A FRACTAL MODEL FOR THE LIGHTNING INDUCED CURRENT ON A TRANSMISSION LINE

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Abstract. This work presents a model of a tortuous lightning channel, and a study of the consequent electromagnetic coupling to a transmission line. We analyse the fractal dimension of the induced current on the line, compared with the fractal dimension of the lightning channel and the impinging electromagnetic field on the line. This comparison confirms our previous studies pointing out that the channel fractal dimension and the electromagnetic field fractal dimension are related (in particular, for typical lightning parameters they are the same). A comparison of simulated fields and line currents with experimental measurements is also attempted.

I. Introduction

Lightning electromagnetic disturbances on electronic devices were investigated in many papers. Most of them are about lightning models which assume a straight vertical discharge channel. A description of a tortuous discharge was presented for the first time in [1]. This model was not adopted in successive work, possibly for the difficulty to define a good algorithm of generation for tortuosity. Only recently, a fractal description of the discharge channel was presented, with encouraging results [2]. In this work, we discuss the effects of the electromagnetic field of a fractal lightning which couples to a transmission line. We evaluate the lightning induced current on a transmission line and we analyse it in terms of fractal dimensions. These results constitute an extension of those presented in [2].

Finally, we compare simulated results with experimental measurements. Both electric field and induced current on a line compare well with experimental measurements. A particularly good agreement between the shape of simulated electric field and experimental measurement (which does not appear for straight models of the lightning channel) was noticed.

II. Model of lightning and induced field

In this section, we briefly present the lightning model adopted in our work, with a specific attention to the geometrical shape of the discharge channel and to the base current model of the return stroke.

The lightning channel is modelled by a fractal curve. Its realization in 3D space is obtained as a composition of two fractal curves in 2D space, which are interpreted as the channel projection in two vertical planes orthogonally oriented. Therefore the channel results in a sequence of rectilinear segments, having different directions and different lengths [2]. We generate the fractal channel by employing Random Midpoint Displacement (RMD) algorithm [3]. The geometric characteristics of the channel are determined by the lightning height, the maximum deviation from lightning striking point (that identifies a cylinder in which the lightning lies), and the fractal dimension of the lightning channel. Those three parameters do not identify a particular lightning channel but a class of lightning channels, because RMD algorithm generates an infinite number of curves with the same fractal dimension. Since the total height of the channel (ground-to-cloud) is assumed to be 6 km, the number of segments that constitute the channel is several hundreds.

We assume double-exponential base current, as commonly employed in the literature. This model allows us to fix the rise and fall time of the current, which are useful parameters to give a fractal description of the lightning induced current on a transmission line. We assume Modified Transmission Line model (MTL) for return stroke current [4]. Lightning current speed is c/3.

The lightning electromagnetic field impinging on the transmission line is the sum of the contributions due to each segment of the fractal channel. According to Green's formulation [5], the lightning current on each segment generates an electric field:

$$\underline{E}_i(\underline{r},\omega) = -j\omega\mu\frac{e^{-jkr_i}}{4\pi r_i}\left[A_i K_{ri}\hat{r} + B_i\left(K_{\theta i}\hat{\theta} + K_{\phi i}\hat{\phi}\right)\right]\cdot \int_{-l_i/2}^{l_i/2} I(l,\omega)e^{jka_i l}\,dl \qquad (1)$$

where geometry is defined in Fig. 1, and:

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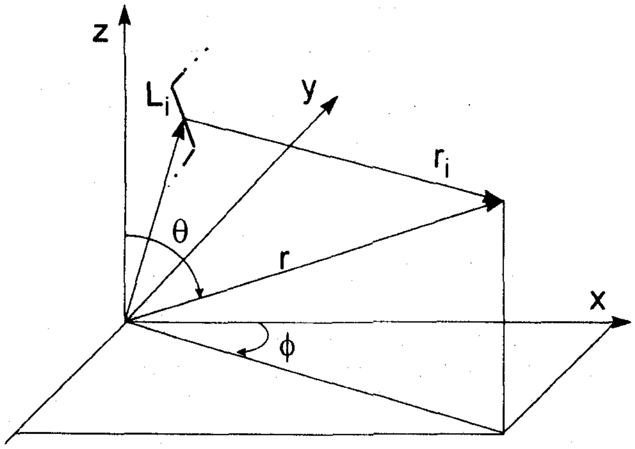

Figure 1. Frame of reference for lightning channel and radiated field.

$K_{ri} = a_i = \hat{r}\cdot\hat{s}_i$; $\quad K_{\theta i} = \hat{\theta}\cdot\hat{s}_i$; $\quad K_{\phi i} = \hat{\phi}\cdot\hat{s}_i$;

$$A(kr) = \frac{2j}{kr} + \frac{2}{k^2 r^2}$$

$$B(kr) = 1 - \frac{A(kr)}{2}$$

and the lightning current model is

$$I(l,\omega) = I_0(\omega) e^{-j(\omega/v)s} e^{-j(\omega/v)l}$$

where v is the lightning current speed and $I_0(\omega)$ is the lightning base current.

A similar formulation can be developed in time domain, starting from an analytical Inverse Fourier Transform of (1).

III. Coupling model

Coupling to transmission line adopted in this work is based on [6]. We have achieved a complete analytical solution of the induced current on a line starting from lightning parameters. The main steps of the coupling model are summarized below.

For each segment of length dx along the transmission line, the effect of the impinging field is described by means of two sources, a series voltage generator due to magnetic field and a shunt current generator due to electric field. The expressions of the generators are:

$$V_h(x) = j\omega\mu_0 h H_y(x, z=0) \qquad (2.a)$$

$$I_{eq}(x) = j\omega C h E_z(x, z=0) \qquad (2.b)$$

In the previous equations, C is the per-unit-length capacitance of the line, h is the line height above ground, H_y is the component of the magnetic field normal to the plane containing the line, and E_z is the component of the electric field across the line. The effect of the distributed generators is reported to the end of the line, and lumped into VL and VR (see Fig. 2):

$$\begin{aligned} V_L &= \frac{K_{v1}}{\beta(1-m^2)}\left[e^{-j\beta mL} + jm\sin(\beta L) - \cos(\beta L)\right] + \\ &+ \frac{K_{v2}}{\beta(1-m^2)}\left[-jme^{-j\beta mL} + jm\cos(\beta L) + \sin(\beta L)\right] \\ V_R &= \frac{-K_{v1}}{\beta(1-m^2)}\left[e^{-j\beta mL}\left(-jm\sin(\beta L) - \cos(\beta L)\right) + 1\right] + \\ &+ \frac{K_{v2}}{\beta(1-m^2)}\left[e^{-j\beta mL}\left(\sin(\beta L) - jm\cos(\beta L)\right) + jm\right] \end{aligned} \qquad (4)$$

with

$$m = \sin\theta\cos\varphi$$

$$K_{v1} = \frac{j\omega\mu_0 H_y(x_0)}{\sin(\beta L)}$$

$$K_{v2} = \frac{-\omega C E_z(x_0) h Z_c}{\sin(\beta L)}$$

where L is the line length, Z_c its characteristic impedance, and β its propagation constant.

Loaded line response can be easily evaluated from open-end equivalent circuit of Fig. 2.

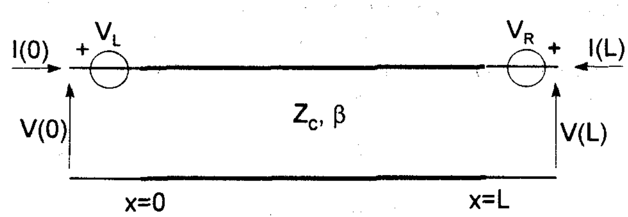

Figure 2. Transmission line with lumped open-end voltage generators equivalent of field coupling.

IV. Fractal description of the lightning induced current on a transmission line

In this section, we analyse the fractal dimension of the impinging electric field and of the current induced on the transmission line with respect to the fractal dimension of the lightning channel. As discussed in [2], the fractal dimension of the channel discharge and the electric field are related. In this paper we point out that fractal dimension of the induced current on the line and the lightning channel fractal dimension are also related.

Because of the possible inaccuracies of the fractal dimension determination in frequency domain (which is based on spectral behaviour), we have used Variation algorithm [7] in time domain that gives more accurate estimates than other methods (i.e., box counting algorithm that is widely adopted, or modified box counting [8]).
Since our formulation gives induced current on the line in frequency domain, we have to perform an Inverse FFT to get time domain response. The use of IFFT is the reason for the inaccuracies of fractal estimation, resulting from Fig. 3. For typical values of lightning channel fractal dimension (1.30 ÷ 1.50), the inaccuracy due to IFFT is small and equivalent to the intrinsic error of fractal measurement algorithm.
In [2], it is discussed how the electric field fractal dimension generated by a lightning depends on the fractal dimension of the lightning channel, on the rise time of the base current and on the current attenuation length in the return stroke model. No other parameters influence the electric field fractal dimension (i.e. base current fall time). We have verified that such result is valid also for the induced current on the transmission line. Moreover, we have found that both resistive load and line length do not influence the current fractal dimension. In Fig. 4, the variation of the fractal dimension of the induced current for lines shorter than the wavelength is explained by the effects of random cancellation noise, since the current in the left load is proportional to *VL-VR,* and *VL≈VR.*
Fig. 5 shows the induced current fractal dimension with respect to the attenuation length Lc of the MTL model, the rise time of the base current and the channel fractal dimension.

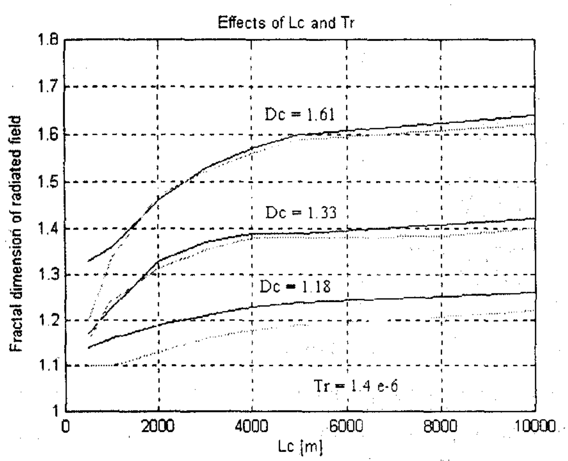

Figure 3. Comparison between evaluated time-domain electric-field fractal dimension (bold line) and time-domain electric-field obtained by Inverse FFT.

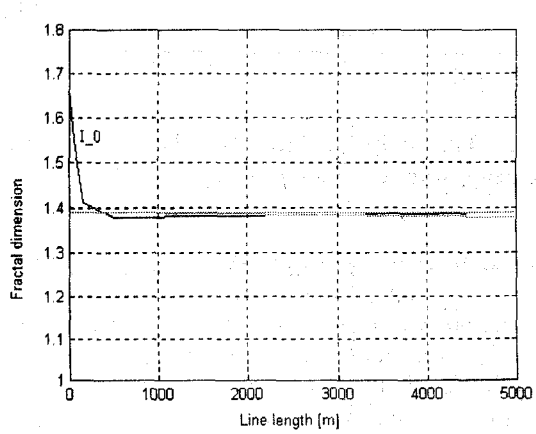

Figure 4. Fractal dimension of the induced current (bold line) and of the generators VL and VR, as a function of line length, with 50-Ω resistive loads.

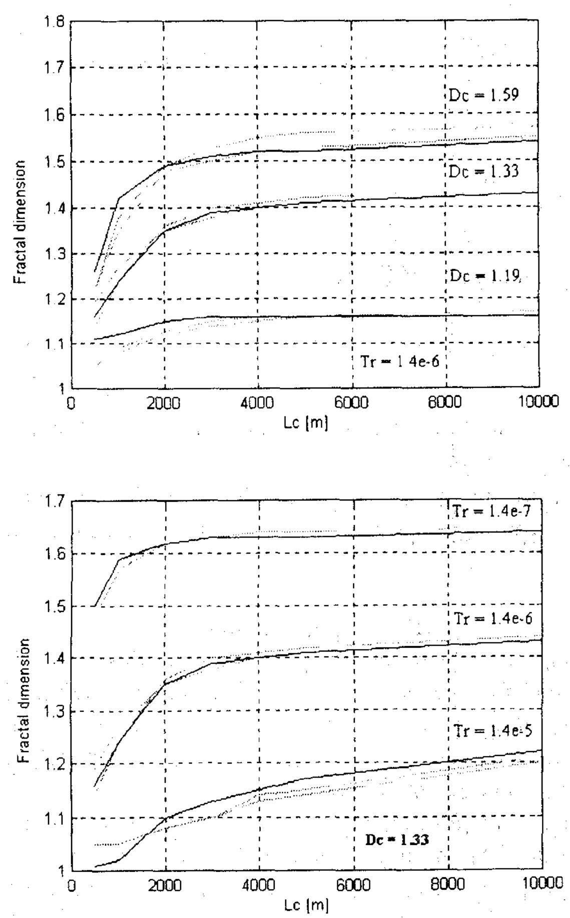

Figure 5. Fractal dimension of the induced current on a transmission line (bold line) and on generators VL and VR, as a functions of the attenuation length Lc of the MTL model, for fractal dimension of the lightning channel Dc (upper panel), and of the base current rise time Tr (lower panel).

No other parameters influence the induced current fractal dimension. Line length is 500 m and both loads are 50 Ω resistive.

V. Comparison with experimental measurements

In this section, we present a comparison between simulated results and experimental measurements (taken from [9]). First of all, we have searched for a fractal channel model that reproduces the experimental electric field. In Fig. 6, our simulated electric field is shown in comparison with the experimental measurement. We have found Lc = 5000 m, base current rise time of 12.5 μs, base current fall time 1 μs, base current intensity 67 kA, channel fractal dimension 1.41, ground-to-cloud lightning height 6000 m.

We have used the above lightning model (that approximates rather well the electric field) to evaluate the induced current on the transmission line of the experimental tests (line length 450 m, height from earth 9.5 m, direction toward x axis, open-end left load, 600 Ω right load). Results are reported in Fig. 7: it can be noticed a good agreement between the two curves, especially in the first 10 μs. Simulated time-domain behaviour is quite similar to experimental curve; also, almost equal peak values are found.

VI. Conclusions

In the previous sections, a prediction of the disturbances generated by a lightning discharge has been discussed. Fractal-channel discharge model has allowed us to achieve good agreement between simulated electric field and measured experimental electric field, whereas rectilinear-channel models are unable to reproduce the fine structure of the measured lightning electric fields.

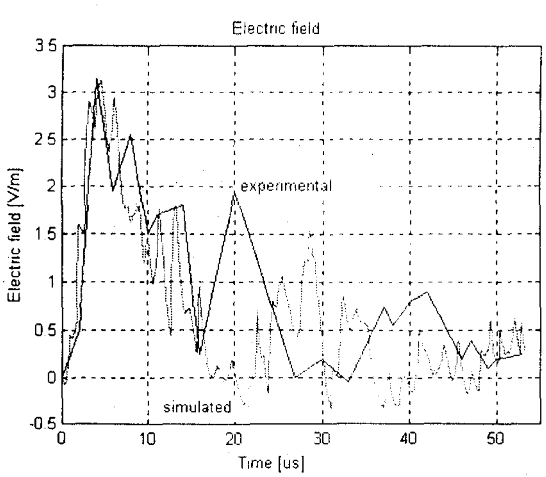

Figure 6. Measured and simulated incident field due to a tortuous channel discharge with MTL model and double exponential base current.

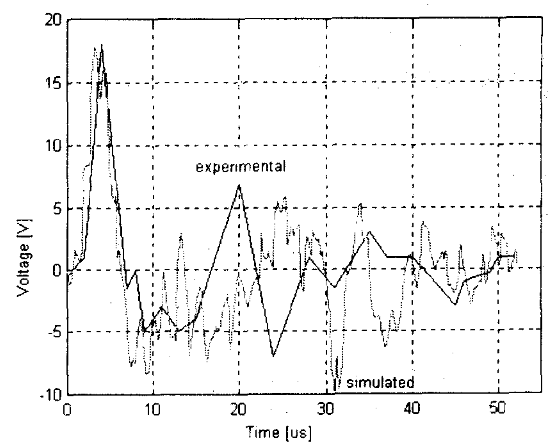

Figure 7. Voltage induced on 600-Ω impedance at the end of the line. Comparison of simulated and experimental results.

We have pointed out that the fractal dimension on the induced current depends only on the base current rise time, on the attenuation length of current discharge in MTL model and on the fractal dimension of the lightning channel.

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