A Stochastic Bin Paking Model for Logistical Capacity Planning

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1 Introduction

Most modern companies are part of international economic networks, where goods are produced under different strategies (made-to-stock or made-to-order), then transported over long distances, and stored for variable periods of time at different locations along the considered network. In this context, having enough capacity to properly perform crucial activities such as the supply, storage and distribution of goods is paramount if a company is to be competitive. It should be noted that these activities are often performed by first consolidating goods in appropriate bins, which are then either stored at warehouses or shipped using multiple vehicles through various transportation modes. Companies thus face the problem of having to plan for enough capacity, expressed here in terms of bins, to be available at different locations throughout their network and for different periods of time. Given that logistical activities are often subcontracted, capacity planning entails negotiations with third party logistics firms (3PL's) to book the needed capacity. The results of these negotiations often take the form of medium term contracts that specify both the capacity to be used (the quantity and type of the bins) and the logistical services to be performed (storage, transportation, bin operations, etc.). Therefore, bin packing models represent important decision support tools for logistics managers, who bear the responsibility for these tactical planning decisions.

Although bin packing optimization problems have been extensively studied, see [2], this research has mainly been conducted under the assumption that all necessary information concerning the different parameters used to model the problems is known and readily available (i.e., deterministic parameters). Given the time lag that usually exists between the capacity planning decisions made and the operational decisions that are taken and that define how the planned capacity is used, such an hypothesis in unlikely to be observed. Therefore, we propose a new stochastic bin packing model, referred to as the stochastic variable size and cost bin packing (SVSCBP) model, that explicitly takes into account the different sources of uncertainty that appear in capacity planning problems. The model is based on a two-stage stochastic programming formulation that separates the tactical capacity planning decisions (i.e., the *a priori* plan), that are taken when certain parameters are unknown and dependent on a random event, from the operational decisions (i.e., the *recourse* decisions), that define the adjustments made whenever the capacity is used and all the information becomes known. To efficiently solve the proposed model, we develop a progressive hedging (PH) based heuristic, which uses as the scenario subproblem solver the algorithms proposed in [1]. The remainder of this abstract is organized in two sections. In section 2, a detailed formulation of the considered problem is provided. We then conclude with section 3 by providing a general description of the solution approach proposed and of the experimental plan that will be conducted.

2 The stochastic variable size and cost bin packing model

Let us consider the problem where a given company must produce a tactical capacity plan by choosing a set of appropriate bins (i.e., first stage decisions), which are defined according to their specific volume and fixed cost. Fixed costs are used here to represent the specific rates offered by the different 3PL's for bins of different sizes. Selected bins are to be available to the company under the terms of a contract specifying the parameters of usage. The capacity plan produced is then repeatedly applied to perform specific logistical operations for a given period of time (e.g., organize a maritime shipment of goods every week for the next semester). In this context, the bins that are included in the capacity plan are chosen in advance and not knowing exactly what items will eventually be packed. Therefore, whenever packing operations are performed (i.e., second stage decisions), it may occur that the planned capacity is insufficient. When such a case occurs, extra capacity must be purchased to properly perform the necessary operations. We thus consider that extra bins may be added during the operational phase (i.e., second stage decisions), provided at a premium by the 3PL's (i.e., fixed costs defined according to the spot market).

This problem can now be formulated as the SVSCBP model. Let $\mathcal{J}$ define the set of available bins in the first stage of the problem where, $\forall j \in \mathcal{J}$, f_j and V_j are respectively the fixed cost and volume associated with bin j. Let set Ω be the sample space of the random event, where $\omega \in \Omega$ defines a particular realization. Let vector ξ contain all stochastic parameters defined in the model and $\xi(\omega)$ be a given realization of this random vector. If we define the first stage variables as follows: $y_j = 1$ if bin $j \in \mathcal{J}$ is selected, 0 otherwise; then the SVSCBP model is formulated as:

$$\min \sum_{j \in \mathcal{J}} f_j y_j + E_{\xi}[Q(y, \xi(\omega))] \tag{1}$$

$$\text{s.t. } y_j \in \{0,1\}, \forall j \in \mathcal{J}, \tag{2}$$

where $Q(y, \xi(\omega))$ equals the extra cost that is paid for the capacity that is added in the second stage of the problem, given the tactical capacity plan y and the realized vector $\xi(\omega)$. The objective function (1) then minimizes the sum of the total fixed cost incurred for the tactical capacity plan and the expected cost associated with the extra capacity added during operations, while constraints (2) impose the integrality requirements on y.

To formulate $Q(y, \xi(\omega))$, we consider the following stochastic parameters in $\xi(\omega)$: $\mathcal{I}(\omega)$, the set of items to be packed; $v_i(\omega)$, $i \in \mathcal{I}(\omega)$, the item volumes; $\mathcal{K}(\omega)$, the set of available bins in the second stage; and $f_k(\omega)$, $k \in \mathcal{K}(\omega)$, the cost of bins available in the second stage. The second stage variables are defined as follows: $z_k = 1$ if bin $k \in \mathcal{K}(\omega)$ is selected, 0 otherwise; and $x_{ij} = 1$ if item $i \in \mathcal{I}(\omega)$ is packed in bin $j \in \mathcal{J}$, 0 otherwise; and $x_{ik} = 1$ if item $i \in \mathcal{I}(\omega)$ is packed in bin $k \in \mathcal{K}(\omega)$, 0 otherwise. We now define function $Q(y, \xi(\omega))$ as follows:

$$Q(y, \xi(\omega)) = \min \sum_{k \in \mathcal{K}} f_k(\omega) z_k \tag{3}$$

$$\text{s.t. } \sum_{j \in \mathcal{J}} x_{ij} + \sum_{k \in \mathcal{K}(\omega)} x_{ik} = 1, \forall i \in \mathcal{I}(\omega) \tag{4}$$

$$\sum_{i \in \mathcal{I}(\omega)} v_i(\omega) x_{ij} \leq V_j y_j, \forall j \in \mathcal{J} \tag{5}$$

$$\sum_{i \in \mathcal{I}(\omega)} v_i(\omega) x_{ik} \leq V_k z_k, \forall k \in \mathcal{K}(\omega) \tag{6}$$

$$x_{il} \in \{0,1\}, \forall i \in \mathcal{I}(\omega), \forall l \in \mathcal{J} \bigcup \mathcal{K}(\omega) \tag{7}$$

$$z_k \in \{0,1\}, \forall k \in \mathcal{K}(\omega). \tag{8}$$

The objective function (3) minimizes the cost associated with the extra bins selected. Constraints (4) ensure that each item is packed in a single bin. Constraints (5) and (6) make sure that the total volume of items packed in each bin does not exceed the bin volume. Finally, (7) and (8) impose integrality requirements on all second stage variables.

3 Solution approach and experimental plan

The first step towards solving model (1)-(2), is to obtain a manageable approximation of function $E_\xi[Q(y, \xi(\omega))]$. Therefore, sampling is applied to obtain a set of representative scenarios (i.e, set $\mathcal{S}$), which are used to approximate the expected cost associated with the second stage. The PH solution approach, as recalled in [3], is then used to solve the approximated problem. Following this approach, scenario decomposition is applied by relaxing the non-anticipativity constraints, which impose that a single tactical capacity plan y is used for all considered scenarios. By doing so, the SVSCBP model can be decomposed by scenarios thus obtaining, $\forall s \in \mathcal{S}$, a deterministic packing problem. This enables the use of the efficient heuristics proposed in [1] to solve these scenario subproblems.

A solution to the approximated problem is then constructed by iteratively applying the following steps: 1) aggregate the scenario solutions obtained to produce a reference point (the average over all scenario solutions is traditionally used as the reference point), 2) using an augmented Lagrangean strategy, penalize the differences observed between the scenario solutions and the reference point (i.e., the objective function of each subproblem is updated in such a way as to induce a single tactical capacity plan for all scenarios), 3) solve anew the updated scenario subproblems. In the presence of integer variables, the PH algorithm may not converge given that the average over all scenario solutions is not necessarily integer. Furthermore, considering that the algorithm solves at each iteration a series of deterministic packing problems (i.e., one for each scenario), the overall solution process may become very time-consuming as the number of iterations increases. To address these issues, a series of algorithmic innovations have recently been proposed in [3]. These innovations include different strategies to update the objective functions of the subproblems, improve the convergence of the algorithm, provide termination criteria or detect cyclic behavior in the solution process. These features are used to design an efficient solution approach for the present problem. It should also be noted that the PH algorithm proposed is implemented using the PySP library, which provides an effective platform on which to develop such procedures, see [4].

Finally, the experimental plan is used to analyze both the impact that diversity (regarding 3PL's) has on capacity planning and the effect of considering different levels of variability and correlation on the stochastic parameters related to items, as well as to measure the consequences of volatility in the spot market.

References

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