Optimization of 5-Axis Milling Processes Using Process Models

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Abstract

Productivity and part quality are extremely important for all machining operations, but particularly for 5-axis milling where the machine tool cost is relatively higher, and most parts have complex geometries and high quality requirements with tight tolerances. 5-axis milling, presents additional challenges in modeling due to more complex tool and workpiece interface geometry, and process mechanics. In this paper, modeling and optimization of 5-axis processes with cutting strategy selection are presented. The developed process models are used for cutting force predictions using a part-tool interface identification method which is also presented. Based on the model predictions and simulations, best cutting conditions are identified. Also, for finish process of a complex surface, machining time is estimated using three machining strategy alternatives. Results are demonstrated by example applications, and verified by experiments.

1 INTRODUCTION

5-axis milling is widely used especially in aerospace, die and mold industries where most parts have complex surfaces. In most of these applications, the part quality is extremely important with tight tolerances. In addition, high productivity is sought due to relatively high cost of these processes. In order to achieve high productivity and required part quality, process models are very good tools where better or optimal parameters can be determined through simulations

Most of the 5-axis milling applications use ball end mills owing to increased contouring capability with better surface finish. 5-axis ball-end process models are required for simulation and optimization. Although there have been several studies on the modelling of ball end milling processes, [1], [2], these have been mostly limited to 3-axis operations. In one of the important works on ball end milling, Lee et al. [1] modelled 3-axis ball end milling using oblique cutting model where cutting parameters are transformed from an orthogonal cutting database. 5-axis milling forces were modelled in only a very few studies mainly using the mechanistic approach i.e. using calibration tests [3]-[5].

In this study, cutting forces in 5-axis ball end milling are modelled by using an oblique cutting model. The force coefficients are identified from orthogonal cutting tests [6]. Once the orthogonal database is obtained, it can be applied to any cutter geometry, unlike mechanistic approach where the force coefficients have to be calibrated for each material and cutting tool pair.

Process models must be integrated with geometrical models in order to simulate and optimize 5-axis processes where complex surfaces are machined. Therefore, surface modeling and tool path generation are other challenges in 5-axis milling applications. Analyses of 5-axis process geometry and tool path generation require powerful geometrical models. Both analytical and discrete methods are used for such purposes. One of the noted studies is the one presented by Choi et al. [7]. Besides curve and surface geometry methods such as Bezier and NURBS splines and surfaces, discrete methods such as Z-Mapping and Octree are also used in modeling and simulation of 3-axis [9]-[11] and 5-axis [3], [12] machining process geometry. Although discrete models are preferred for very complex cases, due to the recent developments in CAD techniques, analytical models can also be applied in many cases. One important requirement for application of process models is the identification of the cutter and workpiece interface along the tool path. Fussel et. al [3] identified tool-workpiece engagements using an extended Z-buffer method. They used swept envelope of the cutter, to determine the intersection between the cutter envelop and Z-buffer elements. Lee et. al [9] estimated depth of cuts by positioning the tool axis coincident with the surface normal at that point. Imania et.

al [10] modeled cutter-workpiece engagement boundaries using a geometric simulation system which uses a commercial solid modeler ACIS$^{©}$ [13] as geometric engine.

In this paper, a practical but powerful method for integration of process models with machining geometry is proposed. In the proposed method, CL file is used as the main information source. Tool position and orientation is used with the analytical information of the workpiece to perform geometrical analysis.

In production, process time and part quality vary conversely, and optimization methods should be used to compromise between them. By optimization methods, cutter orientation, cutting strategy, feed rate and various cutting parameters can be optimized. Lim et al. [11] proposed a model which identifies appropriate cutting strategy combined with feedrate scheduling for 3-axis milling. Ramos et al. [14] investigated the effects of machining strategies on complex surface machining, in a totally experimental manner. Baptista et al. [15] analyzed the effects of machining parameters on surface roughness in 3 and 5-axis machining of complex surfaces experimentally.

In this paper, a methodology to optimize cutter orientations is proposed which is based on process simulations. With respect to simulation results, the desired parameters are optimized using iterative algorithms. Besides, various machining strategies for a sculptured surface are compared from process time aspect where the surface is modeled as Bezier surface.

The paper consists of five sections. In the following section, kinematics and force model are briefly presented for 5-axis processes. In section 3, the process model is integrated with the machining geometry by performing geometrical analysis of the 5-axis processes and simulation techniques used for optimization are explained where optimization methods are given in section 4 section. Finally, experimental results are compared with simulations and conclusions derived in section 5.

2 PROCESS MODEL

An overview of the important steps in the process modeling and optimization is given in Figure 1. Process information parser module, reads tool orientation, tool position from CL file. Workpiece geometry and process information is combined and provided to geometrical analysis module, where the geometrical analysis is performed and cutting conditions are extracted. Since the process model is based on orthogonal to oblique transformation, orthogonal database is also provided to the process mechanics model with cutting conditions. Finally, according to simulation results cutting parameters are optimized with respect to cutting forces using iterative algorithms by optimization module.

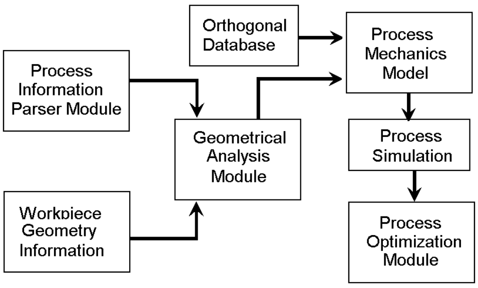


Figure 1: Process optimization progress.

2.1 5-axis milling geometry

In 5-axis milling, in addition to the 3-axis translation, there are 2 rotations of the tool, namely lead and tilt angles, which are defined in section 3. In the analysis of 5-axis machining processes, mainly 3 coordinate systems are used: work coordinate system (WCS), the tool coordinate system (TCS) and the process coordinate system (FCN). Those coordinate systems are shown in Figure 2. WCS consists of (**X**), (**Y**) and (**Z**), where TCS consists of (**x**), (**y**) and tool axis (**z**), finally, FCN consists of feed (**F**), cross-feed (**C**) and surface normal (**N**) vectors as shown in Figure 2. WCS is also the coordinate system of the table type dynamometer used in the experiments. In 5-axis machining of sculptured surfaces, FCN and TCS may continuously vary during process, whereas WCS remains constant. Therefore, the relation between those coordinate systems should be constructed properly. By definition, (**c**) is on a plane perpendicular to (**f**). Those construct tangent plane to the surface at a point. Thus FCN is an orthogonal basis and TCS is rotated form of FCN. Therefore, the transformation procedures given in [7] are valid for FCN and TCS.

Transformation of forces from TCS to FCN is performed as follows:

$$\boldsymbol{T} = \begin{bmatrix} C_l & 0 & S_l \\ S_t S_l & C_t & -S_t C_l \\ -C_t S_l & S_t & C_t C_l \end{bmatrix} \qquad (1)$$

where;

$C_\theta = \cos(\theta), S_\theta = \sin(\theta)$ *l*=*lead* and *t*= *tilt*

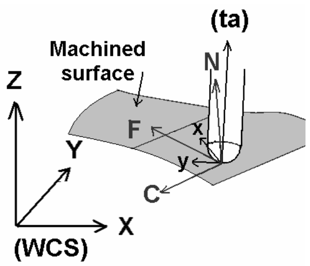


Figure 2: Coordinate Systems.

2.2 5-Axis force model

One of the tools which are required for process simulation and optimization is the 5-axis force model [16] which is introduced in this section. In force model [16], orthogonal database approach developed by Budak, et al.[6] is used. Differential forces, (see Figure 3), are calculated in radial (r), tangential (t) and axial (a) directions as follows:

$$\begin{aligned} dF_{ij}(\varphi_j,K) &= K_{ie}dS + K_{ic} * t(\varphi_j, K) * db \\ &= K_{ie}dS + K_{ic} * t(K)\sin\varphi_j * RdK \end{aligned} \quad (2)$$

Where, dF_{ij} is differential force in direction i at immersion angle j. In (2), φ_j, K_{ie}, K_{ic}, t and dS are immersion angle, edge and cutting force coefficients, chip thickness and infinitesimal cutting edge length respectively. Differential forces are integrated over the engagement domain [17] and cutting forces in TCS are calculated then, transformed to FCN, transformed to WCS [16]. K and immersion angle φ_j is shown in Figure 3.

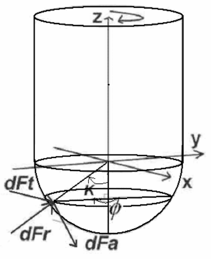


Figure 3: Differential forces on the tool.

3 PROCESS SIMULATION

In order to improve machining processes, it is often required to simulate processes so that optimal conditions can be identified. The force model needed to simulate 5-axis ball end milling is summarized in section 2. The other tool which is strictly needed is geometric model. In this section, simulation techniques and methods for determination of geometrical parameters are presented. Two simulation approaches are considered to serve different purposes. In the first approach, various cutting conditions are simulated individually to have an insight into the process mechanics under different conditions. By doing so, some guidelines, e.g. selection of best combination of lead and tilt angles, may be provided to the process planner beforehand. The second approach considers a whole process where machined surface topology and cutting conditions vary continuously. In this approach, mechanics of the process is simulated along the tool path to predict the variation of forces throughout the process in order to identify how the process can be improved and optimized. In order to perform the simulations explained above, geometrical parameters must be determined. For this purpose, the descriptions of those are given, and then the calculation methods are explained briefly.

3.1 Description of Geometrical Parameters

Having information on geometrical parameters is crucial in machining as it provides an insight into process geometry. From the process geometry aspect, as the tool immerses into the workpiece, the flutes on the cutter start to engage with the part. As explained in the force model, cutting forces are calculated using the engagement boundaries between each flute and workpiece. These boundaries are identified using depth of cuts, lead and tilt angles, and the tool geometry. Depth of cuts are the axial (*a*) and radial (*r*) immersions of the tool into the workpiece, which are shown in Figure 4.

Tool rotations are about (**C**) and (**F**) axis and defined with respect to surface normal. Rotation about (**C**) is named as lead angle, and about (**F)** is named as tilt angle, which are illustrated in Figure 5.

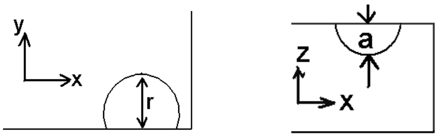


Figure 4: Depth of cuts, (*r*) and (*a*).

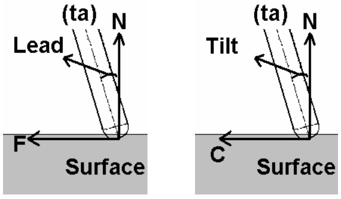


Figure 5: Lead and tilt angles.

Once, the parameters are described, the calculation methods are explained briefly in the following section.

3.2 Geometric Parameter Calculations

Calculation procedure of geometrical parameters is illustrated in Figure 6. The information on the lead and tilt angles and depth of cuts are not given in CL file explicitly. Thus, it can not be used directly in simulations, but it can be used together with the geometrical information of the workpiece to extract the required parameters.

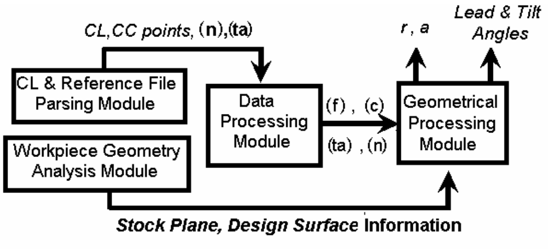


Figure 6: Geometrical calculation procedure

Geometrical parameters are determined in FCN coordinates. Therefore, in order to use the data given in the CL file, first FCN should be established. In this manner, determination of feed (**f**), crossfeed (**c**) and surface normal (**n**) vectors accurately is vital.

In ball end milling processes, the coordinates of the tool tip, i.e. the CL point, is available in the CL file. Since tool has continuously changing spatial motion due to lead and tilt angles in 5-axis, CC differs from CL as shown in Figure 7. Therefore it is required to calculate (**f**) between consecutive CC points. Besides, calculation of CC point requires (**n**). This creates a recursive relation between (**n**) and (**f**). Such a relation could be solved using iterative algorithms. Instead of using those, this is tackled in the following manner. First, a reference file is generated where lead and tilt angles are chosen to be zero and all other parameters being same with the original file. So, in the reference file, CL and CC points are same, and toolaxis (**ta**) is coincident with (**n**). By using this approach, CC and (**n**) can be obtained without using any iterative methods. (**f**) and (**c**) vectors are calculated as follows:

$$(\mathbf{f}) = \frac{\left[(x_{n+1}-x_n);(y_{n+1}-y_n);(z_{n+1}-z_n)\right]}{\mathrm{norm}\left|\,P_n\,P_{n+1}\,\right|} \qquad (3)$$

$$(\mathbf{c}) = (\boldsymbol{n})\,\mathrm{x}\,(\boldsymbol{f}) \qquad (4)$$

where, $P_n=(x_n, y_n, z_n)$, is the *n*th CC point and $P_{n+1}=(x_{n+1}, y_{n+1}, z_{n+1})$ is the *(n+1)*st CC point.

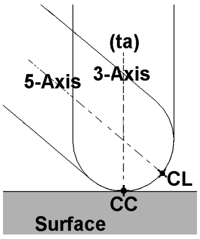


Figure 7: Difference between CC and CL points.

In calculation of the axial depth, it must be kept in mind that the stock material in general differs in roughing and finishing operations, thus different cases should be taken into account. In Figure 8.a, the stock material on a bumped surface is shown for different process steps. After establishing FCN, depths of cuts are determined as follows. In this study, axial depth of cut is calculated as the distance between P_4 and P_6, given in Figure 8.b. The intersection point, i.e. P_6 is determined by line-plane intersection [18]. Finally, (a) is calculated as follows:

$$a = \mathrm{norm}\left|\,P_4\,P_6\,\right| \qquad (5)$$

Whereas, in semi finish and finish operations (*a*) is equal to the left stock material.

In calculation of radial depth of cut, different conditions are considered as well. For example, in first cut (r) is different from the next steps as shown in Figure 10, For first cut (*r*) is determined as the distance RP_1 to RP_2, (see Figure 9). (r) is calculated as follows:

$$r = \mathrm{norm}\left|RP_1\,RP_2\right| \qquad (6)$$

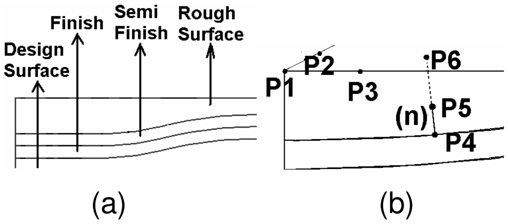
Where RP_1 is calculated as below and RP_2 is calculated in a manner as in (5).

$$RP_1 = P + a.(\boldsymbol{n}) + Ra.(\boldsymbol{c}) \qquad (7)$$

Radial depth of cut in the next steps is calculated as the side step, which is shown in Figure 10. Side step is defined as the distance between each cut steps and calculated as follows.

$$r = \left|(CC_{(k+1),n} - CC_{k,n}).(\mathbf{c})\right| \qquad (8)$$

Where, $CC_{k,n}$ is the nth corresponding point at the kth step.



(a) (b)

Figure 8: Calculation of (*a*)

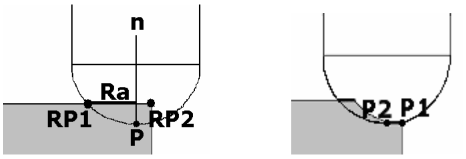
The major difference between 3-axis ball end milling and 5-axis ball end milling is the existence of lead and tilt angles. Lead and tilt angle calculation methods have not appeared in the literature. Thus, the method presented can be considered as the first attempt. Problem of lead and tilt angle calculation resembles an inverse kinematics problem. Although, toolaxis (**ta**), (**F**) and (**C**) are known, (**ta**) is given in WCS coordinates where, (**F**) and (**C**) are given in FCN coordinates. Therefore, (**ta**) should be transformed into FCN coordinates as follows:

$$\begin{bmatrix} ta_f \\ ta_c \\ ta_n \end{bmatrix}_{(FCN)} = \begin{bmatrix} f_X & f_Y & f_Z \\ c_X & c_Y & c_Z \\ n_X & n_Y & n_Z \end{bmatrix} \begin{bmatrix} ta_X \\ ta_Y \\ ta_Z \end{bmatrix}_{(WCS)} \qquad (9)$$

Finally, lead and tilt angles can be calculated as below:

$$\begin{aligned} &\text{lead} = \text{arctan2}(a, \sqrt{b^2 + c^2}) \\ &\text{tilt} = \text{arctan2}(-b, c) \end{aligned} \qquad (10)$$

where, $a = \text{dot}[(\boldsymbol{ta}),(\boldsymbol{f})]$, $b = \text{dot}[(\boldsymbol{ta}),(\boldsymbol{c})]$, $c = \text{dot}[(\boldsymbol{ta}),(\boldsymbol{n})]$. Here, arctan2(y,x) function calculates the corresponding angle value by considering the sign of x and y. So, the angle at the right quadrant is calculated.



(a) (b)

Figure 9: Calculation of (r).

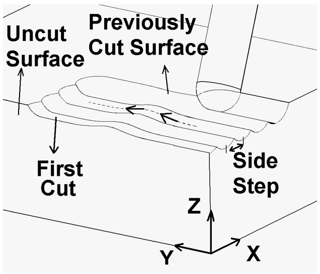


Figure 10: Illustration of cut steps.

3.3 Verification of calculation methods

Method of lead and tilt angle calculation is verified by two tests which are performed at 5 points selected on the tool path generated for the surface shown in Figure 11. In the first test, varying and in the second test constant lead and tilt angles are applied on tool axis. Given and calculated angles are given in Table 1. In the first test tool has an abruptly changing spatial motion. The comparison is shown in Figure 12. Except when there are abrupt changes in tool axis, the calculated values are very accurate. In most of the 5-axis sculptured surface machining processes, tool axis does not change so abruptly, thus the presented method is valid for majority of the applications.

CAM package uses interpolation techniques to calculate the tool axis when varying angles are applied on the tool. The bias between the given and calculated lead, tilt angles in the first case may be due to these interpolation techniques. In order to overcome this problem, plugging similar interpolation techniques in lead and tilt calculation method or establishing an iterative algorithm, which takes the biased values as initial guess may be beneficial.

Pt	Lead/Tilt(deg)			
	Test1		Test2	
	Given	Calculated	Given	Calculated
1	-27/-27	-24 / -24.4	15/-20	14.5 / -20
2	-10/-15	-8 /-12.3	15/-20	14.5 / -20
3	0/-5	-0.7 /-4.5	15/-20	13.9 / -20
4	5/10	3.1 / 7.3	15/-20	14.5 / -20
5	10/13	8.8 / 11.4	15/-20	14.5 / -20

Table 1: Given and calculated lead and tilt values

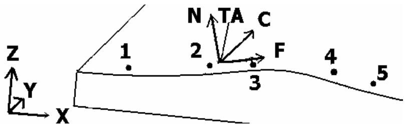


Figure 11: Example surface.

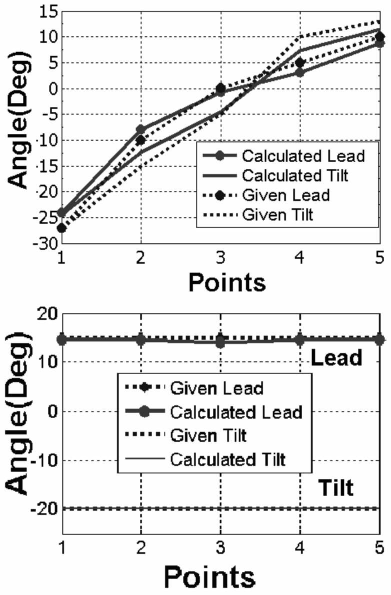


Figure 12: Comparison of given and calculated values

4 PROCESS OPTIMIZATION

In machining, selection of cutting parameters and milling strategy are the two main issues which may benefit from optimization. After tools required for process simulation and optimization are given, optimization techniques developed for process optimization is explained in this section.

From process mechanics point of view, undesired results such as tool breakage or excessive form errors may occur due to high cutting forces if the process parameters are not selected properly. In most of the cases, even when other cutting conditions are known, selection of lead and tilt angles is an important issue in 5-axis milling. It is not straight forward to choose optimal lead and tilt angles due to many possible combinations and their non-linear effects on the process. Therefore, foresight about the effect of those on the process mechanics is required in order to select appropriate values.

In this study, lead and tilt angles are optimized in the following manner. First, an optimization surface given in Figure 13 is generated by simulating a slotting process under conditions given in Table 2. Then, the lead and tilt pair keeping the *xy* resultant force on the tool, i.e. F_{xy} at lowest level is chosen. Point 1 on Figure 13 is chosen for illustration. Finally, the chosen pair is verified by experiments.

Machining strategy is the trajectory of tool on machined surface. Variation of cutting forces is directly related to machining strategy since the cut direction and order of cut steps are defined by it. Besides, process time is related to cutting strategy, which is the main objective to be minimized.

Axial (mm)	1.5
Radial	Slot
Lead (deg)	[-10,-5,..,30]
Tilt (deg)	[-30,5,...,30]
Spindle speed (rpm)	3000 rpm
Feed per tooth (mm/rev)	0.1

Table 2: Simulation Parameters.

In this study, zig, zig-zag and follow periphery strategies are compared with respect to process time. Process time is calculated in the way given in Figure 16. To calculate process time independent of any CAD/CAM package first the surface given in Figure 14 is mathematically modeled (see Figure 15) using Bezier surface method. Then, tool path length and process time is calculated applying the cutting strategy on the surface together with the cutting conditions such as feed rate.

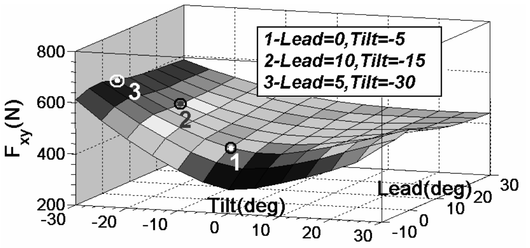


Figure 13: Optimization surface.

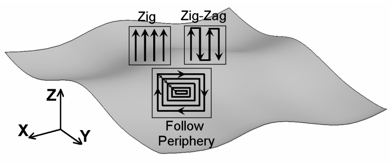


Figure 14: Surface and strategies used in time calculation.

After presenting process optimization methods proposed in this paper, the next section gives the experimental results and comparison of those with simulations.

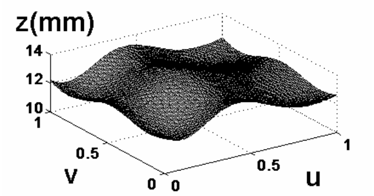


Figure 15: Mathematical model of the surface.

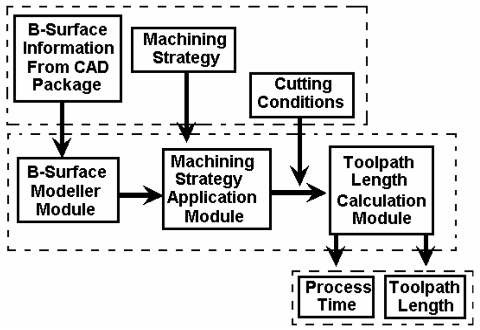


Figure 16: Machining time calculation.

5 EXPERIMENTAL VERIFICATION & SIMULATIONS RESULTS

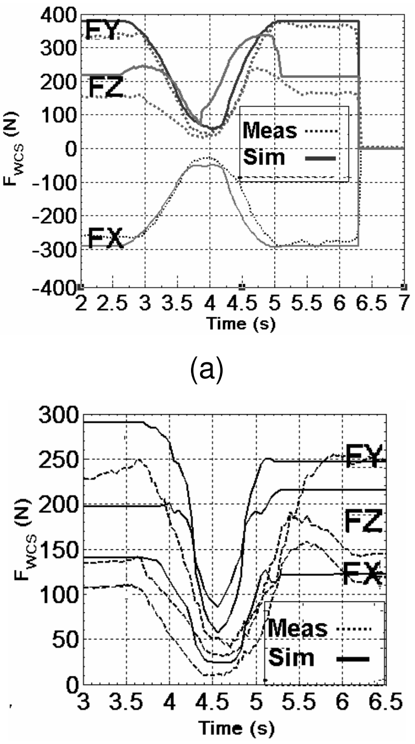
In this section, experiments are explained. Then, experimental results and simulation results are compared. First experiment is machining of a bumped surface and the second one is slotting on a flat surface. Where, detailed information on experimental setup is given in [16]. Finally the results of machining time simulation are given. In the experiments, the orthogonal database for Ti_6Al_4V given in [17] is used. The cutting tool used in the simulations and experiments is a two-flute, 12 mm diameter ball end mill with a helix angle of 30°.

5.1 Experiment 1

This experiment is performed to verify simulation of roughing process generated for a bumped surface. For this case, 1st cut step is slotting where the maximum forces are prone to be observed and the other steps are identical with the 2nd cut first cut step. Therefore, the comparison of measurements and simulations for 1st and 2nd steps are presented. Process parameters are given in Table 3.

Max axial depth (mm)	1.5
Min axial depth (mm)	0.2
Tool axis	Constant, Z
Spindle speed (rpm)	3000
Feed per tooth (mm/rev)	0.1

Table 3: Experiment Parameters.



(a)

(b)

Figure 17: Comparison of measured and simulated forces, FX, FY, and FZ in WCS for first (a) and second (b) steps respectively.

As seen in Figure 17, FX and FY are predicted accurately where some discrepancy is seen in FZ. Since the machine tool used in the experiments does not approach the given feed rates due to default feed rate reduction at curved surfaces, some amount of time lag and negative bias in magnitudes of the cutting forces between the simulations and experiments are also observed. As a result, it can be concluded that, cutting forces are predicted within a reasonable tolerance.

5.2 Experiment 2

This experiment is carried out to verify the lead and tilt optimization method and carried out under conditions given in Table 2.

Point No		1	2	3
Lead (deg)		0	10	5
Tilt (deg)		-5	-15	-30
Fxy max (N)	Sim	404	479	620
	Mea	450	500	630

Table 4: Lead and tilt values for experiment 2

In Table 4, simulation and experiment results are given for points 1, 2 and 3 on Figure 13 where the lowest F_{xy} is predicted at point 1. The simulated and measured forces are in good agreement. Also the trend of F_{xy} is predicted for different lead and tilt angle combinations. So, it can be concluded that by the proposed technique, appropriate lead and tilt angles can be provided to part programmer beforehand.

5.3 Machining time simulation

In this simulation, it is desired to estimate the finishing time of a complex surface with given milling strategies and cutting conditions where simulation conditions and results are given in Table 5 and Table 6 respectively. Three machining strategies named as zig, zig-zag, and follow periphery, are compared.

Feed Rate (mm/min)	4000
Rapid Retract (mm/min)	50000
Number of Steps	100
Points per step	101
Clearance plane	z=10 mm

Table 5 : Simulation conditions

		Zig	Zig-Zag	Follow Perip.
Path length (mm)	Rapid	4200	39	36
	Cut	2200	2200	2012
Machining Time (s)		38	33	30

Table 6: Comparison of machining time among strategies.

As seen in Table 6, though, the lowest process time is calculated in follow periphery, machining time does not change significantly among the strategies. Hence, strategies should be compared from mechanics and part quality aspects. Moreover, the system developed for machining time calculation can be extended to propose new machining strategies by considering also process mechanics. This issue is recently under investigation. Yet, a machining time estimation system is established independent of any CAM packages.

6 CONCLUSIONS

In this paper, methods for simulation and optimization of 5-axis ball end milling processes using process models are presented. Detailed formulation for geometrical analysis methods used to integrate force model [16] with machining geometry to perform simulations are given. With respect to simulation results, cutting parameters are optimized.

Methodology to optimize lead and tilt angles is proposed. The results are verified by slotting process carried out with chosen optimal lead and tilt angle combinations and 2 other combinations. Trend of cutting forces is predicted and optimum lead and tilt angle combinations are determined. Besides, a full process simulation to predict the variation of cutting forces is performed and the predictions are verified by experiments. Using the results of full process simulation feed rate scheduling can be performed. Finally, cutting strategy selection is investigated from process time aspect. The developed method can be extended in order to optimize cutting strategy considering cutting mechanics also, which is recently under investigation.

7 ACKNOWLEDGEMENTS

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