SPARSITY-DRIVEN SPARSE-APERTURE ULTRASOUND IMAGING

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ABSTRACT

We propose an image formation algorithm for ultrasound imaging based on sparsity-driven regularization functionals. We consider data collected by synthetic transducer arrays, with the primary motivating application being nondestructive evaluation. Our framework involves the use of a physical optics-based forward model of the observation process; the formulation of an optimization problem for image formation; and the solution of that problem through efficient numerical algorithms. Our sparsity-driven, model-based approach achieves the preservation of physical features while suppressing spurious artifacts. It also provides robust reconstructions in the case of sparse observation apertures. We demonstrate the effectiveness of our imaging strategy on real ultrasound data.

1. INTRODUCTION

Nondestructive evaluation (NDE) of materials is a critical task in applications including defense, nuclear power, manufacturing, and infrastructure monitoring [1]. Through imaging, one could view the internal structure of homogeneous materials to determine the presence, severity, and characteristics of inhomogeneities, such as cracks. Ultrasound continues to be the imaging modality of choice in many NDE scenarios due to its safety, versatility, and low cost [1]. There are a number of data collection and imaging setups, and here we focus on a monostatic configuration, consisting of a non-focused transducer mechanically scanned to construct a synthetic aperture. At each mechanically scanned position, the transducer sends acoustic pulses and records the scattered waveforms. Given such data, the goal is to reconstruct a 3-D image of the material or that of a 2-D cross section. A conventional technique to reconstruct images is beamforming, which suffers from poor resolution and sidelobe artifacts. One could also consider data inversion through a pseudoinverse operation, which is very sensitive to noise in the data.

A current trend in many imaging applications is to develop and study imaging strategies for the case of *sparse*

apertures, in which the data lie in a small and potentially irregular portion of what would be considered a full aperture of spatial or spectral observation points. In some applications sparse apertures emerge as a result of physical or geometric constraints in the observation scenario (e.g. we cannot place the sensor at a particular location). In other applications, such apertures are of interest, because sensing is viewed as a dear resource, and the goal is to form accurate images with as little data as possible. When data are limited and lie on an irregular grid, conventional imaging strategies suffer severely from degraded resolution and imaging artifacts. For the practical utility of such sparse-aperture sensing scenarios, advanced image formation algorithms that produce enhanced imagery facilitating visual or automatic interpretation of the underlying scenes are needed.

We propose a new approach for ultrasound imaging to produce enhanced images especially in challenging scenarios involving sparse apertures. The primary application that has motivated us is nondestructive evaluation, although the approach could be adapted to other applications as well. Our framework is based on a regularized reconstruction of the underlying reflectivity field based on the scattered ultrasound data. We use nonquadratic regularization functionals which exploit the expected sparsity of the underlying fields. In our previous work, we have applied such sparsity-driven approaches to other wave-based imaging problems such as radar imaging [2]. Such functionals enable the preservation of strong physical features (such as strong scatterers or boundaries between regions with different reflectivity properties), and have been shown to lead to superresolution. We combine such functionals with a data fidelity term based on a physical optics-based linear model of the observation process to formulate an optimization problem for image formation. We solve the resulting optimization problem using efficient numerical algorithms.

There are a number of publications which have relations to our perspective for ultrasound imaging. A Bayesian approach for the nonlinear inverse scattering problem of tomographic imaging using ultrasound probing has been proposed in [3]. In [4], maximum entropy regularization has been used for image reconstruction from sparsely sampled coherent field data. The work in [5] proposes a regularized autoregressive model for spectral estimation, with application to medical ultrasonic radio-frequency images. A statistical deconvolution technique for diffuse ultrasound scattering has been proposed in [6], where sampling techniques are used for inference. Finally the approach in a recent thesis [7], performed indepen-

dently from our work, shares some of the key ideas in this paper.

There are a number of aspects of our work that differentiate it from existing literature. A detailed comparison is beyond the scope of this paper, but some key aspects of our work include: use of ℓ_p-norms for regularization which can seamlessly handle complex-valued data; use of sparsity constraints both on the complex-valued reflectivity field as well as on the gradient of its magnitude; development and use of efficient optimization algorithms matched to the problem structure. Given the previous work by us and others on the use of these types of algorithms in other applications, the contribution of this paper is the adaptation of these ideas to the ultrasound imaging modality through the incorporation of a physics-based forward model, as well as demonstration of the effectiveness of the approach on real, sparse-aperture ultrasound data. In particular, these experiments show how the proposed approach can provide improved resolution, reduced artifacts, and robustness to aperture sparsity as compared to conventional imaging methods.

2. OBSERVATION MODEL FOR ULTRASOUND SCATTERING

The observation model we use for ultrasound scattering is based on a linearization of the scalar wave equation. We use the following Green's function to model the scattered field in space in response to a point source of excitation:

$$G(|\mathbf{r}' - \mathbf{r}|) = \frac{\exp(jk(|\mathbf{r}' - \mathbf{r}|))}{4\pi|\mathbf{r}' - \mathbf{r}|} \qquad (1)$$

where $\mathbf{r}$ and $\mathbf{r}'$ denote the source location and the observation location, respectively, in three-dimensional space, and k is the wavenumber. In this paper we consider a monostatic data acquisition scenario. In specifying the response of a scatterer to an incident field emitted by a transducer, we assume the case of impenetrable scatterers. This is reasonable for a nondestructive evaluation application since inhomogeneities such as cracks act as strong reflectors of ultrasound energy. This leads us to use the physical optics approximation in linearizing the wave equation to obtain the following observation model:

$$y(\mathbf{r}') = 2jk \int G^2(|\mathbf{r}' - \mathbf{r}|)f(\mathbf{r})d\mathbf{r} \qquad (2)$$

where $y(\cdot)$ denotes the observed data and $f(\cdot)$ denotes the underlying, unknown reflectivity field. Note that squaring the Green's function captures the two-way travel from the transducer to the target and back. Also note that the observation model in (2) involves essentially a shift invariant point spread function. We discretize this model and take into account the presence of measurement noise to obtain the following discrete observation model:

$$\mathbf{y} = \mathbf{Tf} + \mathbf{n} \qquad (3)$$

where $\mathbf{y}$ and $\mathbf{n}$ denote the measured data and the noise, respectively, at all transducer positions; $\mathbf{f}$ denotes the sampled unknown reflectivity field; and $\mathbf{T}$ is a matrix representing the observation kernel in (2). In particular, each row of $\mathbf{T}$ is associated with measurements at a particular transducer position. The entire set of transducer positions determines the

nature of the aperture used in a particular experiment, and the matrix $\mathbf{T}$ carries information about the geometry and the sparsity of the aperture.

3. SPARSITY-DRIVEN IMAGING

Given the noisy observation model in (3), the imaging problem is to find an estimate of $\mathbf{f}$ based on the data $\mathbf{y}$. In general this is an ill-posed inverse problem, and its solution requires the incorporation of explicit or implicit prior information or constraints about the underlying field $\mathbf{f}$. One type of generic prior information that has recently been successfully applied in a number of imaging applications involves the *sparsity* of some aspect of the underlying field. In the context of ultrasound imaging for nondestructive evaluation, such sparsity priors could also be a valuable asset, as we expect the underlying homogeneous material to be fairly sparse in terms of both the location of inhomogeneities (e.g. cracks), as well as the boundaries between such inhomogeneities and the homogeneous background.

It has been observed that imposing sparsity directly leads to combinatorial optimization problems, but both empirical and recent theoretical results suggest that this could in practice be achieved by relaxed and tractable nonquadratic optimization formulations, based on e.g. ℓ_p-norms (see, e.g. [8]). This is the strategy we adopt in this paper. In particular we propose to find the reconstructed image $\hat{\mathbf{f}}$ as the minimizer of the following cost functional:

$$J(\mathbf{f}) = \|\mathbf{y} - \mathbf{Tf}\|_2^2 + \lambda_1 \|\mathbf{f}\|_p^p + \lambda_2 \|\nabla|\mathbf{f}|\|_p^p \qquad (4)$$

where $\| \cdot \|_p$ denotes the ℓ_p-norm ($0 < p \leq 1$), ∇ denotes a discrete approximation to the spatial gradient operator, $|\mathbf{f}|$ denotes the vector of magnitudes of the complex-valued vector $\mathbf{f}$, and λ_1, λ_2 are scalar parameters. The first term of $J(\mathbf{f})$ in (4) is a data fidelity term, while the other terms are regularizing sparsity constraints. In particular, the second term has the role of preserving strong scatterers such as cracks while suppressing artifacts (these types of constraints lead to superresolution). The third term has the role of smoothing homogeneous regions while preserving sharp transitions, such as those between cracks and the background. The relative magnitudes of the scalar parameters λ_1 and λ_2 determine the emphasis on each term. In our experimental work, we use the second term as the dominant one, and use values of p around 1. We solve the optimization problem in (4) by adapting efficient iterative algorithms we have developed in our previous work [2] to the ultrasound imaging application.

4. EXPERIMENTS

We present the results of imaging experiments based on data collected at the Large Ultrasound Test Facility (LUTF) [9] at Boston University.

4.1. Data Collection

In our experiments we use a tank full of water as the homogeneous material in which waves propagate. We insert an aluminum object inside this homogeneous medium as the inhomogeneity. The objective of the imaging experiments is
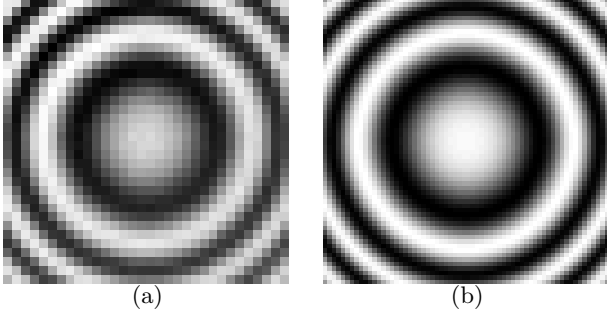
Fig. 1. The point spread function (PSF) of the data collection system at 300 kHz. (a) Measured using a 1 mm diameter spherical scatterer. (b) Theoretical model. (Real parts shown.)

Fig. 2. (a) Location and shape of the cross-section of the aluminum object to be imaged. (b) Real part of the data scattered by the object at 300 kHz.

to reconstruct a 2-D cross section of this object. We use a monostatic arrangement in which a broadband single-element unfocused transducer is mechanically moved on a 64×64 grid of locations (covering a square with a side of 76.8 mm.) at the top of the tank to send and receive acoustic waveforms. We place the object to be imaged at a depth of 175 mm, and time-gate the reflected signals to isolate the response from that depth. The transducer emits a broadband signal, whose two most significant peaks are at 730 kHz and 300 kHz (with the corresponding wavelengths of 2 mm and 5 mm). We transform the time-gated received signal to the frequency domain and extract the response at these two frequencies. Although our framework is suitable for processing multi-channel data, in this paper we focus on processing single-channel data at each of these two frequencies.

In order to experimentally estimate the impulse response of the system to test the validity of the theoretical model described in Section 2, we have first collected data from a spherical aluminum scatterer of 1 mm diameter. Real part of the data measured at 300 kHz through the full 64×64 aperture is displayed in Fig. 1(a). Real part of the point spread function based on the theoretical model in (2) is shown in Fig. 1(b), which is in very well agreement with Fig. 1(a). We use this theoretical model to construct the operator $\mathbf{T}$ in our experiments.

In Fig. 2(a) we illustrate the shape and the location of the U-shaped cross section of the aluminum object to be imaged (our use of this shape is inspired by the experiments in [4]). The length of each side is 12 mm, and the thickness is 2.4 mm. Real part of the measured full-aperture data at 300 kHz in the presence of this object in the tank is shown in Fig. 2(b).

4.2. Results

We first present the results of full-aperture imaging experiments based on the type of scattered data shown in Fig. 2(b). Fig. 3(a) and (b) show the results of two conventional imaging strategies, beamforming and regularized pseudoinverse [10], respectively, at 730 kHz and 300 kHz. At 730 kHz beamforming produces a good reconstruction, however when we reduce the operating frequency to 300 kHz significant resolution loss occurs. Low-frequency operation is of interest because acoustic waves suffer from more attenuation as the frequency is increased. The regularized pseudoinverse approach aims to
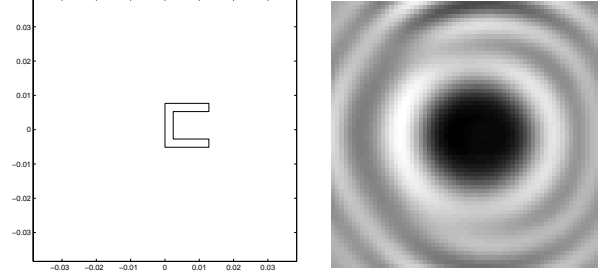
improve upon the pseudoinverse operation (which is unstable in the presence of measurement noise, and consequently not considered here) by solving the problem iteratively and providing regularization through early stopping. Although this gets rid of severe artifacts, the resulting images shown in Fig. 3(b) still do not exhibit the shape of the inhomogeneity very accurately. The reconstructions obtained using our proposed approach are shown in Fig. 3(c) and provide much more accurate images of the U-shaped object, even at the low operating frequency of 300 kHz. In our experiments, we use $p = 1$, and $\lambda_1 \gg \lambda_2$.[1] This relative choice of λ_1 and λ_2 indicates our emphasis on preserving and sharpening the strong scattering from inhomogeneities in the scene while suppressing background artifacts.

Next we consider a sparse aperture, in particular the star-shaped synthetic aperture shown in Fig 4. (Note that the full aperture used in the previous experiments was based on measurements on the 64×64 square region in Fig 4.) The number of data collection points in this sparse aperture is only 6% of the full aperture considered in the previous experiments. The imaging results are shown in Fig. 5. The conventional images shown in Fig. 4(a) and (b) suffer from insufficient resolvability of fine features and sidelobe artifacts caused by the sparsity of the aperture, making it difficult to infer the shape of the inhomogeneity. Our approach is able to suppress such artifacts and recover the shape as shown in Fig 4(c). These results illustrate the robustness of our strategy to data limitations due to the sparsity of the aperture. The experiments we have conducted are based on data carefully collected in a controlled environment, and hence represent a high-SNR scenario. We also expect our imaging strategy to provide improved robustness in low-SNR data collection scenarios.

5. CONCLUSION

We have proposed and demonstrated a sparsity-driven image formation approach for ultrasound imaging with application to nondestructive evaluation. Attractive characteristics of the proposed technique include improved resolvability of fine features, suppression of artifacts, and robustness to the sparsity of the observation aperture. Based on the initial work presented in this paper, a number of directions emerge as potential research topics. First, although the study in this

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[1]We do not specify the actual values as they depend on the scaling of the data in a particular experiment, and hence are not very informative.
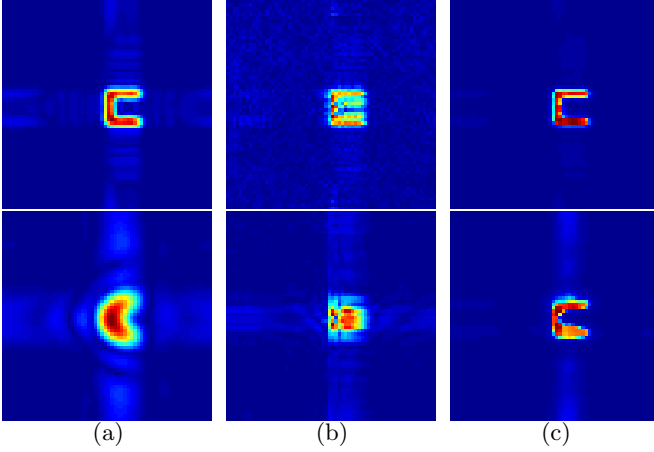
Fig. 3. Reconstructed images from full-aperture data. (Magnitudes shown.) Top: 730 kHz. Bottom: 300 kHz. (a) Beamforming. (b) Regularized pseudoinverse. (c) Proposed method.
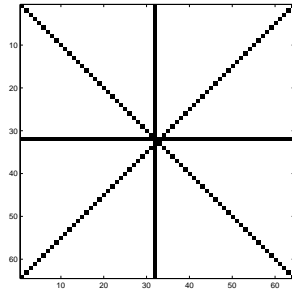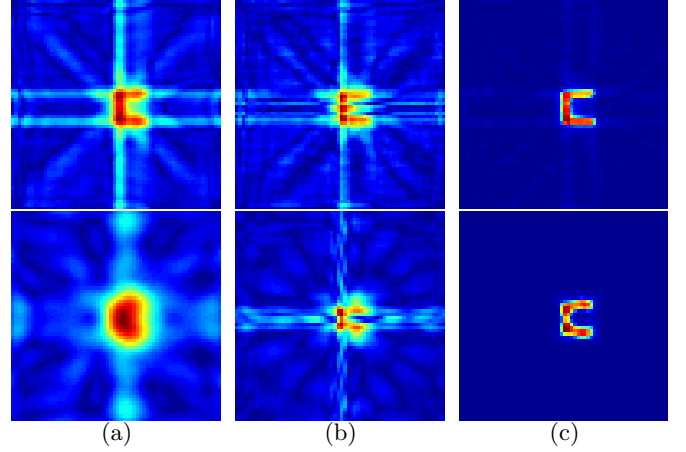

Fig. 5. Reconstructed images from sparse-aperture data. (Magnitudes shown.) Top: 730 kHz. Bottom: 300 kHz. (a) Beamforming. (b) Regularized pseudoinverse. (c) Proposed method.

Fig. 4. Transducer positions used to construct a sparse aperture. Relative to the full-aperture used, this aperture is 6% filled.

paper was limited to monostatic, single-channel data, extension of the developed framework to the multistatic case, as well as to the processing of multi-channel data is straightforward. Our work could also be used for forming 3-D images. For the experimental setup considered in this paper, a linear observation model based on a single-scattering assumption was reasonable, however it might be of interest to generalize the framework to the case of multiple scattering and nonlinear models. It is also worthwhile to characterize the behavior of the proposed approach as the problem becomes more challenging (e.g. through the scene content, frequency of operation, sparsity of the aperture, etc.), and understand how the performance of the proposed approach degrades. Finally, although nondestructive evaluation was the motivating application here, it is of interest to adapt and apply this technique on other ultrasound applications, the most notable one being medical imaging.

6. REFERENCES

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