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Optimization of Broaching Design

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Abstract

Broaching is one of the most recognized machining processes that can yield high productivity and high quality when applied properly. One big disadvantage of broaching is that all process parameters, except cutting speed, are built into the broaching tools. Therefore, it is not possible to modify the cutting conditions during the process once the tool is manufactured. Optimal design of broaching tools has a significant impact to increase the productivity and to obtain high quality products. In this paper, an optimization model for broaching design is presented. The model results in a non-linear non-convex optimization problem. Analysis of the model structure indicates that the model can be decomposed into smaller problems. The model is applied on a turbine disc broaching problem which is considered as one of the most complex broaching operations.

Keywords
Broaching, Machining Optimization, Turbine disc broaching

1. Introduction and Scope

The broaching process is commonly used in the industry for the machining of variety of external and internal features such as keyways, noncircular holes, and fir-tree slots on turbine discs. Broaching can offer very high productivity and part quality when the conditions are selected properly. It has several advantages over other machining processes. For example roughing and finishing of a complex form on a part can be completed in one stroke of the machine which would require many passes with another process such as milling. However, achieving high quality and high productivity for the part needs a well-designed process. In broaching, all process parameters except the cutting speed are predefined during the design of the cutting tool. Therefore, it is not possible to modify cutting conditions after cutters are manufactured unlike other machining processes where depth-of-cut or feed-rate can be changed easily. This makes tool design the single most important aspect of broaching. In this paper, the constraints on the process are discussed and a broaching model is presented with applications to machining of fir-tree forms on turbine discs which is regarded as one of the most difficult broaching operations due to complex geometry, very tight tolerances and difficult-to-machine work material (see Figure 1).

Although widely used in industry, there is very limited literature on broaching. The book by Monday [1] presents the technology of broaching machines, processes and tools in a detailed manner. Collection of the works edited by Kokmeyer [2] has several different broaching applications in the industry demonstrating the effectiveness of the process. Ozturk and Budak [3] [4] used proper cutting models and Finite Element Analysis (FEA) to model the broaching process. In their study the cutting conditions are changed by enumeration until a constraint is met to improve the process. An extension of [3] and [4] can be found in [5] as well. The objective of the current study is to provide a mathematical programming solution for the broaching process to identify optimal designs.

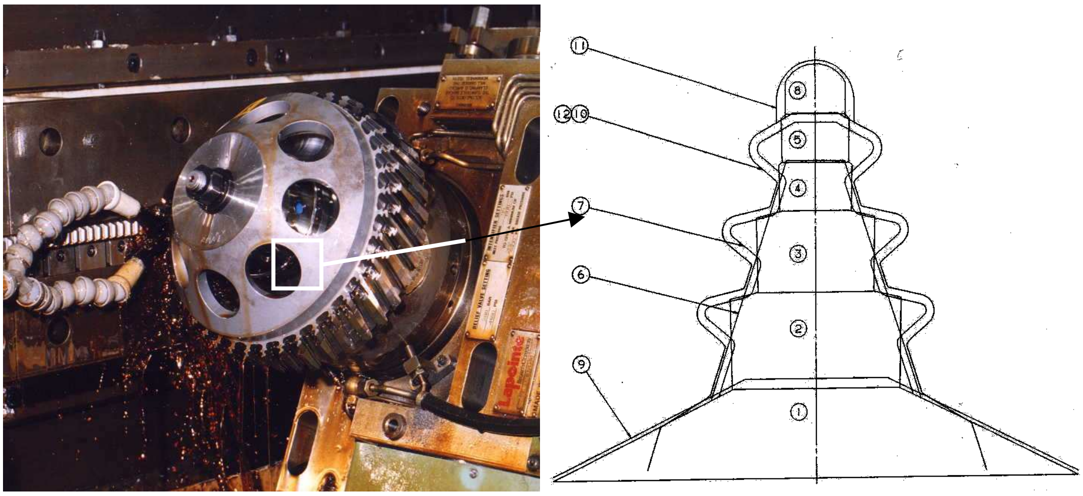

Figure 1: Broaching of fir-tree forms on a turbine disc and a broaching tool with 12 different tool segments.

2. Model

Before describing the model, we would like to define the notation below.

Let $i=1,\ldots,N_s$ denote the index for each section of the broaching tool to be designed with N_s number of sections, then the following can be identified as the main decision variables:

t_i: chip thickness (mm), $i=1,\ldots, N_s$
p_i: pitch of the section (mm), $i=1,\ldots, N_s$

Chip thickness is the most critical design decision. It has considerable effect on cutting forces directly related to power consumption and tooth stresses. The pitch of the tool has an impact on the total tool length and it also affects the stresses in teeth.

The following is a list of other parameters:

A_{1i}: angle of the tooth (degrees)
A_{2i}: rake angle (degrees)
AMP: available machine power (Watts - N m/sec)
b_i: chip width (mm)
B_i: width of the tooth (mm)
c_1, c_2, c_3, c_4 : pitch related constants
K_{tc}, K_{te}, K_{fc}, K_{fe} : cutting constants (units of MPa, N/mm, MPa, N/mm, respectively)
L_{ram}: ram length (mm)
m: is the number of teeth in-cut
n_i: number of teeth in a section
PS: permissible stress (MPa)
SF: safety factor
T_i: top length of the tooth (mm)

V: cutting speed (m/sec)
w: length of the section to be cut (mm)

Objective Function:
Since higher productivity and lower cost are the objectives, it is desired to increase the material removal rate (MRR) which is computed as volume removed per unit time.

There are several constraints that need to be taken into account including power requirements, ram length, tooth stress, chip space, chip thickness.

Power
The Available Machine Power (AMP) capacity of every machine is limited. As the cutting forces increases, the power consumption also increases. This constraint ensures that all tangential cutting forces created by teeth in-cut are less than the AMP.

Ram Length
This constraint indicates that total length of the broach sections cannot be higher than the machine's ram length.

Tooth Stress
Broaching forces can be quite high due to large width of cuts which may be required for a given profile. High forces may cause tooth breakage, thus the tooth stress (which is a function of the cutting force) must be less than the permissible stress.

Chip Space
Chip jam is a common problem in broaching. Broach chips can be very short depending on the thickness of the part. If there is not enough curvature in the chip they may get stuck on the surface. This constraint ensures this by setting the ratio of the cut chip volume and the gullet space within the allowable limit.

Chip Thickness
This constraint indicates that chip thickness has limitations in order to prevent rubbing or chipping.

The objective function and the constraints can be summarized as follows:

Objective	Maximize $MRR = \dfrac{w\sum_{i=1}^{N_S} t_i b_i n_i}{w + \sum_{i=1}^{N_S}(n_i - 1)p_i} V$	(1)
Subject to		
Power	$\left(\dfrac{w}{p_i} + 1\right)(K_{tc}t_i + K_{te}) \le \dfrac{AMP}{Vb_i}$	(2)
Ram Length	$\sum_{i:1}^{N_s}(n_i - 1)p_i \le L_{ram}$	(3)
Tooth Stress	$[(K_{tc}t_i + K_{te})^2 + (K_{fc}t_i + K_{fe})^2]^{0.5} b_i (c_{1i}^{\ 0.374} c_{2i}^{\ -0.082} c_{4i}^{\ 0.356} p_i^{\ 0.648} B_i^{\ -1.09} T_i^{\ 0.072} A_{1i}^{\ 0.088}) - PS / SF \le 0$	(4)
Chip Space	$t_i b_i - (0.331) p_i^{\ 1.8929} (1 - c_{4i})^{0.816} (c_{1i})^{1.14} (c_{2i})^{0.026} (c_{3i})^{-0.0891} A_{2i}^{\ 0.0388} \le 0$	(5)
Chip Thickness	$0.012 \le t_i \le 0.065$	(6)
Non-Negativity	$p_i \ge 0$	(7)

This is a non-linear optimization problem that has $2N_s$ decision variables; a non-linear objective function, $3N_s$ non-linear constraints, N_s +1 linear constraints, and N_s bound constraints on the decision variables. As it is illustrated in the numerical computations section, it can be verified that this problem has a non-convex solution space due to constraints (4) and (5). While in general non-convex problems can be difficult to solve, as it will be shown below, a close examination of this problem indicates that under some mild assumptions this problem can be decomposable into N_s simpler optimization problems in which we identify the optimal pitch and the chip thickness for each section separately.

Assumption A: Let C_i $i=1,\dots N_s$ denote the cut area for each section of the broaching tool. It is assumed that the cut area is known.

This assumption is reasonable since each broaching tool would be tailor-made for making a specific cut which the engineer or the designer is familiar with.

Lemma 1: If Assumption A holds, the broaching problem given in equations (1)-(7) is equivalent to the following modified problem with a minimization objective.

$$\text{Minimize } z = \sum_{i=1}^{N_S} \left(\frac{C_i}{t_i} - 1\right) p_i \qquad (1')$$

Subject to

Constraints (2)-(7)

Proof: The proof uses the fact that $C_i = n_i t_i$. Substituting $n_i = \frac{C_i}{t_i}$ into (1) and recognizing that b_i, w, V, and N_s are constant parameters yields $(1')$.

Lemma 2: The broaching optimization problem with objective function $(1')$ can be decomposed into N_s simpler 2 dimensional optimization problems.

Proof: The proof requires recognizing first that the objective function $(1')$ and the left hand side of constraint (3) are the same. Since the minimization problem will identify the minimal possible total length imposed by the sections, comparing the right hand side to the L_{ram} (Ram Length) as in (3) is merely for feasibility reasons. Thus constraint (3) may be removed from the formulation, and a feasibility check can be made after the optimization problem is solved. Since constraint (3) is the only constraint tying all sections together, removal of it makes the broaching optimization problem decomposable.

3. Numerical Computations

Figure 2 illustrates an example for selected parameters for a broaching tool with one section (i.e. N_s =1). As seen from this figure, non-convexity is caused by constraints (4) and (5). Due to non-convexity of the feasible region, application of non-linear optimization algorithms would only guarantee a locally optimal solution. Typical approach to deal with this type of optimization scenarios is to use a multi-start approach in which multiple runs of the optimization is carried, and then the best solution is selected among the identified local optimal solutions. In this study, we leveraged the two-dimensional nature of the decomposed broaching optimization problem and deployed a two step approach to identify the optimal solution as follows: In step 1, we graphed the problem to identify the feasible space and the optimal point. As it can be seen from Figure 2, the optimal solution is determined by constraints (2) and (4). In step 2, we solved the expressions for constraints (2) and (4) simultaneously to identify the coordinates of the optimal point. Note that due to non-linearity, identification of the intersection point of these two constraints is not trivial, but it can be done through numerical techniques. Here, we used the “solver” functionality of MS Excel to identify the solution as t^*=0.0496 mm and p^*=1.6826 mm with an objective function value of z*=58.85.

4. Summary and Conclusions

In this study, we provided a mathematical programming formulation for the broaching design problem. The problem yields a non-linear and non-convex optimization problem. Analysis of the problem structure based on mild assumptions indicate that the original problem (which aims to identify all the design parameters simultaneously) can be decomposed into smaller (thus simpler) optimization problems where parameters for each section are identified independently.

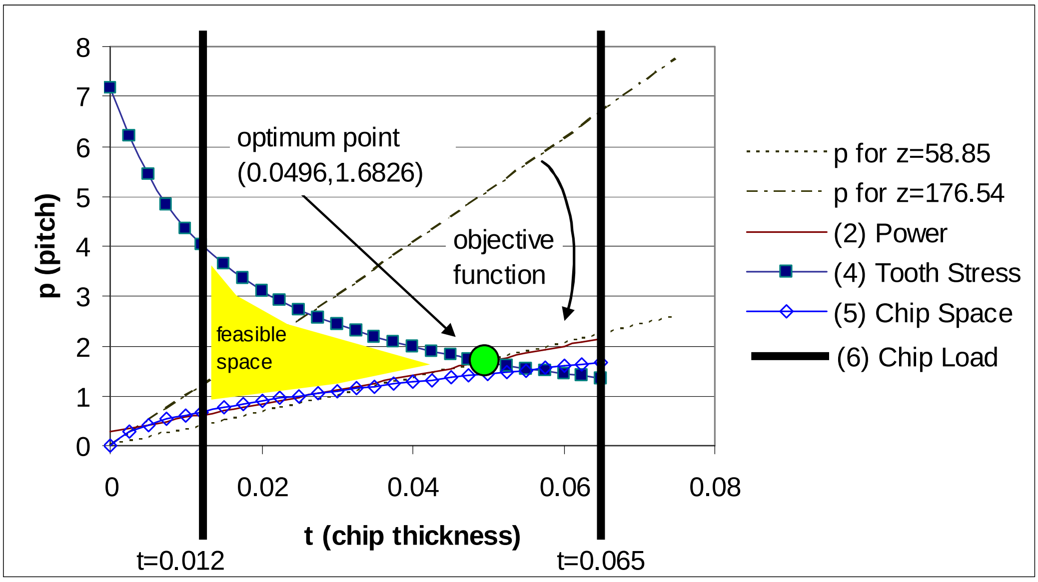

Figure 2. Optimal broaching design for one section.

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