1 Introduction

Over the past decades, active control methods have become valuable tools besides passive methods for attenuating the sound radiation of structures. The goal of a control system is to cancel the response generated by a disturbance or primary source by introducing one or several secondary controlled source(s). In active noise control (ANC) the secondary sources are speakers. This approach has some successful applications but in general its effect is only local, i.e. the exists only a small "quiet zone". A more promising method for global control which was first introduced by Fuller (Fuller, 1990) is active structural acoustic control (ASAC). In this method actuators are directly attached to the structure, and a reduction of the radiated sound is achieved by changing the vibrational behaviour of the structure. Furthermore, often a control system is used with sensors that measure vibrations instead of acoustic pressure. Piezoelectric materials are often used in ASAC as actuator or sensor, mainly because they can be bonded directly to the structure, not requiring a back support.

Since the introduction of ASAC in the early nineties part of the research carried out worldwide has focused on modelling of structural-acoustic systems. With such models, the performance of several control algorithms can be determined already in the design process of a certain product. Furthermore, models are essential if one wants to determine optimal sensor and actuator locations. A structural-acoustic model describes the interaction between the vibration of a structure and the corresponding sound field. In many cases it is allowed use uncoupled models, i.e. it is assumed that the sound field does not influence the structural vibration. For such analyses first the structural vibration due to any disturbance or control inputs is determined and the calculated response is the input for the acoustic analysis. Several methods are available for modelling of an acoustic field, such as the finite element method (FEM) and boundary element method (BEM), but these will not be discussed in this work.

This work aims at the development of models for predicting the structural vibration of typical ASAC systems with piezoelectric actuators and/or sensors. The finite element method is applied for several reasons. FEM models can be used to model complex structures where no analytical models are available. The dynamical behaviour of a structure in the frequency range of interest for ASAC, i.e. up to 1 or 2 kHz, is in general well described by FEM models. Furthermore, the dynamics of piezoelectric actuators and sensors can be included relatively easily. A major drawback of FEM models is the large number of degrees of freedom (DOF). Since control system design
is an iterative procedure with many simulations, a model requiring long simulation times is not suitable for this purpose. A model reduction technique for models with piezoelectric coupling is presented here which drastically reduces the number of DOF in the model. It was found that standard modal superposition does not give an accurate prediction of the response, especially near the anti-resonance frequencies or zeros. It will be shown that this problem can be overcome with the concept of residual flexibility. 

The reduced model is rewritten in state space form which is often used for control system design. The model reduction is validated against experimental results for a test case consisting of a clamped rectangular plate with one surface bonded piezoelectric patch.

2 Finite Element Modelling Approach

In this section the FEM modelling approach of an arbitrary structure with piezoelectric sensors and actuators is presented. The linear FEM equations of motion for a coupled structural-piezoelectric system are given by

\[
\begin{bmatrix}
M_{uu} & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\ddot{u} \\
\dot{\phi}
\end{bmatrix}
+ \begin{bmatrix}
C_{uu} & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\dot{u} \\
\dot{\phi}
\end{bmatrix}
+ \begin{bmatrix}
K_{uu} & K_{u\phi} \\
K_{\phi u} & K_{\phi\phi}
\end{bmatrix}
\begin{bmatrix}
u \\
\phi
\end{bmatrix}
= \begin{bmatrix}
f \\
g
\end{bmatrix},
\]

(1)

where \(u\) is the vector with the nodal structural displacements and rotations, and \(\phi\) is the vector with nodal voltages. Matrices \(M_{uu}\), \(C_{uu}\), and \(K_{uu}\) are respectively the structural mass, damping and stiffness matrix. The piezoelectric coupling arises in the piezoelectric stiffness matrices \(K_{u\phi}\) and \(K_{\phi u} = K_{u\phi}^T\) and the dielectric stiffness matrix \(K_{\phi\phi}\). The external loads are stored in \(f\), i.e. the vector with nodal structural forces, and \(g\), which is the vector with nodal electrical charges. The symbols ‘\(\dot{\ }\)’ and ‘\(\ddot{\ }\)’ denote the first and second time derivatives. The main assumption made in the derivation of Eqs. (1) is that the electrical field behaves quasi-statically. Note that \(u\) contains the nodal displacements of the structure as well as the nodal displacements of the piezoelectric material. There exists various formulations for beam, plate and solid piezoelectric finite elements, of which an overview can be found in Benjeddou (Benjeddou, 2000).

2.1 Model Reduction

In general FEM models contain a large number of degrees of freedom (DOF). This feature makes such models not suitable for the design of a controller, since control system design is often an iterative procedure which requires many simulations. In this section a model reduction technique is presented for a dynamical system described by Eqs. (1). In a control setup piezoelectric materials can be used either as actuator or sensor. In case the patch is used as actuator the electrode potential is prescribed, whereas for a sensor the potential is free. The charge can also be used as sensor signal, but this will not be discussed in this work. It is convenient to divide the vector with the nodal voltages into two parts: \(\phi = \{\phi^p \ \phi^f\}^T\), where \(\phi^p\) is the vector with prescribed nodal voltages, and \(\phi^f\) contains the free nodal voltages. Substitution of this vector into
Eqs. (1) gives
\[
\begin{bmatrix}
M_{uu} & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\ddot{u} \\
\ddot{\phi}
\end{bmatrix}
+ \ldots +
\begin{bmatrix}
K_{uu} & K_{up}^p & K_{uf}^f \\
K_{pu}^p & K_{pp} & K_{pf}^f \\
K_{fu}^p & K_{fp} & K_{ff}
\end{bmatrix}
\begin{bmatrix}
\dot{u} \\
\dot{\phi}
\end{bmatrix}
= \begin{bmatrix}
f \\
g_p \\
g_f
\end{bmatrix}.
\]

For ease of writing the damping forces are omitted in this equation. The reduction method discussed here is a mode superposition method. To perform a modal analysis the electrical DOF are eliminated from the system. With the third row in Eqs. (2) the free voltages can be written in terms of the displacements, the free charges, and prescribed voltages:
\[
\phi' = (K_{\phi\phi}^{ff})^{-1} \left[ g_f - K_{\phi u}^f u - K_{\phi\phi}^p \phi_p \right].
\]

Substitution of this equation into the first row in Eqs. (2) results to an equation of motion in terms of the structural displacements, i.e.
\[
M_{uu} \ddot{u} + C_{uu} \dot{u} + K_{uu}^* u = f^*,
\]
where the effective stiffness matrix $K_{uu}^*$ is defined as,
\[
K_{uu}^* = K_{uu} - K_{\phi u}^f (K_{\phi\phi}^{ff})^{-1} K_{\phi u}^f.
\]

The effective force vector $f^*$ is defined as
\[
f^* = f - K_{\phi u}^f (K_{\phi\phi}^{ff})^{-1} g_f - K_{uu}^{pp} \phi_p,
\]
where $K_{uu}^{pp} = K_{uu}^{pp} - K_{\phi u}^f (K_{\phi\phi}^{ff})^{-1} K_{\phi p}^f$. This equation shows that all applied electrical loads, i.e. nodal charges and prescribed nodal voltages, are transformed to structural loads. It will be clear that once the structural response is determined, the free voltages can be calculated with Eq. (3). If necessary, the nodal charges corresponding with the prescribed voltages can be calculated with the second row in Eqs. (2).

In the mode superposition method, the response is expanded in terms of the undamped eigenvectors or mode shapes (modes) of the problem. In case the undamped free vibration is considered ($f^* = 0$), and harmonic time dependency is assumed ($u = \hat{u} e^{j\omega t}$), Eq. (4) reduces to the generalized eigenvalue problem:
\[
\omega^2 M_{uu} \hat{u} = K_{uu}^* \hat{u},
\]
where $\omega$ is the angular frequency of vibration. The solution of this eigenvalue problem comprises $n$ angular eigenfrequencies $\omega_i$ and corresponding eigenvectors $\hat{u}_i$ ($i = 1 \ldots n$), where $n$ is the total number structural DOF in the model. The matrix with natural eigenfrequencies and the modal matrix with structural responses are defined as
\[
\Omega = \begin{bmatrix}
\omega_1 & 0 & \cdots & 0 \\
0 & \omega_2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \omega_n
\end{bmatrix}, \quad \Psi_u = \begin{bmatrix}
\hat{u}_1 & \hat{u}_2 & \cdots & \hat{u}_n
\end{bmatrix}.
\]
The mode shapes are normalized with respect to the mass matrix, thus satisfying

\[ \Psi_u^T M_{uu} \Psi_u = I, \quad (9) \]

\[ \Psi_u^T K_{uu}^\star \Psi_u = \Omega^2, \quad (10) \]

where \( I \) is the identity matrix. The modal responses of the free voltages follow from Eq. (3) for the unforced case, i.e. \( g^f = \phi^p = 0 \). The modal matrix with electrical responses is thus defined as

\[ \Psi_f = - (K_{\phi^f})^{-1} K_{\phi^u} \Psi_u. \quad (11) \]

Following the method of modal superposition, the solution of Eq. (4) is written as

\[ u = \sum_{i=1}^{n} \hat{u}_i q_i = \Psi_u q, \quad (12) \]

where \( q \) is the column vector with modal participation factors or generalized co-ordinates. Substitution of this solution into Eq. (4) and multiplying through by \( \Psi_u^T \) leads to the generalized equation of motion:

\[ I \ddot{q} + 2 \Xi \Omega \dot{q} + \Omega^2 q = \Psi_u^T f^*, \quad \text{or,} \]

\[ \ddot{q}_i + 2 \xi_i \omega_i \dot{q}_i + \omega_i^2 q_i = \hat{u}_i^T f^*, \quad i = 1 \ldots n. \quad (13) \]

It is here assumed that the damping is classical, which implies that the modal damping matrix \( \Xi \) is diagonal, i.e. \( \Xi = \text{diag}(\xi_i) \) with \( \xi_i \) the modal damping ratio for mode \( i \). The notation in Eqs. (13) and (14) is not valid if the damping is not classical, although a transformation to generalized co-ordinates is still possible. Often it is the goal to determine the response in a limited frequency band \( \omega \in [0, \omega_b] \). Conveniently, it is not necessary to take into account all modes of the system for modal superposition. A good estimate of the response in the frequency range of interest is obtained when only a small number of mode shapes is taken into account. When \( m \) eigenfrequencies and eigenvectors are included with \( m \ll n \), the structural response in the frequency domain is approximated by

\[ \hat{u} \approx \sum_{i=1}^{m} \frac{\hat{u}_i \hat{u}_i^T f^*}{-\omega^2 + 2j \xi_i \omega_i \omega + \omega_i^2}. \quad (15) \]

### 2.2 Residual Flexibility

A consequence of truncating the modal expansion is that it can lead to errors in prediction the response near the anti-resonance frequencies, or in control theory referred to as zeros (Preumont, 1999). This is because the mode shapes with eigenfrequencies outside the frequency range of interest also contribute to the frequency response in the range \([0, \omega_b]\). This contribution is especially significant in the off-resonance regions. The concept of \textit{residual flexibility} improves the accuracy of the truncated expansion. It is most easily explained when considering the frequency domain response. The exact solution of Eq. (4) when all variables show harmonic time dependency can be written as

\[ \hat{u} = \sum_{i=1}^{m} \hat{u}_i \hat{q}_i + \sum_{i=m+1}^{n} \hat{u}_i \hat{q}_i, \quad \text{where} \quad \hat{q}_i = \frac{\hat{u}_i^T f^*}{-\omega^2 + 2j \xi_i \omega_i \omega + \omega_i^2}. \quad (16) \]
In the case of standard modal reduction, the second right-hand-side term is neglected (see Eq. (15)). Since the maximum frequency in the range of interest \([0, \omega_b]\) is much smaller than the natural eigenfrequencies for modes satisfying \(i > m\), the system response is well approximated by

\[
\hat{u} \approx \sum_{i=1}^{m} \frac{\hat{u}_i \hat{u}_i^T f^*}{-\omega_i^2 + 2j\xi_i \omega_i + \omega_i^2} + \sum_{i=m+1}^{n} \frac{\hat{u}_i \hat{u}_i^T f^*}{\omega_i^2}.
\]  

(17)

In this approximation the high frequency modes \((i > m)\) contribute statically to the system response, whereas the low frequency modes \((i \leq m)\) respond dynamically. The second right-hand-side term is called the residual flexibility. In general only the eigenfrequencies and corresponding mode shapes for \(i = 1 \ldots m\) are calculated when a modal analysis is performed with a FEM package. The residual flexibility can be expressed in terms of the static response and low frequency mode contributions. The modal expansion of the static response simply follows after inserting \(\omega = 0\) into Eq. (16). Now the approximate solution becomes

\[
\hat{u} \approx \sum_{i=1}^{m} \frac{\hat{u}_i \hat{u}_i^T f^*}{-\omega_i^2 + 2j\xi_i \omega_i + \omega_i^2} + \hat{u}_0 - \sum_{i=1}^{m} \frac{\hat{u}_i \hat{u}_i^T f^*}{\omega_i^2}.
\]  

(18)

Solution \(\hat{u}\) is now written in terms of modes \(i = 1 \ldots m\) and the static response \(u_0\). So the cost for a more accurate approximation is that a static response analysis has to be performed. The approach was here explained for a structure which has no rigid body modes. For a discussion on systems with rigid body modes, the reader is referred to Preumont (Preumont, 1999). In matrix-vector notation, Eq. (18) reads

\[
u \approx \tilde{\Psi}_u \tilde{q} + \hat{u}_0 \tilde{\Omega}^{-2} \tilde{\Psi}_u^T f^* ,
\]  

where \(\tilde{\Omega}^{-2} = \text{diag}(\omega_i^{-2})\). In this equation the \(^\sim\) symbol indicates that only modes \(1 \ldots m\) are included. If this equation is inserted into Eq. (3) and some terms are rearranged, the following expression for the response of the free voltages is found:

\[
\phi^f \approx \tilde{\Psi}_f \tilde{q} + \phi^f_0 - \tilde{\Psi}_f \tilde{\Omega}^{-2} \tilde{\Psi}_u^T f^* ,
\]  

(20)

where \(\phi^f_0\) is the vector with the static response of the free voltages.

### 2.3 State Space Representation

The foregoing model is rewritten in state space form since this form is more convenient for control system design. The general state space representation of any linear dynamical system is given by

\[
\dot{x} = Ax + Bv,  \quad (21)
\]

\[
y = Cx + Dv.  \quad (22)
\]

The modal participation factors are used to define state vector; i.e. \(x = \{\tilde{q} \tilde{q}^T\}^T\). The input vector containing all external inputs acting on the system, i.e. forces, charges and prescribed voltages is defined as \(v = \{f \ g^f \phi^p\}^T\). The system matrix and input matrix are defined as

\[
A = \begin{bmatrix} 0 & I \\ -\tilde{\Omega}^2 & -2 \tilde{\Xi} \tilde{\Omega} \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 \\ \tilde{\Psi}_u^T & \tilde{\Psi}_f^T & -\tilde{\Psi}_u^T \tilde{K}_{u_0}^p \end{bmatrix}. \quad (23)
\]
Here the output vector is defined as \( y = \{ u \ \phi \}^T \). The nodal displacement vector is here an output, but the velocity or acceleration can also be used. The corresponding output and feedthrough matrix are given by

\[
C = \begin{bmatrix} \Psi_u & 0 \\ \Psi_f & 0 \end{bmatrix}, \quad D = \begin{bmatrix} U_0 \\ \Phi_0 \end{bmatrix} - \begin{bmatrix} \Psi_u \\ \Psi_f \end{bmatrix} \Omega^{-2} \begin{bmatrix} \Psi_u^T \\ \Psi_f^T \end{bmatrix} - \begin{bmatrix} \Psi_u^T K \phi \end{bmatrix},
\]

(24)

The residual flexibility is accounted for in the feedthrough matrix. In matrices \( U_0 \) and \( \Phi_0 \) every column \( i \) represents the static response when input \( i \) is equal to one whereas all other inputs are zero. In general, the number of inputs and outputs are much smaller than the number of DOF in the model. In the foregoing this aspect was not accounted for, i.e. the modal matrices contain the response of all DOF. The state space model can be written more compact when only those parts of the modal matrices are included which correspond with input or output DOF. The results presented in the next section were determined with a state space model implemented in such a way.

3 Validation

The setup for validation of the modelling approach is a clamped rectangular aluminium plate with one surface bonded piezoelectric patch (see Figure 1). The plate has dimensions \( 490 \times 245 \times 1.2 \text{ mm}^3 \). The piezo patch is located at the plate center and has dimensions \( 50 \times 30 \times 1.0 \text{ mm}^3 \). The material properties of the plate and piezo patch are given in Appendix A. In this section the open loop responses for two load cases are considered. In the first case the plate is excited in the transverse direction by a point force at \((x, y) = (70, 154) \text{ mm}\). The second load case is a voltage applied to the upper electrode of the piezoelectric patch whereas the electrode on the plate surface is grounded. In the latter case the transfer to the structural response at some discrete locations is considered. When the system is excited by the point force also the electric potential across the patch is measured (open circuit measurement).

![Figure 1: Test setup: clamped plate with one surface bonded piezoelectric patch.](image)

3.1 Numerical model

The model was constructed in the commercial finite element package ANSYS. The plate is modelled with linear quadrilateral shell elements (\texttt{SHELL63}). In ANSYS no shell type elements with piezoelectric coupling are available, and therefore the piezo patches are modelled with linear cubic solid elements (\texttt{SOLID5}). In order to couple the solid and shell elements coupling conditions are introduced equivalent with perfect
bonding between the plate and the piezo patch. The state space formulation of the reduced model has been implemented in MATLAB/SIMULINK. Besides the eigenfrequencies and modeshapes also piezoelectric stiffness matrices are required to define the reduced model. A number of routines to import the ANSYS model data and results were implemented in MATLAB.

A total of 15 modes are used in the reduced model, i.e. the state space representation has 30 states. Furthermore a static analyses is required for every input to determine the residual mode. It is noted that two different models are used to determine the response for the two load cases. For the first load case (primary excitation) the voltage on the top electrode of the piezo patch are coupled but free. This boundary condition is realized in practice if a measurement instrument with infinite electrical impedance is used. When the plate is excited by the piezo patch, short circuit boundary conditions are introduced, i.e. all nodal voltages on the top electrode are set to zero.

3.2 Experimental setup

The experimental setup used to validate the modelling approach is shown in Figure 2. The plate is clamped in a aluminium frame with 20 clamping bolts. An electromagnetic shaker introduces the point force on the plate. A force transducer measures the force which is applied to the plate. The out of plane vibration of the plate is measured with an accelerometer. Note that the acceleration signal can be easily integrated in the frequency domain to obtain the displacement. When the piezo patch serves as actuator, it is driven by a voltage amplifier. The SIGLAB DSP system is used for data acquisition.

3.3 Results

In this section simulated and measured transfer functions for the two load cases are compared. The frequency range of interest which is considered includes the first nine eigenfrequencies. In all figures three transfer functions are compared; (i) simulated with the reduced model including residual flexibility (blue, dashed line), (ii) simulated with the reduced model using the standard truncation (green, dash-dot line), (iii) measured (red, solid line). It is noted that no model updating techniques were used to improve the correspondence between model and measurement, i.e. in the simulations the material properties were used according to the specifications of the suppliers.

The results for the first load case are shown in Figure 3. Figure 3(a) compares the transfer function between the point force and the transverse displacement at the
location where the force is applied. The figure shows that neglecting the contribution of high frequency modes is allowed. The difference between the results for both models is very small, although it is visible near the anti-resonance frequencies. Note that an anti-resonance is present between every pair of resonances since the response location matches the excitation location. The figure also shows that the models are in reasonable agreement with the measurement. The resonance frequencies do not exactly correspond, and the shift between model and measurement is not equal for all modes.

It was found during the measurements that the results are very sensitive to the torque applied to the clamping bolts. Furthermore, measured transfer function for a setup consisting only of the plate showed the same differences. It is therefore suggested that the mismatch between model and experiment is not due to unmodelled interaction between the plate and the piezoelectric patch (such as the bonding layer).

Figure 3: Transfer functions between point force and out-of-plane displacement (a,b) and voltage measured on top electrode of piezo patch (c).

Figure 3(a) compares the transfer between the point force and the transverse displacement at the center of the piezo patch (also plate center). According to the model only three resonance frequencies are visible in this transfer function. This is because all other modes in the frequency range of interest have a node line through this response point. The measurement results show that the modes are still excited in practice. This is because the experimental setup is not exactly symmetric, mainly because of the mass introduced by the force transducer. For this load case the piezoelectric patch was used as sensor. The response of the voltage to the point force is shown in Figure 3(c). Again
the correspondence between the simulation and experimental results are good.

The results for the second load case are shown in Figure 4. The experimental results were determined with a setup without the shaker and force transducer. A comparison of the results for this load case and the results shown in Figure 3 reveals that the overall vibration level is about two orders of magnitude smaller for this load case. In other words, the secondary input (piezo voltage) that is required to suppress the response due to the primary input (point force) is relatively high. This is a well known disadvantage of piezoelectric actuators. Figures 4(a) and 4(b) show the good correspondence between the experimental and simulation results. In Figure 4(a) it can be seen that the effect of residual flexibility is negligible for this transfer function. However, in Figure 4(a) there is a large difference between the simulation results. Here the residual flexibility effect is considerable. The difference between the models introduced by the residual flexibility is given by

$$\Delta \hat{u} = u_0 - \sum_{i=1}^{m} \hat{u}_i \hat{u}_i^T \frac{f^*}{\omega_i^2},$$

where the first right-hand-side term is the static response and the second right-hand-side term is the contribution of the low frequency modes to the static response. This equation shows that the residual flexibility effect will be significant if the static response is of the same order as the dynamic response, and, the static response is not well expanded in terms of the lower modes. Since for this load case the static response is the much larger near the point location than near the piezo patch location, the residual flexibility effect is more clearly visible in Figure 3(a).

The results presented in this section indicate that the models are in good correspondence with the measurements, and thus the models can be applied for control system design in active structural acoustic control.

4 Conclusions

An approach for the dynamical modelling of structural vibration of structures with piezoelectric actuators and/or sensor was presented. The finite element method is applied, mainly because this method is suitable to model complex structures. A simu-
lation model suitable for control system design is obtained after the FEM equations of motion are reduced with a mode superposition method. The contribution of high frequency modes to the low frequency response is accounted for by the residual flexibility. The modelling approach was validated for a test case consisting of a clamped rectangular plate with one piezoelectric patch. The residual flexibility can improve the model, especially when transfer functions are considered with coinciding response and excitation location. The results show that the models are in good correspondence with the measurements, and thus the models can be applied for control system design in active structural acoustic control.

References


A TEST CASE PROPERTIES

The results presented in this paper were obtained with the dimensions and material parameters given in Tables 1 and 2.

Table 1: Plate properties

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimensions ($l_{p,x} \times l_{p,y} \times t_p$)</td>
<td>490 × 245 × 1.2 mm$^3$</td>
</tr>
<tr>
<td>Density</td>
<td>2710 kg m$^{-3}$</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>70.0 · 10$^9$ N m$^{-2}$</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.3</td>
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</table>

Table 2: Piezo patch properties

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimensions ($l_{pe,x} \times l_{pe,y} \times t_{pe}$)</td>
<td>50 × 30 × 1.0 mm$^3$</td>
</tr>
<tr>
<td>Location ($x_{pe}, y_{pe}$)</td>
<td>220, 107.5 mm</td>
</tr>
<tr>
<td>Density</td>
<td>7760 kg m$^{-3}$</td>
</tr>
<tr>
<td>Elasticity coeff. ($S_{11}^E, S_{33}^E, S_{12}^E, S_{13}^E, S_{44}^E, S_{66}^E$)</td>
<td>1.68, 1.90, −0.57, −0.71, 5.10, 4.50 (· 10$^{-11}$) m$^2$ N$^{-1}$</td>
</tr>
<tr>
<td>Piezoelectric coeff. ($d_{31}, d_{33}, d_{15}$)</td>
<td>−2.14, 4.23, 6.10 (· 10$^{-10}$) m V$^{-1}$</td>
</tr>
<tr>
<td>Dielectric coeff. ($\epsilon_{11}, \epsilon_{22}$)</td>
<td>9.82, 7.54 (· 10$^{-9}$) F m$^{-1}$</td>
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