Piezoceramics-based Devices for Active Balancing of Flexible Shafts

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Abstract
This paper focuses on vibration control of flexible shafts by means of rotor-fixed piezoelectric materials. The target is to realize compact solutions for the suppression of problematic resonant vibration at so-called flexural critical speeds. For analysis, parametric finite element models of flexible rotors with piezoceramic sheets and strain or displacement sensors are developed, where the number of degrees of freedom is kept low. Several mechanisms which can destabilize flexible rotors are quantized, such as rotor material damping, dissipation of currents induced in rotor-fixed piezoceramics and active feedback control proportional to rotor strain rates. The effectiveness of low frequency feedback and feedforward control for the suppression of the unbalance response is demonstrated using analytic and experimental results. Emphasis is on the interaction between the dynamics of the rotor and that of the connected electronic circuits. The experimental setup which is used for validation is a flexible shaft equipped with piezoceramic sheets and strain sensors. A slipring assembly is used to simplify measurements with, and control of, the sensors and actuators on the shaft and to facilitate the development of compact drive electronics.

1 Introduction

Fastly rotating flexible rotors may exhibit severe bending vibrations and may induce vibration, wear and noise in and near their support structures. These problems can often be solved by improving the rotor balance and support damping by passive means. In cases where these means are exhausted, active control methods for rotor balancing and damping provide a solution. Devices for active vibration control of rotating machinery have become more compact and versatile in the past decades. A relatively new approach to rotor vibration control is based on the use of piezoceramic materials which are bonded to the surface of a flexible rotor. This vibration control solution was investigated by several authors in recent years (e.g. [2], [3]). They considered the following applications:

- suppression of the resonant response to unbalance,
- suppression of forced vibrations caused by a driving motor, and
- stabilization of instable vibration.

The compactness and low power consumption of piezoceramic actuators (and the conformability of fiber composite actuators to curved surfaces) make their use attractive in these applications. However, in both investigations [2] and [3], experiments were performed using costly and wear sensitive slipring assemblies for power transmission between stator and rotor. In addition, the application of piezoceramic materials to flexible rotors has certain drawbacks which have not been analyzed thoroughly yet. There remains therefore some work to be done before this control solution reaches maturity.

It is focused in this document on devices for active balancing of flexible rotors at speeds near so-called flexural critical speeds. (A flexural critical speed is defined as a speed at which the bending vibration response of a rotor to unbalance reaches a maximum. Unbalance is defined as the distribution of the centers of mass of the rotor cross sections times their distances from the nominal rotation axis. Flexural
critical speeds are often different from the frequencies of structural bending modes due to so-called gyroscopic effects, which couple the three-dimensional rotational motion of rotor cross-sections).

For a flexible rotor with attached piezoceramics, the amplitude of bending vibration at a critical speed depends mainly on:

- the amount of unbalance exciting the respective bending mode,
- the amount of mechanical dissipation in bearings and supports, and
- the bending moments induced by active control of rotor-fixed piezoceramics.

Active control of rotor-fixed piezoceramics so as to realize virtual stator damping, modify the rotor unbalance distribution, or both, may effectively reduce unbalance induced vibration at critical speeds.

In addition to forced vibration due to unbalance, flexible rotors may also exhibit instable vibration. (Dissipative mechanisms in elastic media which are rotating with respect to an inertial frame have non-dissipative mass acceleration effects which can lead to instability. Such effects often limit the operating speed of rotating machinery.) Mounting piezoelectric materials on flexible rotors for the purpose of active vibration control may give rise to the following destabilizing mechanisms:

- mechanical dissipation (hysteresis) in piezoelectric materials,
- electric dissipation (resistive loss) of currents induced in the piezoceramics, and
- bending moments induced by feedback control proportional to rotor strain/displacement rates.

In order to avoid instable vibration at speeds exceeding flexural critical speeds, the magnitude of these mechanisms should be determined and be limited by careful design if necessary.

## 2 Finite element models

### 2.1 Two rotor cases

Two different rotor systems are considered (Figure 1, 2), the main properties of which are summed up in Table 1. Rotor 1 is a hollow aluminium shaft of length one meter which is connected by flexible couplings to short shafts that rotate in ball bearings. This system was developed as a down-scaled model of a composite helicopter tail drive shaft and is used also in experiments. The flexible couplings at the ends of this rotor lead to low frequency bending modes with virtually no bearing motion and hence very little damping, which is partly compensated by stiffness reduction and damping augmentation of the bearing supports. The shaft is equipped with piezoceramic sheets and strain sensors. Rotor 2 is supported by ball bearings at its ends and contains two heavy disks connected by a hollow shaft. The model of this rotor is used to investigate heavy rotor systems with inertia concentrated at a few positions. The system contains rotor-fixed piezoceramic sheets and distance sensors. The low frequency bending modes of this rotor are lightly damped because the bearings are assumed to provide no rotational stiffness and damping.

![Figure 1. Cross-sections of finite element models (axes not to scale). a) Rotor 1. b) Rotor 2.](image)
Main properties rotor systems

<table>
<thead>
<tr>
<th></th>
<th>Rotor 1</th>
<th>Rotor 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor: material type, stiffness, loss factor:</td>
<td>Alum., 68·10^9, 5·10^-5</td>
<td>Steel, 210·10^-9 10-10^-3</td>
</tr>
<tr>
<td>Rotor: total length (m), total mass (kg):</td>
<td>1.052, 0.22</td>
<td>0.847, 42.6</td>
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<tr>
<td>Additional inertia: trans.(kg); rot.(kg.m²), {nodes}:</td>
<td>-</td>
<td>20; 0.15, {3, 8}</td>
</tr>
<tr>
<td>Piezoelectric: material, stiffness (N/m), loss factor:</td>
<td>PZT5H, 95·10^9, 8·10^-3</td>
<td>PZT5H, 95·10^9, 8·10^-3</td>
</tr>
<tr>
<td>Piezoelectric: thickness (m), width (m), mass (kg):</td>
<td>500·10^9, 8·10^-3, 28·10^-3</td>
<td>200·10^9, 20·10^-3, 17·10^-3</td>
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<tr>
<td>Piezoelectric: total cap.(F), piezoelectric constant. d_{31}:</td>
<td>270·10^-9, −215·10^-12</td>
<td>1032·10^-9, −215·10^-12</td>
</tr>
<tr>
<td>Piezoelectric actuator placement {node1,node2}:</td>
<td>{7,8},{11,13},{16,17}</td>
<td>{5,6}</td>
</tr>
<tr>
<td>Sensor placement {node(s)}:</td>
<td>{9,10}, {14,15}</td>
<td>{5}</td>
</tr>
<tr>
<td>Bearing stiffness: trans. (N/m), rot. (N/m.rad):</td>
<td>5.5·10^6, 25·10^-7</td>
<td>100·10^6, 0</td>
</tr>
<tr>
<td>Bearing damping: trans. (N/m.s), rot. (N/m.s.rad):</td>
<td>4.0·10^-3, 200·10^-3</td>
<td>4.0·10^-3, 0</td>
</tr>
<tr>
<td>First three bending modes: frequencies (Hz):</td>
<td>26, 107, 234</td>
<td>84, 254, 461</td>
</tr>
</tbody>
</table>

Table 1. Main properties of the considered rotor systems.

Figure 2. First three mode shapes at zero speed. a) Rotor 1. b) Rotor 2.

2.2 Finite element model code

For analysis, a rotor finite element code is implemented in Matlab. First, a set of N₀ nodes is defined at the rotor center line or z-axis. Only transverse bending and translation are considered, hence each node is assigned only four degrees of freedom, two translational (uᵣ and uᵦ) and two rotational (ϕᵣ and ϕᵦ). The degrees of freedom are reordered in a vector x as 2N₀ complex pairs: x_{2n-1} = uᵣ,n + i uᵦ,n, x_{2n} = ϕᵣ,n - i ϕᵦ,n. It is assumed that the rotor speed ω (angular velocity dφᵣ/dt) is prescribed and varies only slowly.

The rotor and stator properties are assumed to be isotropic with respect to the rotation axis, such that the system properties can be specified for both the x- and y-directions using real [2N₀x2N₀] matrices. Stator stiffness and damping matrices Kᵣ and Cᵣ are obtained by specifying stiffness and damping coefficients at the bearing nodes. Mass and stiffness matrices Mᵣ and Kᵣ for the rotor including attached materials are computed using Timoshenko beam element code (see [1]). To take into account additional rotor-fixed components or flexible coupling elements, inertia and stiffness coefficients are added to Mᵣ and Kᵣ where necessary. A gyroscopic matrix Gᵣ is computed from the nodal and beam element rotational inertia. A hysteretic damping matrix Hᵣ of the rotor is computed by multiplying the beam element stiffness matrices by their respective material loss factors η. In addition, a rotor viscous damping matrix Cᵣ is defined for completeness.

From the rotor mass distribution, a nodal mass load vector m is computed, which is multiplied by the gravitational acceleration g transverse to the rotor length axis in order to obtain the rotor mass loading. A distribution of nodal eccentricities of the centers of mass of the rotor cross-sections is assumed in order to obtain a nodal unbalance vector e. This vector is multiplied by the square of the angular velocity to obtain the resulting unbalance excitation. Rotor-fixed strain gauges are included in the model by assuming that these sensors measure surface strains which are linearly dependent on the rotor cross-section rotations at the z-positions of the sensor ends. Placing four equal strain sensors at element n and connecting these to electrode pair k gives rise to a sensor matrix S ([N₀x2N₀]) with nonzero coefficients S_{k,2n-1} = S_{k,2(n+1)} ≠ 0. A distance sensor on the stator at node n gives rise to a coefficient S_{k,2n-1} ≠ 0.
Rotor-fixed piezoceramic sheets are included in the model by assuming that pairs of oppositely poled sheets are mounted at opposite sides of the rotor, such that electrically charging a pair of sheets effectively induces bending moments on the rotor at the sheet ends. Placing four equal sheets at four sides of the rotor at element n (see Figure 3) and connecting these to electrode pair k gives rise to an actuation matrix \( Q_r \) with nonzero coefficients 
\[
Q_{2n,k} = -Q_{2(n+1),k}.
\]
These coefficients are functions of the sheet properties (width w, length l, thickness t, average distance from rotation axis r, Young’s modulus \( E_{11} \), piezoelectric voltage constant \( d_{31} \) and dielectric constant \( e_{33} \)) and are given by:
\[
q_{kk} = \frac{2 w_n r_n d_{31} E_{11,n}}{\tau_n} \sum_{n=1}^{N_v} \sum_{k=1}^{N_q} \frac{E_{11,n}}{n} 
\]
with \( n = 1..N_v \), \( k = 1..N_q \).

The vector of actuator charges is denoted \( q = q_x + iq_y \) (corresponding to actuation voltages \( v = v_x + iv_y \)). Matrix \( Q_r \) is multiplied by the actuator charges \( q \) in order to obtain the actuation force vector. The capacitances of sets of parallel connected actuator sheets give rise to a diagonal capacitance matrix \( C = \text{diag}(C_{kk}) \). Each actuator set is assumed to be connected to a series circuit of a resistance \( R_k \) and an inductance \( L_k \). This gives rise to electric system matrices \( R^q = \text{diag}(R_k^q) \) and \( L^q = \text{diag}(L_k^q) \). For the rotors considered in this document, all actuator sheets at two opposite sides of the rotor are connected in parallel, hence only one pair of electrodes is present (\( N_v = 1 \)), \( q \) and \( v \) are single complex numbers and \( C^q \), \( R^q \) and \( L^q \) are single scalars. (Note that the model does not contain the ‘dielectric stiffening’ effect which arises because strains in piezoceramics give rise to dielectric displacements which on their turn give rise to opposing strains, because its influence on the total rotor stiffness is considered negligible).

\[
\text{Figure 3. Piezoceramic actuators: a) degrees of freedom, b) geometry and c) wiring in y-plane.}
\]

### 2.3 Equations of motion

The following transformation relates vectorial quantities in the stationary (\(^s\)) and rotating (\(^r\)) frames:
\[
x^s = x^r e^{i \omega t} \\
x^s = (x^r + i \omega x^r) e^{i \omega t} \\
x^s = (\dot{x}^r + 2i \omega x^r - \omega^2 x^r) e^{i \omega t}
\]
(2)

The equations of motion of the rotor system in the stationary (inertial) reference frame are given by ([1]):
\[
M^s \ddot{x}^s + (C^s + C^r - i \omega G^r) \dot{x}^s + (K^s + K^r \pm i H^r - i \omega C^r) x^s = e \omega^2 c^{i \omega t} + m g + Q^q q e^{i \omega t}
\]
(3)

The correct sign in front of the hysteretic damping matrix in Equation 3 can be determined only if the rotor exhibits circular motion of which the direction in the rotating frame is known.

Equation 3 can be transformed to the rotating (non-inertial) reference frame to yield:
\[
M^r \ddot{x}^r + (C^r + C^r + i \omega (2M^r - G^r)) \dot{x}^r + (K^r + K^r \pm i H^r + i \omega C^r - \omega^2 (M^r - G^r)) x^r = e \omega^2 c^{i \omega t} + m g e^{i \omega t} + Q^q q
\]
(4)

Note that, if seen from the rotating frame, the unbalance excitation is constant in direction and the piezoelectric forces are constant if the actuator charges are constant, whereas gravity leads to an excitation which rotates backward.

The rotor dynamic model is extended with a charge balance equation which describes the dynamics of the piezoceramic actuator sets with attached series resistances and series inductances:
\[
L^q \ddot{q} + R^q \dot{q} + C^q q - Q^r T x^r = v
\]
(5)
3 Rotor dynamic stability analysis

3.1 Influence of material damping

The dynamics described by the homogeneous Equation 3, with $\mathbf{C}'=[0]$, is expressed in the state space as:

$$
\dot{\mathbf{z}}^s = \mathbf{A}^s \mathbf{z}^s = \begin{bmatrix} \ddot{\mathbf{x}}^s \\ \dot{\mathbf{x}}^s \end{bmatrix}, \quad \mathbf{A}^s = \begin{bmatrix} -\mathbf{M}^{-1}(\mathbf{C}'-i\omega\mathbf{G}') & -\mathbf{M}^{-1}(\mathbf{K}'+\mathbf{K}' \pm i\mathbf{H}') \\ 1 & 0 \end{bmatrix}
$$

The $4N_n$ eigenvalues $\lambda_n$ of matrix $\mathbf{A}'$ can be determined for both signs in front of $\mathbf{H}'$. The sign of the imaginary part $\Im(\lambda_n)$ is equal to the direction of circular bending motion in the stationary frame (forward or backward, where the spin speed $\omega$ is always forward). For each one of the eigenvalues, the correct sign in front of $\mathbf{H}'$ is therefore $\text{sign}(\Im(\lambda_n)-\omega)$, the direction of circular bending motion in the rotating frame. Using this rule, the correct eigenvalues can be determined for any value of the speed $\omega$ (5).

For the two considered rotor systems, the frequencies of forward and backward bending vibrations and their respective decay rates as functions of the spin speed are shown in Figure 3 in so-called Campbell diagrams. Note that the frequencies are nearly constant functions of the speed for Rotor 1, whereas they change considerably for Rotor 2 due its large rotational inertia. The critical speeds can be read from the crossings of the frequency trajectories $\Im(\lambda_n)$ with the diagonal line $\lambda=\omega$. At these speeds, the corresponding decay rates can be seen to change abruptly. This is due to the deformation of the rotor changing direction, such that the effect of rotor hysteretic damping at ones becomes destabilizing. Although both rotors are found to be stable in the selected speed range, it can be seen that the first forward mode of Rotor 1 has only very little damping at speeds above the first critical speed.

To quantify the relative influence of the hysteretic damping in the piezoceramics attached to both rotor systems, the change in decay rate $\Delta \Re(\lambda)$ at the critical speeds is compared with cases where the loss factor of the piezoceramic is set to zero. It is found that the hysteresis in the piezoceramic raises the rotor hysteretic damping at the first two critical speeds by 115% and 135% percent for Rotor 1 and by 3.7% and 1.3% for Rotor 2, respectively. The large destabilizing effect of hysteresis in the piezoceramic in case of Rotor 1 is simply due to the relatively large amount of piezoceramic material which is added to this rotor. For Rotor 2, the change in decay rate due to added piezoceramic material is negligible.

Figure 4. Campbell diagrams showing influence of material damping. a) Rotor 1. b) Rotor 2.
3.2 Influence of electric dissipation

Connecting resistive and/or inductive elements in series with the electrodes of piezoceramics leads to current dissipation upon actuator straining. This dissipation can be maximized for a structural resonance by selecting the resonance frequency of the connected electric circuit equal to that of the structure [4]. However, very large inductors are required to realize low resonance frequencies in the case of low-capacitance actuators. In the following analysis, it is therefore assumed that only resistors are connected to the actuator electrodes ($L_q=0$). The equations of motion of the rotor with resistive damping are obtained by transforming equation 4 to the stationary frame using fictitious charge states $q_s$, as follows:

$$\dot{q} = q^* e^{-i\omega t} \quad \dot{q}^* = (i \omega - (R^q C^q)^{-1}) q^* + R^q Q^T x^*$$  \hspace{1cm} (7)

The dynamic matrix $A^{sq}$ including rotating resistive damping reads:

$$\dot{z}^{sq} = A^{sq} z^{sq} \quad z^{sq} = \begin{bmatrix} x^s \\ q^s \end{bmatrix} \quad A^{sq} = \begin{bmatrix} A_{11} & A_{12} & -M^{-1} Q^f \\ A_{21} & A_{22} & 0 \\ 0 & R^q Q^T & i \omega - (R^q C^q)^{-1} \end{bmatrix}$$  \hspace{1cm} (8)

The lower right cell of the dynamic matrix indicates that the stabilizing and destabilizing effects of electric current dissipation depend on the frequencies $(R^q C^q)^{-1}$ and on the rotor speed. For structures, maximum damping is obtained by selecting resistances $R_q = (1-k_{31})^{0.5}/(\omega \omega_{31})$, with $\omega$ the frequency of the bending mode to be damped and $k_{31}$ the electromechanical coupling constant of the piezoceramic ([4]).

For the rotor systems under consideration, with $k_{31} = 0.38$, the resistances for maximum damping of the first bending mode at standstill are found to be 42kΩ and 2.8kΩ, respectively. The resulting resistive dissipation has almost no effect in the case of Rotor 2. In contrast, it can destabilize the first forward bending mode of Rotor 1 at speeds exceeding the first critical speed (26 rps (revolutions per second)). Figure 5 shows the decay rates for the first two bending modes of Rotor 1 for four values of $R_q$, in the absence of hysteretic damping ($H_q=0$). The following is noted: 1) The second bending mode shows no change in decay rate because it is orthonormal to the actuator distribution. 2) Similar to viscous rotor damping, resistive damping has no effect on the decay rate of the first bending mode at a speed equal to the first critical speed. 3) For $R_q = 42k\Omega$, resistive damping peaks at standstill and at a speed twice the first critical speed. 4) Resistive damping reduces to zero for resistances approaching zero or infinity.

![Figure 5. Influence of resistive dissipation on decay rates for Rotor 1 (no hysteretic damping).](image)

- a) $R_q = 4.2k\Omega$
- b) $R_q = 42k\Omega$
- c) $R_q = 420k\Omega$
- d) $R_q = 4200k\Omega$
### 3.3 Influence of active velocity feedback

The following state space model is used for analysis of active feedback control:

\[
\begin{align*}
\dot{z}^{sq} &= A^{sq}z^{sq} + B^s q^r \\
q^r &= -K^q y^r \\
y^r &= Y^{sr} z^{sq} \\
B^s &= \begin{bmatrix} M^{-1} Q^r e^{i\omega t} & 0 & 0 \end{bmatrix}^T \\
Y^{sr} &= \begin{bmatrix} Sc^{-i\omega t} & -i\omega S e^{-i\omega t} & 0 \\
0 & S e^{-i\omega t} & 0 \end{bmatrix}
\end{align*}
\]

with \( A^{sq} \) the rotor dynamic matrix as given in Equation 8, \( B^s \) the piezoelectric input gain matrix, \( Y^{sr} \) the output gain matrix relating rotor strain measurements \( y^r \) to stationary states \( z^{sq} \) (or relating stationary measurements to their transformed values \( y^r \) in the rotating frame) and \( K^q \) the feedback gain matrix. It follows directly for the closed loop system matrix \( A^{sqk} \):

\[
\dot{z}^i = A^{sqk} z^i = (A^{sq} + A^{qk}) z^i \quad \text{with} \quad A^{qk} = -B^s K^q Y^{sr}
\]

A common method to reduce vibrations of structures with attached piezoceramics is to impose actuator charges proportional to strain or displacement rate \( \dot{y} \). For the considered rotor systems with single pairs of actuators and sensors, a single gain \( k^q \) can be used to define the feedback matrix \( K^q \) for active rotor damping. The feedback system matrix \( A^{qk} \) is expressed in terms of an ‘active damping matrix’ \( C^{qk} \):

\[
A^{qk} = -B^s K^q Y^{sr} = \begin{bmatrix} -M^{-1} C^{qk} M^{-1} i\omega C^{qk} & 0 \\
0 & 0 \end{bmatrix} \quad \text{with} \quad K^q = \begin{bmatrix} k^q & 0 \\
0 & S \end{bmatrix}
\]

Note that this gain matrix has the same destabilizing effect as rotor viscous damping. For stabilization, the gain matrix should instead be chosen so as to obtain virtual viscous stator damping as follows:

\[
A^{qk} = -B^s K^q Y^{sr} = \begin{bmatrix} -M^{-1} C^{qk} & 0 \\
0 & 0 \end{bmatrix} \quad \text{with} \quad K^q = \begin{bmatrix} k^q & i\omega k^q \\
0 & S \end{bmatrix}
\]

Figure 6 shows the decay rates for the first three modes of Rotor 1 and Rotor 2 for Equation 11 and 12, where moderate feedback gains are used. The model also contains hysteretic rotor damping and series resistances \( R^q = 42k\Omega \) and \( R^r = 2.8k\Omega \). Figures 6a and 6b show that active damping in the rotating frame (Equation 11) has a large destabilizing effect on most of the forward modes of both rotors systems. From figures 6c and 6d, it can be concluded that virtual stator damping (Equation 12) has the desired stabilizing effect on the first forward and backward modes of both rotor systems. (It also has a destabilizing influence on the third forward and backward modes of Rotor 1 due to non-collocation of its sensors and actuators).

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**Figure 6. Decay rates.** Equation 11: a) Rotor 1, b) Rotor 2. Equation 12: c) Rotor 1, d) Rotor 2.
4 Analysis of unbalance response reduction

4.1 Active damping using low frequency feedback control

The main stability issues having been analyzed in the previous sections, it is focused in this section on suppression of unbalance induced vibration at critical speeds. The steady state response at speed \( \omega \) can be solved from Equation 4 if the rotor damping, mass loads and state derivatives are set to zero:

\[
(K' + i\omega C^s - \omega^2(M' - G'))x' = e \omega^2 + Q'q
\]  

(13)

The gyroscopic effect may increase the stiffness so as to avoid the actual occurrence of critical speeds ([1]). At low speeds, however, the gyroscopic effect often has only a small influence and the response at a low critical speed \( \omega = \omega_c \) is determined mainly by the stator damping and actuation forces:

\[
K' + i\omega C^s = \omega^2(M' - G') \Rightarrow i\omega C'x' = e \omega^2 + Q'q
\]  

(14)

The inability of passive stator damping \( C^s \) to limit the unbalance response of (flexible) rotors at critical speeds is the main motivation for the active solution considered here. On their turn, due to their limited elastic stiffness, rotor-fixed piezoceramic actuators can increase the dynamic stiffness (the matrix at the left-hand side of Equation 13) only significantly under resonance conditions. The dynamic stiffness is therefore usually dominated by inertia at high speeds, by stiffness at low speeds and by active rather than passive damping at critical speeds. Analog to the effect of stationary damping, the effective component of virtual active stator damping \( (K^s_i - [k^s_i, i\omega k^s_i]) \) in suppressing the unbalance response at critical speeds is the orthogonal stiffness realized by the imaginary part of the feedback gain matrix \( ([0, i\omega k^s_i]) \).

If the rate of change in the rotor speed is small, such as is the case with rotor systems which accelerate not too fast from standstill to their nominal speeds and back, the rate of change in the response to unbalance is small as well. Since the control system operates on the rotor-fixed quantities \( y' \) and \( q' \), low pass filters with corner frequencies significantly lower than the first critical speed can be placed at the input (or alternatively output) of the control system. The advantage of such an arrangement is that the controller can no more destabilize backward modes or non-collocated high frequency modes. For analysis of the controlled system including low pass filters, the state space equation is extended with the complex state \( j' \) (and its equivalent stationary state \( j_0 \)) to denote the low-pass filtered measurement times the feedback matrix, where the pair of low-pass filters is parametrized by resistance \( R^j \) and capacitance \( C^j \):

\[
j' = (i\omega - (R^j C^j)^{-1})j' - R^j k^s S(x' + i\omega x') \Rightarrow 
\]

\[
\begin{align*}
\dot{z}^s &= A^s z^s \\
z^s &= \begin{bmatrix} x' \\ x' \\ j' \end{bmatrix} \\
A^s &= \begin{bmatrix} A_{11}^s & A_{12}^s & -M^s Q' \\
A_{21}^s & A_{22}^s & 0 \\
k^s R^j S & i\omega k^s R^j S & i\omega - (R^j C^j)^{-1} \end{bmatrix}
\end{align*}
\]  

(15)

Figure 7 shows the decay rates for Rotor 1 and 2 for a low pass filter time constant \( (R^j C^j)^{-1} = 1 \). For Rotor 1, the decay rate of the first bending mode is significantly raised at the critical speed, while the destabilizing influence of active virtual stator damping on the third forward and backward modes has become negligible in the considered speed range. For Rotor 2, the decay rates of the first and second forward modes are significantly raised at the respective critical speeds as well.

The response to unbalance at critical speeds is largely inversely proportional to the decay rate. Figure 8 shows the magnitude of the displacement at nodes 10 and 5 in response to the assumed unbalance distribution for Rotor 1 and 2, respectively (see also Table 2). For the flexible Rotor 1, a problematic displacement in the order of several mm is reduced to the order of two hundred \( \mu m \). For the heavy Rotor 2, a displacement in the order of hundred \( \mu m \) is reduced to the order of several \( \mu m \), which should lead to acceptable bearing loads and stator vibration. (In order to suppress the second forward modes more effectively, the feedback gain should be made a function of the speed in the case of Rotor 2, whereas the actuators would have to be rearranged or wired differently in the case of Rotor 1).
Figure 7. Decay rates for first modes for Equation 15 with $(R^j C^j)^{-1} = 1$, moderate gain. No rotor hysteresis nor current dissipation. a) Rotor 1, b) Rotor 2.

Figure 8. Response to unbalance. No feedback: ···. Feedback: —. a) Rotor 1, b) Rotor 2.

<table>
<thead>
<tr>
<th>Rotor system:</th>
<th>Rotor 1</th>
<th>Rotor 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eccentricity distribution (µm) {nodal direction}:</td>
<td>$100{0,0,0,1,1,1,i,i,1,1,1,1,1,-i,-i,1,1,1,0,0,0}$</td>
<td>$2{0,0,1,0,1,0,i,0,0}$</td>
</tr>
<tr>
<td>Mode:</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Modal unbalance $e$ ($10^4$):</td>
<td>1800</td>
<td>530i</td>
</tr>
<tr>
<td>Actuator modal force ($10^{-4}$/V):</td>
<td>79</td>
<td>0</td>
</tr>
<tr>
<td>Voltages compensating unbalance at critical speeds (V):</td>
<td>61</td>
<td>–</td>
</tr>
</tbody>
</table>

Table 2. Modal unbalance and actuation voltages required for unbalance compensation.

4.2 Active balancing using feedforward control

The compensation of unbalance in off-resonance conditions is normally prohibited by the difference in the distributions of unbalance and the displacements that can be realized by actuation. (An exception being the case where unbalance can be compensated by controlling very little degrees of freedom, i.e. the eccentricity and orientation of a single heavy disk on a flexible shaft). Near critical speeds, however, the rotor response to unbalance is dominated by a single bending mode, such that a modal unbalance excitation $e$ can be compensated by a single modal actuation force $q$ (where both $e$ and $q$ are complex numbers in the notation employed). Table 2 shows for Rotor 1 and 2 the distributions of nodal cross-section eccentricity, and, for the first three modes, the modal unbalance $e$ and modal actuation force per volt $q$ (where the mode shapes of the undamped system at zero speed are employed for computation of these modal parameters). Note that the eccentricities of the light-weight Rotor 1 are rather large along the shaft length, while the much smaller eccentricities of Rotor 2 are located mainly at the centers of the disks.
The actuation voltages required for compensation of modal unbalance (see Table 2) are found to be very reasonable, as voltages in the \pm 200V range can safely be used with piezoceramic sheets of 500\,\mu m thickness and piezoceramic fiber actuators of 200\,\mu m thickness.

The magnitude of modal unbalance can usually be considered constant during a run. An effective approach to the suppression of unbalance induced vibration at critical speeds is therefore to estimate unbalance at speeds not too far nor too near to critical speeds, in order to subsequently charge the actuators to constant voltages so as to compensate the unbalance excitation of the respective mode. Algorithms for this approach were developed and analyzed using time domain simulations and a reduced modal model of Rotor 1. The approach and simulation results are not further detailed in this document. Instead, an example of the results is given in section 5.2 for the corresponding experiment.

### 4.3 Active balancing using self-powered actuators

An interesting question is whether the small constant actuation charges which are required for active balancing could be generated by storing charges induced by harmonic straining of the piezoceramics due to the rotor deflection under its own weight. The extraction of electric power from cyclically strained piezoelectric elements is known as power harvesting (see [5]). The maximum amount of power \( P \) which can be generated from bending of the rotor under its own weight is approximately equal to:

\[
P = \frac{1}{4} \omega k_{31}^2 2 E^g
\]

with \( E^g \) is the maximum strain energy in the piezoceramic material due to the gravity load and \( k_{31} \) the electromechanical coupling constant of the piezoceramics. The strain energy \( E^g \) is easily computed using the finite element model. It is found that at a speed of 60 rps, 0.56mW and 0.96mW of electric power can be extracted from Rotor 1 and Rotor 2, respectively. The magnitude of these values indicates that the deflection of flexible rotors under gravity is not an ideal strain source for power harvesting, the frequency and magnitude of periodic straining usually being rather small. Yet, at the considered speed of 60 rps, in the ideal case of power harvesting in the absence of resistive losses, the actuators on both shafts could in principle all be charged to 200V within 9.6 and 21.5 seconds for Rotor 1 and Rotor 2, respectively. Hence, active balancing using self-powering actuators might be feasible in certain cases. For example, the sensors and control system could be placed on the stator with an external power source, while the charge state of the rotor-fixed actuators could be optimized for minimum unbalance by regulating it through low power optical or electromagnetic interference mechanisms. Experimental work on power harvesting and high voltage generation is described in Section 5.3. As a last point, it should be noted that power harvesting has as a drawback a destabilizing effect on forward modes which is at most equal to that of resistive dissipation optimized for the same speed (see section 3.2). Power harvesting is therefore not advisable in the case of lightly damped rotors with a relatively large amount of attached piezoelectric material, as it may significantly reduce the maximum speed of stable operation.

### 5 Experiments

#### 5.1 Experimental setup

The experimental setup which is used to investigate active balancing and power harvesting for Rotor 1 is shown in Figure 9. The hollow aluminium shaft is driven by an electric motor and drives a slipring assembly. The piezoelectric actuators and strain gauge bridges on the shaft are connected by means of the slipring assembly to high voltage amplifiers and strain gauge amplifiers, respectively, which on their turn are connected to a dSpace\textsuperscript{\textregistered} control system. For reference measurements and calibration of the strain sensors, laser distance sensors are placed at midshaft to measure the rotor displacements. The motor speed is controlled from dSpace\textsuperscript{\textregistered} such that any speed profile can be generated.
The first natural frequency of the rotor is 24.6Hz, which is near to the predicted value of 26Hz. The midshaft displacement introduced by the actuators is 1.15µm/V, where the finite element model predicts 1.13µm/V. In the experiments described in this section, the rotor is slowly accelerated from standstill to a speed of 50 rps in 30 seconds. At the first critical speed, the displacement response to unbalance at midshaft exceeds 10mm and is therefore limited by a catcher bearing to 4mm (see Figure 10a). (It is noted that catcher bearings are used also with the full scale helicopter tail drive shafts. They can give rise to cutting damage of the thin-walled drive shafts and hence to dangerous situations, especially if they are improperly mounted. An active but safe solution could avoid the shaft to hit the catcher bearing at all.)

5.2 Suppression of the unbalance response

A single experiment with the suppression of unbalance induced vibration at the first critical speed is described in this section. The approach used is to: a) estimate the modal unbalance $\varepsilon$ from the strain measurements using an inverse modal model while a scheduled gain $\delta_e(\omega)$ is nonzero, b) apply feedback control according to Equation 15 (low-frequency virtual stator damping) while a scheduled gain $\delta_d(\omega)$ is nonzero and c) apply feedforward control (active modal balancing) on the basis of estimate $\varepsilon$ while a scheduled gain $\delta_b(\omega)$ is nonzero. The results are shown in Figure 10b), with the midshaft displacements at the bottom, the applied voltages at the top and the gains as a function of speed in the center of the figure. The vibration response at the critical speed is reduced from far more than 3800µm to less than 120µm: a reduction of 97%. This magnitude could not have been reduced much further, because the midshaft displacement corresponding to the estimated modally balanced state was 105µm.

5.3 Power harvesting and self-charging of the actuators

To investigate power harvesting, the circuit shown in Figure 11a) is used. While the shaft is rotating at a speed of 60 rps, the voltage over the storage capacitors in this circuit is measured for resistances in a range of 30kΩ to 80kΩ. Figure 11b) shows that a resistance of 70kΩ maximizes the dissipated power at 0.48mW. This corresponds quite accurately to the 0.56mW computed in Section 4.3, considering that no voltage drop over the diodes was taken into account in the prediction. Figure 10c) shows the unbalance response in the presence of electric dissipation. Note that the shaft is only marginally stable in the considered speed range. Slightly reducing the stator damping by modifying the bearing supports actually led to instability at speeds exceeding 40 rps, a risk which was noted at the end of Section 4.3.

For the realization of self-powering active balancing systems, voltage multiplier circuits are considered to be promising components. The circuit in Figure 11c is implemented using ceramic capacitors of 100nF. At a speed of 60 rps, this circuit charges the actuators to ±90V within 20 seconds. During an acceleration from standstill to 30 rps within 15 seconds, the actuator voltages reach only ±40V. Further research is conducted to optimize these circuits and to combine them with sensor solutions.
Figure 10. Unbalance response for a) uncontrolled rotor, b) controlled rotor and c) uncontrolled rotor with electric current dissipation.

Figure 11. a) Circuit for power dissipation measurement. b) Voltage and dissipated power as functions of resistance. c) Voltage multiplier circuit for self-charging of actuators.
6 Conclusions

Two rotor systems with rotor-fixed piezoceramic actuators are analyzed, the first rotor being light-weight and flexible, the second rotor being heavy and more stiff in bending. Numerical stability analysis is performed to determine the relative importance of hysteretic damping in the piezoceramics and of resistive dissipation of electric charges induced in the piezoceramics. It is found that these dissipative mechanisms have a significant effect on the first rotor system, while they have a negligible effect on the second rotor system. Next, stability analysis is performed for the cases of active feedback proportional to rotor strain or displacement rates expressed in either the rotating or stationary frame. It is found that active rotor damping is destabilizing at speeds higher than the first critical speed, while active virtual stator damping is always stabilizing for systems with collocated actuators and sensors. Using computations and experiments, a combination of low-frequency feedback control (virtual stator damping) and feedforward control (modal balancing) is demonstrated to be very effective for the suppression or avoidance of unbalance induced vibration. Finally, computations and experiments are used to determine the amount of electric power which can be generated from cyclic straining of the piezoceramics during rotation of the rotor. It is concluded that self-powering systems for active balancing might be feasible in certain cases.

References


