Formation Control of Multiple Robots Using Parametric and Implicit Representations

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Abstract. A novel method is presented for formation control of a group of autonomous mobile robots using parametric and implicit descriptions of the desired formation. Shape formation is controlled by using potential fields generated from Implicit Polynomial (IP) representations and the control for keeping the desired shape is designed using Elliptical Fourier Descriptors (EFD). Coordination of the robots is modeled by linear springs between each robot and its nearest two neighbors. This approach offers more flexibility in the formation shape and scales well to different swarm sizes and to heterogeneous systems. The method is simulated on robot groups with different sizes to form various formation shapes.

Keywords: Shape Formation; Robot Coordination; Autonomous Mobile Robots.

1 Introduction

The role of the autonomous robots in our lives is increasing in many fields. There are many applications that a group of multiple robots can achieve much better than a single robot. There has been significant research on groups of autonomous mobile robots which can be used for applications including tasks such as exploration [1], surveillance [2], search and rescue [3], mapping of unknown or partially known environments [4], distributed manipulation [5], and transportation of large objects [6]. An important issue in these tasks is the flexibility in the definition of the working area of robots.

This paper introduces a shape formation method which is highly flexible for various tasks. The robots are initially positioned randomly in a defined area. The aim is to make the robots to form and keep a desired shape. During this formation, the robots are also desired to coordinate among themselves to avoid collisions and keep the desired distance between each other.

Our proposed method is based on a parametric description of the desired formation by using elliptic Fourier descriptors (EFD) and its implicit polynomial (IP) representation [7]. Implicit polynomial description of the curve is used to

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define an algebraic distance between each robot and the desired curve in the first phase in which robots are attracted to form the desired shape. The parametric description is used for trajectory tracking in the second phase in which the robots are controlled to keep the formation. Besides these controls, a coordination control to keep the desired distances between the neighboring robots is also introduced. In this work, each mobile robot is modeled as a point particle.

The main advantage of this approach over other approaches is its flexibility on the desired curve. The desired curve can be any reasonable closed curve. Some other methods are rigid in formation constraints [8][9]. Another advantage of our method is that it offers more flexibility on the number of robots in the group and the heterogeneousness of the swarms compared to other methods such as [10][11].

The presented method is simulated in Matlab. The simulation program is written to be modular so that the simulations can be done with any desired number of robots and with any desired closed curve. The performance of the proposed method has been verified by simulations with different sizes of the swarms.

Organization of this paper is as follows: Representation of the free-form curves is presented in Section 2. Designing a control for shape formation, keeping the formation and coordination of robots are presented in Section 3. Section 4 presents the simulation results and discussions. Conclusions are presented in Section 5.

2 Parametric and Implicit Representation of Formation

It is known that closed-bounded curves can be represented by elliptic Fourier descriptors (EFD) [12]. The elliptic Fourier description of a closed curve is given as follows:

$$x(t) = a_o + \sum_{k=1}^{n} (a_k \cos kt + b_k \sin kt) \ . \tag{1}$$

$$y(t) = c_o + \sum_{k=1}^{n} (c_k \cos kt + d_k \sin kt) \ . \tag{2}$$

In this equation, n is a positive number which represents the number of the harmonics used to represent the closed curve. As it is well-known, the efficiency of the representation increases with increasing number of harmonics. On the other hand, the computational cost increases with the number of the harmonics. Therefore, an optimal decision on the number of the harmonics should be made when modeling a complex curve by considering both the efficiency in the description and the computational cost.

The desired formation shape can be represented both by a parametric function using elliptic Fourier descriptors and by an implicit function, $F(x, y) = 0$, obtained through the implicitization of EFDs using the method detailed in [7]. The implicit function is good for producing potential functions to be used for formation shaping and the parametric function is employed for keeping the formation.

The representation of implicit curve has the form $F(x, y) = \sum_{0 \leq i+j \leq d} a_{ij} x^i y^j = 0$ where a_{ij} are the coefficients and d is the degree of the polynomial respectively. As detailed in [7], the resulting implicit curve has degree $d = 2n$ where n is the number of harmonics used in the parametric representation of the curve.

3 Control of Mobile Robots

In this study, the robots are modeled as point particles. The kinematic model for the i^{th} robot is as follows:

$$\begin{pmatrix} \dot{x}_i \\ \dot{y}_i \end{pmatrix} = \begin{pmatrix} u^i_{total} \\ v^i_{total} \end{pmatrix} . \tag{3}$$

where $\dot{x}_i$ and $\dot{y}_i$ are the velocities of the particle in the x and y directions respectively with respect to the world coordinate frame. In this control, $\begin{pmatrix} u^i_{total} \\ v^i_{total} \end{pmatrix}$ is obtained by the sum of the control components designed for the formation control and coordination control, namely

$$\begin{pmatrix} u^i_{total} \\ v^i_{total} \end{pmatrix} = \begin{pmatrix} u^i_{formation} \\ v^i_{formation} \end{pmatrix} + \begin{pmatrix} u^i_{coord} \\ v^i_{coord} \end{pmatrix} . \tag{4}$$

where $\begin{pmatrix} u^i_{formation} \\ v^i_{formation} \end{pmatrix}$ is formation component of the control. The design of this formation component changes with respect to the phase of the formation, if the robots are in the phase to form the desired shape or to keep the formation. $\begin{pmatrix} u^i_{coord} \\ v^i_{coord} \end{pmatrix}$ is the coordination component of the control.

3.1 Formation Control Using Implicit Polynomial Potential Functions

Initially, robots are randomly positioned in the defined area and the aim is to form the desired formation shape. For the design of the formation component of control input, the implicit polynomial representation of the curve is used. The position error function between the i^{th} robot and the curve is given by the algebraic distance to the curve using the implicit equation as

$$e^i_{form} = F(x_i, y_i) . \tag{5}$$

where e^i_{form} is the position error with respect to the desired curve defined for i^{th} robot and x_i and x_i are the x and y position of the robot with respect to the world coordinate frame. The control will be designed to force the error to decrease exponentially as

$$\dot{e}^i_{form} = -\lambda e^i_{form} . \tag{6}$$

where λ is a positive number. Substituting $e^i_{form} = F(x_i, y_i)$ into the equation above yields

$$\dot{F}(x_i, y_i) = -\lambda F(x_i, y_i) \ . \tag{7}$$

Using chain rule of differentiation, we have

$$F_x(x_i, y_i)\dot{x}_i + F_y(x_i, y_i)\dot{y}_i = -\lambda F(x_i, y_i) \ . \tag{8}$$

which can be rewritten as

$$\begin{pmatrix} F_x(x_i, y_i) & F_y(x_i, y_i) \end{pmatrix} \begin{pmatrix} \dot{x}_i \\ \dot{y}_i \end{pmatrix} = -\lambda F(x_i, y_i) \ . \tag{9}$$

Taking the kinematic model of the robots into account, the optimal formation control can be determined using pseudo inverse, namely

$$\begin{pmatrix} u^i_{formation} \\ v^i_{formation} \end{pmatrix} = -\lambda \frac{1}{||\nabla F(x_i, y_i)||^2} F(x_i, y_i) \begin{pmatrix} F_x(x_i, y_i) \\ F_y(x_i, y_i) \end{pmatrix} \ . \tag{10}$$

3.2 Keeping Formation Using Elliptic Fourier Descriptors

After the robots reach the desired formation, a new control is designed to keep the robots in the desired formation. In the design of this control component, the parametric representation of the desired formation is used. The error function for this phase is defined as

$$e^i_{form} = \begin{pmatrix} x^*_i(t) \\ y^*_i(t) \end{pmatrix} - \begin{pmatrix} x_i(t) \\ y_i(t) \end{pmatrix} \ . \tag{11}$$

$$\Rightarrow \dot{e}^i_{form} = \begin{pmatrix} \dot{x}^*_i(t) \\ \dot{y}^*_i(t) \end{pmatrix} - \begin{pmatrix} \dot{x}_i(t) \\ \dot{y}_i(t) \end{pmatrix} = \begin{pmatrix} \dot{x}^*_i(t) \\ \dot{y}^*_i(t) \end{pmatrix} - \begin{pmatrix} u^i_{formation} \\ v^i_{formation} \end{pmatrix} \ . \tag{12}$$

where $x^*_i(t)$ and $y^*_i(t)$ are the desired positions and $x_i(t)$ and $y_i(t)$ are the actual positions.

The control is designed to make the error decrease exponentially, namely:

$$\dot{e}^i_{form} = -\lambda e^i_{form} \ . \tag{13}$$

which yields

$$\begin{pmatrix} u^i_{formation} \\ v^i_{formation} \end{pmatrix} = \lambda e^i_{form} + \begin{pmatrix} \dot{x}^*_i(t_i) \\ \dot{y}^*_i(t_i) \end{pmatrix} \ . \tag{14}$$

3.3 Coordination Control

The robots are appointed to be in coordination besides the shape formation task. The aim of this coordination is to avoid collisions and keep a desired distance between each robot and its neighbors. The control input of a robot for this coordination is modeled as the sum of forces of linear springs between this robot and its two nearest neighbors as in Figure 1.

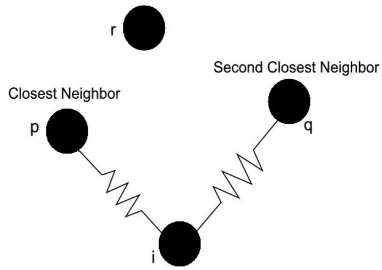

Fig. 1. Modeling of coordination control

The spring is proposed to have a normal length which is equal to the desired distance. The spring produces a force linearly proportional to the difference between the actual distance and the desired distance between the robots. The force of the spring is on the direction of the vector from the i^{th} robot to its neighbor as below:

$$\begin{pmatrix} u^i_{coord} \\ v^i_{coord} \end{pmatrix} = k(d_{desired} - d^{ip}_{actual}) \begin{pmatrix} x_i - x_p \\ y_i - y_p \end{pmatrix} + k(d_{desired} - d^{iq}_{actual}) \begin{pmatrix} x_i - x_q \\ y_i - y_q \end{pmatrix} . \tag{15}$$

where k is an adaptable spring constant; the spring constant is different when the robot is in shape formation or keeping formation phases. p and q are the indices for the robots that are the nearest two neighbors of i^{th} robot. d^{ip}_{actual} and d^{iq}_{actual} are the actual distances of the robot i from the robots p and q respectively. (x_p, y_p) and (x_q, y_q) are the x and y position coordinates of the robots p and q with respect to the world coordinate frame.

4 Simulation Results and Discussions

The simulations are performed in Matlab. The program is written to be modular so that the simulations can be carried out with any desired number of robots. Two different simulations will be presented. First one is a simulation of a single robot to see the efficiency of the proposed formation control. The second one is done with 5 and 6 robots on two different desired patterns to see the success of the control in both formation and coordination.

In simulations, parameters are chosen to be $\lambda = 3$, $k_{ShapeFormation} = 2$, $k_{KeepFormation} = 1$ and $d_{desired} = 0.1$.

4.1 Simulation Results for a Single Robot

In the first simulation, the proposed method is applied on a single robot. Desired pattern is represented by a Fourier descriptor function using 7 harmonics and an implicit polynomial with a degree of 14 using the methods of Section 2. The initial position of the robot is: $x = 2.5$, $y = -2$. The route of the robot can be seen in Figure 2.

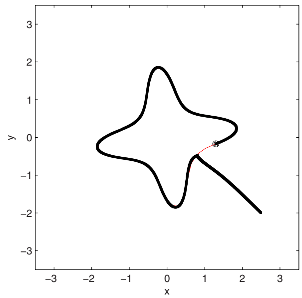

Fig. 2. Route of mobile robot

It is clearly seen that the method is successful to enable the robot to reach the desired formation curve and moves on it.

4.2 Simulation Results for 5 and 6 Robots

In this simulation, the proposed method is applied on a group of 5 and 6 robots, respectively. The desired pattern for 5 robots is an ellipse generated by a Fourier descriptor with 1 harmonic and a corresponding implicit polynomial of degree 2. Ellipse formation is shown in Figure 3. The desired pattern for 6 robots is a more complicated star shape represented by 7 harmonics and a corresponding implicit polynomial of degree 14. Formation results are depicted in Figure 4. Examination of these figures reveals the fact that although the initial positions of the robots are far away from the desired formation curve, proposed method

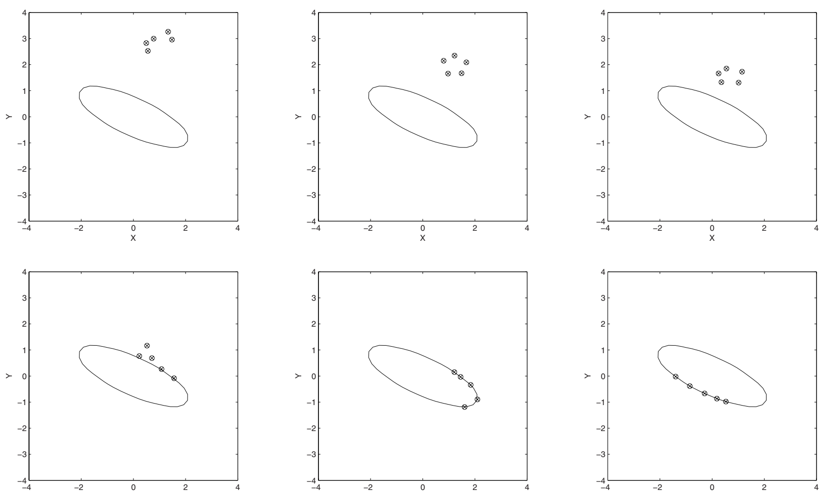

Fig. 3. Desired formation (ellipse) with 5 robots

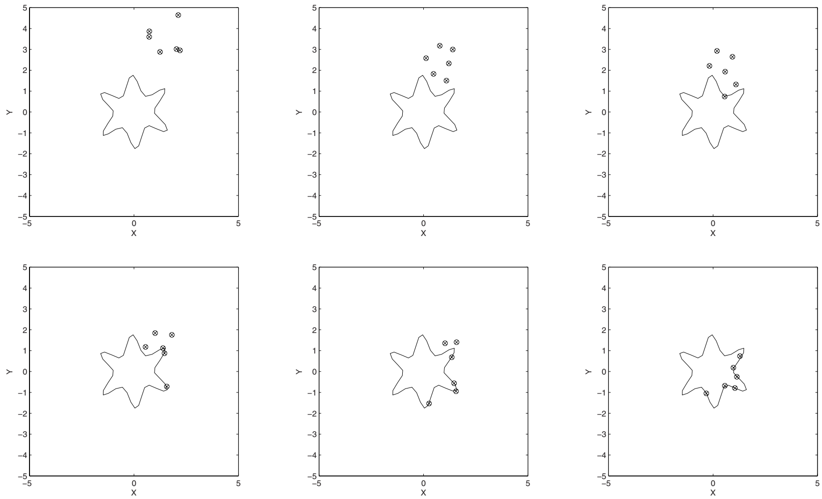

Fig. 4. Desired formation (star shape) with 6 robots

enables robots to achieve and maintain the desired formation while keeping good coordination with each other.

5 Conclusions

We have now presented a novel method for a group of mobile robots to form arbitrary desired shapes using parametric and implicit descriptions of the formation. Proposed method introduces flexibility on the desired shape and it can scale well to different swarm sizes.

We are planning to work on possible extensions of this work with non-holonomic finite size robots and implement them on actual robots.

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