

**SENSORLESS CONTROL OF  
INDUCTION MACHINE**

**By  
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**Submitted to the Graduate School of Engineering and Natural Sciences  
in partial fulfillment of  
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**SABANCI UNIVERSITY  
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**SENSORLESS CONTROL OF  
INDUCTION MACHINE**

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**DATE OF APPROVAL:** .....

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## ABSTRACT

AC drives based on fully digital control have reached the status of a maturing technology in a broad range of applications ranging from the low cost to high performance systems. Continuing research has concentrated on the removal of the sensors measuring the mechanical coordinates (e.g. tachogenerators, encoders) while maintaining the cost and performance of the control system. Speed estimation is an issue of particular interest with induction motor electrical drives as the rotor speed is generally different from the speed of the revolving magnetic field. The advantages of sensorless drives are lower cost, reduced size of the machine set, elimination of the sensor cable and reliability. However, due to the high order and nonlinearity of the IM dynamics, estimation of the angle speed without the measurement of mechanical variables becomes a challenging problem. Variety of solutions has been proposed to solve this problem in the literature.

In this thesis work, by combining the variable structure systems and Lyapunov designs a new sensorless sliding mode observer algorithm for induction motor is developed. A Lyapunov function is chosen to estimate the rotor flux of an induction motor under any initial condition based on the principle that the aim of the vector control of IM is to keep the rotor flux magnitude constant from zero to nominal speed. Additionally, an observer estimating the rotor speed and the rotor time constant of the machine simultaneously has been proposed that stems from the flux estimation. The proposed method is very suitable for closed loop high-performance sensorless drives and it is believed that with its new approach it will help many researchers in their further work in the field of sensorless vector control of IM.

The proposed algorithm has been tested and verified via simulation and experimental results on an IM in the graduate laboratory of Mechatronics at the Sabanci University.

## ÖZET

Sayısal kontrol yöntemlerine dayalı asenkron ve senkron motor sürücüleri düşük ve yüksek performanslı sistemlerin yer aldığı çok geniş bantlı uygulamalarda olgunlaşan teknoloji seviyesine ulaşmıştır. Bu alandaki mevcut araştırma motorun mevcut maliyetini ve performansını iyileştirirken motor mekanik koordinatlarını ölçen sensörlerin (takojenaratör, enkoder v.b.) sistemden ayrılmasına yöneliktir. Endüksiyon motoru sürücülerinde rotor hızı motor manyetik alanı dönme hızından farklı olduğundan hız kestirimi ayrı bir önem arz etmektedir. Sensörsüz sürücülerin en önemli avantajları düşük maliyet, makine ebatlarında azalma, sensör kablosunun çıkarılması ve güvenilirliktir. Buna rağmen, endüksiyon motorunun yüksek dereceli ve doğrusal olmayan dinamikleri sistemin mekanik durumlarını ölçmeden yapılan hız kestirimlerini zorlayıcı bir problem haline getirmektedir. Literatürde bu problemi çözmek için birçok yöntem önerilmiştir.

Bu tezde, değişken yapılı sistem ve Lyapunov tasarım yöntemleri kullanılarak endüksiyon motoru için yeni bir kayan kipli gözlemleyici modeli önerilmiştir. Bunun için, vektör kontrolünün manyetik alan büyüklüğünün sıfır hızdan anma hızına kadar sabit tutulması prensibine dayalı olarak motor akısının başlangıç koşullarından bağımsız olarak kestirilmesi için bir Lyapunov fonksiyonu seçilmiştir. Ayrıca akı kestiriminden faydalanarak motor shaft hızı ve zaman sabitini kestirilmesi için bir gözlemleyici önerilmiştir. Önerilen gözlemleyiciler kapalı-çevrim yüksek performans sensörsüz sürücüler için çok uygundur ve önerilen bu yeni fikrin bu alanda çalışan birçok araştırmacıya ileriki sensörsüz vektör kontrol alanındaki çalışmalarında yardımcı olacağına inanılmaktadır.

Önerilen algoritma Sabancı Üniversitesi Mekatronik yüksek lisans laboratuvarında benzeşim ve deneylerle test edilmiş ve doğrulanmıştır.

*To my “endless love” Tuba*

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## TABLE OF SYMBOLS

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$R$	Dissipation resistance
$\psi(t)$	Flux linkage
$T_e$	Electrical torque
$T_l$	Load torque
$\theta$	Angular position and speed
$L_s$	Stator inductance
$L_r$	Rotor inductance
$L\sigma$	Leakage inductance
$L_m$	Magnetizing inductance
$R_s$	Stator resistance
$R_r$	Rotor resistance
$i$	Phase winding current
$u_s, u_r, i_s, i_r$	Stator and rotor voltages and currents
$\omega_g$	Arbitrary frame of reference rotational speed
$\omega_e$	Rotor flux angular speed
$\omega_s$	Stator flux angular speed
$\omega_r$	Shaft speed
$\alpha - \beta$	Stationary frame of reference
$d - q$	Rotor flux frame of reference
$K$	Fictitious variable for the rotor flux observer

## TABLE OF ABBREVIATIONS

e.m.f.	Electro motive force
IM	Induction machine
V.S.S	Variable structure systems
SMC	Sliding Mode Controller
SMO	Sliding Mode Observer
HVAC	Heating, ventilation, and air conditioning
DC	Direct current
AC	Alternative current
DSP	Digital signal processor
LPF	Low pass filter
SVPWM	Space Vector Pulse Width Modulation

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## **1 INTRODUCTION**

Today the industrial processes require advanced high performance, low cost control techniques to control the torque and accurate position and low speed for their operations in the application areas like appliances (washers, blowers, compressors), HVAC (heating, ventilation and air conditioning), industrial servo drives (Motion control, Power supply inverters, Robotics), .Automotive control (electric vehicles).Asynchronous motors are based on induction. The least expensive and most widely spread induction motor is the squirrel cage motor. They are known as the “work horses” of the industry. The wires along the rotor axis are connected by a metal ring at the ends resulting in a short circuit. There is no current supply needed from outside the rotor to create a magnetic field in the rotor. This is the reason why this motor is so robust and inexpensive. Previously the circuitry for driving the induction motors were too complicated and expensive to apply to the daily life, DC drives were dominating the market. During the last few years the field of controlled electrical drives has undergone rapid expansion due mainly to the advantages of semiconductors in both power and signal electronics and resulting in micro-electronic microprocessors and DSPs. These technological improvements have enabled the development of really effective AC drive control with ever lower power dissipation hardware and ever more accurate control structures. Thanks to these factors, the control of AC machine acquires every advantage of DC machine control and frees itself from the mechanical commutation drawbacks.

### **1.1 Problems In The Control of Induction Machine**

In the high performance control of AC drives a technique called “field oriented control” is used. The aim in this technique is to decouple the torque and flux of the machine resulting in a high performance independent control of the torque and flux in

the transient and steady state operation as for the separately excited DC machine. To decouple the components of the stator current that are controlling the flux and torque independently the rotor flux position should be measured or estimated. The measurement of the rotor flux is not an easy task, special winding design and additional sensors should be added to the plant to be controlled which causes reliability problems and increase in the cost. Thus, due to the problems explained above the main approach in getting the rotor flux position is to construct flux observers.

Removal of the sensors that are measuring the mechanical coordinates of the system is one of the other main ongoing research due to the advantages like reliability, low cost, maintenance and operation in the harsh environment of these sensorless structures not only in the field of electrical drives but also in the field of dynamic control. However, due to the high order (5th) and nonlinearity of the IM dynamics, estimation of the angle speed and rotor flux simultaneously without the measurement of mechanical variables becomes a challenging problem.

## **1.2 Literature Survey On Flux Observers**

Many researchers proposed their solution to solve the problems of sensorless control. Most of them are purely based on machine flux model. There are in general two flux based methods, voltage and current model of induction machine. In the literature, generally both voltage and current models of induction machine have been used together for flux estimation and then from those speed has been estimated [16][17]. Both current and voltage models of induction machine are needed to get flux information. Those methods imply the estimation of the time-derivative with subsequent integration. However, implementation of an integrator for motor flux estimation is no easy task. A pure integrator has dc drift and initial value problems. To solve the problems, the pure integrator has replaced by a low pass filter (LPF). To estimate exactly stator flux in a wide speed range, the LPF should have a very low cutting frequency. However, there still remains the drift problem due to the very large time constant of the LPF. A digital filter was proposed to solve the drift problem [18]. In [17][19], open loop observer structures based on voltage model of the induction motor are proposed and integration problem is attempted to be avoided by using different

programmable and/or digital low-pass filter structures. The proposed programmable low pass filter (LPF) has phase compensator to estimate exactly stator flux and solves the dc drift problem associated with a pure integrator and a LPF in a wide speed range.

One approach to the sensorless control problem is to consider the speed as an unknown constant parameter and to use the techniques of adaptive control to estimate this parameter [20][21][22]. This idea is that the speed changes slowly compared to the electrical variables. This approach was first formulated by Shauder [23] and with some modification proposed by Peng and Fukao [21].

Sliding mode control theory, due to its order reduction, disturbance rejection, strong robustness and simple implementation by means of power converter, is one of the prospective control methodologies for electrical machines. The basic concepts and principles of sliding mode control of electrical drives were demonstrated in [11] and some aspects of the implementation are illustrated in [12]. Furthermore, sliding mode observers [1][2][3][4][7][8][12] have been proposed for estimating the states of the control system. Sliding mode observers also have the same robust features as the sliding mode controllers. Zaremba [2] suggested a sliding mode speed observer in d-q coordinate with stability and robustness analysis for the system with constant speed. Benchaib et al. [4] proposed a control and observation of an induction motor using sliding mode technique. The observer model is a copy of the original system, which has corrector gains with switching terms. Parasiliti et al. [5] presents an adaptive sliding mode observer for sensorless field oriented control of induction motors. The observer detects the rotor flux components in the stationary reference frame by motor mechanical equations. An additional relation obtained by a Lyapunov function let us identify the motor speed. Şahin [1] proposed a convergence term for the rotor side of the observer by assuming that the estimated speed is equal to the actual one. Yan et al. [7] proposed a full order observer adding convergence terms to the rotor side but here the systematic how to find the convergence terms have not been mentioned. Dal [8] presented a new control selection with chattering free sliding modes [13] in the observer structure s.t. the calculated control can be directly used in the rotor side dynamics of the observer. Stability analysis was also given in this paper. In [10] by combining the variable structure systems and Lyapunov designs a new sliding mode observer algorithm for induction motor is developed. A Lyapunov function is chosen to determine the speed and rotor resistance of an induction motor simultaneously based on the assumption that the speed is an unknown constant parameter.

In this thesis, by combining the variable structure systems and Lyapunov designs a new sliding mode observer algorithm for induction motor is developed. The proposed method offers a solution for the initial condition mismatch and integration problem which has been discussed a lot in the literature. This method uses measurement of the stator currents and stator voltages to estimate the derivative of the rotor flux. Then a using the property of the vectors rotor flux and its derivative being orthogonal a convergence term is derived to compensate for the initial condition mismatch in the estimation of the rotor flux. Then using the estimated flux information, speed and rotor time constant of the motor is estimated.

In the thesis first the state space vector model of the induction motor and the principles of the control of the electrical drives and the vector control theory is covered. Also the variable structure systems and the sliding mode control theory (SMC) is explained in this chapter. The proposed observer design and stability analysis is given in the third chapter. And finally the performance of the proposed method is investigated and verified via simulation and experimental results given in the last chapter.

## **2 MODELING AND CONTROL OF THE INDUCTION MACHINE**

### **2.1 State Space Vector Modeling and Dynamics of Electrical Machine**

For the purpose of understanding and designing torque controlled drives, it is necessary to know the dynamic model of the machine subjected to control. Such a model should be valid for any instantaneous variation of all the states (stator voltages, stator currents, rotor fluxes etc.) describing the performance of the machine under both transient and steady state operation and this kind of a model can be easily obtained by the utilization of space vector theory [14]. The advantages of such a model is that it is physically more understandable, the modern control theory stems from this space vector modeling of the machine since the time dependencies of the inductances of the machine are removed during the transformation from 3 phase variables to 2 phase space vector variables and in the literature the observers required to estimate the unknown or unmeasured states of the machine are constructed based on this theory. During this chapter specifically the application of state space vector theory on the asynchronous electrical machine will be dealt since it is the plant on which the thesis work is done, but the same procedure can be easily adapted to the synchronous machine without so much work.

#### **2.1.1 Dynamics of Electromechanical Energy Conversion Systems**

The dynamic equations of motion of the electromechanical system can be determined from the basic physical laws such as [25]:

- Faraday's law and Kirchoff's law for electrical subsystem,
- d'Alambert's principle for the mechanical subsystem.

For quasi-static (low frequency) and low velocity operation of an electromechanical system the equations of motion may be expressed in terms of lumped parameters. Electrical subsystem does not have any energy storing elements and consists solely on the sources and dissipative (resistive) elements. So, it can be simply modeled as a resistive electrical circuitry supplied from the input sources and connected to the inputs of the coupling subsystem.

For all rotating machines mechanical subsystem can be modeled as a single cylinder rotating around its axis.

As a result of the energy conversion, at the input terminals of the coupling two subsystems which are the electrical and the mechanical subsystems two different reactions are present:

- the induced voltage at the electrical terminals;
- the mechanical forces at the mechanical terminals .

This separation of the coupling system reaction is a starting point for the derivation of the equations of motion for overall system.

When an electromechanical system coupled with single electrical and single mechanical inputs is investigated; Electrical subsystem is represented by the resistance connected in series with input source and to the electrical terminals to the coupling system. The mechanical subsystem is represented by the cylinder, which can freely rotate around its axis. The cylinder motion is influenced by the generated torque and the mechanical torque applied from the source of the mechanical energy. No losses are assumed in the mechanical subsystem.

If  $u$  is the source voltage,  $R$  is the resistance of the dissipation in the electric subsystem,  $\psi(t)$  is the flux linkage representing the coupling field,  $d\psi(t)/dt$  is the induced electromotive force (e.m.f.) at the electric terminal of the coupling system, then current entering the coupling subsystem through electrical terminal, can be calculated as:

$$\frac{d\psi(t)}{dt} + Ri = \frac{d(Li)}{dt} + Ri = u \quad (2.1)$$



The generated torque can be expressed as:

$$T_e = \frac{1}{2} \frac{\partial L}{\partial \theta} i^2 = \frac{1}{2} \psi \frac{\partial i}{\partial \theta} \quad (2.2)$$

The mechanical subsystem exhibit simple rotational motion around the cylinder axis and, from d'Alambert's principle, equations of motion can be written as:

$$\frac{d\theta}{dt} = \omega \quad (2.3)$$

$$\frac{d(J\omega)}{dt} = T_e - T_L \quad (2.4)$$

where  $J$  is a moment of inertia of the rotor,  $T_L$  is a torque applied to the system from the mechanical sources usually called load torque.

The set of the equations (2.1) and (2.4) describes the behavior of electromechanical energy converters with one electrical and one mechanical terminal. Usual construction of the rotating electrical machines is such that there are more than one electrical terminals and only one mechanical terminal. Application of the developed mathematical model to such systems requires simple transformation into vector notation, while keeping the same form of all expressions. So, the general mathematical model of an electromechanical converter with rotational motion,  $n$  electrical inputs and one mechanical input to the coupling field can be written as:

$$\frac{d\bar{\psi}(t)}{dt} + \bar{R}\bar{i} = \bar{u} \quad (2.5)$$

where  $\bar{\psi}^T = [\psi_1 \ \psi_2 \dots \ \psi_m]$  is the linkage flux vector,  $\bar{u}^T = [u_1 \ u_2 \dots \ u_m]$  is input the voltage vector,  $\bar{i}^T = [i_1 \ i_2 \dots \ i_m]$  is input current vector,  $\bar{R} = \text{diag}\{R_{ii}\}$  ( $i=1, \dots, n$ ) is the diagonal resistance matrix. For electrically linear system linkage flux vector can be expressed as linear function of the input current  $\bar{\psi}^T = \bar{L}\bar{i}$  where matrix  $\bar{L} = \{L_{ij}\}$  ( $i, j=1, \dots, n$ ), represent inductance matrix of the machine, and the mathematical model can be written as  $\frac{d[\bar{L}\bar{i}]}{dt} + \bar{R}\bar{i} = \bar{u}$ . Generated torque can be expressed as :

$$T_e = \frac{1}{2} \bar{\mathbf{i}}^T \frac{\partial \mathbf{L}}{\partial \theta} \bar{\mathbf{i}} = \frac{1}{2} \psi^T \frac{\partial \bar{\mathbf{i}}}{\partial \theta} \quad (2.6)$$

The motion of the mechanical subsystem is described by the equations (2.3) and (2.4). For the application of this model the components of the linking flux vector and the resistance matrix  $R$  must be calculated. The calculation of matrix  $R$  and matrix  $L$  is

not going to be mentioned here instead the obtained matrix  $\mathbf{R}$  and  $\mathbf{L}$  for a 3-phase smooth air-gap machine will be given as below and an explanation about why it is required to model in 2-phase space vector will be given.

An electrical machine consists of the stationary part, called the stator, and cylindrical rotating part, referred as the rotor. Electro-magnetically, a machine consists of two or more sources of magnetic excitation, which can be an electrical winding or permanent magnet, coupled magnetically by means of the magnetic circuit. The magnetic circuitry consists of the stator, air gap and rotor.

Following the 3-phase model of the smooth air-gap machine will be given;

$$\mathbf{R} = \begin{bmatrix} \mathbf{R}_s & 0 \\ 0 & \mathbf{R}_r \end{bmatrix}; \mathbf{L} = \begin{bmatrix} \mathbf{L}_{ss} & \mathbf{L}_{sr} \\ \mathbf{L}_{rs} & \mathbf{L}_{rr} \end{bmatrix} \quad (2.7)$$

where indexes "s" and "r" denote stator and rotor parameters respectively, index "sr" and "rs" denote mutual inductances stator to rotor and rotor to stator respectively.

Assumed symmetry of the stator and rotor windings is represented by the fact that all phase resistances on the stator are equal. The same property can be applied to the resistances of the phase windings on rotor. The resistance matrices for both stator and rotor windings are diagonal and, if the resistance of the stator winding is  $R_s$  the resistance of the rotor winding is  $R_r$ , resistance matrices can be written as  $\mathbf{R}_s = \text{diag}\{R_s\}_{3 \times 3}$  for stator circuit and  $\mathbf{R}_r = \text{diag}\{R_r\}_{3 \times 3}$  for rotor circuit.

The inductance matrix  $\mathbf{L}$  has four terms: matrix  $\mathbf{L}_{ss}$  represents the inductances of the stator windings, matrix  $\mathbf{L}_{rr}$  represents the inductances of the rotor windings and matrices of mutual inductances between stator and rotor windings  $\mathbf{L}_{sr}$  and  $\mathbf{L}_{rs}$ . The windings on the stator are stationary to each other so the self and mutual stator-to-stator inductances for all windings are constant. The same apply to the self and mutual rotor-to-rotor inductances of the windings. Denoting self inductance of the stator winding by  $L_s$  and  $L_r$  the self inductance of the rotor windings, the matrices  $\mathbf{L}_{ss}$  and  $\mathbf{L}_{rr}$  can be expressed as:

$$\mathbf{L}_{ss} = L_s \begin{bmatrix} 1 & \cos 2\pi/3 & \cos 4\pi/3 \\ \cos 2\pi/3 & 1 & \cos 2\pi/3 \\ \cos 4\pi/3 & \cos 2\pi/3 & 1 \end{bmatrix} = L_s \mathbf{T} \quad (2.8)$$

$$\mathbf{L}_{rr} = L_r \begin{bmatrix} 1 & \cos 2\pi/3 & \cos 4\pi/3 \\ \cos 2\pi/3 & 1 & \cos 2\pi/3 \\ \cos 4\pi/3 & \cos 2\pi/3 & 1 \end{bmatrix} = L_r \mathbf{T} \quad (2.9)$$

Matrix T is equal for both stator and rotor due to the symmetry of the windings. Due to the angular rotation of the rotor, and the windings attached to it, the relative position of the corresponding stator and rotor windings changes and mutual stator-to-rotor and rotor-to-stator inductances are function of the angle formed between axes of the corresponding windings. Matrix of the mutual stator-to-rotor inductances  $L_{sr}$ , and rotor-to-stator inductances  $L_{rs}$ , can be expressed as:

$$\mathbf{L}_{sr} = L_{sr} \begin{bmatrix} \cos \theta & \cos(\theta + 2\pi/3) & \cos(\theta + 4\pi/3) \\ \cos(\theta + 2\pi/3) & \cos \theta & \cos(\theta + 2\pi/3) \\ \cos(\theta + 4\pi/3) & \cos(\theta + 2\pi/3) & \cos \theta \end{bmatrix} = L_{sr} \mathbf{T}_{st} \quad (2.10)$$

$$\mathbf{L}_{rr} = L_{sr} \mathbf{T}_{sr}^T \quad (2.11)$$

Here matrix  $\mathbf{T}_{sr}$  depends on the mutual position of the stator-to-rotor windings. The elements of this matrix are periodic function of the angular position of the rotor. Mathematical model (2.3), (2.4) and (2.5), for this machine, with  $\mathbf{i}_s^T = [i_{sa} \ i_{sb} \ i_{sc}]$  stator current vector,  $\mathbf{i}_r^T = [i_{ra} \ i_{rb} \ i_{rc}]$  rotor current vector,  $\mathbf{u}_s^T = [u_{sa} \ u_{sb} \ u_{sc}]$  stator voltage vector,  $\mathbf{u}_r^T = [u_{ra} \ u_{rb} \ u_{rc}]$  rotor voltage vector, becomes:

$$\begin{bmatrix} \mathbf{L}_{ss} & \mathbf{L}_{sr} \\ \mathbf{L}_{rs} & \mathbf{L}_{rr} \end{bmatrix} \begin{bmatrix} \frac{d\mathbf{i}_s}{dt} \\ \frac{d\mathbf{i}_r}{dt} \end{bmatrix} + \begin{bmatrix} \mathbf{R}_s & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{R}_r \end{bmatrix} \begin{bmatrix} \mathbf{i}_s \\ \mathbf{i}_r \end{bmatrix} + \begin{bmatrix} \frac{\partial \mathbf{L}_{ss}}{\partial \theta} & \frac{\partial \mathbf{L}_{sr}}{\partial \theta} \\ \frac{\partial \mathbf{L}_{rs}}{\partial \theta} & \frac{\partial \mathbf{L}_{rr}}{\partial \theta} \end{bmatrix} \begin{bmatrix} \mathbf{i}_s \\ \mathbf{i}_r \end{bmatrix} \frac{d\theta}{dt} = \begin{bmatrix} \mathbf{u}_s \\ \mathbf{u}_r \end{bmatrix} \quad (2.12)$$

$$J \frac{d\omega}{dt} + T_L = \frac{1}{2} \begin{bmatrix} \mathbf{i}_s & \mathbf{i}_r \end{bmatrix} \begin{bmatrix} \frac{\partial \mathbf{L}_{ss}}{\partial \theta} & \frac{\partial \mathbf{L}_{sr}}{\partial \theta} \\ \frac{\partial \mathbf{L}_{rs}}{\partial \theta} & \frac{\partial \mathbf{L}_{rr}}{\partial \theta} \end{bmatrix} \begin{bmatrix} \mathbf{i}_s \\ \mathbf{i}_r \end{bmatrix} \quad (2.13)$$

The analysis of the electromagnetic torque generation (2.6) reveals that the torque generated in the examined structures is due to the change of the self-inductance and/or mutual inductances as a function of the angular position of the rotor. The configuration of the magnetic circuitry of electrical machines determines the dependence of the inductances of the windings, located at the stator and rotor structures, as function of the angular displacement of the rotor.

Here complexity of the equations of motion due to the time varying mutual inductances between stator and rotor windings is apparent. Further analysis of the dynamics of the machine with smooth air gap using this mathematical model is very complicated even using computers.

If the inductance matrices given in (2.8), (2.9), (2.10), (2.11) is investigated, it can be easily realized that they depend on the rotor angular position which causes the

parameters of the mathematical description to be time varying for all operating conditions except steady state operation with zero speed.

The state variables were selected to be a winding's variables for the model given above. In the analysis of the dynamical systems transformations of variables is common tool to simplify the mathematical models and to make the analysis of the system simpler. The same procedure can be applied to the analysis of the electrical machines to overcome the time varying inductance problem in the modeling of the machine.

During the simplification of the modeling of the asynchronous machine in state space following assumptions are made:

- Air-gap flux distribution is sinusoidal
- Motor magnetic circuit is operating in linear region without saturation
- Stator windings are symmetric and star-connected and the neutral point between the phases is electrically isolated
- Number of pole pairs in the stator windings is taken as 1, but the results can be easily adapted to more than 1 pole pairs.
- Number of stator and rotor turns is assumed to be equal
- Skin effect and the eddy current losses are neglected.

### **2.1.2 State Space Vector Modeling of the Asynchronous Machine**

All the 3-phase state variables (voltages, fluxes, currents) related to the rotor and the stator circuit of the induction machine (IM) can be transformed to orthogonal space vectors which are a combination of an imaginary and a real part. 2-phase equivalent orthogonal components (e.g. 2-phase winding variables) of 3-phase rotor and stator winding variables are obtained by this transformation. For this transformation, proper frames of reference have to be chosen. There are generally 3 frames of reference existing which can be seen in figure 1 from which proper one is chosen so as to be used for the modern control approaches. These are:

- Stationary frame of reference: The real axis of this frame is selected as collinear with one of the phases of the stator windings.
- Rotor frame of reference: This frame of reference is rotating with the electrical speed ( $\omega_e$ ) of the rotor where this speed is given as. Here  $p$  is the number of pole pairs and  $n$  is the rotor mechanical speed.

- Synchronously rotating reference frames: These frames can be chosen collinear with one of the stator flux vector, the rotor flux vector or the magnetizing flux vector.

Instead of all the frames of reference given above a general frame of reference can be used and with the required transformation between the axes all the frames of reference mentioned above can be easily obtained.

### 2.1.3 Transformation of three phase variables to two phase variables

All the states defined previously (voltages, currents, fluxes) of three phase asynchronous motor with symmetric windings supplied from a symmetric three phase source can be transformed to either one of the frames of reference mentioned above or a general reference frame rotating with an arbitrary angular velocity of  $\omega_g$ . To explain such general transformation; the transformation of stator currents ( $i_{sa}$ ,  $i_{sb}$ ,  $i_{sc}$ ) of the machine from three phase to any reference frame will be explained and if the same procedure is applied to all the remaining states then the 2 phase model of the machine can be easily obtained.

First step in transforming from 3-phase variables to a general frame of reference is to transform the 3-phase variables to the stationary frame of reference explained above. There is a  $120^\circ$  phase shift between each phase of the stator currents. If one of the phase currents is taken collinear with the  $\alpha$ -axis of the stationary frame of reference as shown in figure 2.1 below, following equation can be written from figure 1 for the stator current vector.

$$\bar{i}_s = i_{sa} + i_{sb} \cdot e^{j2\pi/3} + i_{sc} \cdot e^{-j2\pi/3} = i_{sa} + a \cdot i_{sb} + a^2 \cdot i_{sc} \quad (2.14)$$

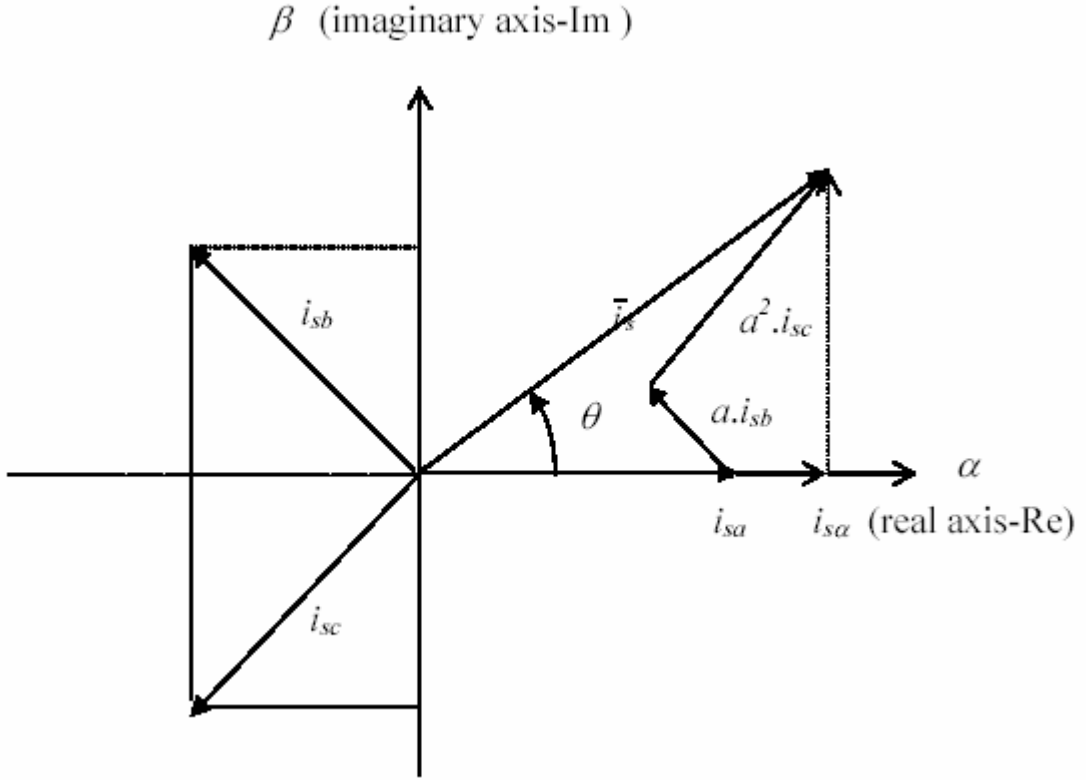


Figure 2.1:  $\alpha$ - $\beta$  stationary reference frame with stator current

From Figure 2.1 for stator current vector following is obtained:

$$\vec{i}_s = i_{sa} - \frac{1}{2}i_{sb} - \frac{1}{2}i_{sc} + j\frac{\sqrt{3}}{2}i_{sb} - j\frac{\sqrt{3}}{2}i_{sc} = i_{s\alpha} + ji_{s\beta} \quad (2.15)$$

(2.15) can also be given in matrix form as:

$$\begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \cdot \begin{bmatrix} i_{sa} \\ i_{sb} \\ i_{sc} \end{bmatrix}; i_{s\alpha, \beta} = K_s \cdot i_{sa, b, c} \quad (2.16)$$

Here  $\vec{i}_s$  is constant magnitude stator current vector rotating with the velocity of  $\omega_e$  where  $\omega_e = 2\pi \cdot f = p \cdot \omega_s$  where  $p$  is the number of pole pairs and  $\omega_s$  is the velocity of synchronously rotating frame of reference.  $K_s$  used in (2.16) is the matrix gain used to enable power invariant transformation for 3/2 phase transformation. The inverse of the transformation (2.16) is also valid.

$$\begin{bmatrix} i_{sa} \\ i_{sb} \\ i_{sc} \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & 0 \\ -\frac{1}{3} & \frac{\sqrt{3}}{2} \\ -\frac{1}{3} & -\frac{\sqrt{3}}{2} \end{bmatrix} \cdot \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix}; i_{sa, b, c} = K_s^{-1} \cdot i_{s\alpha, \beta} \quad (2.17)$$

In vector control applications all the states in the rotor and the stator circuits must be defined on the same frame of reference. An operator of  $e^{-j\theta_g}$  is used for this transformation. Here  $\theta_g$  is the angle for general frame of reference shown in fig.2.2 below.

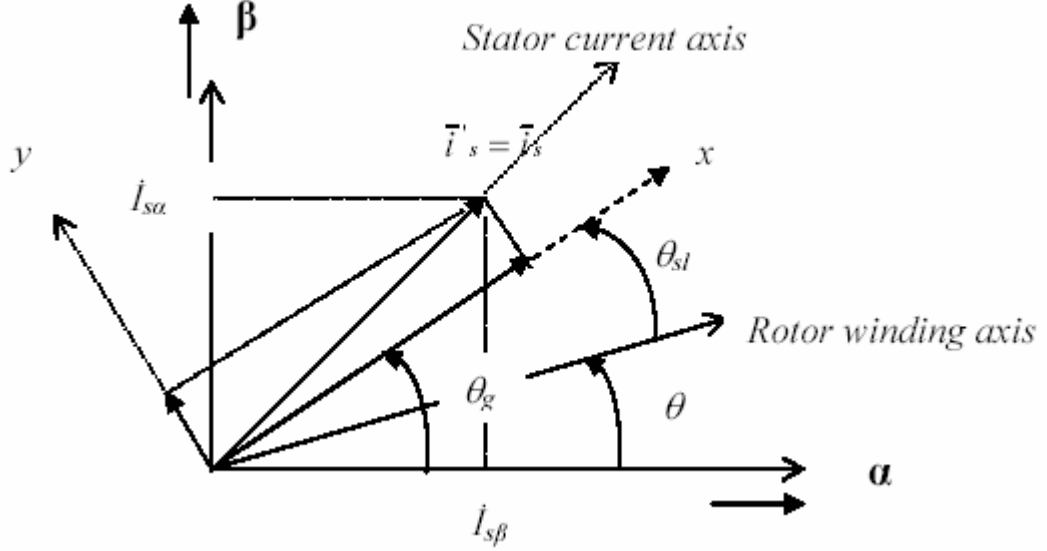


Figure 2.2: ( $\alpha$ - $\beta$  stationary frame) to ( $x$ - $y$  arbitrary frame) transformation

$$\bar{i}_s = i_{sx} + j i_{sy} \quad (2.18)$$

For the transformation in fig.2.2 following is used:

$$\bar{i}_s' = \bar{i}_s \cdot e^{-j\theta_g} = (i_{s\alpha} + j i_{s\beta}) \cdot e^{-j\theta_g} = \bar{i}_s \cdot (\cos \theta_g - j \sin \theta_g) = i_{sx} + j i_{sy} \quad (2.19)$$

Here we know that  $\|\bar{i}_s'\| = \|\bar{i}_s\|$ , the transformation in (2.19) can be given in matrix form as:

$$\begin{bmatrix} i_{sx} \\ i_{sy} \end{bmatrix} = \begin{bmatrix} \cos \theta_g & \sin \theta_g \\ -\sin \theta_g & \cos \theta_g \end{bmatrix} \cdot \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix}; \quad i_{s_{x,y}} = T(\theta_g) \cdot i_{s_{\alpha\beta}} \quad (2.20)$$

Here  $T(\theta_g)$  is the transformation matrix from the stationary frame of reference to general arbitrary frame of reference. Also the inverse transformation ( $x$ - $y$  to  $\alpha$ - $\beta$ ) can be obtained by inverting the given matrix  $T(\theta_g)$  as:

$$\begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix} = \begin{bmatrix} \cos \theta_g & -\sin \theta_g \\ \sin \theta_g & \cos \theta_g \end{bmatrix} \cdot \begin{bmatrix} i_{sx} \\ i_{sy} \end{bmatrix}; \quad i_{s_{\alpha\beta}} = T(\theta_g)^{-1} \cdot i_{s_{x,y}} \quad (2.21)$$

As mentioned before the same transformation is valid for the transformation of the stator voltage, stator flux, rotor current and rotor flux. Since the transformed variables and the original variables are describing the same system the power entering the system

must be the same regardless of the variables used to describe the system. Thus  $K_s$  value given in (16) and (17) can be calculated from the following equality:

$$S_B = 3 \cdot U_{B(abc)} \cdot I_{B(abc)} = \frac{3}{2} U_{B(\alpha\beta)} \cdot I_{B(\alpha\beta)} \quad (2.22)$$

Here  $U_{B(abc)}$  and  $I_{B(abc)}$  are the 3 phase rms voltages and currents of the machine.

Then if the transformations explained above (e.g.  $\mathbf{abc} \rightarrow \alpha\text{-}\beta \rightarrow \mathbf{x}\text{-}\mathbf{y}$ ) for a general frame of reference rotating with the velocity  $\frac{d}{dt}\theta_g = \omega_g$  are applied to the model given

in. (11), (12) and (13), then by defining the parameters  $L_s = \frac{3}{2}L_{ss}$ ,  $L_r = \frac{3}{2}L_{rr}$ ,  $L_m = \frac{3}{2}L_{sr}$  mathematical model of the asynchronous machine can be written in the form:

$$\frac{d\psi_{s\alpha}}{dt} = -R_s i_{s\alpha} + \omega_g \psi_{s\beta} + u_{s\alpha} \quad (2.23)$$

$$\frac{d\psi_{s\beta}}{dt} = -R_s i_{s\beta} - \omega_g \psi_{s\alpha} + u_{s\beta} \quad (2.24)$$

$$\frac{d\psi_{r\alpha}}{dt} = -R_r i_{r\alpha} + (\omega_g - \omega) \psi_{r\beta} + u_{r\alpha} \quad (2.25)$$

$$\frac{d\psi_{r\beta}}{dt} = -R_r i_{r\beta} - (\omega_g - \omega) \psi_{r\alpha} + u_{r\beta} \quad (2.26)$$

The components of the flux linkages vector  $\bar{\psi}_s^T = [\psi_{s\alpha} \ \psi_{s\beta}]$ ,  $\bar{\psi}_r^T = [\psi_{r\alpha} \ \psi_{r\beta}]$  depend on the stator  $\mathbf{i}_s^T = [i_{s\alpha} \ i_{s\beta}]$  and rotor  $\mathbf{i}_r^T = [i_{r\alpha} \ i_{r\beta}]$  currents in the following way:

$$\bar{\psi}_s = \mathbf{L}_s \mathbf{i}_s + \mathbf{L}_m \mathbf{i}_r \quad (2.27)$$

$$\bar{\psi}_r = \mathbf{L}_m \mathbf{i}_s + \mathbf{L}_r \mathbf{i}_r \quad (2.28)$$

Electromagnetic torque can be expressed as:

$$T = \{\bar{\psi}_s \times \mathbf{i}_s\} = -\{\bar{\psi}_r \mathbf{i}_r\} \quad (2.29)$$

and mechanical motion can be described as:

$$J \frac{d\omega}{dt} + T_L = T; \frac{d\theta}{dt} = \omega \quad (2.30)$$

Model (2.23)-(2.30) does not have any time varying parameters but it is still nonlinear due to the presence of the product of different variables. This model is written in the orthogonal frame of references rotating with angular velocity  $\omega_g$  with respect to the stator stationary frame of reference. In the analysis of the smooth gap electrical machines three frame of references are important as previously mentioned:

- Stationary frame of references defined by  $\omega_g = 0$ . All electrical variables appears to have the same angular frequency equal to the frequency of the supply;



- Rotating frame of references stationary with respect to the rotor  $\omega_g = \omega$ . All electrical variables have angular frequency equal to the difference between angular frequency of supply and rotor angular frequency;
- Rotating frame of references stationary with respect to any vector of electrical coordinates (flux or current or voltage). Steady state values of all electrical variables appear to be DC quantities. In the design of the control systems the frame of references with one axis collinear with the rotor flux vector is dominating since this transformation is the only one with full decoupling of flux and torque components. It is known as (d-q) frame of reference.

After these transformations the model of the machine reduces to sixth order. By selecting  $\mathbf{u}_r = 0$  model describes the behavior of squirrel cage induction machine which is the plant used in the simulations and the experiments. Finally, considering the stator current and the rotor flux the model for the squirrel cage IM in the general frame of reference-after some manipulation of (2.23)-(2.30)-becomes:

$$\begin{bmatrix} \dot{i}_{sx} \\ \dot{i}_{sy} \\ \dot{\psi}_{rx} \\ \dot{\psi}_{ry} \end{bmatrix} = \begin{bmatrix} -\left(\frac{R_s}{L_\sigma} + \frac{R_r L_m^2}{L_\sigma L_r^2}\right) & \omega_g & \frac{R_r L_m}{L_r^2 L_\sigma} & \frac{\omega L_m}{L_r L_\sigma} \\ -\omega_g & -\left(\frac{R_s}{L_\sigma} + \frac{R_r L_m^2}{L_\sigma L_r^2}\right) & -\frac{\omega L_m}{L_r L_\sigma} & \frac{R_r L_m}{L_r^2 L_\sigma} \\ \frac{L_m R_r}{L_r} & 0 & -\frac{R_r}{L_r} & (\omega_g - \omega) \\ 0 & \frac{L_m R_r}{L_r} & -(\omega_g - \omega) & -\frac{R_r}{L_r} \end{bmatrix} \begin{bmatrix} i_{sx} \\ i_{sy} \\ \psi_{rx} \\ \psi_{ry} \end{bmatrix} + \frac{1}{L_\sigma} \begin{bmatrix} u_{sx} \\ u_{sy} \\ 0 \\ 0 \end{bmatrix} \quad (2.31)$$

$$J \frac{d\omega}{dt} + T_L = T; \frac{d\theta}{dt} = \omega \quad (2.32)$$

where

$$T_e = \frac{3}{2} \cdot p \cdot (\psi_{rx} \cdot i_{sy} - \psi_{ry} \cdot i_{sx}) \quad (2.33)$$

$$\text{In (2.31)} \quad L_\sigma = \frac{L_s L_r - L_m^2}{L_r}$$

Finally with (2.31), (2.32) and (2.33) 2-phase state space vector model of the squirrel cage induction machine (IM) is completed. Following chapter is dealing with the principles of vector control and its usage for the induction motor.

## 2.2 Vector Control of Induction Machine

The main goal in field oriented vector control of an induction machine is to control the torque and flux independently-by decoupling them through required transformation which is explained previously- as in the separately excited DC machine [14]. In a separately excited DC machine the magneto-motive-force  $F$  (MMF) generated by the armature current  $i_a$  and the flux  $\psi_r$  generated by the field current  $i_f$  are orthogonal. The reaction of the rotor to the main flux  $\psi_r$  is compensated by the additional windings supplied by the armature current. Thus the  $90^\circ$  angle and the independence between  $i_a$  and  $\psi_r$  held. Flux  $\psi_r$  is not only independent from  $i_a$  but also from the rotor speed  $\omega_r$ . Due to this fact, the electromagnetic torque  $T_e$  of a DC machine depends on the vector product of these two vectors both in the transient and steady state operation where the flux is in its linear region of operation. Since these two vectors are orthogonal the electromagnetic torque can be given as follows:

$$T_e = k_d \cdot \psi_f \cdot i_a \quad (2.34)$$

In (2.34)  $k_d$  is the DC machine torque constant. If (2.34) is carefully investigated, it can be deduced that when the flux magnitude is kept constant then the electromagnetic torque of the machine can be controlled very fast by changing the armature current. Thus fast dynamics torque response can be easily obtained.

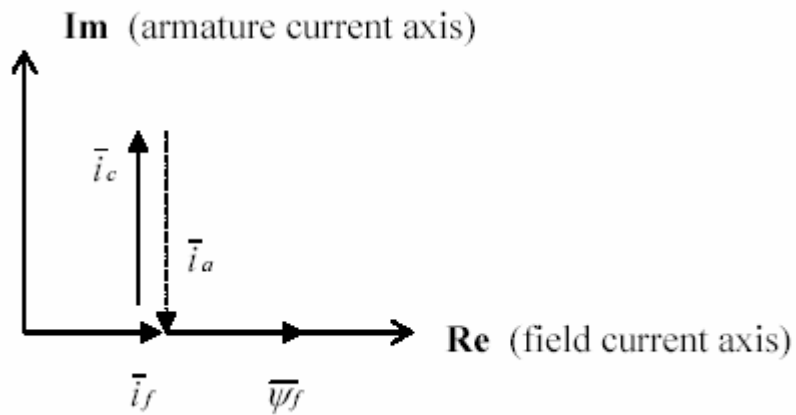


Figure 2.3: Armature, field currents and flux vectors for DC machine

For the squirrel cage asynchronous induction motor (IM) the stator flux plus the rotor current consecutively the rotor flux are generated by the stator current. Following figure shows the instantaneous orientation of the stator current and the rotor flux in the stationary and rotating frames of reference.

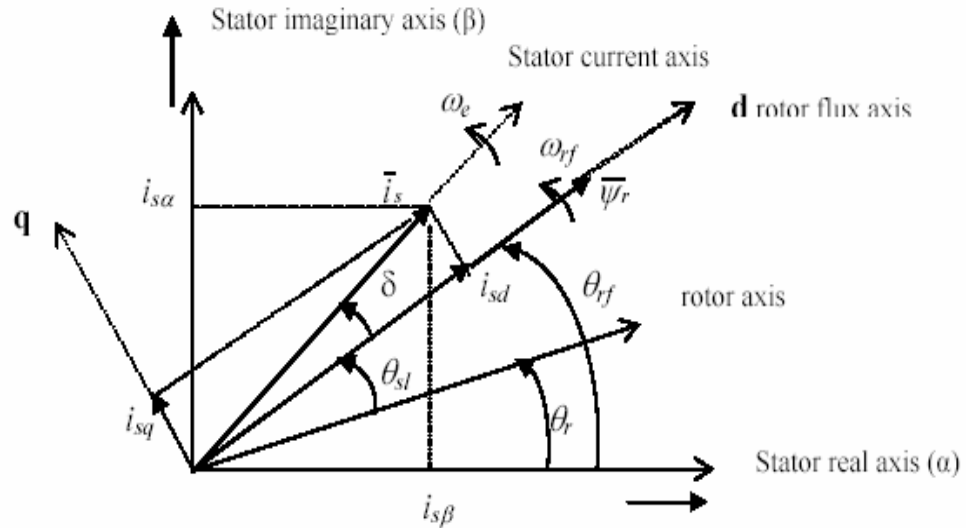


Figure 2.4: Rotor flux and stator current in the stationary ( $\alpha$ - $\beta$ ) and the rotary frame of reference (d-q)

Here  $\omega_e = \omega$  is the stator current angular velocity and  $\delta$  is the angle between stator current and the rotor flux  $\psi_r$  and generally known as the load angle and in steady state this angle is constant (e.g. rotor flux angular velocity  $\omega_{rf} =$  stator flux angular velocity  $\omega_s$ ). Thus both the rotor and the stator flux rotate with the same speed. Note that  $\omega_s = \omega_e / p$  where  $p$  is the number of poles in the stator windings.

As explained previously in the transformations and can be seen in figure.2.4: if the stator current is moved to the rotor flux rotating frame of reference (d-q) via proper transformation then  $i_{sd}$  and  $i_{sq}$  components of the stator current become dc quantities. For this case the electromagnetic moment of the IM is given as follows:

$$T_e = k_m \cdot \|\bar{\psi}_r\| \cdot \|\bar{i}_s\| \cdot \sin \delta \quad (2.35)$$

where  $k_m$  is the induction machine torque constant and  $\|\bar{\psi}_r\|$  and  $\|\bar{i}_s\|$  are the magnitudes of the rotor flux and stator current vectors consecutively. (2.35) is similar to (2.34) but although for DC motor it is easy to control the flux and the armature current independently for the IM this is a very hard task to accomplish. Because there is no

simple way to obtain the rotor flux information in the squirrel cage IM and the flux vector  $\bar{\psi}_r$  and the stator current  $\bar{i}_s$  are coupled. If the rotor flux position and the magnitude could be determined, then the rotor flux and the motor torque can be controlled independently with the two components of the stator current  $i_{sd}$  and  $i_{sq}$  given in figure 2.4 by transforming the stator current in the rotor flux frame of reference.

The electromagnetic moment of the IM in the  $d$ - $q$  reference frame can be given from the eqn. (2.33) as following:

$$T_e = \frac{3}{2} \cdot p \cdot (\psi_{rd} \cdot i_{sq} - \psi_{rq} \cdot i_{sd}) \quad (2.36)$$

From (2.28) it can be written that:

$$\bar{\psi}_r = \mathbf{L}_m \mathbf{i}_s + \mathbf{L}_r \mathbf{i}_r \quad (2.37)$$

and since in  $d$ - $q$  reference frame the rotor flux  $\psi_r$  :

$$\begin{aligned} \bar{\psi}_r &= \psi_{rd} + j \psi_{rq} \\ \psi_{rd} &= \mathbf{L}_m \mathbf{i}_{sd} + \mathbf{L}_r \mathbf{i}_{rd} \\ \psi_{rq} &= \mathbf{L}_m \mathbf{i}_{sq} + \mathbf{L}_r \mathbf{i}_{rq} \end{aligned} \quad (2.38)$$

Since the  $d$ -axis is collinear with the rotor flux vector then the  $q$  component of the rotor flux should be zero from eqn (2.38.a) we have:

$$\mathbf{i}_{rq} = -\frac{\mathbf{L}_m}{\mathbf{L}_r} \mathbf{i}_{sq} \quad (2.39)$$

then substituting (2.39) in (2.36) the following is obtained:

$$T_e = \frac{3}{2} \cdot p \cdot \frac{L_m}{L_r} \cdot \psi_{rd} \cdot i_{sq} \quad (2.40)$$

since  $\bar{\psi}_r = \psi_{rd}$  (e.g the vector itself is the  $d$  component) the following is written:

$$T_e = \frac{3}{2} \cdot p \cdot \frac{L_m}{L_r} \cdot \bar{\psi}_r \cdot i_{sq} \quad (2.41)$$

After a careful observation it could be deduced that (2.35) and (2.41) are the same. Then from (2.31) 4<sup>th</sup> row it can be written that;

$$u_{rq} = R_r \cdot i_{rq} + \frac{d\psi_{rq}}{dt} + (\omega_{fr} - \omega) \cdot \psi_{rd} = 0 \quad (2.42)$$

where  $\omega_{sl} = (\omega_{rf} - \omega)$  ( $\omega_{sl}$  is the slip frequency in rad / s)

Substituting (2.39) into (2.42) with  $\psi_{rq} = 0$  and calculating  $\omega_{sl}$  :

$$\frac{d\theta_{sl}}{dt} = \omega_{sl} = \omega_{rf} - \omega = \frac{R_r \cdot L_m}{L_r \cdot \psi_r} \cdot i_{sq} \quad (2.43)$$

From (2.43) it can be written for the rotor flux angular velocity ( $\omega_{rf}$ ) that;

$$\frac{d\theta_{rf}}{dt} = \omega_{rf} = \omega + \omega_{sl} = \omega + \frac{R_r \cdot L_m}{L_r \cdot \psi_r} \cdot i_{sq} \quad (2.44)$$

Here  $\omega$  is the rotor angular velocity and  $\omega_{rf}$  is the angle between the stationary reference frame and  $d$ -axis of the rotating reference frame. From (2.31) 3<sup>rd</sup> row it can be written that:

$$u_{rd} = R_r \cdot i_{rd} + \frac{d\psi_{rd}}{dt} - (\omega_e - \omega) \cdot \psi_{rq} = 0 \quad (2.45)$$

Then if the flux  $\psi_{rd} = \|\overline{\psi}_r\|$  in the motor is kept constant then the rate of change of the flux becomes zero (e.g.  $\frac{d\psi_{rd}}{dt} = 0$ ). Then combining these considering that  $\psi_{rq} = 0$ , (2.45) becomes  $R_r \cdot i_{rd} = 0$ . Here since  $R_r$  can not be zero then  $i_{rd} = 0$ . Then from (2.38.a) it can be obtained that:

$$\psi_r = \psi_{rd} = L_m \cdot i_{sd} \quad (2.46)$$

which means that rotor flux can be controlled with the d-component of the stator current. Then considering both of the (2.41) and (2.46) it can be concluded that with the d and q components of the stator current rotor flux and electromagnetic torque of the induction machine can be controlled independently as for the separately excited DC motor where stator current components  $i_{sd}$  and  $i_{sq}$  correspond  $i_f$  (field current) and  $i_a$  (armature current) of the DC motor consecutively. Controlling both the electromagnetic torque and the rotor flux independently is the main principle of the field oriented control of the IM. To achieve this goal the magnitude and the position of the rotor flux should be determined precisely. The required flux vector can be obtained through measurement or through proper observer design by estimation. In the measurement techniques first the air-gap flux is measured with hall effect sensors or by special windings in the stator then the rotor flux is calculated from the air-gap flux. [15]. In these techniques special motor design is required to mount the flux sensors and special windings which causes an increase in the cost and time spent in the design. Also the vulnerability and temperature dependence of these sensors are the other drawbacks. All of these drawbacks of the sensors prevent the IM to be used in the high performance servo systems[15]. As mentioned before another way to get the rotor flux information is to design a proper observer. The developments in the digital signal processors and the power electronic devices enabled the observers to draw more attention in the high performance control of induction machine drives. There are actually two types of flux

observers that those including a mechanical sensor (e.g. encoder, tachometer) to measure the mechanical coordinates (e.g. position and/or speed of the shaft) of the motor and those having no mechanical sensors in estimating both the speed and the rotor flux of the motor. The IM drives that includes no mechanical sensors called “Sensorless Induction Machine Drives” in the literature. In this thesis as mentioned previously a novel sensorless flux, speed, rotor time constant observer is designed. The details about the observers design and the simulation and experimental results with the designed observer will be given in the 3rd chapters. Following is the general structure for a sensorless induction machine drive;

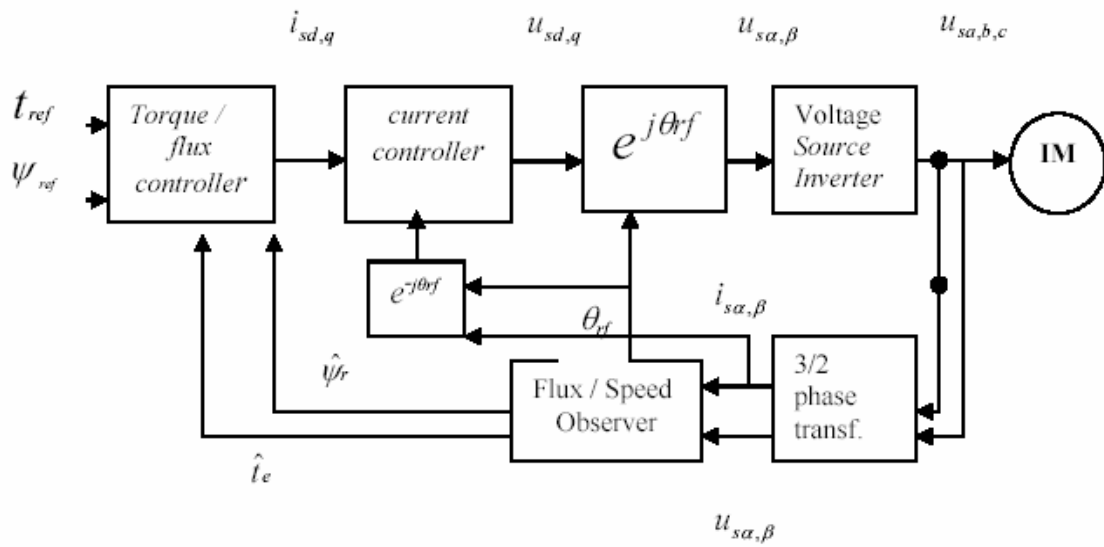


Figure 2.5: Block diagram for the sensorless torque / flux control of the IM

The position - the angle  $\theta_{rf}$  - of the rotor flux vector is given is calculated following:

$$\theta_{rf} = \tan^{-1} \left( \frac{\hat{\psi}_{r\beta}}{\hat{\psi}_{r\alpha}} \right) \quad (2.47)$$

Here  $\hat{\psi}_{r\alpha}$  and  $\hat{\psi}_{r\beta}$  are the components of the rotor flux in the stationary frame of reference as in figure 2.4. Thus after the correct estimation of the rotor flux; the required measured currents is converted from 3 to 2 phase as in (2.16) and with the flux angle  $\theta_{rf}$  and the transformation  $i_{sd,q} = T(\theta_{rf}).i_{s\alpha,\beta}$  the  $d$ - $q$  currents are obtained to be used in the current controller in figure 2.5. After obtaining the reference stator voltages from the current controller in  $d$ - $q$  frame; through well known transformation as in (2.20), the reference voltage in stationary frame of reference is obtained and applied to the motor. In the thesis space vector pulse width modulation (SVPWM) technique is used in the

implementation of the control while applying the reference voltage through a voltage source inverter which will be explained later in the implementation details. Again in this control scheme given in figure 2.5 the required torque of the motor is estimated through the following:

$$\hat{T}_e = \frac{3}{2} \cdot p \cdot \frac{L_m}{L_r} \cdot \hat{\psi}_r \cdot i_{sq} \quad (2.48)$$

To have high performance IM electrical drives the following properties should be held (Murphy and Turnbull 1982, [14]):

- High acceleration, high torque / inertia ratio,
- High power density (e.g. maximum output power per mass of the motor),
- Four region of operation
- Fast transient response,
- Short period overload capability,
- Low speed operation without ripple in the torque produced,
- Zero speed torque control

When the model of the IM given in (2.31), (2.32), (2.33) is investigated it can be easily realized that the model is a 5<sup>th</sup> order (2 order for the stator current dynamics, 2 order for the rotor flux dynamics and 1 degree for the mechanical dynamics), highly nonlinear and coupled system. Also the motor parameters vary considerably mostly with temperature and magnetic saturation, the supply voltage, supply frequency and load changes. Especially, the rotor resistance is effected too much from the temperature variations. Thus, traditional linear control techniques are not enough for the high performance low speed control of the IM. For the high performance control of the IM, the flux magnitude and angle should be determined precisely considering the effects of the motor parameter changes. Many researches have different approaches to this problem which was surveyed previously. In this thesis work with its well known robustness against parameter changes and unmodelled uncertainties and the property of order reduction in control design the sliding mode control (SMC) approach is used not only for the control system design but also for the observer design –sliding mode observer (SMO). In the next chapter the principles of sliding mode control will be explained.

## 2.3 Variable Structure Systems (VSS)

In the design of variable structure systems (VSS) the principle in designing the control algorithm is to enforce the system to have sliding motion on a predefined manifold in the state space [11]. Due to the switch in the control during the control process these systems are named as the VSS systems. To indicate the importance of the sliding motion or “sliding mode” in the VSS these control systems are called “Sliding Mode Control”. The most important property of these systems is their robustness and invariance. The robustness and the invariance property stands for the insensitivity against the external disturbance, parameter uncertainties and modeling error or uncertainties. (Hung et al 1993). With these emerging properties the SMC has many application areas including the linear time invariant, linear time varying, nonlinear systems such as robotics, vibration control, power converters and electrical drives and etc.

### 2.3.1 Structure and Fundamentals of VSS

For a given n-dimensional state space representation (2.49) of any system the VSS control system design has two steps.

$$\dot{x} = f(x,t) + B(x,t)u \quad (2.49)$$

here  $x$  is the  $(n \times 1)$  dimensional state and  $u$  is the  $(m \times 1)$  control input vectors of the given system.

- There exists  $m$  number of switching functions  $\sigma_i(x)$  designed representing the desired dynamics of the control system which is lower dimensional than the original system. Here  $\sigma_i(x)$  is the  $i$ th element in the switching vector  $\sigma(x)$ . Thus the first step in the design is selecting these functions properly in the intersection of surfaces.
- The second step is to find the discontinuous control to enforce sliding mode in the intersection of the surfaces selected in the first step.

Partitioning of the overall motion into two motions of lower dimensions – the first motion precedes sliding mode within a finite interval (e.g. achieving the desired



dynamics in a finite time) and the second motion is sliding mode with the desired properties – may simplify the design procedure considerably.

### 2.3.2 Switching Function (Sliding Surface)

The structure of the VSS is determined by the switching function  $\sigma(x)$ . Each switching function  $\sigma_i(x)$  defines a linear function between the states and their derivatives of the system passing through the origin of the plane determine by these states as shown in figure 2.6 below. These lines dividing the phase plane into two called the switching line. Additionally, these lines defines the set of points where  $\sigma(x)=0$  in the phase plane. This set is known as sliding or switching surface.

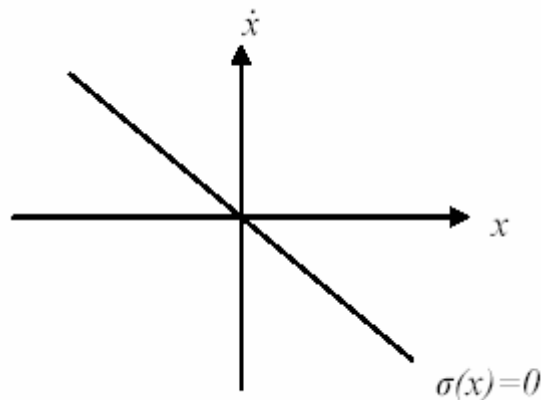


Figure 2.6: Sliding Surface

The aim in the sliding mode control is to push the system states to sliding surface and enforcing them to stay in this manifold. When the system states reaches the sliding manifold the error in the states converges to zero with the dynamics of the switching manifold to the states of the system under consideration [11].

Variable Structure Systems are originally defined in continuous-time for dynamic systems described by ordinary differential equations with discontinuous right hand side. In such a system so-called sliding mode motion can result. This motion is represented by the state trajectories in the sliding mode manifold and high frequency changes in the control. The fact that motion belongs to certain manifold in state space with a dimension lower than that of the system results in the motion equations order reduction. This enables simplification and decoupling design procedure to be employed. For sliding

mode application the equations of motion and the existence conditions shall be solved first [25].

### 2.3.3 Equations of motion

Here again is the system under consideration;

$$\dot{x} = f(x, t) + B(x, t)u \quad (2.50)$$

where  $B(x, t)$  is an  $n \times m$  matrix,  $x \in \mathbf{R}^n$ ,  $u \in \mathbf{R}^m$ ,  $f: \mathbf{R}^1 \times \mathbf{R}^n \rightarrow \mathbf{R}^n$ . For such a system boundary-layer [11] regularization enables substantiation of so called equivalent control method that is used in deriving the sliding mode equations. In accordance with this method, in (2.50) the control  $u$  should be replaced by the equivalent control, which is the solution to  $\dot{\sigma} = Gf(x, t) + GB(x, t)u_{eq} = 0$ ,  $G = \{\partial \sigma / \partial x\}$ . where  $\sigma = 0$ ,  $\sigma \in \mathbf{R}^m$  is defining Sliding Mode Manifold while  $\sigma_i = 0$  describe so-called switching surfaces. For  $\det GB \neq 0$  equivalent control for system (2.50) in manifold  $\sigma = 0$  can be calculated as  $u_{eq} = -(GB)^{-1}Gf$ , and sliding mode equation is:

$$\dot{x} = [I - (GB)^{-1}G]f, \quad \sigma = 0. \quad (2.51)$$

From  $\sigma = 0$ ,  $m$  components ( $x_2 \in \mathbf{R}^m$ ) of the state vector  $x$  may be determined as functions of the rest ( $n-m$ ) components ( $x_1 \in \mathbf{R}^{n-m}$ ) as  $x_2 = \sigma_0(x_1)$ ,  $\sigma_0 \in \mathbf{R}^m$  and the order of the sliding mode equation (2.51) may be reduced by  $m$ :

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, \sigma_0(x_1)), \quad x_1 \in \mathbf{R}^{n-m}. \\ x_2 &= \sigma_0(x_1) \end{aligned} \quad (2.52)$$

### 2.3.4 Existence and Stability of Sliding Modes

To derive sliding mode existence conditions in analytical form the projection of the system motion on manifold.

$$S = \{x \in \mathbf{R}^n : \sigma(x) = 0\}; \sigma \in \mathbf{R}^m \quad (2.53)$$

should be analysed. The conditions of existence of multidimensional sliding modes are in close relation with the convergence of motion toward the manifold  $S$ . Hence the solution of the considered task is based on the methods of the theory of

stability which are applied to the equations of projection of motion onto the subspace  $S$  expressed as:

$$\dot{\sigma} = Gf(x, t) + GB(x, t)u_{eq} = 0, \quad G = \frac{\partial \sigma}{\partial x}. \quad (2.54)$$

The domain  $S$  is the domain of existence of the sliding mode if there exists such a continuously differentiable function  $V(\mathbf{x}, \boldsymbol{\sigma}, t)$  that:

- For each  $\mathbf{x}$ ,  $\boldsymbol{\sigma}$  and  $t$  that function is defined positively with respect to  $\boldsymbol{\sigma}$  and

$$\inf_{\|\boldsymbol{\sigma}\|=R} V = h_R \quad \sup_{\|\boldsymbol{\sigma}\|=R} V = H_R \quad h_R = 0 \quad \text{for } R = 0$$

( $h_R, H_R$  depend only on  $R$ )

The time derivative of the function  $V$  is a negative upper boundary on the set of all points of the sphere  $\|\boldsymbol{\sigma}\|=R$ , with the exception of the points on the surfaces of discontinuity, where this derivative is undetermined.

Since standard method for obtaining the function of Lyapunov for non linear systems does not exist, for arbitrary matrices in (2.54) no standard solution of finding the function  $V$  is known.

### 2.3.5 VSS Control System Design

Equations of motion in the sliding mode (2.51) depend on the  $m \times n$  elements of matrix  $G$ , which consists of the gradients of function  $\sigma$ . Therefore, this motion can be influenced by changing the positions of the switching surfaces within the system state space. The synthesis of control in the systems with sliding modes is performed in the following order. The first step consists of the selection of switching surfaces so that the motion in the sliding mode has the required properties. The second step consists of the selection of such control input that the sliding mode exists on the entire domain  $S$ . Finally, the third task can be formulated as the finding of conditions for the state to reach the domain  $S$  from any initial position.

Considering that the right side of the sliding mode equations (2.51) is continuous, the first stage of the synthesis is considered as the task of the synthesis of continuous

control. The second and the third stage of the synthesis are near to the task setting, because they consist of providing the stability of the system origin (2.54) in both small and large. It is necessary to consider the method of selecting the controls which provide for the existence of the sliding mode the intersection of all switching surfaces. The main difficulty in solving the formulated task is the lack of a universal method which would permit the splitting of the domain  $S$  into sub domains in which the sliding mode exists and into sub domains where this motion does not take place. At the same time, for a special form of the matrix  $GB$  it is possible to obtain the sufficient conditions for the existence of the sliding mode. In spite of the fact that reduction to such special cases can be done only through an appropriate selection of the matrix  $G$ , which has to provide for the required character of the sliding mode, these two tasks can be solved independently.

### 2.3.5.1 Invariant Transformations

The possibility of such separate consideration of the relevant tasks results from invariance of the sliding mode equations as compared with linear transformations of the switching manifold or control vectors given in the form:

$$\sigma^* = R_\sigma \sigma \quad u^* = R_u u \quad (2.55)$$

If the matrices  $R_\sigma$  and  $R_u$  are non singular and continuously differentiable, the motion in the sliding mode will not change if already selected switching surfaces  $\sigma_i = 0$  are replaced by new ones  $\sigma_i^* = 0$  ( $\sigma_i^*$  - components of vector  $\sigma^*$ ), or if the surfaces  $\sigma_i = 0$  have the discontinuities of vector component  $u^*$  instead of  $u$ . In the same time, the equations describing the projection of the system motion on the subspace  $S^* = \{x \in R^n : \sigma^* = 0\}$ , in the first case,  $S = \{x \in R^n : \sigma = 0\}$ , in the second case, depend on  $R_\sigma$  or  $R_u$  respectively.

As a result of the demonstrated invariance for any matrix  $G$ , as already defined by the required equations of the sliding mode, by selecting  $R_u$  or  $R_\sigma$  it is possible to obtain the matrix preceding the control  $u$  or  $u^*$  of an arbitrary form. In the method of diagonalization, permitting that the matrix preceding the control is reduced to diagonal form,  $R_u$  and  $R_\sigma$ , are selected in the following way:

$$R_\sigma = Q(GB)^{-1} \quad R_u = Q^{-1}(GB) \quad (2.56)$$

where  $Q$  - is an arbitrary diagonal matrix with nonzero elements.

In order to provide for the sliding mode in any point of the intersection  $\sigma=0$ , in the method of diagonalization it is necessary to select the components of vector  $\mathbf{u}$  with the discontinuities on the surfaces  $\sigma_i^* = 0$  or the components of vector  $u^*$  with the discontinuities on the surfaces  $\sigma_i=0$  in compliance with (2.55).

The task that the domain  $S = \{x \in R^n : \sigma = 0\}$  be reached from arbitrary initial conditions can be solved in the analog method. Let a linear transformation of the switching surfaces be used:

$$R_\sigma = (GB)^T \quad (2.57)$$

Select a function of Lyapunov as a positively definite quadratic form:

$$V = \frac{\sigma^T \sigma}{2} \quad (2.58)$$

The derivative of the function (2.58) on the system trajectories (2.50) is then equal to:

$$\frac{dV}{dt} = \sigma^T (Gf + GBu) = \sigma^{*T} [(GB)^{-1} Gf + u] \quad (2.59)$$

In order to get a negatively definite derivative of the function  $V$  it is enough to select the components of vector  $\mathbf{u}$  with discontinuities on surfaces  $\sigma_i^*=0$ . with amplitudes of the control vector components satisfying relations  $|u_i| > |d_i(x,t)|$ ,  $d_i(x,t)$ - the  $i$ -th component of vector  $[(GB)^{-1} Gf]$ . With the last condition fulfilled,  $V$  and  $dV/dt$  have opposite signs, the origin of the subspace  $S^* = \{x \in R^n : \sigma^* = 0\}$  (or, due to regularity of  $R_\sigma$ , the origin of the subspace  $S = \{x \in R^n : \sigma = 0\}$ ) is stable in large accordingly.

In this way, the procedure of synthesis represents the selection of the required discontinuity surfaces and such a transformation of these surfaces (or the control vectors) for which it is possible to provide the existence of the sliding mode.

### 2.3.5.2 Decoupling

Decreasing dimensionality of the equations of motion with sliding mode and its independence on control permits to decouple the design problem into two independent ones of lower dimensionality, which make it easier to apply different synthesis methods

for high-dimensional systems. The design procedure of the sliding mode control will demonstrated on system (2.50) with control (2.52). This system can be represented in the regular form

$$\frac{dx_1}{dt} = f_1(x_1, x_2, t) \quad (2.60)$$

$$\frac{dx_2}{dt} = f_2(x_1, x_2, t) + B_2(x_1, x_2, t)u; x_1 \in R^{n-m}; x_2 \in R^m; \text{rank}B_2 = m \quad (2.61)$$

In (2.60) vector  $x_2$  can be formally taken as control. The design of the desired motion is reduced to selection of the  $m$ -dimensional control for  $(n-m)$  dimensional system. Suppose the solution can be expressed as:

$$x_2 = \sigma_1(x_1); \quad \sigma_1(x_1) \in R^m \quad (2.62)$$

The dimension of vector  $x_2$  is the same as the dimension of the control vector  $u$ , so in the sliding mode vector  $x_2$  can be forced to track its reference  $\sigma_1(x_1)$ . Selecting the switching manifold  $\sigma(x_1, x_2) = \sigma_1(x_1) - x_2$  equivalent control can be calculated as:

$$u_{eq} = B_2^{-1}(G_1 f_1 + f_2); \quad G_1 = \frac{\partial \sigma_1}{\partial x_1} \quad (2.63)$$

And vector  $x_2$  will track its reference  $\sigma_1(x_1)$ . The stability of the sliding mode can be proved as follows. Let scalar function  $F(x_1, x_2, t)$  be the upper bound of  $B_2 u_{eq}$ . Then the time derivative of the Lyapunov function  $V = \sigma^T \sigma / 2$  on the trajectories (2.60), (2.61) with control  $u = -kF(x_1, x_2, t)B_2^{-1} \text{sign} \sigma; \text{sign} \sigma = [\text{sign} \sigma_1 \quad \dots \quad \text{sign} \sigma_m]^T; k > 1$  is negative-definite. That testifies the convergence of the state to the origin of the subspace  $S$ .

One of the main problems in sliding mode control is the ‘‘chattering’’. Chattering is the phenomenon of finite frequency, finite amplitude oscillations appearing in many sliding mode implementations. These oscillations are caused by the high-frequency switching of a sliding controller exciting unmodelled dynamics in the closed loop. Another cause of chattering appears from the digital implementations of control that has fixed sampling rate. In this thesis in the control and the observer design and implementation the chattering free sliding mode control based on the Lyapunov stability criteria approach which will be explained further has been used.

### 2.3.6 Chattering Free Sliding Mode Control

As usual as for the continuous time systems the control system design procedure for the sampled data systems begins with a selection of the candidate Lyapunov function and the form which the time derivative of the candidate Lyapunov function should satisfy. From these two selections the control input is determined. In sampled data systems the satisfaction of the stability conditions is determined at the moment renewed control is applied (usually the beginning of the sampling interval) and at the end of the sampling interval in order to select the sampling interval and allowed computational delay.

Reconsidering the plant given in (2.50):

$$\dot{x} = f(x, t) + B(x)u \quad (2.64)$$

With rank (B) =m,  $x \in IR^n$ ,  $u \in IR^m$ . As mentioned above In VSS control, the goal is to keep the system motion on the manifold  $S$  which is defined as:

$$S = \{x : \sigma(x, t) = 0\} \quad (2.65)$$

The solution to achieve this goal can be calculated from the requirement that  $\sigma(x, t) = 0$  is stable. The control should be chosen such that the candidate Lyapunov function satisfies Lyapunov stability criteria. The aim is to force the system states to the sliding surface defined by:

$$\sigma = G(x^d - x) \quad (2.66)$$

Firstly, a candidate Lyapunov function selected:

$$v = \frac{\sigma^T \sigma}{2} > 0 \quad \text{and} \quad \dot{v} = \sigma^T \dot{\sigma} < 0 \quad (2.67)$$

It is aimed that the derivative of the Lyapunov function is negative definite. This can be assured if we can somehow make sure that:

$$\dot{v} = -\sigma^T D \sigma < 0 \quad (2.68)$$

$D$  is always positive definite.

Therefore (2.67) and (2.68) satisfy the Lyapunov conditions. From equation (2.67) and (2.68) can be written:

$$\dot{\sigma} = -D \sigma \quad (2.69)$$

If the equation (2.69) is taken and equated to zero, the resulting control is called as equivalent control. In other words, the control that makes the derivative of the sliding function equal to zero is called as equivalent control. Derivative of (2.66):

$$G \dot{x} - G(f(x,t) + B u_{eq}) = 0 \quad (2.70)$$

As a result, the equivalent control can be written in the following form:

$$u_{eq} = -(GB)^{-1} G(f(x,t) - \dot{x}) \quad (2.71)$$

From derivative of (2.66) and using (2.71):

$$\frac{d\sigma}{dt} = (GB)(u_{eq} - u) \quad (2.72)$$

then, another equation for equivalent control can be written as given below:

$$u_{eq}(t) = u(t) + (GB)^{-1} \frac{d\sigma}{dt} \quad (2.73)$$

By using the definition given by (2.64) and (2.66) in (2.69):

$$G \begin{pmatrix} \dot{x} \\ x - x \end{pmatrix} = G \begin{pmatrix} \dot{x} \\ x - f(x,t) - B(x)u \end{pmatrix} = -D\sigma \quad (2.74)$$

the control obtained as:

$$u = (GB(x))^{-1} \left( G \begin{pmatrix} \dot{x} \\ x - f(x,t) \end{pmatrix} + D\sigma \right) \quad (2.75)$$

using (2.63) the equivalent control can be written as:

$$u(t) = u_{eq}(t) + (GB)^{-1} D\sigma \quad (2.76)$$

In (2.76) the resulting control action is continuous (the equivalent control is continuous and function  $\sigma(\mathbf{x},t)$  is continuous by assumption) and  $\mathbf{u}(\mathbf{x},t) = \mathbf{u}_{eq}(\mathbf{x},t)$  for  $\sigma(\mathbf{x},t)=0$ . In the implementation of algorithms (2.75) or (2.76) full information about system dynamics and external disturbances is required (for equivalent control calculation). Because of this, these algorithms are *not practical* for application. They are used here as intermediate results to show the procedure in the development of simpler and more useful control strategies.

By looking (2.73) an estimation for  $u_{eq}$  can be made using the property that  $u(t)$  is continuous and can not change too much in a short time as given below:

$$\hat{u}_{eq}(t) = u(t - \delta t) + (GB)^{-1} \frac{d\sigma}{dt} \quad (2.77)$$

where  $\delta t$  is a short delay time.



This estimation is also consistent with the logic that  $u_{eq}$  is selected as the average of  $u$ . By putting the equation (2.77) into the equation (2.76), the last form for the controller becomes:

$$u(t) = u(t - \delta t) + (GB)^{-1} \left( D\sigma + \frac{d\sigma}{dt} \right) \quad (2.78)$$

This form of expressing the control input is very instructive. It shows that in order to force the system to reach  $\varepsilon$ -vicinity of sliding mode manifold (2.65) and to stay within  $\varepsilon$  boundary layer the control input should be modified by the term  $(GB)^{-1} \left( D\sigma(x, t) + \frac{d\sigma(x, t)}{dt} \right)$  at every instant of time. The control (2.78) takes the value of the equivalent control for  $\sigma(x, t) = 0$ .

Algorithm (2.78) can be easily modified for the application in the discrete time systems *with no computational delay*. In such a system relations between measured and computed variables are depicted in Fig.2.7 below where measurement taken *before* the calculation of new value of the control input are denoted as  $\bullet(kT^-)$  and all variables immediately *after* new value of the control input is applied are denoted by  $\bullet(kT^+)$ , (from now on all variables will be written shorter so  $\sigma(kT)$  means  $\sigma(x(kT), kT)$ ). Note that all continuous functions and variables satisfy  $\bullet(kT^-) = \bullet(kT^+)$

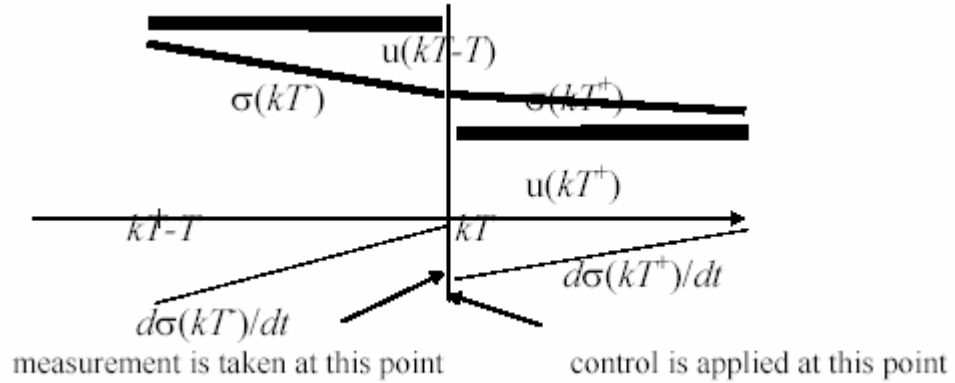


Figure 2.7: The relations between measured and calculated variables for discrete time systems without computational delay

By taking into account the relationship depicted in Fig.2.7, algorithm (2.78) can be rewritten in the following form:

$$\mathbf{u}(kT^+) = \mathbf{u}(kT^-) - (GB)^{-1} \left( D\sigma(kT^-) + \frac{d\sigma(kT^-)}{dt} \right) \quad (2.79)$$

By using Euler interpolation:

$$\mathbf{u}(kT^+) = \mathbf{u}(kT^-) - \frac{(\mathbf{GB})^{-1}}{T} ((\mathbf{DT} + 1)\sigma(kT^+) - \sigma(kT^-)) \quad (2.80)$$

Finally with (2.80) the chattering free sliding mode controller is obtained that is proper for implementation in digital systems of which will be further used in the control system design of the IM below:

## 2.4 Control System Design for IM

The dynamical model and the structure of the induction motor supplied from a switching power converter is given below:

$$\frac{d\mathbf{z}}{dt} = \mathbf{f}_z(\mathbf{z}, \mathbf{i}_{dq}) \quad (2.81)$$

$$\frac{d\mathbf{i}_{dq}}{dt} = \mathbf{f}_i(\mathbf{z}, \mathbf{i}_{dq}, \boldsymbol{\psi}) + \mathbf{B}_u(\mathbf{i}_{dq}, \mathbf{z}, \boldsymbol{\psi})\mathbf{u}_{dq} \quad (2.82)$$

$$\frac{d\boldsymbol{\psi}}{dt} = \mathbf{f}_\phi(\boldsymbol{\psi}, \mathbf{z}, \mathbf{i}_{dq}) \quad (2.83)$$

$$\mathbf{u}_{dq} = \mathbf{F}(\theta_r)\mathbf{A}_M\mathbf{s}_{sw} \quad (2.84)$$

In this model  $\mathbf{z}^T = [\theta \ \omega]$ , (2.81) represents the mechanical motion, (2.82) represents stator current dynamics, (2.83) represents rotor flux dynamics and (2.84) represents the switching converter dynamics.

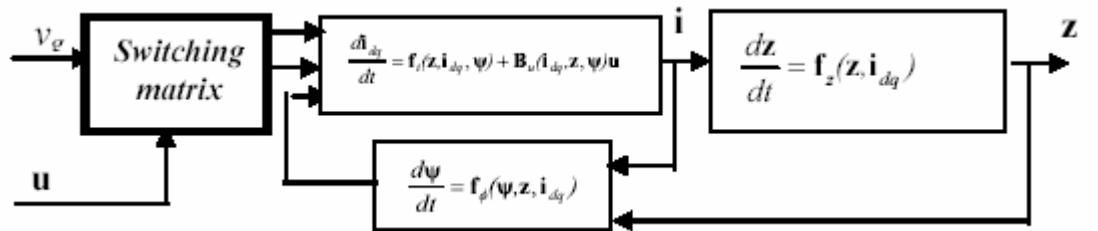


Figure 2.8: Dynamical Structure of three phase induction machine

From the structure of the IM given above the solution for the motion control may be decomposed into two problems:

- The first dealing with the control of the electromagnetic subsystem of the machine with the goal to generate necessary instantaneous torque / flux (e.g. inner loop current controller) to satisfy the requirements of the mechanical motion(outer loop).
- The second dealing with the mechanical motion of the rotor and desired flux dynamics of the motor with control inputs (the instantaneous electromagnetic torque / flux) and the load torque as the disturbance.

### 2.4.1 Current Controller

Current control is based on the sliding mode existence in the manifold  $\boldsymbol{\sigma}^T = [\mathbf{i}^{ref}(t) - \mathbf{i}]^T = 0$  where vector  $\boldsymbol{\sigma}^T = [\sigma_d \quad \sigma_q]^T$  with  $\sigma_d = i_d^{ref}(t) - i_d$ ,  $\sigma_q = i_q^{ref}(t) - i_q$  and  $i_d^{ref}(t)$ ,  $i_q^{ref}(t)$  are continuous functions to be determined later. Design of the current controller is based on the system description (2.82)  $d\mathbf{i}_{dq}/dt = \mathbf{f}_{dq} + \mathbf{B}_{udq}\mathbf{u}_{dq}$  where matrix  $\mathbf{B}_{udq}$  is diagonal. The structure of function  $\mathbf{f}_{dq}$  and matrix  $\mathbf{B}_{udq}$  could be easily found from mathematical models given in the previous chapter. The time derivative of  $\boldsymbol{\sigma}^T = [\sigma_d \quad \sigma_q]^T$  is determined as:

$$\frac{d\boldsymbol{\sigma}}{dt} = \frac{d\mathbf{i}_{dq}^{ref}}{dt} - \frac{d\mathbf{i}_{dq}}{dt} = \frac{d\mathbf{i}_{dq}^{ref}}{dt} \mathbf{f}_{dq} - \mathbf{B}_{udq}\mathbf{u}_{dq}; \quad \mathbf{u}_{dq}^T = [u_d \quad u_q] \quad (2.85)$$

Equivalent control can be calculated as  $\mathbf{B}_{udq}^{-1} [d\mathbf{i}_{dq}^{ref}/dt - \mathbf{f}_{dq}] = \mathbf{u}_{eq}$  and (2.85) is expressed as:

$$\frac{d\boldsymbol{\sigma}_{dq}}{dt} = \mathbf{B}_{udq} [\mathbf{u}_{eq} - \mathbf{u}_{dq}(S_i)] \quad , i = (1, \dots, 9) \quad (2.86)$$

Following design procedure explained before for the voltage vectors  $\mathbf{u}_{dq}^T = [u_d \quad u_q]$  we have the following control inputs (e.g. output of the current controller) for the current vectors  $\mathbf{i}_{dq}^T = [i_d \quad i_q]$ :

$$\mathbf{u}_{dq}(kT^+) = \mathbf{u}_{dq}(kT^-) - \frac{(\mathbf{GB})^{-1}}{T} ((\mathbf{DT} + 1)\boldsymbol{\sigma}(kT^+) - \boldsymbol{\sigma}(kT^-)) \quad (2.87)$$

where  $\boldsymbol{\sigma}^T = [\mathbf{i}^{ref}(t) - \mathbf{i}]^T = 0$  and  $(\mathbf{GB})^{-1} = \frac{1}{L_\sigma}$

the obtained control voltages are then applied to the motor via proper transformation and space vector pulse width modulation technique which will be

explained in the implementation further.  $T$  is the sampling time for the control implementation.

After designing the proper current controllers next step is – by following the same procedure with the model of the IM obtained previously – to design controllers for speed and flux for completing the control system and to obtain the reference currents for the current controllers.

### 2.4.2 Speed / Flux Controller

Following the same procedure previously explained the required sliding surfaces for the reference currents of the speed and flux controllers can be as follows:

$$\mathbf{i}_{dq(k)}^{ref} = \mathbf{i}_{dq(k-1)}^{ref} - (\mathbf{GB}_i \mathbf{T})^{-1} ((\mathbf{E} + \mathbf{T}\mathbf{D})\boldsymbol{\sigma}_{dq(k)} - \boldsymbol{\sigma}_{dq(k-1)}); \quad \mathbf{G} = \{\partial \boldsymbol{\sigma} / \partial \mathbf{x}\} \quad (2.88)$$

$$\text{where } \mathbf{i}_{dq}^{ref} = [i_d^{ref} \quad i_q^{ref}]^T \quad \boldsymbol{\sigma}_{dq} = [\sigma_d \quad \sigma_q]^T$$

$$\sigma_q = C_\omega \Delta\omega; \Delta\omega = \omega^{ref} - \omega \quad ; \quad \sigma_d = \sigma_\phi(\phi, \phi^{ref}) = \Delta\phi; \Delta\phi = \phi^{ref} - \phi$$

The realization of the control explained above requires information on the sliding functions and the plant gain matrix. The speed / flux controller completes the control system design for the induction machine, the only remaining part is the design of the flux observer required to decouple the currents controlling the flux and torque of the machine independently which will be mentioned in the following chapter. Figure 2.9 below indicates the overall sliding mode sensorless control structure of the induction motor.

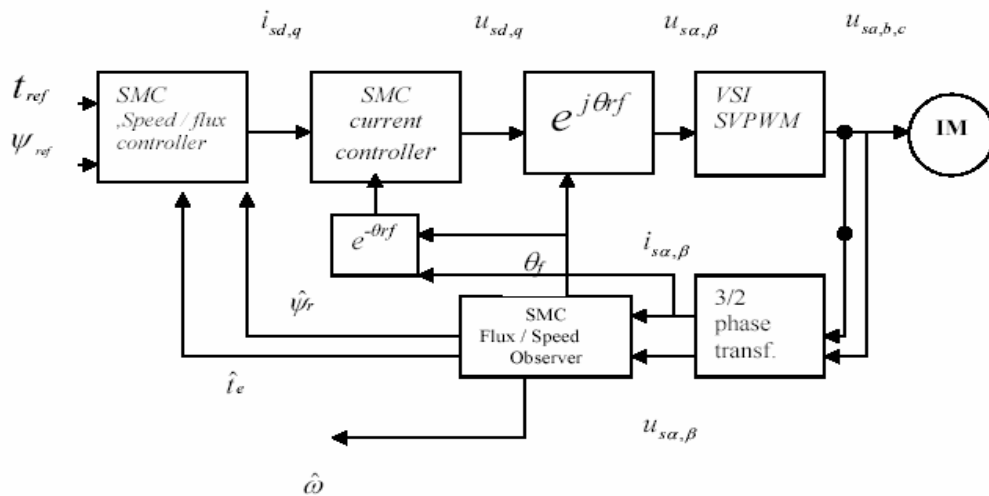


Figure 2.9: Overall structure of the control system for IM

### 3 OBSERVER DESIGN

All the control system design methods for the IM electrical drive mentioned in the previous section were developed under the assumption that the rotor flux and the speed of the machine is available. Estimation or measurement of the rotor flux is the core point for the vector control of the IM to be used in the required transformations mentioned previously in order to decouple and control the torque and the flux of the machine independently. In practice, the rotor speed is available sometimes via mechanical sensors and the rotor flux can only be obtained via special techniques (e.g. special windings, sensors etc.) not proper for the standard type IM widely used in the industry or via proper observers. Due to the additional cost and the problems of reliability and ruggedness of these sensors, observer design is preferred by many researchers to estimate the rotor speed and rotor flux. In this chapter using the sliding mode control approach and Lyapunov design explained in the previous chapter a new sliding mode flux / speed observer is developed. The performance of the proposed observer is investigated and verified via simulations and experiments.

#### 3.1 Proposed Flux / Speed Observer

The structure of the proposed observer can be seen in figure 3.1 below. For the observer design the motor model given in the previous chapter in the stationary frame, for the stator current observer chattering free-SMC and for the flux / speed observer Lyapunov design methodology are used. The observer proposed is of the full order and designed in two levels. First level is to design a stator current observer so that the estimated stator current tracks the measured one and the error in the current vector ( $\Delta \mathbf{i}_s$ ) is used as the input for the current observer controller. The control input calculated in the controller ( $\mathbf{u}_{\psi_{\alpha\beta}}$ ) for the stator current observer is then directly applied

to the rotor flux and speed observer. This control has the information about the rotor flux, rotor time constant (e.g.  $\frac{1}{x_r} = \frac{L_r}{R_r}$ ) and rotor speed inside. For the second level an important geometrical approach in vector control of IM drives is used to estimate the flux / speed observer shown in fig.3.1. The details about these observers are given below.

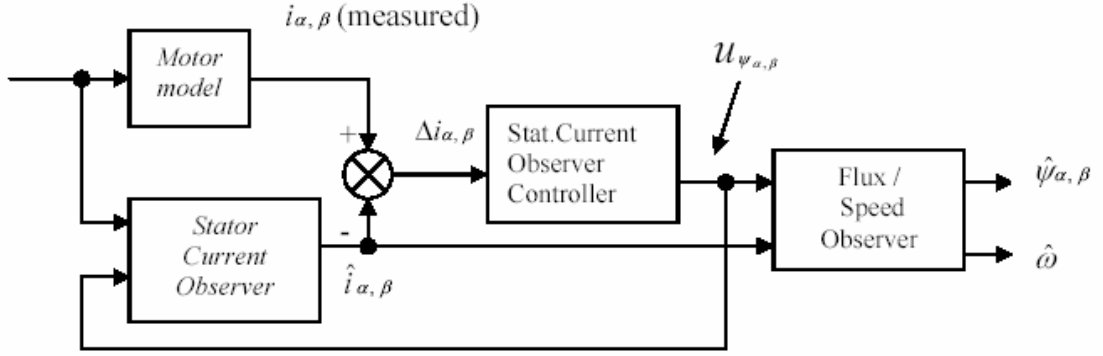


Figure 3.1: Structure of the proposed observer

### 3.2 Motor Model

From the previous chapter (SSVM) the electrical side model of the IM can be written in stationary reference frame according to explained stator current observer and controller structure given in figure1 as follows:

$$\frac{di_{s\alpha}}{dt} = \frac{1}{L_\sigma} \left[ \frac{L_m}{L_r} \left( \frac{R_r}{L_r} \psi_{r\alpha} + \omega \psi_{r\beta} - \frac{R_r}{L_r} L_m i_{s\alpha} \right) - R_s i_{s\alpha} + u_{s\alpha} \right] \quad (3.1)$$

$$\frac{di_{s\beta}}{dt} = \frac{1}{L_\sigma} \left[ \frac{L_m}{L_r} \left( -\omega \psi_{r\alpha} + \frac{R_r}{L_r} \psi_{r\beta} - \frac{R_r}{L_r} L_m i_{s\beta} \right) - R_s i_{s\beta} + u_{s\beta} \right] \quad (3.2)$$

$$\frac{d\psi_{r\alpha}}{dt} = - \left( \frac{R_r}{L_r} \psi_{r\alpha} + \omega \psi_{r\beta} - \frac{R_r}{L_r} L_m i_{s\alpha} \right) \quad (3.3)$$

$$\frac{d\psi_{r\beta}}{dt} = - \left( -\omega \psi_{r\alpha} + \frac{R_r}{L_r} \psi_{r\beta} - \frac{R_r}{L_r} L_m i_{s\beta} \right) \quad (3.4)$$

$$u_{\psi\alpha} = \frac{R_r}{L_r} \psi_{r\alpha} + \omega \psi_{r\beta} - \frac{R_r}{L_r} L_m i_{s\alpha} \quad (3.5)$$

$$u_{\psi\beta} = -\omega \cdot \psi_{r\alpha} + \frac{R_r}{L_r} \cdot \psi_{r\beta} - \frac{R_r}{L_r} L_m i_{s\beta} \quad (3.6)$$

In (3.1)-(3.4)  $u_{s\alpha}, u_{s\beta}$  and  $i_{s\alpha}, i_{s\beta}$  are the stator voltages and the currents consecutively,  $\psi_{r\alpha}, \psi_{r\beta}$  are the rotor flux components in  $\alpha$ - $\beta$  reference frame,  $\omega$  is the rotor electrical speed and  $u_{\psi\alpha}, u_{\psi\beta}$  are the defined variables in the stator current dynamics- including the rotor flux, the most varying variables; rotor time constant ( $1 / x_r = L_r / R_r$ ) and rotor speed – which is equal to the negative of the rotor flux dynamics. In (1)-(6)  $u_{s\alpha}, u_{s\beta}$  and  $i_{s\alpha}, i_{s\beta}$  are the measurable variables, inductances of the motor are known. After some manipulation, (5) and (6) can be written as a vector product following:

$$\begin{bmatrix} u_{\psi\alpha} \\ u_{\psi\beta} \end{bmatrix} = \begin{bmatrix} \psi_{r\alpha} - L_m i_{s\alpha} & \psi_{r\beta} \\ \psi_{r\beta} - L_m i_{s\beta} & -\psi_{r\alpha} \end{bmatrix} \begin{bmatrix} R_r \\ L_r \\ \omega \end{bmatrix} \quad (3.7)$$

From (3.7) it can be realized that if one of the parameter vectors  $[x_r \ \omega]^T$  or the rotor flux vector components  $[\psi_{r\alpha} \ \psi_{r\beta}]^T$  is known, then the other component can be easily calculated. Then the stator and the rotor side of the observer are designed as in the model considering this fact.

### 3.3 Observer Model

$$\frac{d\hat{i}_{s\alpha}}{dt} = \frac{1}{L_\sigma} \left( \frac{L_m}{L_r} u_{\psi\alpha} - R_s \cdot \hat{i}_{s\alpha} + u_{s\alpha} \right) \quad (3.8)$$

$$\frac{d\hat{i}_{s\beta}}{dt} = \frac{1}{L_\sigma} \left( \frac{L_m}{L_r} u_{\psi\beta} - R_s \cdot \hat{i}_{s\beta} + u_{s\beta} \right) \quad (3.9)$$

$$\frac{d\hat{\psi}_{r\alpha}}{dt} = -u_{\psi\alpha} \quad (3.10)$$

$$\frac{d\hat{\psi}_{r\beta}}{dt} = -u_{\psi\beta} \quad (3.11)$$

Here “ $\wedge$ ” indicates the observed variables.  $u_{\psi\alpha}, u_{\psi\beta}$  are the control inputs for the designed observer which are chosen according to the (3.7) explained previously and these variables are directly used both in the stator current and rotor flux observers.

### 3.4 Sliding Mode Observer Controller (Stator Current Observer)

The aim of this controller is to suppress the effects due to the stator side dynamics by enforcing the error in the observed and the measured stator current ( $\Delta i_s$ ) to zero. Due to this reason the stator current observer is built depending on the current error dynamics. From (3.1), (3.8) and (3.2), (3.9) the error dynamics is calculated as follows:

$$\frac{d\Delta i_{s\alpha}}{dt} = \frac{1}{L_\sigma} \left[ \frac{L_m}{L_r} \left( \left( \frac{R_r}{L_r} \psi_{r\alpha} + \omega \psi_{r\beta} - \frac{R_r}{L_r} L_m i_{s\alpha} \right) - u_{\psi\alpha} \right) - R_s \Delta i_{s\alpha} \right] \quad (3.12)$$

$$\frac{d\Delta i_{s\beta}}{dt} = \frac{1}{L_\sigma} \left[ \frac{L_m}{L_r} \left( \left( -\omega \psi_{r\alpha} + \frac{R_r}{L_r} \psi_{r\beta} - \frac{R_r}{L_r} L_m i_{s\beta} \right) - u_{\psi\beta} \right) - R_s \Delta i_{s\beta} \right] \quad (3.13)$$

Here,  $\Delta i_{s\alpha} = i_{s\alpha} - \hat{i}_{s\alpha}$  ;  $\Delta i_{s\beta} = i_{s\beta} - \hat{i}_{s\beta}$

To obtain the required control vectors ( $u_{\psi\alpha}, u_{\psi\beta}$ ), the chattering free sliding mode control approach, which was explained previously, is used. The sliding surface (manifold) for this control is defined as follows:

$$\sigma(x) = G \Delta i_{\alpha, \beta} = G (i_{\alpha, \beta} - \hat{i}_{\alpha, \beta}) \quad (3.14)$$

In (3.14) taking the derivative of both sides:

$$\frac{d(\sigma(x))}{dt} = \dot{\sigma}(x) = G (\dot{i}_{\alpha, \beta} - \dot{\hat{i}}_{\alpha, \beta}) \quad (3.15)$$

As it was explained in the previous chapter, the necessary condition for the switching function in holding Lyapunov stability is to choose it as  $\dot{\sigma}(x) = -D \sigma(x)$  (chattering free SMC). The states (e.g. stator current) and their derivatives in (3.12), (3.13) become linearly dependent by this selection. Then using the definition of equivalent control – the control which drives the system to sliding manifold given in the previous chapter – the equivalent control can be calculated by substituting (3.8) and (3.9) in (3.15):



$$\frac{d(\sigma(x))}{dt} = \dot{\sigma}(x) = G \left( \dot{i}_{\alpha, \beta} - \frac{1}{L_{\sigma}} \left( \frac{L_m}{L_r} u_{\psi\alpha\beta} - R_s \cdot \hat{i}_{s\alpha\beta} + u_{s\alpha\beta} \right) \right) \quad (3.16)$$

In (3.16) if the system is defined as in (2.50), from (3.8) and (3.9):

$$\hat{x} = \begin{bmatrix} \hat{i}_{\alpha} & \hat{i}_{\beta} \end{bmatrix}^T, \quad f(\hat{x}, t) = -\frac{R_s}{L_{\sigma}} \hat{x} + \frac{1}{L_{\sigma}} \begin{bmatrix} u_{s\alpha} & u_{s\beta} \end{bmatrix}^T \quad (3.17)$$

$$B = \frac{L_m}{L_{\sigma} L_r}, \quad u = \begin{bmatrix} u_{\psi\alpha} & u_{\psi\beta} \end{bmatrix}^T$$

using the definition of the “equivalent control” (e.g.  $\dot{\sigma}(x) = 0 \Rightarrow u_{\psi\alpha\beta} = u_{\psi eq}$ ). For this case, the stator currents are used as reference signals and the error in the estimation ( $\Delta \mathbf{x} = \mathbf{x}^r - \hat{\mathbf{x}}$ ) becomes the error function for the controller designed. Then using the chattering free sliding mode approach explained previously the control input for the observer controller can be found from (3.16) as follows:

$$\begin{bmatrix} u_{\psi\alpha} \\ u_{\psi\beta} \end{bmatrix}_{(t)} = \begin{bmatrix} u_{\psi\alpha} \\ u_{\psi\beta} \end{bmatrix}_{(t-T)} + \frac{L_{\sigma} L_r}{L_m T} \begin{bmatrix} (DT+1) \cdot \Delta \dot{i}_{\alpha(t)} - \Delta \dot{i}_{\alpha(t-T)} \\ (DT+1) \cdot \Delta \dot{i}_{\beta(t)} - \Delta \dot{i}_{\beta(t-T)} \end{bmatrix} \quad (3.18)$$

Here  $T$  is the sampling period for the observer implementation. The control vector calculated in (3.18) depends on the previous values of the control and the error in the estimation and drives the system to equilibrium point in which the error in the estimation and its derivative is enforced to converge to zero (e.g.  $\Delta i_{\alpha, \beta} \rightarrow 0$  and  $\Delta \dot{i}_{\alpha, \beta} \rightarrow 0$ ). In other words when the sliding mode occurs in the switching surface the error and its derivative converges to zero. Then from (3.12), (3.13) and the definition of equivalent control, again substituting  $\Delta i_{\alpha, \beta} \rightarrow 0$  and  $\Delta \dot{i}_{\alpha, \beta} \rightarrow 0$  it can be found that:

$$u_{\psi\alpha eq} = \left( \frac{R_r}{L_r} \cdot \psi_{r\alpha} + \omega \cdot \psi_{r\beta} - \frac{R_r}{L_r} L_m \dot{i}_{s\alpha} \right) \quad (3.19)$$

$$u_{\psi\beta eq} = \left( -\omega \cdot \psi_{r\alpha} + \frac{R_r}{L_r} \cdot \psi_{r\beta} - \frac{R_r}{L_r} L_m \dot{i}_{s\beta} \right) \quad (3.20)$$

For the chattering free sliding mode control, the equivalent controls obtained in (3.19) and (3.20) equal to the controls calculated in (3.18). If the rotor flux can be estimated correctly, from the controls in (3.19), (3.20) and the relation (3.7) the rotor speed and rotor time constant ( $L_r / R_r$ ) can be estimated without any problem. This will

be explained later after the design of rotor flux observer. Another observation from (3.19) and (3.20) that after some manipulation the following relation can be obtained:

$$\begin{bmatrix} \hat{\psi}_{r\alpha} \\ \hat{\psi}_{r\beta} \end{bmatrix} = \begin{bmatrix} \frac{R_r}{L_r} & -\omega \\ \omega & \frac{R_r}{L_r} \end{bmatrix} \begin{bmatrix} u_{\text{req}\alpha} + \frac{R_r}{L_r} L_m \dot{i}_{s\alpha} \\ u_{\text{req}\beta} + \frac{R_r}{L_r} L_m \dot{i}_{s\beta} \end{bmatrix} \quad (3.21)$$

Here if the rotor speed and rotor time constant are measured / known and since the stator currents are measured, then rotor flux can be calculated (estimated) from (3.21) easily. Experiments about this approach are shown in the next chapter. But, since this approach requires a mechanical sensor to measure the speed for flux estimation it can only be used as a reference for the sensorless scheme which is the main goal in this thesis.

### 3.5 Rotor Flux Observer

For the rotor side of the observer model given in (3.10), (3.11) after a quick observation it can be easily realized that following the convergence of the estimated stator current to its real value there is nothing unknown left. The control given in (3.18) is equal to the negative of the rotor flux derivative. Thus, theoretically the rotor flux can be estimated through direct ideal integration of the negative of the control in (3.18). But, although the effects of the stator side dynamics can be suppressed by the stator current controller; due to the problems of initial condition mismatch, integration and the variation of the inductances of the machine the estimation of the correct rotor flux is problematic. Thus, the error in the flux estimation should be avoided.

The success of an observer depends on its error dynamics, so the error dynamics of the rotor flux should be checked. From (3.3), (3.4) and (3.10), (3.11) the error dynamics for the flux is found as follows:

$$\frac{d\Delta\psi_{s\alpha}}{dt} = -\left(\left(\frac{R_r}{L_r}\psi_{r\alpha} + \omega\psi_{r\beta} - \frac{R_r}{L_r}L_m i_{s\alpha}\right) - u_{\psi\alpha}\right) \quad (3.22)$$

$$\frac{d\Delta\psi_{s\beta}}{dt} = -\left(\left(-\omega\psi_{r\alpha} + \frac{R_r}{L_r}\psi_{r\beta} - \frac{R_r}{L_r}L_m i_{s\beta}\right) - u_{\psi\beta}\right) \quad (3.23)$$

The error dynamics in (3.22) and (3.23) are also included in the stator current error dynamics and from the dynamics of this observer it is obtained that when the states reach the sliding surface (3.19) and (3.20) are valid. Then using (3.19), (3.20), (3.22) and (3.23) it is obtained that:

$$\frac{d\Delta\psi_{s\alpha}}{dt} = 0; \frac{d\Delta\psi_{s\beta}}{dt} = 0 \quad (3.24)$$

In (3.24) for the error in the derivative of the flux to be zero does not mean that flux error is zero. Due to this reason an additional convergence term to compensate for the error in the flux estimation caused by the initial condition mismatch and direct integration is unavoidable.

### 3.6 Convergence Term

To design the convergence term a geometrical approach is used. Note that in the vector control of IM the aim is to control the flux and the torque of the machine independently with two orthogonal components of the stator current. Also remember that another important fact is that the flux magnitude should be kept constant so that the fast torque response of the machine can be obtained by changing the stator current as for the separately excited DC machine. And finally in vector space any vector and its derivative should be orthogonal. Keeping in mind these facts following statement about the *desired(actual)* rotor flux vector and its derivative can be written in the stationary reference frame.

$$\psi_r \perp \dot{\psi}_r \quad (3.25)$$

(3.25) state that the rotor flux in the machine and its derivative is orthogonal. Then from (3.25) and orthogonality property of the vectors following can be written for the rotor flux and its derivative:

$$\psi_{\alpha} \cdot \dot{\psi}_{\alpha} + \psi_{\beta} \cdot \dot{\psi}_{\beta} = K = 0 \quad (3.26)$$

(3.26) will be the basis for the sensorless flux estimator. (3.26) is nothing but just the simple scalar product of the flux vector and its derivative. Mathematically, this can be written as follows:

$$\psi_r \bullet \dot{\psi}_r = K = 0 \quad (3.27)$$

The orthogonality of the two vectors can be seen in the figure below:

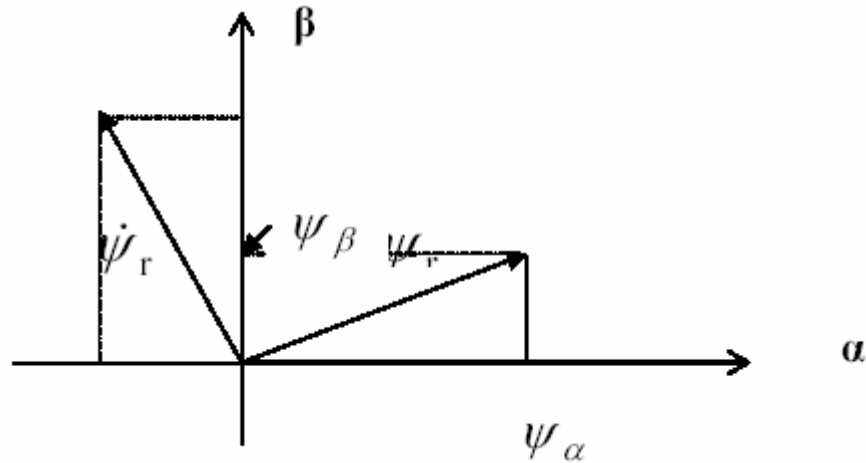


Figure 3.2: Rotor flux and its derivative in ( $\alpha$ - $\beta$ ) reference plane

As explained previously; from the control calculated from the stator current tracking the position and the magnitude of the derivative of the rotor flux is well known. Here the problem exists in the estimation of the flux itself due to the problems explained previously. To solve this problem an additional term should be designed for the rotor flux to converge to its real value. This additional term is added to the rotor flux observer in (3.10) and (3.11) as follows:

$$\frac{d\hat{\psi}_r}{dt} = -u_v + f(\Delta\psi_r) \quad (3.28)$$

In (3.25) all the state variables are in vector form with two components in stationary frame. The function  $f(\Delta\psi_r)$  should be determined s.t. the problems of initial condition mismatch in the integration and variation of inductances can be suppressed. When the rotor flux converges to its real value, then this function should converge to zero.

In obtaining the convergence terms the relation given in (3.26) will be used. First let the relation between the estimated and the actual flux as follows:

$$\psi_\alpha = \hat{\psi}_\alpha + \Delta\psi_\alpha \text{ and } \psi_\beta = \hat{\psi}_\beta + \Delta\psi_\beta \quad (3.29)$$

(3.26) can be written for the estimated flux as follows:

$$\hat{\psi}_\alpha \cdot \dot{\psi}_\alpha + \hat{\psi}_\beta \cdot \dot{\psi}_\beta = \hat{K} \quad (3.30)$$

From (3.10) and (3.11) the control calculated can be substituted for the derivative of the flux:

$$-(\hat{\psi}_\alpha \cdot u_{\psi\alpha} + \hat{\psi}_\beta \cdot u_{\psi\beta}) = \hat{K} \quad (3.31)$$

Then if  $\hat{\psi}_r \rightarrow \psi_r$ , then  $\hat{K} \rightarrow K \rightarrow 0$  (e.g. when flux converges to actual one estimated scalar product converges to zero). The situation in (3.30) can be seen in the figure below

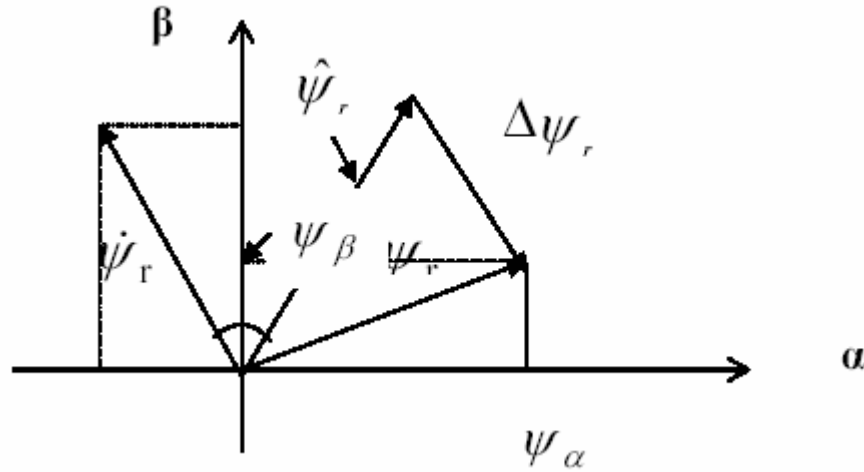


Figure 3.3: Estimated and actual flux and the derivative in stationary frame

To investigate the dynamics of the change of the variable  $K$  in (3.26) taking the derivative of both sides, the following relation is obtained:

$$\dot{K} = \dot{\psi}_\alpha^2 + \dot{\psi}_\beta^2 + \psi_\alpha \cdot \ddot{\psi}_\alpha + \psi_\beta \cdot \ddot{\psi}_\beta \quad (3.32)$$

Then an observer for  $K$  can be constructed s.t. when the estimated  $\hat{K}$  converges to its actual value  $K$  then the estimated flux will converge to its real value. The observer constructed is as follows:

$$\dot{\hat{K}} = u_{\psi\alpha}^2 + u_{\psi\beta}^2 - \hat{\psi}_\alpha \cdot \dot{u}_{\psi\alpha} - \hat{\psi}_\beta \cdot \dot{u}_{\psi\beta} + L\Delta K \quad (3.33)$$

In (3.33) when constructing the observer since the derivative of the flux is known from stator current observer it is substituted with the negative of the control  $u_\psi$

calculated previously (measurable in the sense of reduced order observers). Double derivative of the flux is also required and this can be calculated by taking the derivative of the first derivative via euler method and again this will be equal to the negative of  $\dot{i}_{\psi}$  which can be found using (3.18) as follows:

$$\begin{bmatrix} \dot{i}_{\psi\alpha} \\ \dot{i}_{\psi\beta} \end{bmatrix}_{(t)} = \frac{L_{\sigma}L_r}{L_m} \begin{bmatrix} (DT+1) \cdot \Delta i_{\alpha(t)} - \Delta i_{\alpha(t-T)} \\ (DT+1) \cdot \Delta i_{\beta(t)} - \Delta i_{\beta(t-T)} \end{bmatrix} \quad (3.34)$$

Also we can calculate error in the observer state as:

$$\Delta K = K - \hat{K} = 0 - \hat{K} = -\hat{K} \quad (3.35)$$

To check for the convergence of the designed observer, the error dynamics should be investigated. For a positive constant  $L$  to be chosen and relations (3.32), (3.33), (3.34) and (3.35) the error dynamics becomes:

$$\Delta \dot{K} = -\Delta \psi_{\alpha} \dot{i}_{\psi\alpha} - \Delta \psi_{\beta} \dot{i}_{\psi\beta} - L\Delta K \quad (3.36)$$

Then using Lyapunov design methodology; select the positive definite Lyapunov candidate function  $V$  as:

$$V = \frac{1}{2} \left( \Delta \psi_{\alpha}^2 + \Delta \psi_{\beta}^2 + \Delta K^2 \right) \geq 0 \quad (3.37)$$

(3.37) is zero only at the equilibrium point where all the errors in the flux and the fictitious variable  $K$  are zero. Taking the derivative of  $V$  becomes:

$$\dot{V} = \Delta \psi_{\alpha} \Delta \dot{\psi}_{\alpha} + \Delta \psi_{\beta} \Delta \dot{\psi}_{\beta} + \Delta K \Delta \dot{K} \quad (3.38)$$

Substituting (3.36) into (3.38) and from the previous knowledge that  $\Delta \dot{\psi}_{\alpha,\beta} = 0$  (3.38) becomes:

$$\dot{V} = -L\Delta K^2 + \Delta K \left( -\Delta \psi_{\alpha} \dot{i}_{\psi\alpha} - \Delta \psi_{\beta} \dot{i}_{\psi\beta} \right) \quad (3.39)$$

From Lyapunov Stability Criteria it is known that the time derivative of the selected function should be negative definite. In order for the function in (35) to be negative definite choosing the adaptive law as:

$$\Delta \psi_{\alpha} = -\Delta \psi_{\beta} \frac{\dot{i}_{\psi\beta}}{\dot{i}_{\psi\alpha}} \quad (3.40)$$

It is obtained that:

$$\dot{V} = -L\Delta K^2 \leq 0 \quad (3.41)$$

(3.41) means that under the chosen adaptive law (3.40), the selected Lyapunov function  $V$  in (3.37) is delaying until the fictitious variable error  $\Delta K$  becomes zero:

$$\Delta K = 0 \quad (3.42)$$

When this variable converges to its real value, substituting (3.42) in (3.36) it can be found that:

$$\Delta\psi_\alpha \dot{u}_{\psi\alpha} + \Delta\psi_\beta \dot{u}_{\psi\beta} = 0 \quad (3.43)$$

Also from (3.26), (3.30), (3.42), (3.10) and (3.11) it can be derived that:

$$\Delta\psi_\alpha \cdot u_{\psi\alpha} + \Delta\psi_\beta \cdot u_{\psi\beta} = \Delta K = 0 \quad (3.44)$$

Combining (3.43) and (3.44) in matrix form:

$$\begin{bmatrix} u_{\psi\alpha} \\ \dot{u}_{\psi\alpha} \end{bmatrix} \Delta\psi_\alpha + \begin{bmatrix} u_{\psi\beta} \\ \dot{u}_{\psi\beta} \end{bmatrix} \Delta\psi_\beta = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (3.45)$$

If the analysis is made, it can be concluded that there exists only the trivial solution for the (3.45) which is a system of 2 eqns. with 2 unknowns.

$$e.g. \rightarrow \Delta\psi_{\alpha,\beta} = 0 \text{ and } \hat{\psi}_r \rightarrow \psi_r \quad (3.46)$$

The trivial solution for the system in (3.45) can be proved by assuming  $u_{\psi\alpha}, u_{\psi\beta}$  in the form below:

$$u_{\psi\alpha} = \|A\| e^{j(\omega t + \phi)} \text{ and } u_{\psi\beta} = \|A\| e^{j(\omega t + \phi + \pi/2)} \quad (3.47)$$

In (3.47)  $\|A\|$  is the amplitude  $\phi$  is the phase and  $\omega$  is the rotation angular speed of the signal. The derivatives ( $\dot{u}_{\psi\alpha}, \dot{u}_{\psi\beta}$ ) of the control can be easily calculated.

Thus, to sum up for the observer in (3.33) under the adaptive law (3.40) the estimated rotor flux converges to its actual value with the additional convergence terms added to the two components ( $\alpha$ - $\beta$ ). After the correct estimation of the rotor flux the last step is to estimate the rotor speed and rotor time constant which is given in the next part.

### 3.7 Rotor Speed / Time Constant Observer

For the estimation of the rotor speed ( $\omega$ ) and time constant ( $L_r / R_r$ ), (3.7) can be used. From (3.7) using the estimated flux obtained from the flux observer, the equivalent control from the stator current observer controller and the measured currents; following can be written:

$$\begin{bmatrix} u_{v\alpha} \\ u_{v\beta} \end{bmatrix} = \begin{bmatrix} \hat{\psi}_\alpha - L_m i_{s\alpha} & \hat{\psi}_\beta \\ \hat{\psi}_\beta - L_m i_{s\beta} & -\hat{\psi}_\alpha \end{bmatrix} \begin{bmatrix} \hat{R}_r \\ L_r \\ \hat{\omega} \end{bmatrix} \quad (3.48)$$

$$u_v = A(\hat{\psi}_r, i_s, L_m) b\left(\frac{\hat{R}_r}{L_r}, \hat{\omega}\right) \quad (3.49)$$

Taking the inverse of matrix  $A$  above the rotor time constant and the speed can be calculated as:

$$\begin{bmatrix} \hat{R}_r \\ L_r \\ \hat{\omega} \end{bmatrix} = \begin{bmatrix} \hat{\psi}_\alpha - L_m i_{s\alpha} & \hat{\psi}_\beta \\ \hat{\psi}_\beta - L_m i_{s\beta} & -\hat{\psi}_\alpha \end{bmatrix}^{-1} \begin{bmatrix} u_{v\alpha} \\ u_{v\beta} \end{bmatrix} \quad (3.50)$$

Using the relation that  $A^{-1} = A^T / \det(A)$ :

$$b\left(\frac{\hat{R}_r}{L_r}, \hat{\omega}\right) = A^{-1}(\hat{\psi}_r, i_s, L_m) u_v \quad (3.48)$$

$$\begin{bmatrix} \hat{R}_r \\ L_r \\ \hat{\omega} \end{bmatrix} = \frac{1}{\left(\hat{\psi}_\alpha^2 + \hat{\psi}_\beta^2 - L_m i_{s\alpha} \hat{\psi}_\alpha - L_m i_{s\beta} \hat{\psi}_\beta\right)} \begin{bmatrix} -\hat{\psi}_\alpha & -\hat{\psi}_\beta + L_m i_{s\beta} \\ -\hat{\psi}_\beta & \hat{\psi}_\alpha - L_m i_{s\alpha} \end{bmatrix} \begin{bmatrix} u_{v\alpha} \\ u_{v\beta} \end{bmatrix} \quad (3.49)$$

(3.49) completes the design of the full order observer and finally with the designed observers, the overall sensorless vector control scheme shown in figure (2.9) has been finished. The performance of the overall sensorless control design scheme and the proposed observer structure is verified via simulation and experiments.



## 4 SIMULATION AND IMPLEMENTATION RESULTS

### 4.1 Implementation Issues

The proposed control scheme shown in figure 2.9 is implemented. In this scheme there is the IM as a working plant with the given data in Table1, voltage source inverter (VSI) operating with the space vector pulse width modulation technique (SVPWM), a tachogenerator to measure the speed, the DSPACE1103 board which includes TMS320F240 digital signal processor to calculate the proposed control and observer schemes and 2-LEM current sensors to measure the AC current going through the stator windings. Also the 3-phase transformer and variac are used to isolate the system galvanically from the mains. The processor works in 100 $\mu$ s sampling time, the system was tested with no-load.

$P_n = 370 W$	$R_s = 24.6 \Omega$
$I_n = 1.5 A$	$R_r = 16.9 \Omega$
$F_n = 50 Hz$	$L_m = 1.46 H$
$N_{rpm} = 2280$ $rpm$	$L_s = 1.499 H$
$\# \text{ of poles} = 2$	$L_r = 1.499 H$

Figure 4.1: Table for the nominal parameters of the IM plant

## 4.2 Space Vector Modulation

Pulse Width Modulation technique is used to generate the required voltage or current to feed the motor or phase signals.[24] This method is increasingly used for AC drives with the condition that the harmonic current is as small as possible and the maximum output voltage is as large as possible. Generally, the PWM schemes generate the switching position patterns by comparing three-phase sinusoidal waveforms with a triangular carrier.

In recent years, the space vector theory demonstrated some improvement for both the output crest voltage and the harmonic copper loss. The maximum output voltage based on the space vector theory is  $2/\sqrt{3} = 1.155$  times as large as the conventional sinusoidal modulation. It enables to feed the motor with a higher voltage than the easier sub-oscillation modulation method. This modulator allows having a higher torque at high speeds, and a higher efficiency.

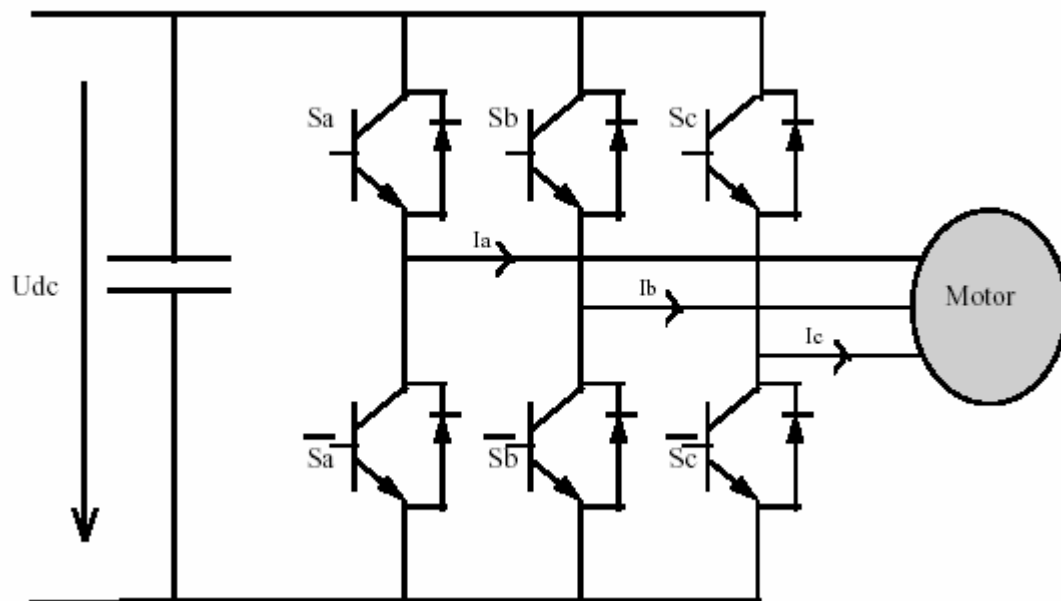


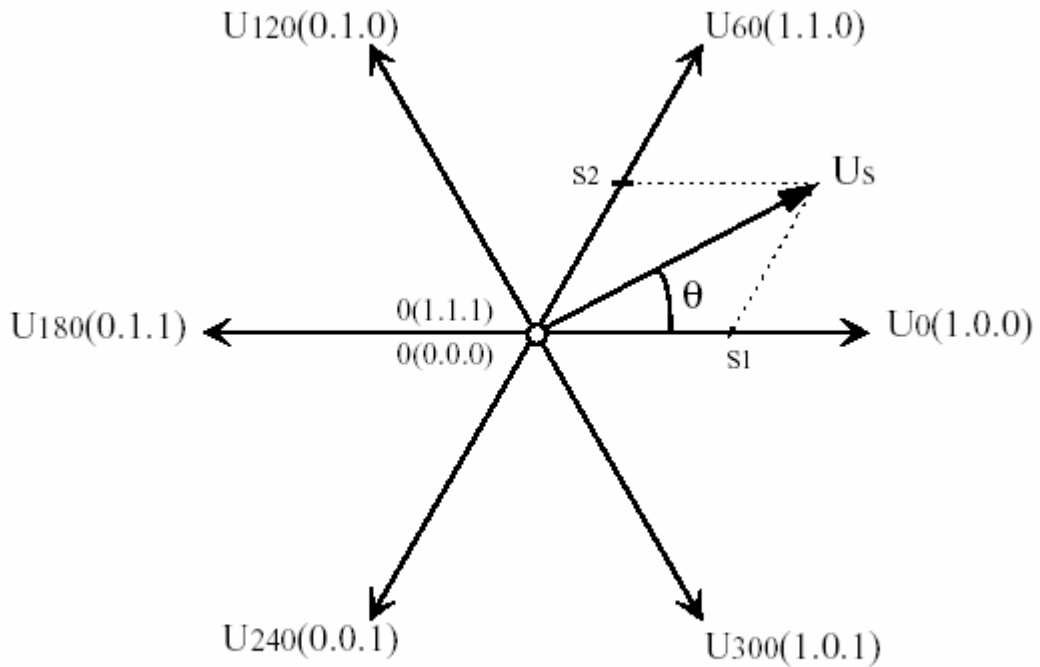
Figure 4.2: A Three Phase Inverter Fed by 3 PWM Signals  $S_a$ ,  $S_b$ ,  $S_c$  and Their Respective Complementary  $S_{a'}$ ,  $S_{b'}$ ,  $S_{c'}$ .

For a better understanding of the space vector process and to represent the switching state of the inverter we define a switching function  $S_a$  for phase A as follows:  $S_a = 1$  when the upper transistor of phase A is on, and  $S_a = 0$  when the lower transistor of phase A is on. Similar definitions can be made for phase B and C.

The signals  $Sa' Sb' Sc'$  (complementary) controlling the lower transistors, are the opposite of  $Sa Sb Sc$  with an addition of dead-bands. For our case the dead-band is added by the hardware. Thus there is no need to for soft dead-band implementation.

Dead-band is the name given to the time difference between the commutations of the upper and lower transistor of one phase. The two transistors of each phase are then never conducting at the same time. The aim of the dead-band is to protect the power devices during commutation by avoiding conduction overlap and then high transient current.

In the following graph vectors,  $U_{xxx}$  are represented with their corresponding switching states between brackets,  $U_{xxx} (Sa, Sb, Sc)$ .



### Space Vector Combination of $\vec{i}$

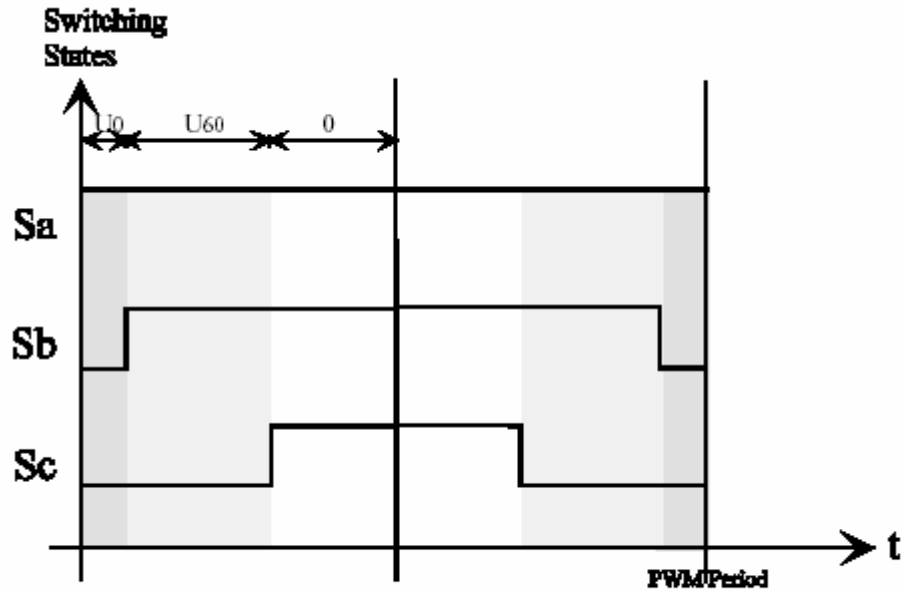
Figure 4.3: Space Vector combination of  $\vec{i}$

In the space vector theory the motor voltage vector is approximated by a combination of 8 switching patterns of the 6 power transistors shown in figure 2 above.  $U_s$  is decomposed as follows:

$$\vec{U}_s = |\vec{U}_s| e^{j\theta} = s_1 \vec{U}_x + s_2 \vec{U}_y \quad (4.1)$$

where  $U_x$  and  $U_x$  are two consecutive vectors. The third vector  $O$  (0.0.0 ) or  $O$  (1.1.1 ) is chosen in a way to minimize the number of switching commutations. This can be expressed with the formula:

$$T\bar{U}_s = s_1\bar{U}_x + s_2\bar{U}_y + (T - s_1 - s_2)\bar{O} \quad (4.2)$$



*PWM Sstates with  $0 \leq \theta \leq 60$  deg*

Figure 4.4: PWM S states with  $0 \leq \theta \leq 60$  deg

In the above case which is a symmetrical PWM generation, the first half period of a PWM is built with the two PWM configurations  $U_0$  and  $U_{60}$  characterized by the switching states (0,0,1) and (1,1,0) and the vector  $O$  (1,1,1). The second half of the period has the same sequence but inverted related to time. This PWM scheme describe a vector  $U_s$  with an angle  $\theta$  as  $0 \leq \theta \leq 60$  deg .

The torque / speed and flux control loops are executed in every 100  $\mu$ s. The system is operated under no-load. Low speed control is challenging, with the proposed control scheme and open-loop flux observer [12] very good results were obtained (0.1 rpm). The controller parameters for the  $i_{dq}$  current, speed and flux controllers are given in Tables 2, 3 and 4 consecutively below.

$C_i = 10$	<i>Multiplied with error</i>
$D_i = 1000$	<i>Sliding manifold slope</i>
$Ku_i = 0.01$	<i>Integrator gain</i>
$T = 1e-4 \text{ s.}$	<i>Sampling time</i>

Figure 4.5: Current Controller Parameters

$C_\psi = 7$	<i>Multiplied with error</i>
$D_\psi = 500$	<i>Sliding manifold slope</i>
$Ku_\psi = 1$	<i>Integrator gain</i>
$T = 1e-4 \text{ s.}$	<i>Sampling time</i>

Figure 4.6: Flux Controller Parameters

$C_w = 1$	<i>Multiplied with error</i>
$D_w = 500$	<i>Sliding manifold slope</i>
$Ku_w = 0.0005$	<i>Integrator gain</i>
$T = 1e-4 \text{ s.}$	<i>Sampling time</i>

Figure 4.7: Speed Controller Parameters

Following are the experimental results of the torque / speed and flux control explained previously with the given controller parameters.

### 4.3 Experimental Results

#### 4.3.1 Torque Flux Control

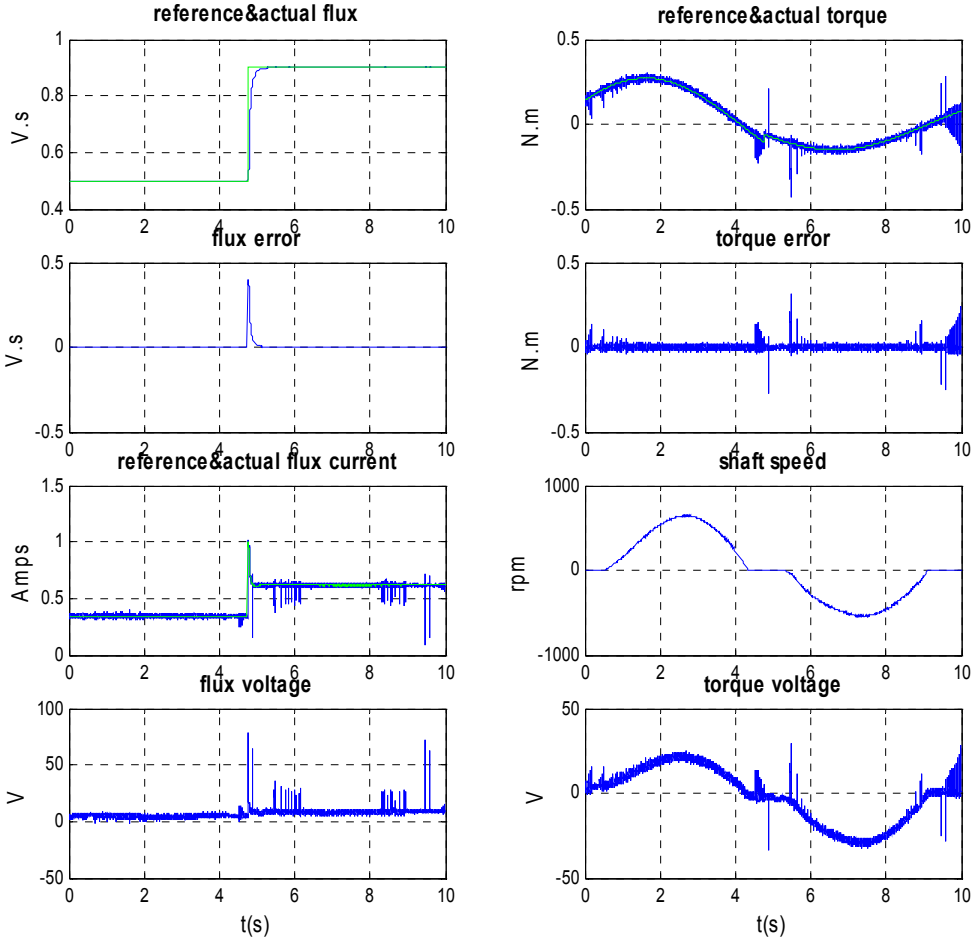


Figure 4.8: 0.1 Hz sinusoidal torque reference and 0.5 to 0.9 V.s step flux reference

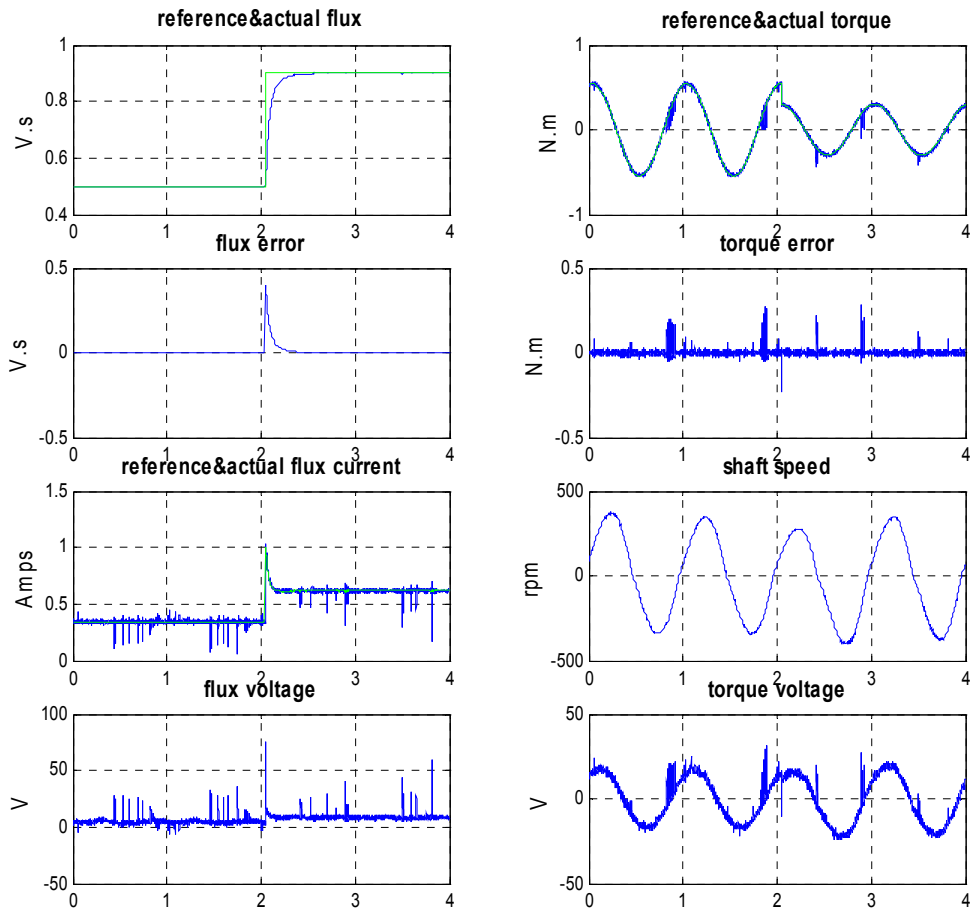


Figure 4.9: 1 Hz sinusoidal torque reference and 0.5 to 0.9 V.s step flux reference

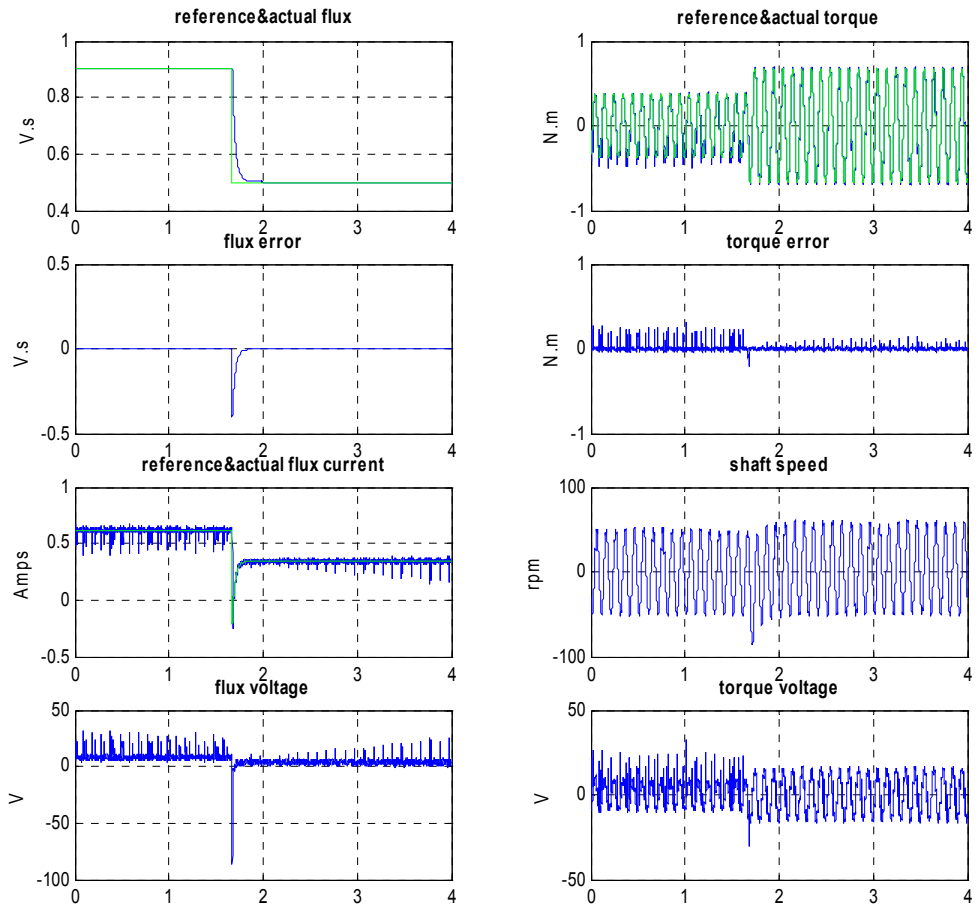


Figure 4.10: 10 Hz sinusoidal torque reference and 0.9 to 0.5 V.s step flux reference



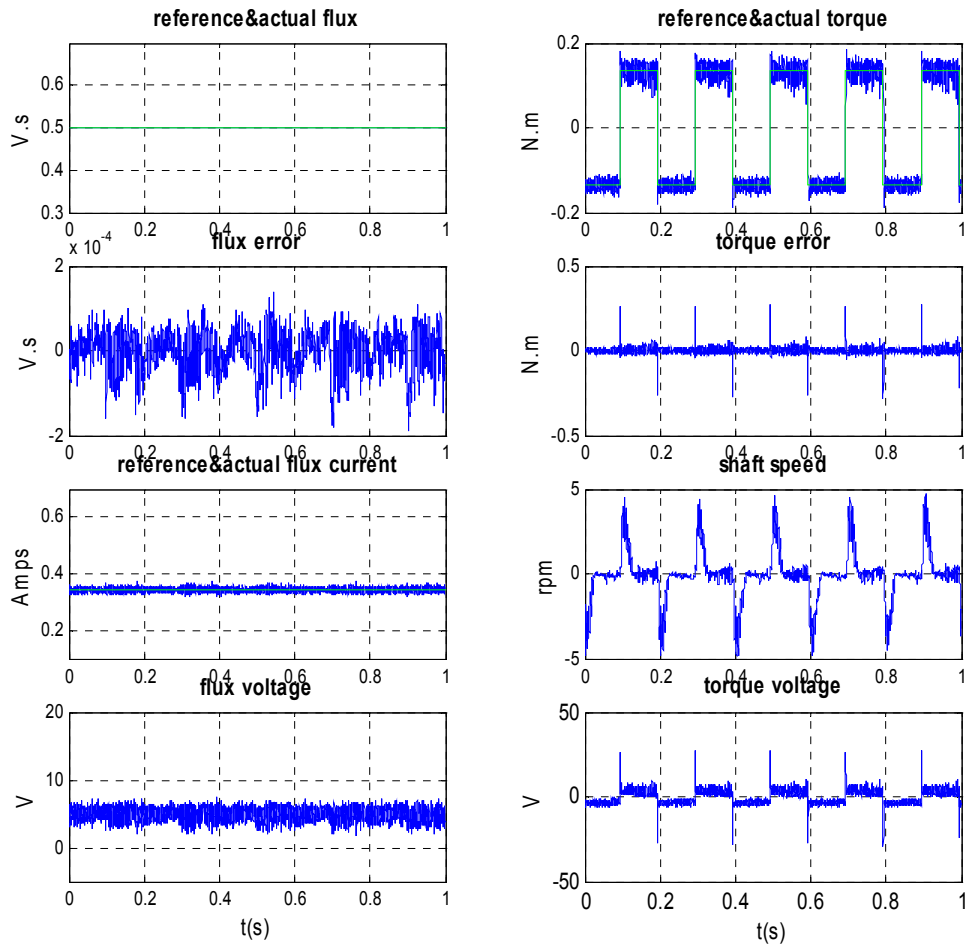


Figure 4.11: 10 Hz pulse torque reference and 0.5 V.s constant flux reference

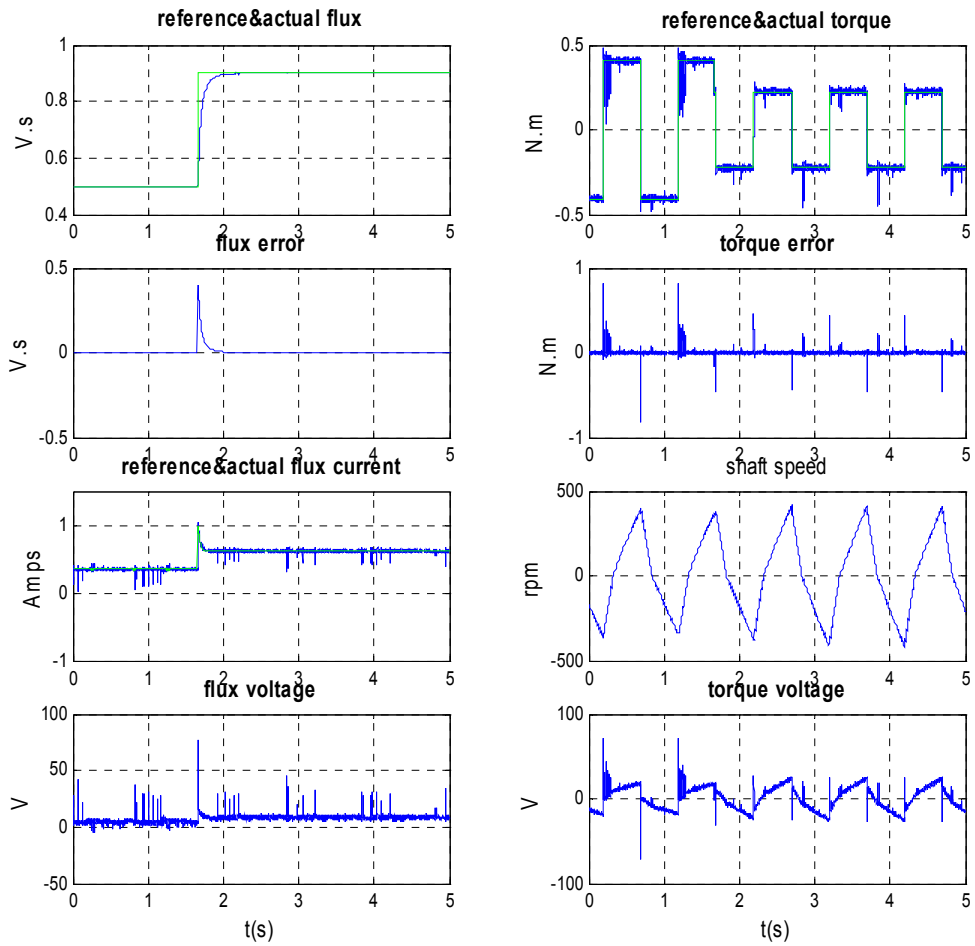


Figure 4.12: 2 Hz pulse torque reference and 0.5 to 0.9 V.s step flux reference

### 4.3.2 Speed Flux Control

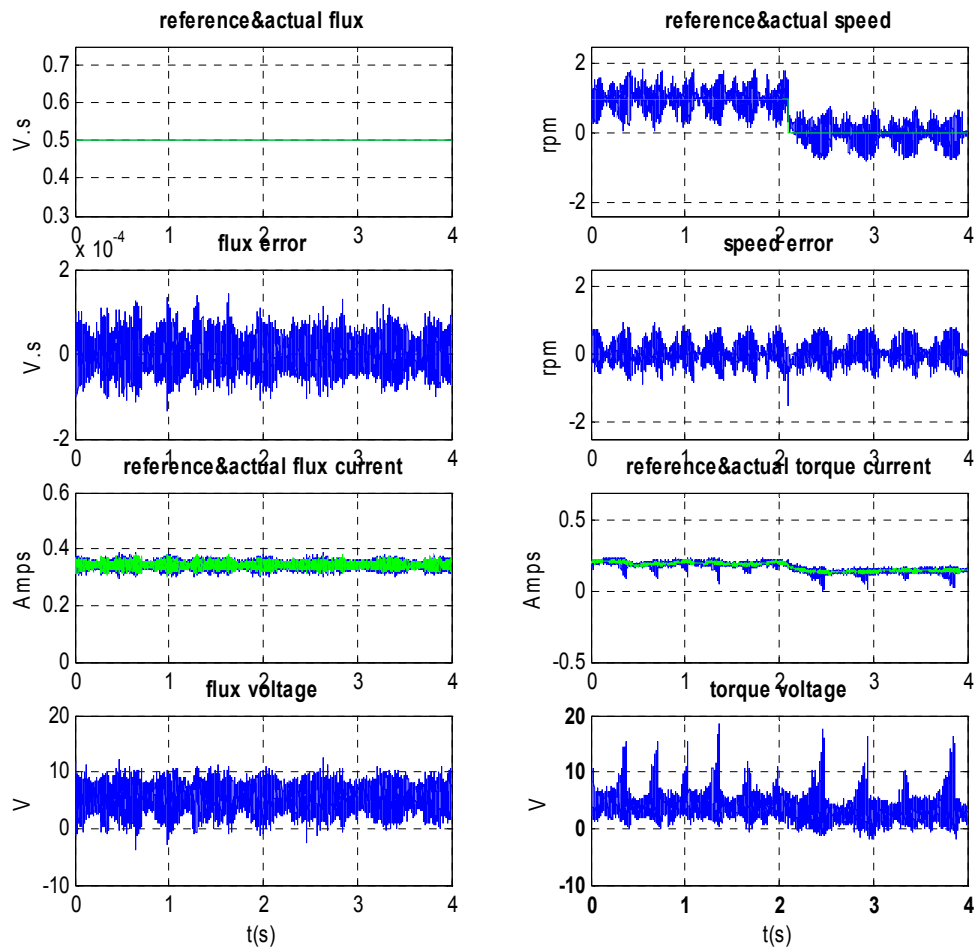


Figure 4.13: 1 rpm speed reference and 0.5 V.s constant flux reference

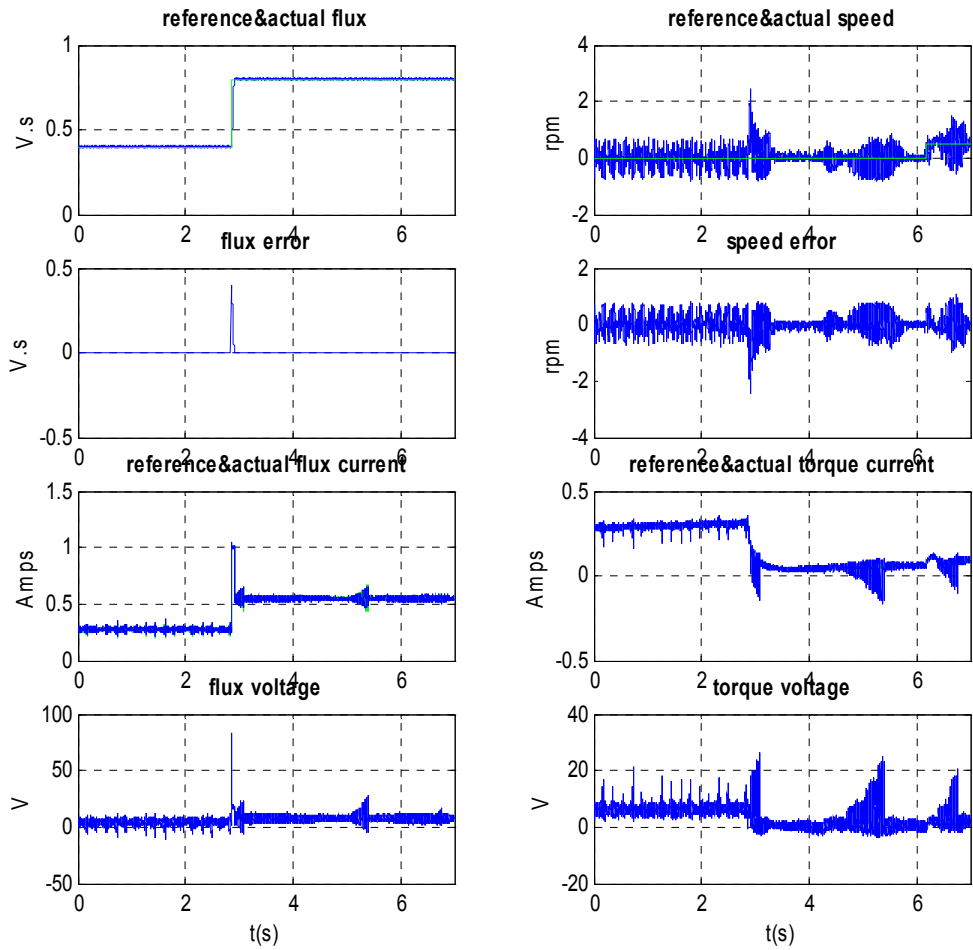


Figure 4.14: 0.5 rpm step speed reference and 0.5 to 0.9 V.s step flux reference

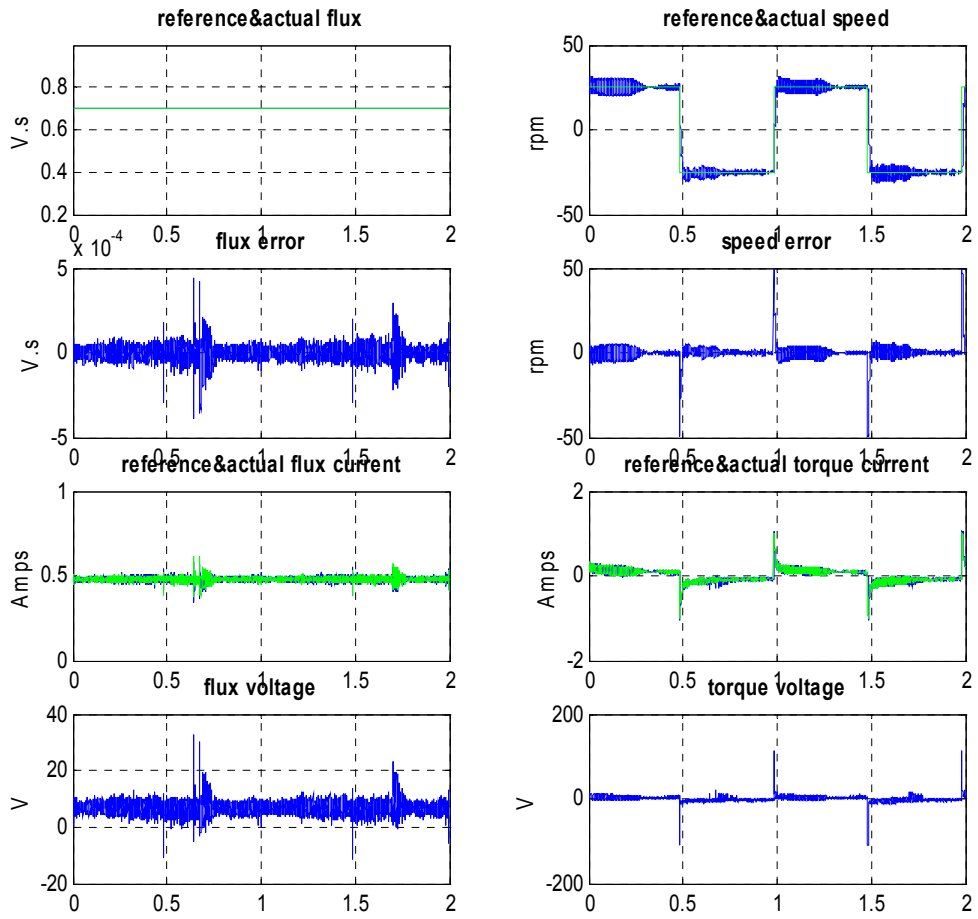


Figure 4.15: 25rpm pulse speed reference and 0.7 V.s flux reference

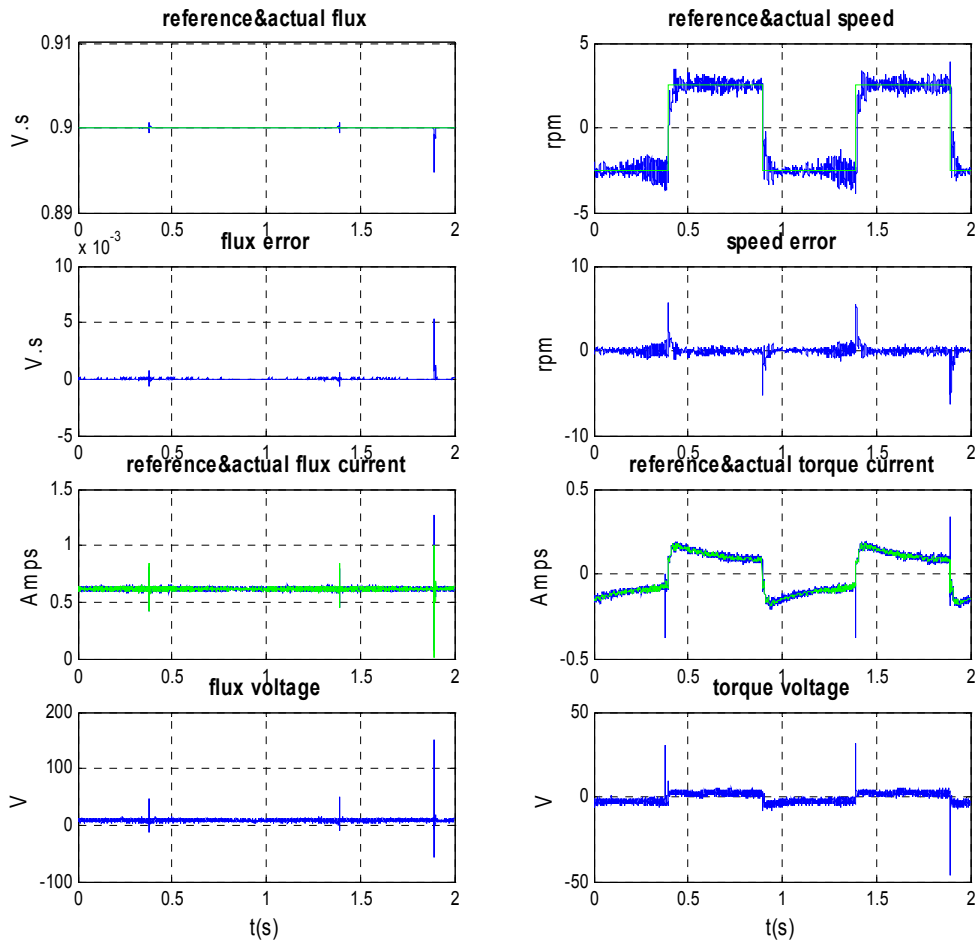


Figure 4.16: 2.5 rpm pulse speed reference and 0.9 V.s flux reference

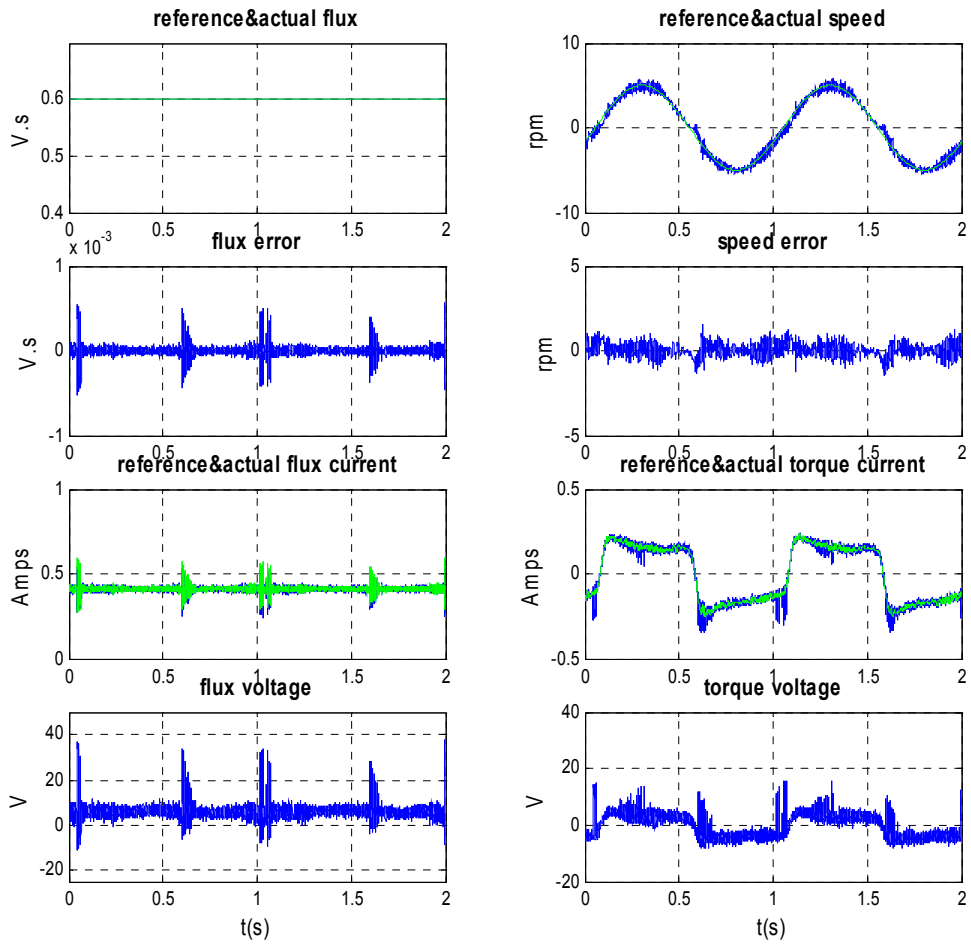


Figure 4.17: 1 Hz sinusoidal speed reference and 0.6 V.s flux reference

## 4.4 Simulation Results

The proposed control scheme was first simulated using MATLAB. In the simulation, pulsewidth modulation (PWM) technique is not used and the dynamics in the voltage-source inverter are also ignored. The proposed sensorless algorithm was tested under no-load, big initial condition for the flux (comparing to the magnitude of the actual flux) and low speed where control is very challenging. Outer loop control was not shown in the simulation results since the main goal was to look for the convergence of the rotor flux, speed and time constant to converge to its actual values. The motor was driven open loop. Here are the simulation results obtained.

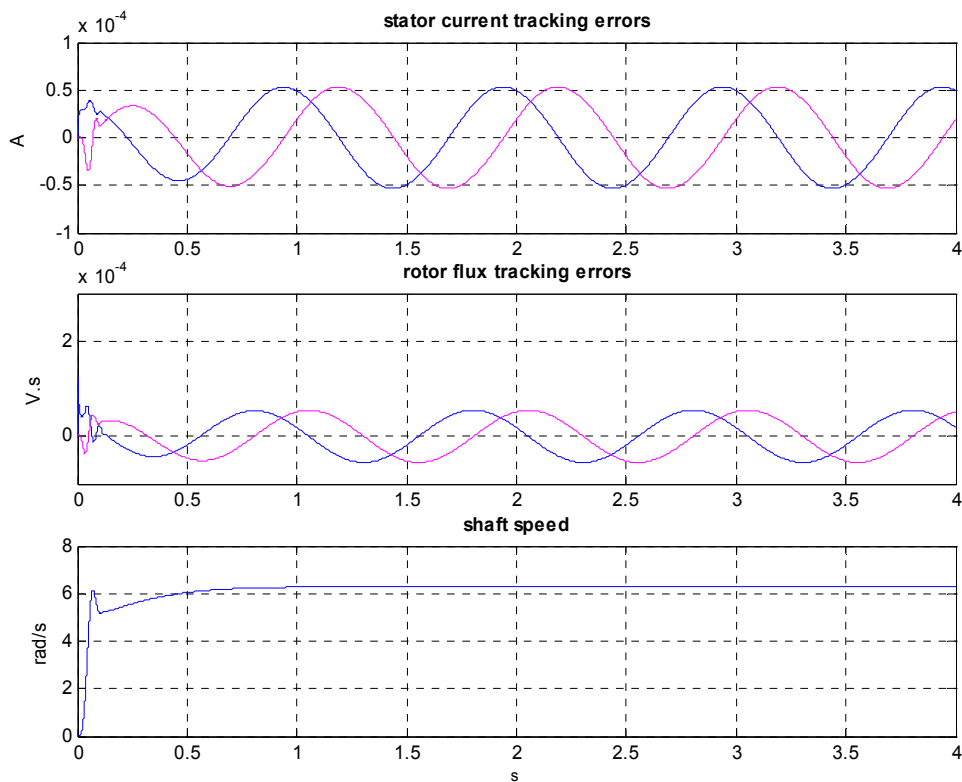


Figure 4.18: 1 Hz, 100 V stator voltage



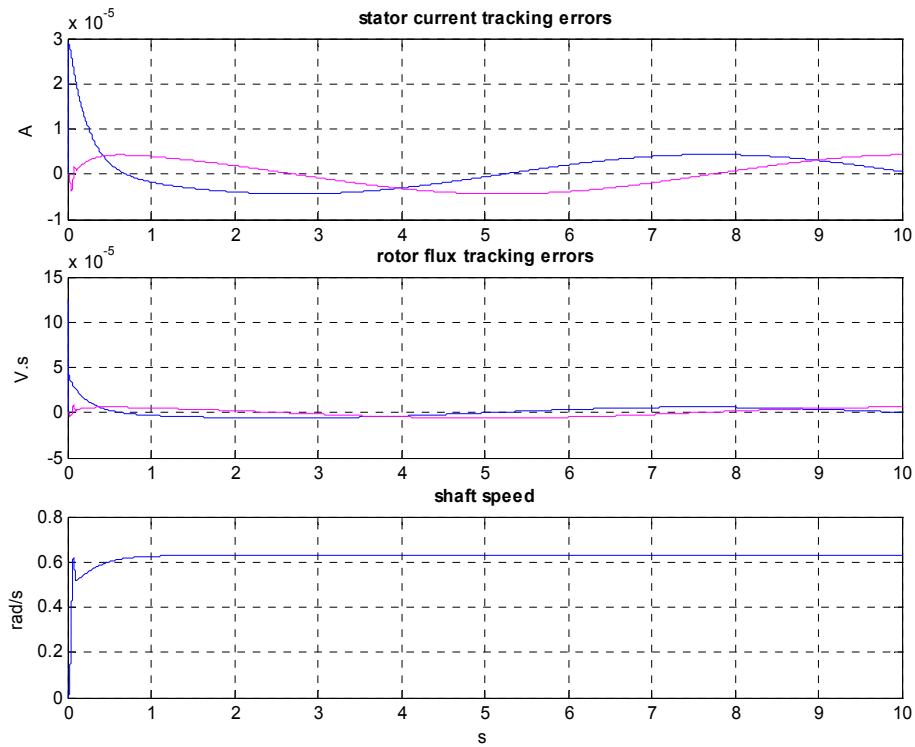


Figure 4.19: 0.1 Hz, 100 V stator voltage

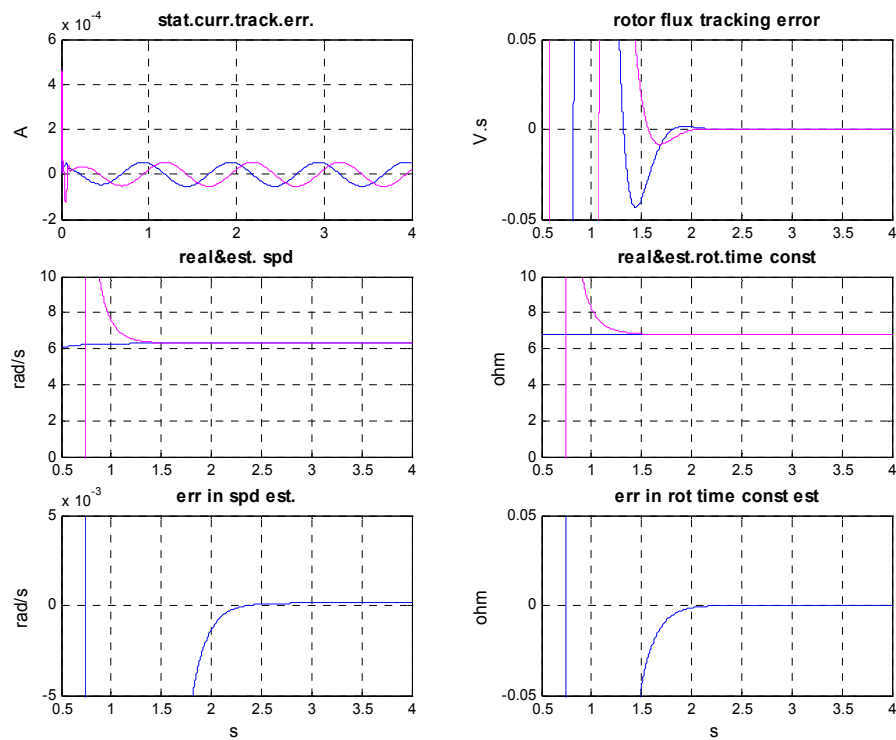


Figure 4.20: Sensorless observer results under no load, low speed conditions

## 5 CONCLUSION

In this thesis, a novel adaptive sliding-mode observer/controller algorithm has been developed for the estimation of the rotor flux, the angle speed and the rotor time constant and for the torque/speed and flux control of an IM without measurement of any mechanical variables, such as speed and torque. Stability analysis of the observer has been performed, which shows that, for any initial condition and from zero to nominal speed, the asymptotic stability of the observer can be achieved. The system is also implemented using a DSP and showed its performance through laboratory experiments. The algorithm has been also tested with simulations which show the convergence of the estimated flux, speed and rotor time constant even for low speed operation. The experiments show that the proposed sliding-mode controller/observer works well for a wide speed range, and the controller itself is very promising for a low-speed situation.

The rotor flux is estimated through the adaptive law and using the flux estimated and the sliding mode current observer, the rotor speed and time constant observer is constructed. The rotor time constant estimation is useful to overcome the rotor resistance variation, which is the most varying parameter with temperature. The overall scheme is very useful in sensorless operation in different environmental conditions, and it is cost and maintenance free.

Also, an experimental system with a speed sensor to estimate the flux is constructed to be used as a reference for the sensorless scheme. In the future the convergence rate of the estimated flux should be improved although it is below 1 s by revising the designed observer structure.

The idea developed in this thesis is a novel approach and it is expected to be helpful for the researchers to develop new algorithms who are working in the field of sensorless control of IM.

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