Sliding Mode Based Piezoelectric Actuator Control

Asif Sabanovic, Khalid Abidi

***Abstract*— In this paper a control of method for a piezoelectric stack actuator control is proposed. In addition briefly the usage of the same methods for estimation of external force acting to the actuator in contact with environment is discussed. The method uses sliding mode framework to design both the observer and the controller based on an electromechanical lumped model of the piezoelectric actuator. Furthermore, using a nonlinear differential equation the internal hysteresis disturbance is removed from the total disturbance in an attempt to estimate the external force acting on the actuator. It is then possible to use this external force estimate as a means of force control of the actuator. Simulation and experiments are compared for validating the disturbance and external force estimation technique. Some experiments that incorporate disturbance compensation in a closed-loop SMC control algorithm are also presented to prove the effectiveness of this method in producing high precision motion.**

***Index Terms*— Hysteresis, Microactautors, Micropositioning, Piezoelectric actuator, Sliding-mode control**

I. Introduction

The use of piezoelectric actuators for accurate and stable control of manipulator position and/or force is greatly facilitated by model-based control system analysis and design. Inherent nonlinearities composed primarily of hysteresis in piezoelectric actuators pose as an obstacle to these objectives. By far, open-loop techniques have not been successful in providing good results due to the difficulties involved in modeling the actuator precisely. In [1], disturbance compensation based on a hysteresis model was used, however, unmodeled disturbances required the addition of a robust controller like H_∞.

Hysteresis is an inherent non-linearity in all piezoelectric actuators. This hysteresis non-linearity is usually 15-20% of the output thereby greatly reducing the performance of the actuators. In [2] and [3] models were made based on the physics of the actuators and these models proved to be effective in modeling the behavior of these actuators under different excitations. Additionally, the models in [2] and [3] define the hysteresis behavior as existent in the electrical domain of the actuator and is between voltage and charge. In [3], a simple differential equation was used to model the voltage-charge hysteresis behavior. This model proved simple to implement in real-time applications due to the simplicity of the equation representing the hysteresis.

In this paper the sliding mode methods are applied in the design of a high- accuracy piezo actuator control. The solution proposed here combines the sliding mode controller and the disturbance rejection method in order to achieve high accuracy in the actuator trajectory tracking. For the disturbance estimation a sliding mode observer based disturbance compensation method is used here. By manipulating model of a piezo actuator in a form where nonlinearities due to hysteresis are presented as an additive disturbance acting together with external force to the mechanical system a simple second order observer is designed to estimate lumped disturbance. Furthermore, using this concept an approach in the external force acting on the actuator is explored. In this the hysteresis model presented in [3] is used. Based on the force estimation a sliding mode force controller was designed.

This paper is organized as follows. In section II a suitable model of a piezo actuator, based on already known results, is presented. In the section III the sliding mode based observer design is presented. In section IV experimental results verifying theoretical works are presented.

II. Electromechanical Model

A. Description of the overall model

The model used in this work is described in [2]. The model proved to be a fairly accurate representation of the electromechanical behavior of the piezoelectric actuator. It is described schematically in Fig. 1.a. and Fig. 1.b.

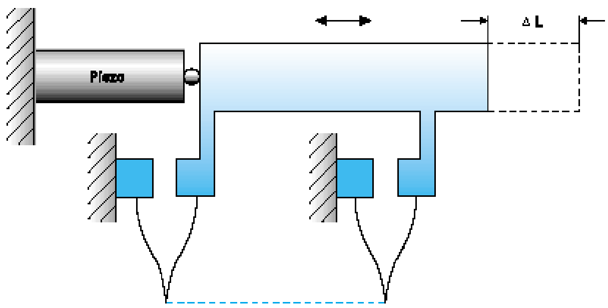

Fig. 1.a Structure of a piezoelectric manipulator.

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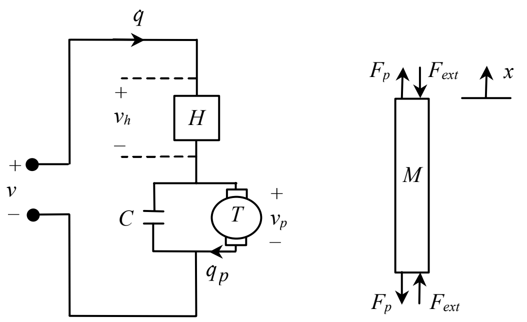

Fig. 1. b. Electromechanical Model of the piezoelectric actuator

The electromechanical lumped model of the piezoelectric actuator can be defined mathematically by equations (1) to (6) given below, [2].

$$v_p = v - v_h \tag{1}$$

$$v_h = H(q) \tag{2}$$

$$q = Cv_p + q_p \tag{3}$$

$$q_p = Tx \tag{4}$$

$$F_p = Tv_p \tag{5}$$

$$m_p\ddot{x} + c_p\dot{x} + k_p x = \underbrace{Tv}_{F} - \underbrace{Tv_h - F_{ext}}_{F_{dis}} \tag{6}$$

Meanings of the terms defined in equations (1) to (6) are as follows: v stands for total voltage across the piezoelectric actuator, v_p stands for voltage due to the piezoelectric effect, v_h stands for voltage due to the hysteresis effect, H is a hysteresis function, T stand for electromechanical transformation ratio, q stands for total charge in the piezoelectric actuator, q_p stands for charge transduced due to mechanical motion, F_p stands for force due to piezoelectric effect, F_{ext} stands for external forces acting on the actuator, F stands for the control force, F_{dis} stands for the lumped disturbance and m_p, c_p, k_p stand for equivalent mass, damping and stiffness. The structure of model (6) is showing that, from the mechanical motion the hyteresis may be perceived as a disturbance force that satisfy matching conditions. This means that the sliding mode based control should be able to reject the influence of the hysteresis nonlinearity on the mechanical motion. At the same time it is obvious that the lumped disturbance consisting of the external force acting on the system and the hystereis can be estimated, thus allowing the application of the disturbance rejection method in the overall system design. In order to make use of the system (1)-(6) a suitable hysteresis model should be developed.

B. Description of the hysteresis mode

In the piezo systems the hysteresis is appearing between voltage and charge. It has been shown that it can be modeled using a first-order differential equation proposed in [3] and [4]. In [4], it has been experimentally verified that this differential equation is suitable for describing electric hysteresis such as that in piezoelectric actuators. The model for the hysteresis effect is given by

$$\dot{q} = \alpha|\dot{v}_h|(av_h - q) + b\dot{v}_h \tag{7}$$

Where: $a = \dfrac{q_c}{v_{h,c}}$ is constant found from loop center point, $b = \dfrac{q_{\max} - q_{\min}}{2A}$ is average slope of the hysteresis loop and $\varepsilon = \dfrac{4}{3}(a-b)\alpha A^3$ is loop area for small sinusoidal inputs from which α can be found. Now system (1)-(7) may be used for the simulation purposes and hysteresis model (7) can be used in the estimating of the external force acting on piezo actuator in the case that lumped disturbance is known. In the following section we will discuss a sliding mode based approach in designing the lumped disturbance and the external force observers.

III. Disturbance Observer and External Force Estimation

A. Total disturbance estimation

The structure of the observer is based on (6) and it is proposed that all the plant parameter uncertainties, nonlinearities and external disturbances can be represented as a single disturbance. The displacement x and the supply voltages v are measurable quantities. Hence, the nominal structure of the plant can be defined as follows

$$\begin{aligned} &m_N\ddot{x} + c_N\dot{x} + k_N x = T_N v - F_d \\ &F_d = T_N v_h + F_{ext} + \Delta T(v + v_h) + \Delta m\ddot{x} + \Delta c\dot{x} + \Delta kx \end{aligned} \tag{8}$$

Here m_N, c_N, k_N and T_N are the nominal plant parameters, assumed to be known and constant, while Δm, Δc, Δk and ΔT are the uncertainties of the plant parameters assumed to be bounded and continuous. Since x and v_{in} are measured the proposed observer is designed as a position tracking system and is of the following form

$$m_N\ddot{\hat{x}} + c_N\dot{\hat{x}} + k_N\hat{x} = T_N v - T_N u_c \tag{9}$$

Here $\hat{x}$ is the estimated position v_{in} is the plant control input and u_c is the observer control input. If $\hat{x}$ can be forced to track x then we expect that from the controller output we can determine the $F_d / T_N = u_c$. In the observer let us select the sliding manifold as $\sigma = (\dot{x} - \dot{\hat{x}}) + C(x - \hat{x})$. In order to determine the controller structure let us select the Lyapunov function as $v_L = \sigma^2/2$ and select the derivative of the Lyapunov function as $\dot{v}_L - D\sigma^2$ with $D > 0$. Equating the above results and simplifying

$$\dot{v}_L = \sigma\dot{\sigma} = -D\sigma^2 \Rightarrow \dot{\sigma} + D\sigma\big|_{\sigma\neq 0} = 0 \qquad (10)$$

If we plug $\sigma = \left(\dot{x} - \dot{\hat{x}}\right) + C\left(x - \hat{x}\right)$ in (10) and simplify we get

$$\left(\ddot{x} - \ddot{\hat{x}}\right) + \left(C + D\right)\left(\dot{x} - \dot{\hat{x}}\right) + CD\left(x - \hat{x}\right) = 0 \qquad (11)$$

Here the roots –C and –D, define the transients of the closed-loop control system. If we subtract (9) from (8) and plug the result into (11), we get

$$u_{ceq} = \frac{1}{T_N}\left\{F_d + \left[c_N - m_N\left(C + D\right)\right]\left(\dot{x} - \dot{\hat{x}}\right) + \left[k_N - m_N CD\right]\left(x - \hat{x}\right)\right\} \qquad (12)$$

Where u_{ceq} is the control that will keep system motion in manifold $\dot{\sigma} + D\sigma = 0$. From (12), it can be seen that as $\sigma \to 0$ then $\hat{x} \to x$ and $T_N u_{eq} \to F_d$. One can easily find that the control to satisfy the condition $\sigma(\dot{\sigma} + D\sigma) = 0$ when $\sigma \neq 0$ is given by

$$u_c = u_{ceq} + D\sigma \qquad (13)$$

For discrete-time applications the following control is used

$$u_{(k)} = u_{(k-1)} + K_u\left(D\sigma_{(k)} + \frac{\sigma_{(k)} - \sigma_{(k-1)}}{T_s}\right) \qquad (14)$$

here K_u is a design parameter, which can be tuned to optimize the controller, and T_s is the sampling interval of the discrete-time control. The block diagram of Fig. 2 best describes the observer implementation.

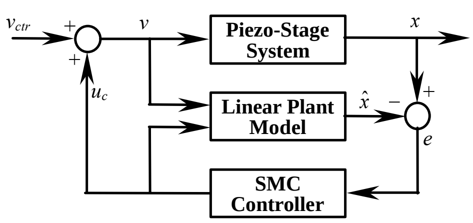

Fig. 2. Observer implementation and disturbance feedback

From (11), when $\sigma \to 0$ then $\hat{x} \to x$ thus disturbance can be determined as $T_N u_{ceq} = \hat{F}_d$. If $u_c = \hat{F}_d / T_N$ is feedback to the system (8) then resulting system is reduced to a nominal plant (15) where v_{ctr} is new control input to the system with compensated disturbance.

$$m_N \ddot{x} + c_N \dot{x} + k_N x = T_N v_{ctr} \qquad (15)$$

Note that compensated system is a second order system with constant parameters.

B. External force estimation

If we refer to structure (8), the disturbance term is as $F_d = T_N v_h + F_{ext} + \Delta T\left(v + v_h\right) + \Delta m\ddot{x} + \Delta c\dot{x} + \Delta kx$. If the terms related to the uncertainty of the plant parameters are small in comparison to the others then

$$F_d \cong T_N v_h + F_{ext} \qquad (16)$$

In Fig. 3 the method defined above is represented in a block-diagram.

From (16) we see that knowledge of the hysteresis term can give the external force. From the model of the plant it is possible to estimate the hysteresis term, $T_N v_h$, which will be called F_{hys}. Hence, the external force acting on the actuator can be found from $F_{ext} = F_d - F_{hys}$. The structure of the disturbance observer and the external force observer is depicted in Fig. 3. Actually the same observer is used in both cases so the dynamics of both observers is the same what is very important in the overall control system design.

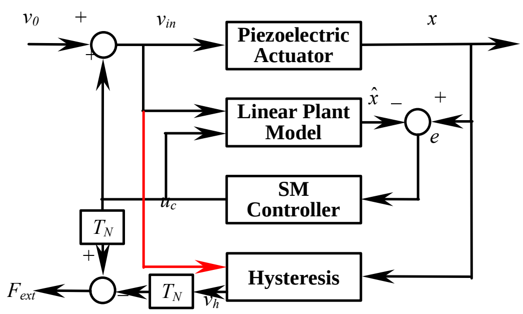

Fig. 3. Block-diagram for external force estimation

C. Control system design

The overall control system design can be implemented based on the uncompensated system (8) or compensated system (15). As mentioned before the difference is obvious – in the design based on uncompensated system (8) the robust controller capable of compensating large disturbance and the nonlinearity of the system should be implemented. This require robust controller and sliding mode based control is very good candidate.

By selecting the sliding manifold as $\sigma_x = \left(\dot{x}^{ref} - \dot{x}\right) + C_o\left(x^{ref} - x\right)$. Following the same steps as for observer design one can easily find that the closed loop behavior of the system with control (14) and $\sigma = \sigma_x$ is described by

$$\left(\ddot{x}^{ref} - \ddot{x}\right) + \left(C_o + D_o\right)\left(\dot{x}^{ref} - \dot{x}\right) + C_o D_o\left(x^{ref} - x\right) = 0 \qquad (17)$$

This controller is suitable for either uncompensated plant (8) or compensated plant (15). When controlling uncompensated plant the control should compensate the disturbance what requires higher control values and thus we may expect that the system may exhibit higher control error comparing with the case when disturbance is compensated by the disturbance feedback and controller must control a nominal second order plant with compensated disturbance and the variation of the plant parameters. The structure of the control system is depicted in Fig. 4.

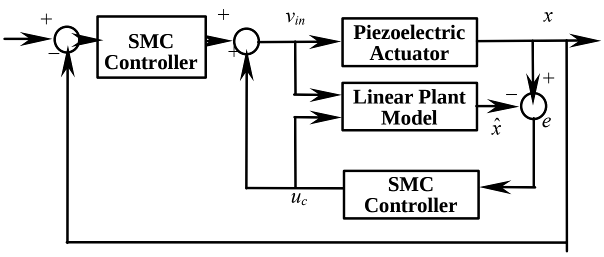

Fig. 4 The structure of the closed loop system

In the experiments we will be showing difference in the control error while implementing both structures – a single loop system with sliding mode controller (14) and a system with disturbance compensation and controller (14). In both cases the control is continuous thus the problem of the possible chattering due to the sliding mode existence is avoided.

IV. Experimental Results

Extensive experiments are conducted in order to confirm validity of the sliding mode control design. The experimental setup consists of a PSt150/5/60 stack actuator ($x_{\max} = 60\,\mu\text{m}$, $F_{\max} = 800\,\text{N}$, $v_{\max} = 150\,\text{Volt}$) produced by Piezomechanik connected to SVR150/3 low-voltage, low-power amplifier. The parameters of the controller are kept constant for all experiments C=100, D=800, T=100microsecondsd.

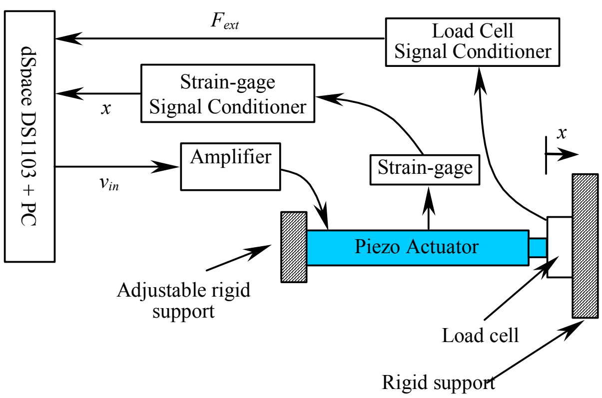

Fig. 5. Simplified structure of the experimental setup

The piezoelectric actuator has built-in strain-gages for position measurement. Force measurement is accomplished by the help of a load cell that is placed against the actuator as shown in Fig. 5.

The entire setup is connected DSP based dSPACE DS1103 module hosted in a PC with dSPACE software Control Desk v.2.0. In Fig. 6 a simplified structure of the experimental setup is shown. All measurement and control is run in dSPACE card. Due to the large noise from the position measurement a simple cut-off filter has been applied in the system.

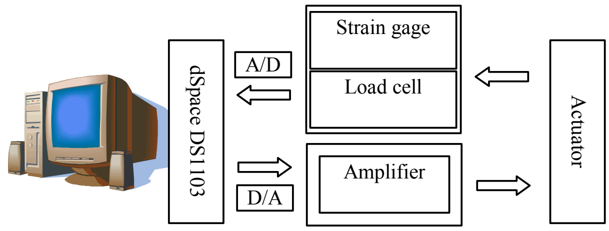

Fig. 6. Simplified structure of the experimental setup

The experiments are conducted in order to confirm:

a – the design of the disturbance observer and the possibility to achieve open loop operation of the piezo actuator;

b – the closed loop behavior of the system with controller (14) for both uncompensated (8) and compensated (15) system in order to see the contribution of the disturbance observer on the tracking error;

c – the validity and accuracy of the force estimation based on the disturbance observer and the hysteresis model.

A. Results with the disturbance observer

Experiments were carried out with the disturbance observer in an attempt to test its capacity of estimating the disturbances acting on the system. The piezo actuator had been supplied so that the output position has trapezoidal form as depicted in

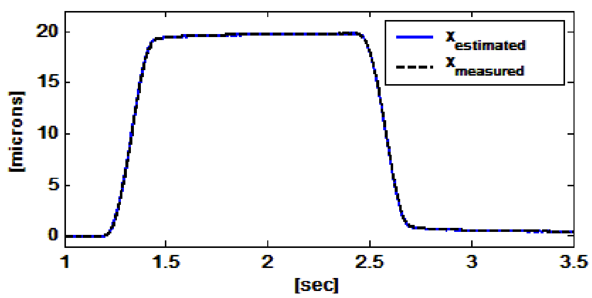

Fig. 7a. Measured and estimated position

Fig. 7.a. Fig. 7.a shows the measured and estimated position while Fig. 7.b shows the estimation error for a trapezoidal voltage input. Both figures show that the observer position is able to track the measured actuator position nicely. The

tracking error is 0.2µm and could be improved in the SMC framework.

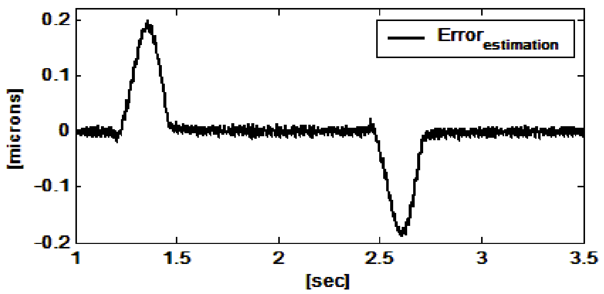

Fig. 7b. Observer estimation error

In order to verify the disturbance compensation a open loop behavior of the actuator under sinusoidal supply. In Fig. 7.c., the response of the actuator with disturbance compensation along with uncompensated system response and the output of the model of the nominal system (15) under the same input are depicted. The comparison shows effectiveness of the disturbance observer and the effective linearisation of the piezo actuator dynamics since the outputs of the compensated system and the ideal model are close to each other while output of the uncompensated plant exhibits very large error in comparison with output of the model.

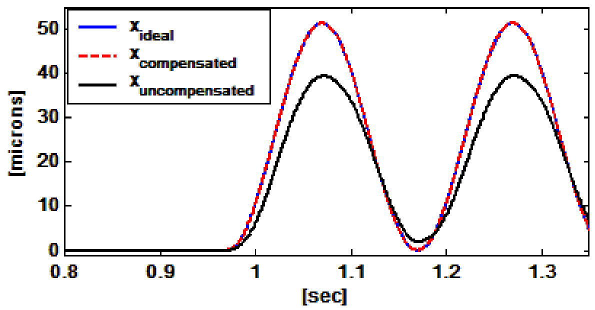

Fig. 7.c. Response to a harmonic voltage input for uncompensated system, ideally compensated nominal system model (1%) and uncompensated system

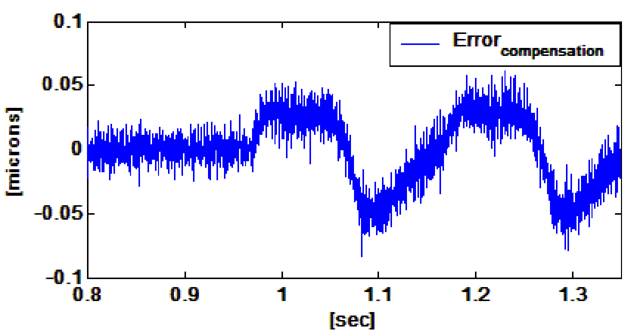

Fig. 7. d. Response to a harmonic voltage input and compensation error for a harmonic voltage input

Error between the compensated and uncompensated system is clearly visible in Fig. 7.c. This shows that the disturbance compensation produces very good result even system works in open loop.

This is expected result since disturbance compensation is known method but it confirmation that sliding mode observer is appropriate for the application and it does not cause any chattering while dynamic error is kept below 50 nanometers (Fig. 7.d.). This indicate the possibility of the application of such a system in a open loop actuation

The closed loop operation with controller (14) is depicted in Fig. 8. The system (15) with compensated disturbance is subject to the sinusoidal reference $x_{ref} = 24 + 24\sin(10\pi t)$. The closed loop system behavior is depicted in Fig. 8.a.

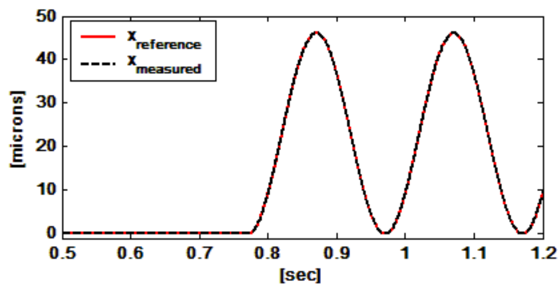

Fig. 8.a The closed loop system response

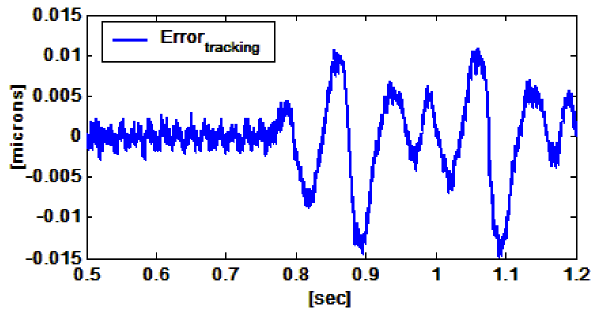

Fig. 8.b The closed loop system response and the control error for closed loop response depicted in Fig.7.c.

The control error of the closed loop system for trajectory tracking as shown in Fig. 8.a. is depicted in Fig. 8.b. The error is now in the range of 10 nanometers and can be improved by better tuning of the controller parameters.

In order to compare the behavior of the sliding mode control with disturbance compensation and without it the same experiment had been executed and results are depicted in Fig. 9. The results show the effectiveness of the sliding mode controller (14) in position tracking for system without disturbance observer. As expected the usage of the disturbance observer contributes to the lowering the control error. That can easily be

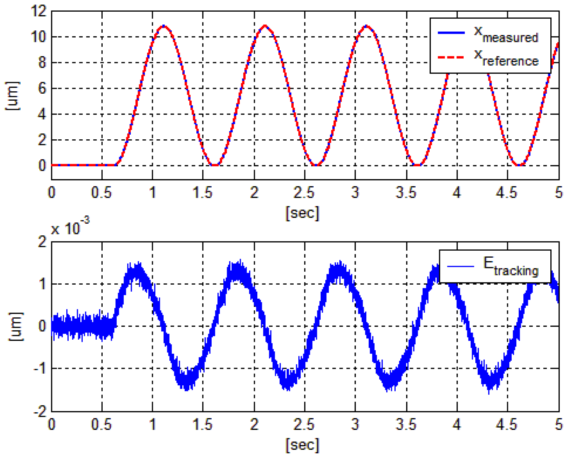

Fig. 9.a. The trajectory tracking for system without disturbance observer and controller (14)

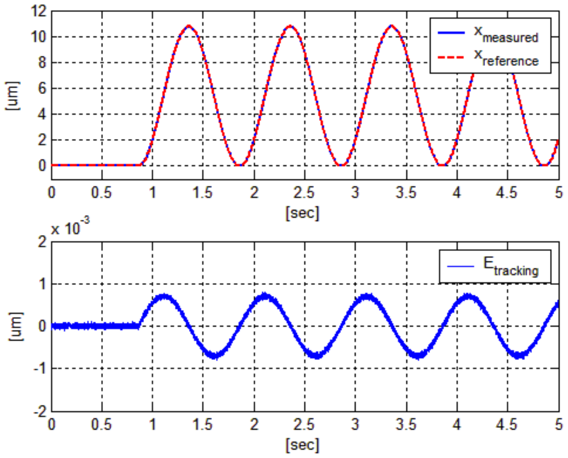

Fig. 9.b. The trajectory tracking for system with disturbance observer and controller (14)

B. Results with external force estimation

The external force estimation method is applied experimentally. The results in Fig. 10.a and Fig.10.b show that the method works nicely for a smooth sinusoidal force.

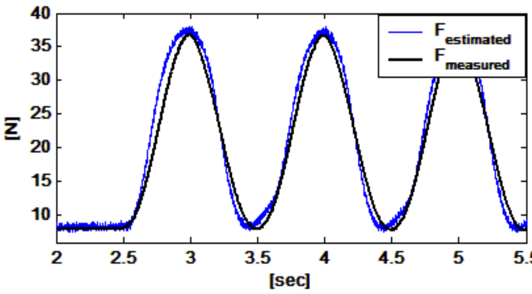

Fig. 10.a. Experimental estimation of harmonic force and estimation error of harmonic force

Due to the lag in the hysteresis estimate there is a lead in the force estimation. It is most certain that any improvements in the hysteresis estimation should improve the estimation of the external force acting on the actuator.

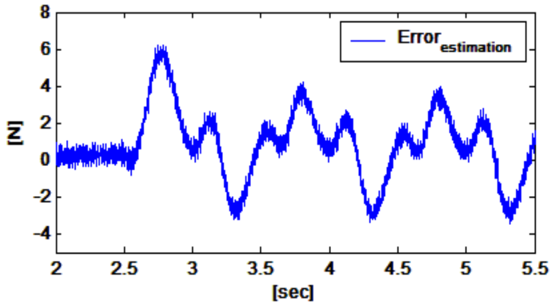

Fig. 10.b.. Experimental estimation of harmonic force and estimation error of harmonic force

V. CONCLUSION

A sliding mode control of a piezo actuator is discussed. In order to have high precision control in the presence of large measuring noise the sliding mode controller alone and its combination with sliding mode based disturbance observer is analyzed. The observer is designed in the SMC framework and is based on a lumped electromechanical model of the piezoelectric actuator. The observer proved successful in compensating the disturbances acting on the actuator and the nonlinearity due to the hysteresis. Addition of the disturbance compensation to a closed-loop control scheme provided good results and should open the way for high precision tracking with piezoelectric actuators. The hysteresis term was removed from the disturbance signal and the remaining term gave some insight into the problem of external force estimation. Work is currently in progress to improve the hysteresis model so that a more accurate estimation of external force is possible.

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