_1, p_2, p_3) + F$$
$$\Pi_3(p_1, p_2, p_3) = \pi_3(p_1, p_2, p_3) - F$$

Each one of the three firms maximizes its profit function with respect to its price. After solving the system of best responses, we arrive at the equilibrium prices which are conditional on the per-unit price of the input:

$$\tilde{p}_1(w) = \frac{3}{2(3 + \beta)} + \frac{3\beta(1 + \beta)}{2(5\beta + 6)(3 + \beta)} w$$
$$\tilde{p}_2(w) = \frac{3}{2(3 + \beta)} + \frac{\beta(9 + 5\beta)}{2(5\beta + 6)(3 + \beta)} w$$
$$\tilde{p}_3(w) = \frac{3}{2(3 + \beta)} + \frac{7\beta^2 + 21\beta + 18}{2(5\beta + 6)(3 + \beta)} w$$

All three downstream prices increase in the per-unit wholesale price w (prices are strategic complements) and they are asymmetric for any $w \neq 0$.

Given the downstream equilibrium prices, firm 3 will agree to purchase the input from firm 2 as long as

$$\Pi_3(\tilde{p}_1(w), \tilde{p}_2(w), \tilde{p}_3(w)) \geq 0 \Rightarrow F \leq \pi_3(\tilde{p}_1(w), \tilde{p}_2(w), \tilde{p}_3(w)) \equiv \tilde{F}$$

In other words, firm 3 will accept the contract offer as long as its equilibrium profit is non-negative. Since firm 2 is assumed to have all the bargaining power, it is going to capture the whole variable profit of firm 3 via the fixed fee, $\tilde{F}$, and set the per unit wholesale price to maximize the sum of the two firms variable profits (conditional on the next stage equilibrium

prices). The optimal per-unit wholesale price maximizes the total profit of firm 2, i.e.

$$
\begin{aligned}
\tilde{w} \; &= \underset{w}{\operatorname{argmax}} \quad \Pi_2(\tilde{p}_1(w), \tilde{p}_2(w), \tilde{p}_3(w)) + \Pi_3(\tilde{p}_1(w), \tilde{p}_2(w), \tilde{p}_3(w)) \\
&= \underbrace{\tilde{p}_2(w) q_2(\tilde{p}_1(w), \tilde{p}_2(w), \tilde{p}_3(w))}_{\text{firm 2's downstream profit}} \\
&\quad + \underbrace{\tilde{p}_3(w) q_3(\tilde{p}_1(w), \tilde{p}_2(w), \tilde{p}_3(w))}_{\text{firm 3's downstream profit}}
\end{aligned}
$$

and is given by

$$
\tilde{w} = \frac{9\beta(5\beta + 6)(1 + \beta)}{2(7\beta^4 + 81\beta^3 + 252\beta^2 + 324\beta + 162)}
$$

Thus, whenever

$$
\underbrace{\pi_2(\tilde{p}_1(\tilde{w}), \tilde{p}_2(\tilde{w}), \tilde{p}_3(\tilde{w}))}_{\text{Profit when selling to firm 3}} \quad > \quad \underbrace{\pi_2(\hat{p}_1, \hat{p}_2)}_{\text{Profit whenrefusing to sell to firm 3}}
$$

firm 2 is better off selling to firm 3. Otherwise, firm 2's best response involves a refusal of supplying the input to firm 3. The model has a single parameter, β, that determines the best response of firm 2 in the case when firm 1 refuses to sell. One can then show that for $\beta > 142.033$, firm 2 obtains a higher profit by not selling the input to firm 3. Basically when the downstream products are close enough substitutes, both VIFs not selling the input to firm 3 is a Nash equilibrium of the one-shot game.

Let us proceed with the characterization of the equilibria of the one-shot game with two part tariff contracts. We have shown earlier that the one-stage game always has an equilibrium at which both VIFs prefer to sell the input to their downstream competitor. When the substitutability of downstream products is low enough (when $\beta < 142.033$ with non-linear contracts), firm 2 actually prefers to sell the input to firm 3 whenever firm 1 refuses to do so. By doing so, firm 2 would earn a strictly higher profit than firm 1. In response, firm 1 can offer a contract with a slightly lower fixed fee, capture the upstream market, and make a higher profit than before. Naturally firm 2 will revise its contract offer and the VIFs will undercut each other until it is no longer profitable to do so. Somewhat surprisingly, it turns out that this upstream competition does not lead to both of them offering the input at marginal cost—a point also noticed by Bourreau et. al. (2010). At this equilibrium, the wholesale price must be set in order to maximize the sum of the profits of the contracting VIF and firm 3, conditional on the outcome of the subsequent downstream price competition. This implies that the equilibrium wholesale price equals $\tilde{w}$ found above. When the VIFs compete for selling the input to firm 3, they will set the wholesale price at $\tilde{w}$, i.e. $w^* = \tilde{w}$, in order to maximize joint profits (of the contracting VIF and firm 3) and then use the fixed fee, F, to share these profits with firm 3.

In the equilibrium where both VIFs prefer to sell, the non-contracting VIF, say firm 1, will not have any incentives to undercut its upstream rival if

$$\Pi_1(p_1(w^*), p_2(w^*), p_3(w^*)) = \Pi_2(p_1(w^*), p_2(w^*), p_3(w^*))$$

which in turn implies that the equilibrium fixed fee must be

$$F^* = \underbrace{p_1(\tilde{w})q_1(p_1(\tilde{w}), p_2(\tilde{w}), p_3(\tilde{w}))}_{\text{Downstream profits of firm 1}}$$
$$- \underbrace{\left(p_2(w^*)q_2(p_1(\tilde{w}), p_2(\tilde{w}), p_3(\tilde{w})) + \tilde{w}q_2(p_1(\tilde{w}), p_2(\tilde{w}), p_3(\tilde{w})) \right)}_{\text{Downstream profits of firm 2}} < 0$$

In short, at the equilibrium both VIFs will make identical offers. Furthermore, they will be indifferent between supplying the downstream competitor and not supplying it. A direct implication of this is that the equilibrium fixed fee must be negative. Since $w^* > c$, the VIFs need to compensate firm 3 by sharing some of the variable profits. Negative fixed fees are not so uncommon in practice: in retailing, slotting allowances constitute fixed payments from upstream producers to downstream distributors made in order to access the downstream distribution network.

A comparison of the equilibrium profits with how much the VIFs would have earned were they to refuse to sell to firm 3 yields

$$\Pi_i^*(p_1(w^*), p_2(w^*), p_3(w^*)) < \hat{\Pi}_i(\hat{p}_1, \hat{p}_2)$$

where $i \in \{1, 2\}$. Obviously, both VIFs would be better off under an agreement to collectively refuse to supply the input to the downstream rival. In a repeated game framework, such an implicit agreement can be sustained by trigger strategies and a sufficiently high discount rate. Hence, for $\beta < 142.033$, an observation of a collective refusal to supply can only be sustained as part of a collusive equilibrium.

Proposition 2 *With two part tariff upstream contracts, an observation of a collective refusal to sell to firm 3*

1. *can be sustained as the subgame perfect equilibrium of a one-shot game for $\beta \geq 142.033$,*

2. *can only be sustained as part of a collusive equilibrium of infinitely repeated game otherwise.*

Proof. See preceding discussion as well as the comparisons of profits under various scenarios presented in the Appendix.

Our findings summarized in proposition 2 suggest that whether an observation of collective refusal to supply is collusive or not depends on the underlying substitutability between the final goods in the downstream market. When products are close substitutes a collective refusal supply may arise as a consequence of competition in good faith, while when products are sufficiently differentiated the only way it could arise is when firms (tacitly) collude. These results also point to a rather troubling scenario which may come before a competition authority. Suppose initially that the products are sufficiently differentiated, i.e. $\beta < 142.033$, and hence both firms compete in supplying the input to their downstream competitor—the do not collude to collectively refuse to supply. Consider now an external exogenous event after which the value of β increases to some level $\bar{\beta} > 142.033$. According to our results, following such an event the two upstream firms may very well stop supplying the input to firm 3 in the Nash equilibrium of a one-shot game. In this sequence of events, there is nothing that suggests the existence of collusive behavior. However, competition authorities may often interpret such changes in upstream supply behavior as indicative of firms achieving a collusive agreement. Our results point out the importance of considering other external events which may induce such a change in the upstream conduct.

Höffler and Schmidt (2008) solve for the equilibrium of the downstream price competition game in a similar setup where the contracting VIF can only offer linear tariffs. Although they do not consider the issue of a refusal to supply in their paper, it is straightforward to show that for $\beta > 26.766$ both firms refusing to supply is a Nash equilibrium. On the other hand, they also show that there always exists an equilibrium with linear upstream contracts, where both VIFs offer a contract that entails marginal cost pricing, i.e. $w^* = 0$. It is also straightforward to show that both firms are collectively better off if they refuse supplying the input to the downstream competitor. Therefore, with only linear contracts feasible, an observation of collective refusal to deal is collusive for $\beta < 26.766$ while it may arise as a result of competition otherwise.

In the next proposition, we contrast our model with two part tariffs with that of Höffler and Schmidt (2008) and establish that the type of contracts that are employed by the VIFs plays a major role in deciding whether observed collective refusals to sell is due to collusive behavior.

Proposition 3 *Whenever $26.766 < \beta < 142.033$, an observation of collective to refusal to sell*

1. *is collusive if firms have access to two part tariff upstream contracts,*

2. *can be the result of competition in good faith if firms can only use linear wholesale prices.*

Proof. See preceding discussion.

According to Proposition 3, we can conclude that the downstream competitor will be fore-closed less often when two-part tariff contracts are employed, assuming that the the VIFs compete in good faith. Alternatively, when we observe collective refusals to deal, it is more likely to be a result of concerted practices, if firms could employ more sophisticated contracts in upstream competition.

An inherent difficulty a competition authority would face in this case is the following. Given an observation of a collective refusal to deal, there will be no contracts that are observable, and hence, the competition authority must assess the nature of feasible contracts in each case, prior to reaching a conclusion of collusive behavior by the upstream parties.

In summary, the antitrust implications of our results are that, when products are highly differentiated, a collective refusal to deal is collusive and anticompetitive, and should be sanctioned. However as mentioned earlier, the degree of product differentiation (or its inverse, the degree of product substitutability) that would deem such an action anticompetitive strongly depends on the type of contracts available to the VIFs.

4.1 More Than One Downstream Competitor

Like in the case of mobile phone services industry, there could be potentially more than one downstream firm that requires the upstream input in order to compete in the downstream market. We are now going to extend the model of the preceding section to introduce a second downstream competitor, firm 4. For the sake of notational simplicity, we assume that the VIFs cannot make discriminatory offers to the downstream firms.

Let us once again start by deriving the best-response of a VIF, say firm 2, when its rival VIF, firm 1, does not supply the input to their downstream competitors. What makes this case more complicated is that firm 2 can find it optimal to offer a contract to either, both, or none of the downstream firms. The following lemma simplifies the analysis of this situation considerably.

Lemma 3 *Suppose that firm 1 does not offer a contract to either downstream firm. Then, if firm 2 finds it profitable to supply the input at all, it will find it optimal to sell it to both downstream firms.*

Proof. See Appendix.

The next step in the analysis involves a comparison of the profits of firm 2 when it chooses to supply the input and when it does not. Supplying the input to the downstream firms allows firm 2 to extract some of the benefits from increased variety that consumers enjoy. On the other hand, it intensifies the downstream price competition.

Proposition 4 *For $\beta > 345.96$, there is a subgame perfect Nash equilibrium of the one shot game with two downstream competitors where neither VIF supplies the input.*

Proof. See Appendix.

In other words, when the downstream products are sufficiently close substitutes, both downstream competitors may be foreclosed. Note that the critical value of β is significantly higher when there is a second potential downstream firm. This makes an observation of refusal to supply by both VIFs much more likely to be the result of a collusive agreement when there are two, rather than one, downstream firms. This finding poses an interesting complication in terms of antitrust policy. Suppose that the antitrust authority observes that the VIFs do not supply the input to any downstream rivals, or equivalently, no other firms compete in the downstream market. Conducting an economic analysis of the relevant market, the authority obtains a reliable estimate the corresponding substitution parameter, β. The question is: what critical value of β should this estimate be compared against? In order to be able to answer that question, one needs to know how many downstream firms would be active in the market, were the VIFs willing to supply the input. Unfortunately, without a clear answer to this counterfactual, one cannot assert a certain type of conduct on the part of the VIFs.

A full characterization of the equilibria of the one-shot game is a challenging task which will not be taken up in this paper. When $\beta < 345.96$, the VIFs will naturally compete at the upstream level. Given wholesale contracts, the downstream firms must play a game of choosing their suppliers. Suppose that one downstream firm contracts with a VIF. The downstream competition and especially the conduct of the VIFs will depend on the supplier choice of the other downstream firm. This is a situation where the analysis is more involved and is left for future research.

5 Conclusion

Antitrust policy towards horizontal agreements, concerted practices and coordination is closely informed by advances in economic theory of the last few decades, especially of game theory and the theory of tacit collusion. The basic tenet of the current approach has been that behavior that can be construed as the equilibrium strategy of a stage game is considered as competitive and is not condemned by antitrust law. By contrast, behavior that cannot be construed as an equilibrium in the stage game but which can be thought to be supported by history-dependent strategies raises competitive or regulatory concerns.

Within the boundaries of this basic approach, the general conclusion that one can derive out

of the analysis presented in the previous sections is that the antitrust or regulatory assessment of an observation of collective refusal to supply is not a straightforward task. In order to ascertain whether an observation of collective refusal to supply reflects the equilibrium of a static game or the collusive equilibrium of a repeated game hinges on not only straightforward parameters such as the degree of substitution between differentiated products, but one would also need to consider counterfactuals such as, were the VIFs to sell, what type of contracts the VIFs would use with their downstream competitors, and the likely number of downstream firms they would contract with. The analysis clearly suggests that a *per-se* condemnation of collective refusal to supply is not a correct anti-trust policy. Similarly, given the basic regulatory framework towards electronic communications, the mere observation of collective refusal to supply can not be taken as evidence of collective dominance and cannot provide grounds for the imposition of regulatory remedies.

The analysis raises some additional questions. For example, we have shown above that there are examples where the stage game has multiple equilibria, where in one equilibrium, the sale of input occurs at prices that are equal to marginal costs and in the other collective refusal to supply obtains (albeit in good faith). How should competition policy react to such a situation? The current analytical tools at the disposal of competition policy does not seem to provide a satisfactory answer to this question. The implications for regulatory policy are also interesting. Consider again the case where refusal to supply is a Nash equilibrium of the stage game, but there is another equilibrium where both VIFs sell, and consumer welfare is higher. Hence a regulatory policy that forces VIFs to sell would improve welfare. However, strictly speaking, the current regulatory regime, towards electronic communications markets in the European Union does not allow such a policy. This is because the imposition of remedies requires a market analysis that concludes with an identification of a set of firms that are collectively dominant. Collective dominance, in turn, relies on a presumption that the collective refusal to deal is sustained through threats of retaliation, which is not the case when collective refusal to deal is an equilibrium of the static game. This conclusion seems to suggest that the way in which modern oligopoly theory informs competition and regulatory policy leaves some unanswered questions.

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Appendix

Proof of Lemma 2

The market inverse demand function is given by $P(Q) = AQ^{-1}$, with $A > 0$. Suppose that firm 1 does not offer a contract to firm 3. Then firm 2 can make a take-it-or-leave-it offer to firm 3, capturing all of the latter's gross profit via the fixed fee. The profit functions of the three firms are given by

$$\pi_1(q_1, q_2, q_3) = (AQ^{-1} - c)q_1$$

$$\pi_2(q_1, q_2, q_3) = (AQ^{-1} - c)q_2 + (w - c)q_3 + F$$

$$\pi_3(q_1, q_2, q_3) = (AQ^{-1} - w)q_3 - F.$$

The second stage quantity competition game leads to Cournot equilibrium strategies given by

$$q_1^*(w) = q_2^*(w) = \frac{2wA}{(w + 2c)^2} \qquad q_3^*(w) = \frac{2A(2c - w)}{(w + 2c)^2}.$$

Given that the optimal fixed fee equals $F = A(q_1^*(w) + q_2^*(w) + q_3^*(w))^{-1} - w)q_3^*(w)$, firm 2 will choose w in order to maximize the sum of the variable profits of itself and firm 3. The optimal wholesale price turns out to be $w^* \geq 2c$ which in turn implies $q_3^*(w^*) = 0$. Given that firm 2 effectively does not supply the input, the best response of firm 1 is similarly not to supply. Both firms earn duopoly profits of $\frac{A}{4}$.

Proof of Proposition 1

Let us assume first that the inverse demand is given by $P = A - q_1 - q_2 - q_3$, with $A > 0$. The two VIFs can produce the input good at a constant marginal cost of c. Suppose that VIF 1 refuses to sell to firm 3, and hence, its profit is simply given by

$$\pi_1(q_1, q_2, q_3) = (A - q_1 - q_2 - q_3 - c)q_1.$$

Let us consider the case, where VIF 2 and firm 3 agree on a wholesale contract which has a per unit fee, w, and a fixed payment of F. Given this wholesale contract, the profit of VIF 2 is given by

$$\pi_2(q_1, q_2, q_3, w) = ((A - q_1 - q_2 - q_3 - c)q_2 + (w - c)q_3 + F$$

while the profit function of firm 3 is given by

$$\pi_3(q_1, q_2, q_3, w) = ((A - q_1 - q_2 - q_3 - w)q_3 - F.$$

Given the sequence of events, and the information structure, we can represent this strategic interaction between the firms as an (equivalent) extensive for game, where first VIF 1 makes

a decision on q_1 without observing w, q_2 and q_3. Neither VIF 2 or firm 3 observe q_1. VIF 2 makes a take-it-or-leave-it offer in the form of a two part tariff to firm 3, which in equilibrium accepts this offer. The two firms, being obviously aware of the details of the contract, i.e. w and F, proceed by non-cooperatively choosing their outputs. Moreover, VIF 2 and firm 3 decide on their output levels given w but without knowing q_1. In a way, they choose their outputs as a best response to the observed w and their belief about q_1. In choosing its contract offer, VIF 2 takes into account the effects of this offer in subsequent output choices of itself and firm 3. However, at this stage VIF 2 also best responds to its belief on q_1. When VIF 1 makes it output choice it best responds to its beliefs about w, q_2 and q_3. In equilibrium, all beliefs turn out to be correct.

We start by finding the best response output choices of VIF 2 and firm 3 for a given value of w and a guess of firm 1's output, q_1^*. It is easy to verify that the best response choices are given by

$$q_2(q_1^*, w) = \frac{A + w - q_1^* - 2c}{3} \qquad q_3(q_1^*, w) = \frac{A - 2w - q_1^* + c}{3}.$$

In choosing its contract, VIF 2 can insure acceptance by setting $F = \pi_3(q_1^*, q_2(q_1^*, w), q_3(q_1^*, w), w)$. This choice of the fixed fee, implies that in choosing w, VIF 2 will maximize the sum of the profits of itself and firm 3, i.e.

$$w = \quad \text{argmax}_w \, \pi_2(q_1^*, q_2(q_1^*, w), q_3(q_1^*, w), w) + \pi_3(q_1^*, q_2(q_1^*, w), q_3(q_1^*, w), w)$$

$$= \quad \text{argmax}_w \, \big(q_2(q_1^*, w) + q_3(q_1^*, w), w)\big)\big(A - q_1^* - q_2(q_1^*, w) - q_3(q_1^*, w) - c\big).$$

It is easy to verify that the optimal choice of w for VIF 2 conditional on the subsequent equilibrium output choices is given by

$$w(q_1^*) = \frac{A - q_1^* + c}{2}.$$

It is straightforward to show that at this wholesale price, firm 3 will optimally produce no output while VIF 2 itself produces

$$q_2(q_1^*, w(q_1^*)) = \frac{A - q_1^* + c}{2}$$

which is exactly the best response function of VIF 2 when it competes with VIF 1 in a duopoly. Also, note that when $q_3 = 0$, the best response of VIF 1 to a believed output q_2^* of VIF 2 is given by

$$q_1(q_2^*) = \frac{A - q_2^* + c}{2}.$$

Finding the values of q_1^* and q_2^* which are best responses against each other, we find the standard Nash equilibrium output levels in a Counot duopoly. Therefore, both firms refusing to sell any input to firm 3 is a Nash equilibrium in a one-shot game.

Proof of Lemma 3

The utility function of a representative consumer over four products is given by:

$$U(q_1, q_2, q_3, q_4; \gamma) = q_1+q_2+q_3+q_4-\frac{1}{2}(q_1+q_2+q_3+q_4)^2-\frac{2}{1+\beta}(q_1^2+q_2^2+q_3^2+q_4^2-\frac{1}{4}(q_1+q_2+q_3+q_4)^2)$$

Utility maximization leads to the downstream demand functions:

$$q1 = \frac{1}{4}(1-p_1)+\frac{1}{16}p_2\beta-\frac{3}{16}p_1\beta+\frac{1}{16}p_3\beta+\frac{1}{16}p_4\beta$$

$$q2 = \frac{1}{4}(1-p_2)-\frac{3}{16}p_2\beta+\frac{1}{16}p_1\beta+\frac{1}{16}p_3\beta+\frac{1}{16}p_4\beta$$

$$q3 = \frac{1}{4}(1-p_3)+\frac{1}{16}p_2\beta+\frac{1}{16}p_1\beta-\frac{3}{16}p_3\beta+\frac{1}{16}p_4\beta$$

$$q4 = \frac{1}{4}(1-p_4)+\frac{1}{16}p_2\beta+\frac{1}{16}p_1\beta-\frac{3}{16}p_4\beta+\frac{1}{16}p_3\beta$$

Suppose VIF 2 sells the input to both downstream firms at (w, F). The profit functions of the four firms will become:

$$
\begin{aligned}
\Pi_1(p_1, p_2, p_3, p_4) &= p_1 q_1 \\
\Pi_2(p_1, p_2, p_3, p_4) &= p_2 q_2 + w(q_3 + q_4) + 2F \\
\Pi_3(p_1, p_2, p_3, p_4) &= (p_3 - w)q_3 - F \\
\Pi_4(p_1, p_2, p_3, p_4) &= (p_4 - w)q_4 - F
\end{aligned}
$$

The equilibrium strategies of the downstream price competition are

$$
\begin{aligned}
p_1^* &= \frac{4(2w\beta^2 + 7\beta + 2w\beta 8)}{(7\beta + 8)(3\beta + 8)} \\
p_2^* &= \frac{14w\beta^2 + 28\beta + 24w\beta + 32}{(7\beta + 8)(3\beta + 8)} \\
p_3^* &= \frac{17w\beta^2 + 28\beta + 44w\beta + 32w + 32}{(7\beta + 8)(3\beta + 8)} \\
p_4^* &= \frac{17w\beta^2 + 28\beta + 44w\beta + 32w + 32}{(7\beta + 8)(3\beta + 8)}
\end{aligned}
$$

Taking these equilibrium prices into account, in the first stage VIF 2 chooses w and F in order to maximize its profit. The optimal per-unit wholesale price is given by

$$w^* = \frac{2\beta(105\beta^2 + 232\beta + 128)}{(51\beta^4 + 664\beta^3 + 1968\beta^2 + 2304\beta + 1024)}$$

and finally, VIF 2's profit becomes:

$$\Pi_2(w^*) = \frac{(101\beta^3 + 350\beta^2 + 432\beta + 192)}{(51\beta^4 + 664\beta^3 + 1968\beta^2 + 2304\beta + 1024)}$$

A comparison of this profit to the one with three downstream products found in Section 3 yields that for all β, VIF 2 prefers to deal with both downstream firms rather than one.

Proof of Proposition 4

Since Lemma 3 has already established that when VIF 1 does not supply the input, VIF 2 will contract with both downstream firms rather than with only one, we just need to compare the profit of VIF 2 found above to the duopoly profits earned when neither VIF supplies the input. For $\beta > 345.96$, the duopoly profit is greater, meaning that the best response of VIF 2 is not to supply the input. For $\beta < 345.96$, the profit from selling to firms 3 and 4 is greater, meaning that the best response of VIF 2 is to supply the input to them. This proves that only for $\beta > 345.96$ does there exist a Nash equilibrium of the one-shot game where neither VIF supplies the input.