### **Analytical Stability Models for Turning and Boring Operations**

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**Abstract:** In this paper an analytical model for stability limit predictions in turning and boring operations is proposed. The multi-dimensional model includes the 3D geometry of the processes. In addition a model for the chip thickness at the insert nose radius is also proposed to observe the effect of the insert nose radius on the chatter stability limit. Chatter experiments are conducted for both turning and boring in order to compare with analytical results and good agreement is observed.

Keywords: chatter stability limit, insert nose radius, turning stability, boring stability.

### 1. INTRODUCTION

Chatter has been one of the most important problems in metal cutting operations due to the poor surface quality it leaves on the surface, and high cutting forces resulting in low dimensional quality. The self excited chatter vibrations have been studied by many researchers in the last half a century. The early work focused on the "micro" aspects of dynamic cutting process more, such as force coefficients, process damping etc., whereas "macro" aspects, such as effect of machine tool dynamics, vibration modes etc., were considered in later works. The modeling of chatter stability started with analysis of orthogonal cutting where a very simple process model was considered in order to understand its fundamental mechanisms (Tlusty, 1958; Tobias, 1963). The approach proposed by Tlusty, which is still used widely, reduces the multi-dimensional dynamic system in 1D by resolving and orientating the process dynamics in one direction using a simplified orthogonal cutting model and tool geometry. Reducing a 2D or multi-D cutting system, which can only be accurately represented as an eigenvalue problem, into a single algebraic equation would result in inaccurate stability predictions as presented in several works. Later, the stability of other processes such as milling, where the process geometry and the system dynamics are more complex, were also investigated (Minis et al., 1993, Budak et al., 1998, Altintas et al., 1995). The multi-dimensional approach suggested by Minis et al. (1993) and Budak et al. (1998) eliminate the errors due to 1D approximation, which is applied for analytical stability predictions in milling operations. In addition to the analytical models, with the advent of the computers, numerical models have also been used heavily for stability analysis of chatter vibrations.

Although turning is one of the most common machining processes, and is usually considered as a simple case for chatter stability, one cannot find a analytical stability model in the literature which considers the correct physics and the geometry of the process. The orthogonal or similar simplified stability models are approximations which may yield erroneous results as they do not consider the practical aspects of the process such as tool angles and cutting tool nose radius. In one of the works, Kuster (1990) developed a model for the dynamic chip thickness in boring analytically, and use crosscoupling terms in order to reflect the effect of deformation at one direction on the other. However, Kuster used an oriented approach, i.e. one dimensional, to model the dynamic system. Later, Rao et al. (1999) used Budak's multi-directional approach (Budak et al., 1998) to model the stability in turning, and calculated the dynamic chip area using a cross coupling term, which includes the effect of vibrations in one direction on the chip area in the other direction. Clancy et al. (2002) added tool wear and process damping modeling to the study above. However, in these studies the cross coupling term made the modeling complicated, solution is made numerically. Chandiramani et al. (2006) employed a multi-dimensional approach to model the turning dynamic system, however, the turning geometry was over simplified, and the stability solution was obtained numerically. (Lazoglu et al., 2002) and (Atabey et al., 2003) proposed an analytical model for force prediction in boring, and using time domain numerical solutions they predicted the chatter vibrations and modulated workpiece topography.

In this paper, a multi-dimensional stability model is proposed for stability analysis of turning and boring processes. Three dimensional geometry of the process is considered by including the tool angles as well as the cutting insert nose radius into the model. The paper is organized as follows. The basic stability model for turning operations and the procedure to obtain stability lobes is presented first. On the following section, the stability model for boring operations including the nose radius effect is derived in detail. The analytical results then compared with experiment results at the last section before conclusions.

#### 2. STABILITY MODEL FOR TURNING

In this section the stability model for turning operations that includes the cutting geometry and transfer functions of the workpiece and the cutting tool is proposed. The model is formulated in detail which excludes the effect of the nose radius and is acceptable for rough turning operations with inserts having relatively small radii. This basic modeling is also a basis for boring stability model. The turning stability model that includes the insert nose radius effect can be found in detail in (Ozlu et al., 2006).

In order to determine the stable cutting conditions for turning operations, a relationship between the dynamic chip thickness and dynamic turning forces must be established as a starting point. Figures 1.a and 1.b show different views of a turning process and insert where the depth of cut b, the chip thickness h, and cutting angles  $\alpha$ 

(the normal rake angle), i (the inclination angle) and c (side edge cutting angle) define the geometry of the process. The latter two angles are measured on the rake face. Since the dynamic displacements in z-direction do not affect the dynamic chip thickness they can be neglected in the formulation. Then, the modulated chip thickness in terms of lathe coordinates (x, y) can be written as follows.

$$h_m(t) = f \cos c + \left(x_c^1 + x_w^1 - x_c^0 - x_w^0\right) \cos c + \left(-y_c^1 - y_w^1 + y_c^0 + y_w^0\right) \sin c \tag{1}$$

where, f represents the feed per revolution,  $(x_c^{\ l}, x_w^{\ l})$  and  $(y_c^{\ l}, y_w^{\ l})$  are the cutter and workpiece dynamic displacements for the current pass, respectively, and  $(x_c^{\ o}, x_w^{\ o})$  and  $(y_c^{\ o}, y_w^{\ o})$  are the cutter and workpiece dynamic displacements for the previous pass in x and y directions, respectively. The feed term in Equation 1 represents the static part of the chip thickness which does not contribute to the regeneration mechanism. Therefore, it can be ignored for the purpose of stability analysis. Hence, the dynamic chip thickness in turning can be defined as follows.

$$h(t) = \Delta x \cos c - \Delta y \sin c \tag{2}$$

where

$$\Delta x = x_c^1 + x_w^1 - x_c^0 - x_w^0$$
  

$$\Delta y = y_c^1 + y_w^1 - y_c^0 - y_w^0$$
(3)

 $\tau$  is the delay term which is equal to one spindle revolution period in seconds.



Figure 1; Schematic description of (a) chip thickness, (b) cutting angles, and (c) turning forces in turning.

Equations (1-4) and the cutting force coefficients can be used to express the relationship between the dynamic chip thickness in the lathe coordinates as follows.

$$\begin{cases} F_x \\ F_y \end{cases} = \frac{b}{\cos c} \begin{bmatrix} -\cos c & \sin c \\ \sin c & \cos c \end{bmatrix} \begin{bmatrix} K_f \\ K_r \end{bmatrix} [\cos c & -\sin c] \begin{cases} \Delta x \\ \Delta y \end{cases}$$
(4)

It should be noted here that the cutting force coefficients  $K_f$  and  $K_r$  can be directly obtained from calibration tests, as well as using the mechanics of cutting method proposed by Armarego et al. (1985) and Budak et al. (1996). The stability limit solution of Equation 4 can be found in detail in (Budak et al. 1998) and in (Ozlu et al. 2006). Basically, the solution results in the following set of equations,

$$b_{\rm lim} = -\frac{1}{2}\Lambda_R \left(1 + \kappa^2\right) \tag{5}$$

$$\kappa = \frac{\Lambda_I}{\Lambda_R} = \frac{\sin \omega_c \tau}{1 - \cos \omega_c \tau} \tag{6}$$

$$\varepsilon = \pi - 2\psi, \ \psi = \tan^{-1}\kappa$$
 (7)

$$\omega_c \tau = \varepsilon + 2k\pi, \ n = 60/\tau \tag{8}$$

where,  $b_{\text{lim}}$  is the stability limit,  $\Lambda$  is the eigenvalue of the dynamic system,  $\varepsilon$  is the phase difference between the inner and outer modulations, k is an integer corresponding to the number of waves in a period, and n is the spindle speed in rpm. The stable depth of cut of the system can be obtained using Equation 5 for different chatter frequencies around the most flexible structural mode. Then, the corresponding spindle speeds can be determined from Equation 8 for different lobes, i.e. k=1,2,3...etc. Thus, the stability diagram of the dynamic system can be obtained by plotting the stable depth of cut vs. the corresponding spindle speeds for different lobes (Budak et al. 1998; Altintas 2000).

### 3. STABILITY MODEL FOR BORING PROCESS

The stability model for boring operations is similar to the turning operations except a few differences. Firstly, since the stable depth of cuts in boring are comparable to the insert nose radius, the effect of insert nose radius becomes critical. Secondly, the boring process coordinates are different which results in modification of the dynamic chip thickness relationship (see Figure 2.a). Thus, in order to formulate the stability in boring operations, an insert nose model is proposed. Then, a similar procedure is followed in order to obtain the dynamic system equations. It is also shown that the stability limit for boring operations reduces to a 1D equation even including the nose radius effect.

### **3.1. Insert Nose Model**

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In order to model the insert nose, 2D trapezoidal elements are used to mesh the chip thickness (Figure 2.b). Each element is defined by its height  $b_e$ , side edge length  $b_{di}$  and angular orientation  $\theta_i$  as follows (Figure 2.c).

$$b_{e} = b_{nose} / n$$

$$b_{nose} = r - r \sin c$$

$$i = 1, ..., n$$
(9)

$$b_{di} = b_e / \cos \theta_i \qquad \qquad i = 1, \dots, n \tag{10}$$

$$\theta_i = \frac{\pi}{2} - \tan^{-1} \left( \frac{r - r \sin c}{n s_i} \right) \qquad i = 1, \dots, n \tag{11}$$

$$s_{i} = \sqrt{r^{2} - \left(r - \frac{i}{n}\left(r - r\sin c\right)\right)^{2}} - \sum_{j=1}^{i-1} s_{j} \qquad i=1,...,n \text{ and } s_{0} = 0$$
(12)

where *r* is the nose radius.



Figure 2; (a) Schematic description of chip thickness and lathe coordinates in boring, (b) meshing of the chip thickness by trapezoidal elements, (c)defining parameters of an element and cutting forces acting on it.

In this model,  $n+1^{st}$  element represents the straight cutting edge of the insert. As it can be seen from Figure 2.a, its angular orientation is equal to the side edge cutting angle, i.e  $\theta_{n+1} = c$ .

It should be noted here that the element heights do not have to be equal, although equal heights bring smoother mesh and faster solution algorithm. By using  $\theta_i$ , the dynamic chip thickness for the  $j^{\text{th}}$  element, similar to the turning case, can be defined as follows (Figure 2.c).

$$h_{i}(t) = \Delta x \cos \theta_{i} + \Delta y \sin \theta_{i}$$
<sup>(13)</sup>

## 3.2. Stability Limit Solution for Stable Depth of Cuts Higher Than the Nose Radius

In order to model the dynamic system's stability, the relationship between the dynamic boring forces and the dynamic chip thickness is defined. Then, the problem is reduced to a 1D eigenvalue problem by the help of a reduced transfer function matrix, and is solved analytically.

Similar to the turning model, the relationship between the dynamic forces and the chip thickness in lathe coordinates can be written as follows:

$$\begin{cases} F_{jx} \\ F_{jy} \end{cases} = b_j \begin{bmatrix} A_j \end{bmatrix} \begin{cases} \Delta x \\ \Delta y \end{cases} \qquad j = 1, 2, \dots, n+1$$
 (14)

where,

$$b_{j}=b_{e} \qquad j=1,2,\dots,n$$
  
$$b_{j}=b_{m} \qquad j=n+1$$

and  $[A_j]$ 's are the directional coefficient matrices which are defined for each element as follows.

$$\begin{bmatrix} A_j \end{bmatrix} = \begin{bmatrix} A_{j11} & A_{j12} \\ A_{j21} & A_{j22} \end{bmatrix} = \begin{bmatrix} \cos \theta_j & \sin \theta_j \\ -\sin \theta_j & \cos \theta_j \end{bmatrix} \begin{bmatrix} K_f \\ K_r \end{bmatrix} \left[ \cos \theta_j / \cos \theta_j & \sin \theta_j / \cos \theta_j \right]$$
(15)

Note that in Equation 14 the dynamic displacements  $\Delta x$  and  $\Delta y$  are the total dynamic displacement of the insert in cut, and can be defined as follows

$$\begin{cases} \Delta x \\ \Delta y \end{cases} = \left(1 - e^{-i\omega_c \tau}\right) \left[G(i\omega_c)\right] \left[\begin{cases} F_{1x} \\ F_{1y} \end{cases} + \begin{cases} F_{2x} \\ F_{2y} \end{cases} + \dots \begin{cases} F_{n+1x} \\ F_{n+1y} \end{cases}\right] e^{i\omega_c t}$$
(16)

where the transfer function matrix  $[G(i\omega_c)]$  is assumed to include only the transfer function in *y*-direction, because at almost all of the boring operations the tool and the workpiece is much more rigid at the *x*-direction. Therefore, the transfer function matrix is given as,

$$\left[G(i\omega_c)\right] = \begin{bmatrix} 0 & 0\\ 0 & \phi_{yy} \end{bmatrix}$$
(17)

Substituting Equation 16 into Equation 14 the dynamic elemental force for the  $j^{th}$  element can be obtained as follows.

$$\left[F_{j}\right]e^{i\omega_{c}t} = b_{j}\left(1 - e^{-i\omega_{c}\tau}\right)\left[A_{j}\right]\left[G(i\omega_{c})\right]\sum_{p=1}^{n+1}\left[F_{p}\right]e^{i\omega_{c}t}$$

$$(18)$$

As it can be seen from the above equation there are now (n+1) equations to solve. The first (n) equations that models the nose radius have the same depth of cut  $b_e$ , which is known. However the last equation that models the straight edge, has the depth of cut  $b_m$  which is to be solved for stability. Adding up all the equations we get,

$$\sum_{p=1}^{n+1} \left[ F_p \right] e^{i\omega_c t} = \left( 1 - e^{-i\omega_c \tau} \right) \left\{ b_m \left[ A_m \right] + b_e \sum_{p=1}^n \left[ A_p \right] \right\} \left[ G(i\omega_c) \right] \sum_{p=1}^{n+1} \left[ F_p \right] e^{i\omega_c t}$$
(19)

Let define matrix [*C*] as follows.

$$\begin{bmatrix} C \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} = \left( e^{-i\omega_c \tau} - 1 \right) \begin{bmatrix} B \end{bmatrix}$$
(20)

where,

$$\begin{bmatrix} B \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = b_m \begin{bmatrix} A_m \end{bmatrix} + b_e \sum_{p=1}^n \begin{bmatrix} A_p \end{bmatrix}$$
(21)

The solution of Equation 19 is possible if and only if its determinant is equal to 0.

$$\det[[I] + [C][G(i\omega_c)]] = 0$$
(22)

Solution of Equation 22 results in the following,

$$C_{22} = -1/\phi_{yy}$$
(23)

Letting  $C_{22}$  be  $\Lambda$  and rewriting Equation 19 the below is obtained:

$$\sum_{p=1}^{n+1} F_{py} e^{i\omega_c t} = \Lambda \phi_{yy} \sum_{p=1}^{n+1} F_{py} e^{i\omega_c t}$$
(24)

Now the problem reduces to a 1D eigenvalue problem, and Equation 20 reduces to the following.

$$\Lambda = \left(e^{-i\omega_c \tau} - 1\right)B_{22} \tag{25}$$

 $A_{j22}$  can also be calculated from Equation 15 as follows.

$$A_{j22} = \left(-\sin\theta_j + \cos\theta_j K_r\right) \tan\theta_j \tag{26}$$

And at the chatter frequency  $B_{22}$  should reach to its stability limit as follows.

$$B_{22\,\text{lim}} = \frac{\Lambda_R + i\Lambda_I}{\cos\omega_c \tau - i\sin\omega_c \tau - 1} \tag{27}$$

Since  $B_{22\lim}$  should be a real number, the imaginary part of Equation 27 has to vanish yielding,

$$B_{22\,\text{lim}} = -\frac{1}{2}\Lambda_R \left(1 + \kappa^2\right) \tag{28}$$

where,  $\kappa$  is defined in Equation 6 which then results in Equations 7 and 8 in order to obtain a relationship between the chatter frequency and spindle speed. Substituting Equation 28 into Equation 21 the following is obtained,

$$b_{m,\lim} = \left[ -\frac{1}{2} \Lambda_R \left( 1 + \kappa^2 \right) - b_e \sum_{p=1}^n A_{p22} \right] / A_{m22}$$
(29)

Note that  $b_{m,\lim}$  is the limiting stable depth of cut for only the straight edge. Thus, the stable depth of cut of the dynamic system  $b_{\lim}$  can be obtained by adding up the rest of the insert in cut as follows:

$$b_{\lim} = b_{m,\lim} + nb_e \tag{30}$$

Once the stability limit is obtained, the stability lobes can be derived using the method described in section 2.

# **3.3. Stability Limit Solution for Stable Depth of Cuts Smaller Than the Nose Radius**

The solution method presented in Section 3.2. is also applicable for the case where the stable depth of cut of the dynamic system is inside the nose section of the insert. However, a step by step search is needed, i.e. the method presented above should be applied for each element incrementally until instability is obtained. For instance, at the  $j^{\text{th}}$  element Equation 29 takes the following form:

$$b_{j,\text{lim}} = \left[ -\frac{1}{2} \Lambda_R \left( 1 + \kappa^2 \right) - b_e \sum_{p=1}^{j-1} A_{p22} \right] / A_{j22}$$
(31)

If the elemental stable depth of cut  $b_{i,\lim}$  is smaller than  $b_e$ , the solution is obtained, otherwise the solution is continued with the  $(i+1)^{st}$  element. Again, once the stability limit is obtained, the stability lobes can be derived using the method presented in section 2.

### 4. EXPERIMENTS

Chatter tests were conducted in order to obtain the absolute stability limit of the dynamic system, in both turning and boring operations. The stability lobes in turning and boring operations are very narrow compare to milling stability lobes due to the lower spindle speeds and single cutting tooth. Thus, only the absolute stability limits are considered in the experiments.

A conventional manual lathe is used in the experiments, which allows for specific spindle speeds, i.e. 700, 1000, 1400 rpm. A modal test setup is used to measure the transfer functions of the workpiece and the tool. In addition, a frequency measurement setup was prepared in order to measure the chatter frequency. In all experiments, coated carbide triangular inserts having zero rake angle are used. The desired inclination and rake angles are provided using the ground insert seats. The side edge cutting angle in the turning experiments is obtained by rotating the tool holder from its clamped end. The workpiece material was AISI 1040 steel, and an existing orthogonal database was used for the cutting force coefficients. A feed rate of 0.08 mm/rev is used for all tests.

### **4.1. Turning Chatter Experiments**

In turning chatter experiments main aim was to verify the proposed stability model. Therefore, the analytically derived stability diagram is compared with experiments for the case where the workpiece is the most flexible component of the dynamic system. The workpiece diameter was 39 mm and the length was 75 mm. The modal parameters of the workpiece are measured as natural frequency of 770 Hz, damping of 0.025, and stiffness of  $6.6 \times 10^6$  N/m. A 0.4 mm nose-radius insert with 5° of rake, 5° inclination (provided by the insert seats) and 30° of side edge cutting angle is used during the experiments. Cutting coefficients values of  $K_t$ =1240 MPa and  $K_r$ =44 MPa are used in the simulations. In the analytical calculation of the stability lobes the insert nose is also taken into account. The method used is based on the stability model presented in this paper with an additional solution procedure that includes nose radius effect which can be found in detail at (Ozlu et al. 2006).



Figure 3; Turning chatter experiment results vs. analytically solved stability lobes, where "O" represents stable cut, "•" represents chatter, and "—" is the analytical solution. And cut surface of a stable and unstable cut.

The result of the experiments and analytically derived stability limit diagram can be found in Figure 3. The difference at 700 and 1000 rpm's can be due to process damping. The predicted stability limit and experimental results show reasonable agreement.



Figure 4; FFT of the chatter sound measured for 1.2 mm depth of cut at 1000 rpm.

An example of a finished surface of stable and unstable cut for the tests conducted in 1400 rpm can also be seen in Figure 3. Another way to identify the unstable cut is to measure the sound data. An example of a measured chatter sound can be found in Figure 4.

### 4.2. Boring Chatter Experiments

The main objective of the boring chatter experiments is to obtain the absolute stability limit for different nose radiuses; 0.4, 0.8 and 1.2 mm in this case. In boring chatter experiments, the boring tool is selected to be more flexible than the workpiece, as it is the most common problem in practice. All the cutting angles are selected to be  $0^{\circ}$ . The tool is clamped by a tool radius/length ratio of 1/3 in order to obtain measurable stable depth of cuts. The modal parameters of the boring bar are measured as follows: natural frequency of 3690 Hz, damping of 0.012 and stiffness of  $2.3 \times 10^7$  N/m. Cutting coefficients values of  $K_t$ =1400 MPa and  $K_r$ =0 MPa are used in the simulations.

The absolute stable depth-of-cuts are also obtained analytically by the model in section 3. 20, 30 and 40 elements are used for 0.4, 0.8, and 1.2 mm insert nose radiuses, respectively.



Figure 5; Boring chatter experiment results vs. analytically calculated absolute stable depth of cuts for inserts having different nose radiuses, where "O" represents stable cut, "•" represents chatter and "—" is the analytical solution. And cut surface of a stable and unstable cut

The analytically derived absolute stability limits vs. experimental results for inserts having 0.4, 0.8 and 1.2 mm nose radius can be seen in Figure 5. The analytically calculated absolute stability limit for the insert having 0.4 mm nose radius is around 8 mm. However, during the tests it was decided to stop at a depth of cut of 1mm in order to avoid high cutting forces and consequently high deformation that the slender boring bar will face, which is also appropriate for a practical case. For other inserts having 0.8 and 1.2 mm nose radius, the results are satisfactory. It should also be noted here that the

trend of the absolute stability limit within insert nose radius is as expected. Increase in the nose radius has the same effect as the increase in side edge cutting angle, i.e. the increase in the nose radius, increases the effect of tool's flexibility on the dynamic system which reduces the absolute stability limit.

An example of a finished surface of stable and unstable cut for the boring tests conducted in 1400 rpm can also be seen in Figure 5. In addition, an example of a measured chatter sound can be found in Figure 6.



Figure 6; FFT of the chatter sound measured for insert having 0.8 mm nose radius and at a 0.4 mm depth of cut with 1000 rpm spindle speed.

### 5. CONCLUSIONS

A multi-dimensional stability model which analytically predicts the stability limit in turning and boring operations is proposed in this study. Another model that meshes the dynamic chip thickness into many elements that helps to include the effect of the nose radius on the stability is also proposed. Combining both models, an analytical stability solution for boring processes in a 1D form is derived. Chatter experiments in turning verify the analytical model. The analytical boring stability model is compared with boring chatter experiments by various insert nose radiuses, and a reasonably good agreement is observed.

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