AN ANT COLONY OPTIMIZATION APPROACH FOR THE MIXED VEHICLE ROUTING PROBLEM WITH BACKHAULS

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ABSTRACT

The Vehicle Routing Problem with Backhauls (VRPB) is a variant of the Vehicle Routing Problem where the vehicles are not only required to deliver goods but also to pick up some goods from the customers. In the mixed VRPB (MVRPB) each customer has either a delivery or a pick-up demand to be satisfied and the customers can be visited in any order along the route. Given a fleet of vehicles and a set of customers with known pick-up or delivery demands MVRPB determines a set of vehicle routes originating and ending at a single depot and visiting all customers exactly once. The objective is to minimize the total distance traversed with the least number of vehicles. A maximum route length restriction may also be imposed on the vehicles.

From a practical point of view MVRPB models situations such as distribution of bottled drinks, chemicals, LPG tanks, etc. In the case of the bottled drinks for instance, full bottles are delivered to customers and empty ones are brought back either for re-use or for recycling. In the chemicals case, some hazardous materials may need to be returned for safe disposal. Regulations or environmental issues may also force companies to take responsibility for their products throughout their lifetime. For this problem, we propose an Ant Colony Optimization (ACO) approach utilizing a new visibility function which attempts to capture the "delivery and pick-up" nature of the problem. We perform an extensive experimental study to compare the performance of the proposed approach with those of the well-known benchmark problems from the literature. Our numerical tests show that the proposed approach provides encouraging results.

Keywords:, vehicle routing with backhauls, ant colony optimization, metaheuristics

INTRODUCTION

The Vehicle Routing Problem with Pickup and Delivery (VRPPD) is a variant of the Vehicle Routing Problem where the vehicles are not only required to deliver goods but also to pick up some goods from the customers. Customers requiring a given demand quantity to be delivered from the depot are called linehauls while customers requiring a given supply quantity to be picked up and transported to the depot are called backhauls. The objective is to minimize the total distance traveled by the vehicles and/or the number of vehicles used subject to vehicle capacity constraint.

Nagy and Salhi (2005) classify the VRPPD into three categories:

i. *Delivery First, Pickup Second* (*VRPB*): the vehicles pick up goods after they have delivered their goods;
ii. *Mixed Delivery and Pickup* (*MVRPB*): linehauls and backhauls can occur in any sequence on a vehicle route; and
iii. *Simultaneous Delivery and Pickup* (VRPSPD): the vehicles simultaneously deliver and pickup goods.

In MVRPB and VRPSPD the objective and constraints are the same as in VRPB except the servicing order of the customers, which makes the former two problems more complicated because of the fluctuating load on the vehicle along the route. In VRPB, the loads of linehaul customers and backhaul customers can be checked separately during the delivery route and pickup route, respectively, to ensure that the vehicle capacity is not exceeded. In MVRPB, however, the decrease or increase on the vehicle load at each customer must be checked depending on whether the customer is a linehaul or backhaul customer, respectively.

In this paper, we address MVRPB where the goods are transported by a fleet of homogeneous vehicles. The delivery and pickup items are identical in the sense that each unit consumes the same amount of vehicle capacity. Each customer has either a delivery or a pickup demand to be satisfied. Items to be delivered are loaded at the depot while picked up items are transported back to the depot. The customers can be visited in any order along the route but they must be serviced exactly once. The objective is to determine a set of vehicle routes visiting all customers such that total distance traversed is minimized. An implicit primary objective is to utilize the minimum number of vehicles. In some cases, a maximum route length restriction may also be imposed on the vehicles.

Although the classical VRPB has been intensively studied in the literature there are very few papers attacking MVRPB. Since our focus is MVRPB we omit the discussion on the VRPB literature and refer the interested reader to Goetschalckx and Jacobs-Blecha (1989), Halse (1992), Toth and Vigo (1997), and Brandão (2006) for details and references.

Casco et al. (1988) propose a load-based insertion method for MVRPB utilizing a penalty cost based on delivery load after the pickup. Salhi and Nagy (1999) extend this method by proposing the cluster insertion of backhauls rather than one insertion at each iteration. They also investigate the case with multiple depots. Nagy and Salhi (2005) first find a solution to the VRP by allowing infeasibilities then modifies this solution to make it feasible for the MVRPB. The proposed approach is also capable of solving multi-depot problems. Recently, Ropke and Pisinger (2006) develop a unified heuristic for a large class of VRPPD based on a large neighborhood search. The proposed heuristic provides very good results.

In this paper, we propose an Ant Colony Optimization (ACO) algorithm for the MVRPB introducing a new visibility function which attempts to capture the "delivery and pickup" nature of the problem. To our knowledge, the only ant colony based approach previously proposed for this problem is in Wade and Salhi (2001). This preliminary work uses the Ant Colony System (ACS) approach of Dorigo and Gambardella (1997); however, the computational results are rather poor compared to those in the literature.

The remainder of the paper is organized as follows: In Section 2, the problem is depicted and a mathematical model formulation is provided. Section 3 is devoted to the introduction of ant systems and the discussion of the proposed algorithm. Section 4 describes the experimental study and discusses the numerical results. Finally, concluding remarks and future research directions are presented in Section 5.

PROBLEM DESCRIPTION

The problem deals with a single depot distribution/collection system servicing a set of customers using a homogeneous fleet of vehicles. Each customer has either a delivery (linehaul) or a pickup (backhaul) demand to be satisfied and is visited exactly once. Goods to be delivered are loaded at the depot and goods picked up are transported back to the depot. The critical feature of the problem is that the customers can be visited in any order along the route. The main reasoning behind visiting backhaul customers after all linehaul customers in VRPB is due to the fact that linehaul customers have precedence over backhaul customers in many real world cases and vehicles are often rear loaded. The latter causes problems when rearranging the items on the vehicle, thus prevents the mixed routes. However, the improved design of vehicles allows side loadings, making the mixed routes a more practical option since that would provide shorter routes.

From a practical point of view MVRPB models situations such as distribution of bottled drinks, chemicals, LPG tanks, etc. In the case of the bottled drinks for instance, full bottles are delivered to customers and empty ones are brought back either for re-use or for recycling. In the chemicals case, some hazardous materials may need to be returned for safe disposal. Regulations or environmental issues may also force companies to take responsibility for their products throughout their lifetime.

Mathematically, MVRPB is described by a set of homogenous vehicles V, a set of linehaul customers L, a set of backhaul customers B, and an undirected graph G (N, A). $N = \{0, \ldots, n+m\}$ denotes the set of nodes where $L = \{1,2, \ldots, n\}$, $B = \{n+1, n+2, \ldots, n+m\}$, 0 is the depot. Each vehicle has capacity Q and each customer (node) i is characterized by its geographical location and its delivery or pickup requests D_i and P_i, respectively. $A = \{(i, j): i, j \in N, i \neq j\}$ denotes the set of arcs that represents connections between the depot and the customers and among the customers. A cost/distance c_{ij} is associated with each arc (i, j). Finally, Q, D_i, P_i, c_{ij} are assumed to be non-negative integers. The objective of MVRPB is to determine a set of routes such that:

i. each vehicle travels exactly one route;
ii. each customer is visited only once by one of the vehicles completely satisfying its demand or supply;
iii. the load carried by a vehicle between any pair of adjacent customers on the route must not exceed its capacity; and
iv. the total distance given by the sum of the arcs belonging to these routes is minimal.

Following the mathematical model of VRPSPD in Dethloff (2001) the 0-1 mixed integer linear programming formulation of MVRPB is as follows:

Decision variables

L_j load of vehicle after having serviced customer $j \in N_C$

π_j subtour elimination variable

$$x_{ijv} = \begin{cases} 1, & \text{if vehicle } v \text{ travels directly from customer } i \text{ to customer } j \\ 0, & \text{otherwise} \end{cases}$$

Model Formulation

Minimize $z = \sum_{i \in N} \sum_{j \in N} \sum_{v \in V} c_{ij} x_{ijv}$ (1)

Subject to

$$\sum_{i \in N} \sum_{v \in V} x_{ijv} = 1 \qquad i \in N_C \qquad (2)$$

$$\sum_{i \in N} \sum_{v \in V} x_{ijv} = 1 \qquad j \in N_C \qquad (3)$$

$$\sum_{i \in N} x_{ikv} - \sum_{j \in N} x_{kjv} = 0 \qquad k \in N_C,\ v \in V \qquad (4)$$

$$l_j \geq \sum_{i \in N} \sum_{j \in N_C} D_j x_{ijv} - D_j + P_j - M(1 - x_{0jv}) \qquad j \in N_C,\ v \in V \qquad (5)$$

$$l_j \geq l_i - D_j + P_j - M\left(1 - \sum_{v \in V} x_{ijv}\right) \qquad i,j \in N_C,\ i \neq j \qquad (6)$$

$$\sum_{i \in N} \sum_{j \in N_C} D_j x_{ijv} \leq Q \qquad v \in V \qquad (7)$$

$$l_j \leq Q \qquad j \in N_C \qquad (8)$$

$$\pi_j \geq \pi_i + 1 - n\left(1 - \sum_{v \in V} x_{ijv}\right) \qquad i,j \in N_C,\ i \neq j \qquad (9)$$

$$\pi_j \geq 0 \qquad j \in N_C \qquad (10)$$

$$x_{ijv} \in \{0,1\} \qquad i,j \in N, v \in V \qquad (11)$$

In the formulation above, N_C denotes the set of customers ($N \setminus \{0\}$) and M is a sufficiently large number (e.g. $M = \max\left\{ \sum_{j \in N_C} (D_j + P_j), \sum_{i \in N} \sum_{j \in N, j \neq i} C_{ij} \right\}$). The objective function (1) minimizes the total distance traveled. Constraint sets (2) and (3) assure servicing each customer exactly once. Constraints (4) make sure that if a vehicle arrives at a customer, then the same vehicle departs from it. The load after servicing the first customer is defined with constraints (5) while the load "en route" is limited with constraints (6). Constraint sets (7) and (8) ensure that the load when leaving the depot and "en route", respectively, does not exceed the vehicle capacity. Constraints (9) are subtour elimination constraints. Constraints (10) are the non-negativity constraints and constraints (11) define the binary variables.

We can prove that MVRPB is *NP*-hard in the following way: let $P_i = 0$ for all $i \in B$, i.e. $B = \emptyset$. Then the problem reduces to VRP, which is known to be *NP*-hard. Thus, MVRPB is also *NP*-hard since VRP is a special case of MVRPB.

DESCRIPTION OF THE PROPOSED ANT COLONY APPROACH

ACO is based on the way ant colonies behave to find the shortest path between their nest and food sources. In the real world, initially all ants wander randomly. When they find food they return to their nest laying down a chemical substance, called pheromone, on their path. If other ants sense the pheromone on a path, they are likely to follow it rather than traveling at random, thus reinforcing the path. Greater level of pheromone on a path will increase the probability of ants following that path.

On the other hand, the pheromones evaporate over time, reducing the chance of other ants to follow the path. The longer the path between the nest and the food source the more the pheromones have to evaporate whereas the shorter the path the faster it is traversed and the more the pheromones are deposited. Thus, the pheromone levels remain higher on the short paths. As a consequence, the level of pheromone laid is basically based on the path length and the quality of the food source.

ACO simulates the above behavior of real ants to solve combinatorial optimization problems with artificial ants. Artificial ants find solutions in parallel processes using a constructive mechanism guided by artificial pheromone and a greedy heuristic known as visibility. The amount of pheromone deposited on arcs is proportional to the quality of the solution generated and increases at run-time during the computation.

The Ant System (AS) is the first ACO algorithm which was applied for solving the Traveling Salesman Problem (Colorni et al., 1991). Some other early applications include the elitist strategy for Ant System (EAS) proposed by Dorigo et al. (1996), rank-based version of Ant System (AS_{rank}) by Bullnheimer et al. (1999), Max-Min Ant System (MMAS) by Stützle and Hoos (1997), and Ant Colony System (ACS) by Dorigo and Gambardella (1997).

Since its first application many implementations of ACO have been proposed for a variety of combinatorial optimization problems such as quadratic assignment problem, scheduling problem, sequential ordering problem, vehicle routing problem and its variants, etc. We skip further discussion of the ACO and refer the interested reader to Dorigo and Stützle (2004) for a complete review and details. Next, we describe our ACO implementation for MVRPB.

Initialization

An initial amount of pheromone τ_0 is deposited on each arc. Dorigo and Gambardella (1997) observed that $\tau_0=n/L_0$, where L_0 is the length of an initial feasible route and n is the number of customers, can generate good routes. We also adopt this initialization of pheromone levels in our algorithm. The initial route is constructed using the nearest-neighbor heuristic: start at the depot and then select the not yet visited closest feasible customer as the next customer to be visited regardless of whether it is a linehaul or backhaul customer. A customer is infeasible if it violates the vehicle capacity. If no feasible customer is available then the route is terminated at the depot and a new route is initiated.

Heuristic Information

In the classical ACO approach the visibility value (heuristic information) between a pair of customers is the inverse of their distance. So, if the distance between two customers i and j is long, visiting customer j after customer i (or vice-versa) will be less likely. In our approach, the visibility function consists of two distinct components. The first component is the Clarke and Wright (1964) savings function as proposed by Doerner et al. (2002) for solving the classical VRP:

$$\eta_{ij}^{1} = d_{i0} + d_{0j} - d_{ij} \qquad (1)$$

where d_{ij} (d_{i0}) denotes the distance between customers i and j (the depot). We incorporate in our visibility function this savings value achieved by serving two customers i and j on the same route instead of serving them on different tours. Since a high value of savings indicates that visiting customer j after customer i is a desired choice the tour length is expected to be shorter if the probability of moving from customer i to customer j increases with η_{ij}^{1}.

The second component depends upon whether the next customer to be visited is a linehaul or a backhaul. In the case of linehaul (backhaul) our visibility is equal to the ratio of delivery to (pickup from) customer j to the average value of all deliveries (pickups) if total deliveries (pickups) of the vehicle so far exceed half of the vehicle capacity; and is equal to 1 otherwise. The idea is to basically give more chance of selection to customers requiring larger delivery (pickup) quantities. Our motivation in doing so stems from the "put first larger items" approach used for the Bin Packing Problem. The reason why we start employing this approach after half of the vehicle capacity is used up is to not adversely affect the influence of the first component. The computation of the second visibility value is as follows:

$$\text{If } j \text{ is a linehaul: } \eta_{ij}^{2} = \begin{cases} \dfrac{D_j}{\overline{D}} & , \text{ if } \sum_{k \in V_q} D_k > \dfrac{Q}{2} \\ 1 & , \text{ otherwise} \end{cases}$$

$$\text{If } j \text{ is a backhaul: } \eta_{ij}^{2} = \begin{cases} \dfrac{P_j}{\overline{P}} & , \text{ if } \sum_{k \in V_q} P_k > \dfrac{Q}{2} \\ 1 & , \text{ otherwise} \end{cases} \qquad (2)$$

Here, $\overline{D}$ ($\overline{P}$) is the average delivery (pickup) and V_q is the set of customers already visited by the associated vehicle q. Note that the first component is static whereas the second depends on the current load of the vehicle.

Route Construction

The route construction process is similar to the pseudo-random proportional rule introduced in ACS. An ant is positioned at each customer and each ant constructs its own route by successively selecting a customer from the feasible candidate customers set N_i^k. For each ant k at each customer i the candidate set N_i^k is formed by taking not yet visited customers that do not violate the vehicle capacity and having the largest attractiveness value. The size of the candidate set is a parameter s. If N_i^k is empty then the ant returns to the depot and starts a new route. This procedure is repeated until all customers are serviced.

The choice of the next customer is based on its attractiveness value, which is a function of the pheromone information (intensity) and heuristic information (visibility):

$$\varphi_{ij} = [\tau_{ij}]^{\alpha} [\eta_{ij}^1]^{\beta_1} [\eta_{ij}^2]^{\beta_2} \tag{3}$$

where α, β_1, and β_2 are parameters to control the relative weight of trail intensity τ_{ij} and visibility η_{ij}^1 and η_{ij}^2. An ant k located at customer i may either visit its most favorable customer or randomly select a customer using the following selection rule:

$$j^k = \begin{cases} \arg\max\limits_{j \in N_i^k} \varphi_{ij} & , \quad \text{if } z \le z_0 \\ J^k & , \quad \text{otherwise} \end{cases} \tag{4}$$

where z is a random variable drawn from a uniform distribution $U[0,1]$ and z_0 ($0 \le z_0 \le 1$) is a parameter to control exploitation versus exploration. J^k is selected according to the following probability distribution:

$$p_{ij}^k = \begin{cases} \dfrac{\varphi_{ij}}{\sum\limits_{l \in N_i^k} \varphi_{il}} & , \quad \text{if } j \in N_i^k \\ 0 & , \quad \text{otherwise} \end{cases} \tag{5}$$

Pheromone Update

The pheromone update includes two steps: pheromone evaporation and pheromone reinforcement. The pheromone evaporation refers to uniformly decreasing the pheromone values on all arcs. The aim is to prevent the rapid convergence of the algorithm to a local optimal solution by reducing the probability of repeatedly selecting certain customers. The pheromone reinforcement process, on the other hand, increases the pheromone values on the arcs belonging to the tour of the best performing ant(s) at each iteration as well as from previous iterations. The aim is to increase the probability of selecting the arcs frequently used by the ants that construct short tours.

In our pheromone update process we adopt a rank-based MMAS strategy. In this strategy, w best-ranked ants of each iteration along with the best-so-far ant are used to update the pheromone trails. The pheromone reinforcement of each ant is proportional to its rank. Our pheromone update rule is as follows:

$$\tau_{ij} \leftarrow (1-\rho)\tau_{ij} + \sum_{r=1}^{w} (w-r)\Delta\tau_{ij}^{r} + w\Delta\tau_{ij}^{bs} \qquad (6)$$

In this formulation, ρ ($0 < \rho \leq 1$) is the pheromone evaporation parameter and $\Delta\tau_{ij}^{r} = 1/L^{r}$ for all arcs (*i, j*) belonging to the tour built by the r^{th} best ant where L^{r} is the length of the corresponding tour. *bs* denotes the best-so-far ant.

Furthermore, if the pheromone level on any arc drops below an explicit lower limit or exceeds an explicit upper limit it is set equal to that limit. In other words, if any $\tau_{ij}<\tau_{\min}$ ($\tau_{ij}>\tau_{\max}$) then $\tau_{ij}=\tau_{\min}$ ($\tau_{ij}=\tau_{\max}$). The aim in using this MMAS approach is to reduce the risk of a premature convergence.

Local Search

After an ant has constructed its tour, a local search is performed in an attempt to further improve the solution. In our algorithm we use the 2-opt and swap procedures sequentially. In 2-opt two customers are exchanged whereas in swap a customer is removed and inserted into a another arc. These procedures are applied both within routes and between different routes.

The outline of our algorithm is given in Figure 1.

```
compute visibility1
initialize pheromone levels
while (maximum number of iterations is not reached)
    for (each ant)
        while (the tour is not completed)
            compute visibility2 and attractiveness
            select the next customer to visit
            update vehicle capacity and candidate list
        end while
        apply local search
    end for
    save the best-so-far solution
    update pheromone levels
end while
```

Figure 1 - Outline of the proposed algorithm

EXPERIMENTAL STUDY

To test the performance of our algorithm we consider the benchmark problems proposed by Goetschalckx and Jacobs-Blecha (1989). This data set consists of 63 instances with the number of customers varying from 25 to 150 and was utilized by Wade and Salhi (2001) as well to test their ant algorithm.

The algorithm is coded using C++. The parameters were set according to initial experimental runs as: z_0=0.5, α=1, β_1=4, β_2=1, ρ=0.1, $\tau_{\max}=(n+m)/\rho L^{bs}$, and $\tau_{\min}= \tau_{\max}/4$. The number of best ants used for the pheromone reinforcement and the size of the candidate list used in the selection of the next customer to be visited are proportional to the number of ants and the number of customers, respectively, and their values are set $w=(n+m)/10$ and $s=(n+m)/5$, respectively. Note that the number of ants is equal to the number of customers. For each problem instance we performed 10 runs, each carried on for 100 iterations.

Table 1 details the results. For comparison, we only use 25 instances for which Wade and Salhi (2001) provided benchmark results for their ant system algorithm. In this table, n_1 and n_2 denote the number of linehaul and backhaul customers, respectively, and Q denotes the vehicle capacity. The results show that our algorithm significantly outperforms that of Wade and Salhi (2001) in all instances. The average improvement is 11%. On the other hand, our average performance is slightly inferior compared to the results of Halse (1992), with an average deviation of 0.66%. Nevertheless, our algorithm improves 10 best-known solutions. We observe that the performance deteriorates as the problem size increases. We also observe that our algorithm is robust in the sense that the average deviation of the average distances from the best achieved distances is 0.86%.

Table 1 – Comparison of computational results

Problem	n_1	n_2	Q	Our Algorithm Avg	Our Algorithm Best	Wade & Salhi Best	% Dev	Halse Best	% Dev
a1	20	5	1550	223748	223088	248049	11.19	227725	2.08
a2	20	5	2550	169500	169500	179744	6.04	169497	0.00
a3	20	5	4050	142058	142034	148775	4.75	142032	0.00
b1	20	10	1600	231756	230813	254091	10.09	233950	1.36
b2	20	10	2600	179348	179258	200828	12.03	182326	1.71
b3	20	10	4000	145702	145702	153377	5.27	145699	0.00
c1	20	20	1800	240781	238103	266925	12.10	242931	2.03
c2	20	20	2600	198409	197448	223760	13.33	197276	-0.09
c3	20	20	4150	165849	164891	180704	9.59	167663	1.68
e1	30	15	2650	222918	220742	250594	13.52	222518	0.80
e2	30	15	4300	191318	191160	204624	7.04	190048	-0.58
e3	30	15	5225	182598	181941	195635	7.53	187793	3.22
g1	45	12	2700	305805	299656	346971	15.79	304106	1.49
g2	45	12	4300	236326	234718	265551	13.14	235220	0.21
g3	45	12	5300	214104	212841	231016	8.54	213757	0.43
g4	45	12	6400	204003	202570	218414	7.82	201875	-0.34
h1	45	23	4000	242001	239586	283141	18.18	235269	-1.80
h2	45	23	5100	219180	216684	238738	10.18	215649	-0.48
h3	45	23	6100	208776	207130	239088	15.43	202971	-2.01
j1	75	19	4400	365833	358946	395709	10.24	337800	-5.89
j2	75	19	5600	323929	317785	362307	14.01	298432	-6.09
j3	75	19	8200	300586	297300	313187	5.34	280070	-5.80
l1	75	75	4400	426154	416515	467021	12.13	412278	-1.02
l2	75	75	5000	384298	380137	424729	11.73	362399	-4.67
m1	100	25	5200	390029	383243	456046	19.00	372840	-2.71
Average							10.96		-0.66

CONCLUSION AND FUTURE RESEARCH DIRECTIONS

In this paper, we address the MVRPB using an ACO algorithm equipped with a new visibility function. The experimental analysis reveals promising results compared to the benchmark results published in the literature. Particularly, our algorithm outperforms the ant system approach of Wade and Salhi (2001). Although we utilize some special mechanisms of ant systems it is worth noting that these results are preliminary in an attempt to gain insights about the structure of the problem, as is the case in Wade and Salhi (2001) as well.

Although a comparison of the computational effort cannot be made we observe that our computation times are quite large: varying from 2.25 seconds for the 25 customer problems to 33 minutes for the 150 customer problems. We can speed up the algorithm in a few ways, e.g. by reducing the number of ants, by performing a selective local search based on the quality of solution generated.

Future work in this area may be dedicated to investigate the visibility function for this special class of VRP, to develop more efficient local search heuristics to improve the solution quality, and to fine tune the algorithm parameter. Moreover, this approach may be easily applied to VRPSPD since it handles both delivery and pickup capacity checks at each customer and to VRPB with little effort.

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