STATISTICAL DESCRIPTION OF SIGNAL PROPAGATION ON RANDOM BUNDLES OF WIRES

S. Salio [*], F. Canavero [*], J. Lefèbvre [**], W. Tabbara [***]
[*]Department of Electronics, Polytechnic of Turin, Italy
[**]Thomson-CSF Communication, Colombes, France
[***]Département de Recherche en Électromagnetisme, LSS/CNRS-SUPELEC, France
(Principle contact: salio@polito.it)

Abstract: The determination of disturbances in random cable bundles requires a statistical approach combined with the solution of non uniform transmission lines. The analysis of such a bundle of wires is typically performed in three steps: the generation of the bundle and its subdivision in a chain of uniform lines, the evaluation of the per-unit-length (p.u.l.) parameters in each cross section, and the solution of the line. In this paper, we propose a fractal generation of cables that is very fast and it is able to generate realistic cables by means of one tuning parameter. Line p.u.l. capacitances and inductances are evaluated through a very efficient implementation of the method of moments with Fourier expansion of the charge on conductors. The electrical simulation of the line is then performed via a chain matrix product of uniform sections.

Since a statistical characterisation of a random non uniform line is extremely time consuming, Kriging method is adopted, because it is based on a very efficient fitting model that is able to describe the real situation with a limited number of simulations.

1. Introduction

Crosstalk, susceptibility and emissions from interconnects are phenomena that are not negligible in modern electronic applications. In a large number of situations, these phenomena are of primary importance because they are the cause of anomalies on electronic equipments that may concern the safety of human beings (for example, cruise controls in avionics and safety systems in road vehicles).

Although the research activity in this field is well developed, the study of EMC effects (interferences, susceptibility, internal crosstalk) in random cables is only at the beginning. The randomness of interconnects having non-uniform cross section has an effect quite important on the determination of disturbances in practical cases. For example, if we turn attention to a specific bundle of wires in a car, it shows a great variability from one car to another because cables are not completely fixed inside the body of the car and because their physical realisation is not deterministic.

This means that it is not sufficient to study a single realisation of a wire but a statistical approach is needed, in order to analyse a great number of realisations of the same interconnect. In fact, the most useful information for the designer is the statistics of the disturbances on a given interconnect, and not simply their value due to a single realisation.

The analysis of randomly twisted bundles of wires requires the following steps:

- the construction of the random non-uniform line
- the evaluation of the p.u.l. parameters (depending on the longitudinal coordinate)
- the computation of the line response
- the statistical elaboration of the results.

The simulated construction of a non-uniform bundle of wires is a critical task, because a detailed description of the physical processes producing the random twisting (some of which are unclear) is unaffordable. Therefore, an empirical model is sought, but the difficulty arises in deciding how it is representative of reality. So far, in the literature, a method is presented, where the random bundle consists of a cascade of uniform lines whose cross-sections are randomly extracted from a given probability distribution, and each cross section is determined in a completely independent way from all the others ([1],[2]). Due to this assumption, the resulting bundle may be far from a realistic one, because it is difficult to control the smoothness degree of wires windings.

In this paper, we present a simple and powerful way to generate random cables with the possibility of controlling the twisting characteristics by means of one parameter. This approach is based on the Random Midpoint Displacement (RMD) method, widely used for the generation of fractal curves.

The second task in order to simulate a non-uniform line is the evaluation of p.u.l. parameters along the line itself. This task is usually very time consuming, because the line cross-section is non homogeneous (due to the presence of wires' dielectric insulation) and non uniform (due to the changes of position of the wires along the line length): the non homogeneity requires the use of 2D numerical methods (e.g., finite elements, finite difference, moment method), and the non uniformity asks for highly repeated applications of the above techniques. In the literature, the analytical formula for

widely spaced wires is often used to reduce the simulation time, but it has clear limitations. In this paper, we present an optimised implementation of the method of moments with Fourier expansion of the charge on conductors and dielectrics [3]. This method gives p.u.l. parameters with high efficiency and good accuracy, even if the conductors are very close each other, because it approximates the charge distribution on circular conductors with circular functions.

The line solution necessary to obtain the frequency behavior of the electrical variables is performed by subdividing the non-uniform line into a chain of uniform segments, and by chain matrix product [4].

Finally, the statistical analysis is in general extremely time consuming, since it requires to repeat the line simulation for a large number of parameters values, randomly chosen (Monte Carlo approach). We adopt an idea that avoids Monte Carlo simulation by computing a limited number (order of tens) realizations of cables and performing a fit of the results. The powerful Kriging [5] method that allows us to efficiently elaborate the statistical results is presented in Sec. 5.

2. Generation of a bundle of wires

In this Section, we introduce our proposed method to generate random cables that resemble to industrial cables and have a degree of randomness easily tuneable with only one parameter. The differences between the method we propose here and the methods available in literature are the iterative process of subdivision of the line and the determination of the position of wires in the cross sections identified by the subdivisions.

Let's start the explanation for a single wire of the bundle. The 3D position of the wire can be constructed from its projections into two orthogonal planes having the z-axis (the one along the line length) in common. These two projections are determined by means of RMD algorithm that generates fractal curves, with the iterative mechanism described in Fig 1 [6]. The algorithm starts with two extreme points in the plane (z,x) (say, (z_1,x_1) and (z_n,x_n) that can be imagined as the connectors points of the considered wire) and evaluates:

$$z_{n/2} = \frac{z_1 + z_n}{2} \qquad (2.1a)$$

$$x_{n/2} = \frac{x_1 + x_n}{2} + d \qquad (2.1b)$$

where d is a parameter extracted from a gaussian distribution having zero mean and variance depending on the fractal dimension and on the iteration number. Then, the algorithm proceeds on subdividing again the intervals on the z axis, and extracting new related positions on the x axis. The result is a set of nodes (z_i,x_i) representing the projections in the (z,x) plane (i is an index belonging to $[1,n]$, and n is the total number of cross sections). The same procedure must be applied to determine the projections of the wire in the (z,y) plane, getting (z_i,y_i). At this stage, the position (x_i,y_i,z_i) of one wire in the bundle in each cross section is defined. If the wire is out of the space assigned to the bundle in one or more cross sections, the position is forced to lay on the boundary of the bundle. If the bundle is not too narrow, this operation is seldom required. To build the entire bundle, the procedure of the generation of a single wire is repeated as many times as the number of wires assigned to the bundle. During the wire assembling process, it may occur that some wires overlap in one or more cross sections. In this case, we proceed in the following manner to avoid wire overlap:

1. the algorithm looks for free positions around overlapping wires and, if a free position is available, moves the conductor that needs the minimum displacement;
2. if overlapping situations still exist, the algorithm moves all wires outside the bundle boundary in order to gain more free space, eliminates overlaps, and moves again the wires inside the bundle, starting from the wire nearest to the centre of the bundle.

The choice of the above algorithm is motivated by its efficiency; in fact, it is very fast and perturbs the wire position as little as possible.

The main advantage of using fractals for random wires is that, through a single parameter, the fractal dimension, we can control the degree of irregularity of the curve described by a wire. The fractal dimension ranges from 1 to 2, where 1 represents a straight rectilinear segment, and 2 refers to a very jagged curve that fills the entire 2D space.

Fig. 2 shows a 10-cm cut of a realisation of a 9 wires bundle having fractal dimension 1.1. The total length of the bundle (2 m) is subdivided into 128 sections of 1.56 cm each. Figs. 3 and 4 differ only for the fractal dimension of the cable that is 1.4 and 1.7, respectively.

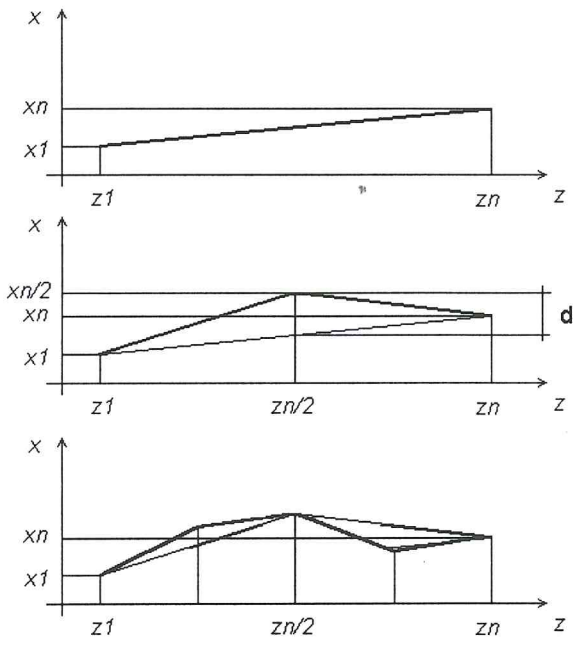

Fig. 1: Random Midpoint Displacement algorithm. The construction of the fractal curve is performed in a finite number of iterations that add details, step by step. The coordinates of a new point between two old ones are determined according to eq. (2.1).

From Figs. 2, 3, 4 it appears that the proposed algorithm is able to generate realistic cables whose degree of twisting is easily controlled by one parameter (the fractal dimension). It is also worth noting that the algorithm is very fast: in fact, the cables of Fig. 2, 3, 4 are generated for their entire length in a few seconds on a Pentium PC.

3. Non uniform line solution

The computation of the p.u.l. parameters is generally a critical task for the accuracy of the results and for the simulation time. The practical case at hand (bundles of wires in close proximity and surrounded by a dielectric layer) imposes that p.u.l. parameters are determined by means of a numerical method, since explicit analytical formula are not available in the literature. A statistical approach to crosstalk evaluation on random cables, as the one we deal with in this paper, requires to repeat the computation of p.u.l. parameters thousands of times (number of cross sections times number of bundle samples), and therefore asks for a very fast software. We limit our simulations to lossless lines, thus requiring only the computation of p.u.l. capacitance and inductance matrices.

In this work, we have adopted the moment method formulation that expands the charge on the wires in Fourier series [3]. This method gives very accurate results because it is based on expansion functions that are a natural choice for the circular contour of the conductors. A good number of harmonics for the charge expansion is about 10-15, even in the worst case of dielectrics in contact.

We have developed an optimised implementation of the above method in MATLAB using its powerful functions on matrices. We are able to evaluate the p.u.l. parameters of a cross-section having 10 insulated conductors in few seconds on a Pentium PC. The same structure, solved with the Finite Element method, takes more than 2 hours on the same PC because this method needs a thoroughly refined mesh with at least thousands triangles, in order to gain the same accuracy. The conventional Moment method with piecewise constant expansion of the charges is not considered because it may produce unphysical results on bundles of circular conductors in close proximity. Also, a finite difference approach is discarded on the obvious consideration that its mesh hardly fits circular conductors.

Once the p.u.l. parameters are evaluated in all cross sections, the non-uniform line is solved by the conventional approach that replaces the line with a cascade of uniform lines and computes its response by means of the product of the chain matrices of the uniform sections [4].

4. Statistical results

The model presented in the previous sections has a large variety of applications, because it allows the prediction of disturbances on real random interconnects.

In this section, we investigate the influence of the fractal dimension of 1-m long cable made of 9 wires on the transfer function between two terminations.

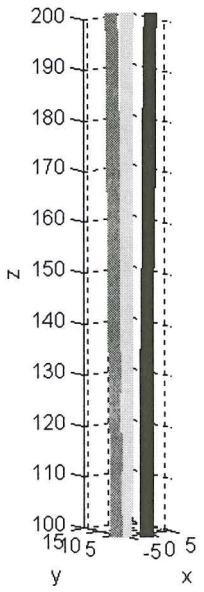

Fig. 2: Plot of 10 cm of a bundle with 9 wires and total length of 2 m (only 3 wires are plotted in order to preserve the readability of the picture). The fractal dimension is 1.1 (wires almost rectilinear). The external diameter of each wire (including dielectric insulation) is 2.30 mm. The bundle is bounded by a cylinder having a radius of 6 mm. Axes quotation is expressed in mm, and the representation is to scale, so that this image is directly comparable to a real bundle.

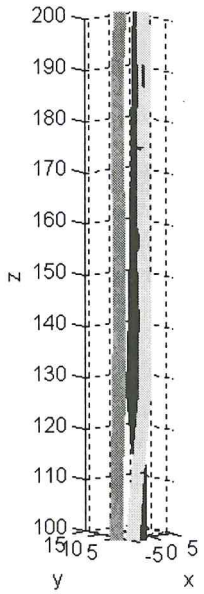

Fig. 3: Same as Fig. 2, except for the fractal dimension that is 1.4. Wires' meandering starts to appear.

We have performed 3 sets of simulations (100 samples each) on bundles of wires having fractal dimensions 1.4, 1.1, and 1.7, respectively.

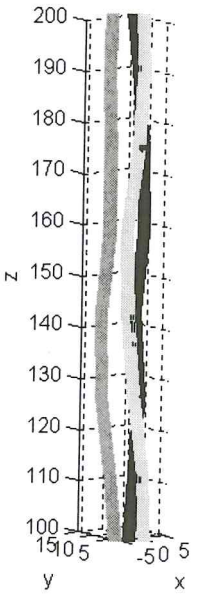

Fig. 4: Same as Fig. 2, except for the fractal dimension that is 1.7. Wires' meandering is well developed.

All simulated cables have the following common characteristics that are representative of real industrial bundles. The radius of the conductors is 0.75 mm, the radius of the dielectric jacket ($\varepsilon_r = 2.6$) surrounding the conductors is 1.15 mm, and the radius of the bundle is 6 mm. The height of the centre of the bundle above the ground plane is 9 mm. The geometry of the first and the last cross-sections of the bundles is plotted in Fig. 5. Each wire is terminated at both ends with a 1-kΩ resistance connected to the ground, except for the left end of wire 1 that is loaded with a 50 Ω resistance in series with a sinusoidal signal source V_s of amplitude 1 V and frequency ranging from 1 kHz to 1 GHz. The output $V_9(0)$ is measured on left end of wire 9 (that is the central conductor of the bundle). We approximate 1 m of the real cable with 64 uniform lines that are much shorter than the minimum wavelength considered; in fact, the length of each segment is 1.56 cm, compared to 30 cm wavelength at 1 GHz.

The results of the three sets of simulations are reported in Figs. 6, 7, 8 and in Table 1 where the amplitude of the transfer function $H = V_9(0)/V_s$ is considered. Fig. 6 collects the transfer functions of the 100 realisations of cables having fractal dimension 1.4. Similar results, obtained for bundles with fractal dimensions of 1.1 and 1.7, are not shown due to space limitations. Instead, we concentrate on a further elaboration, aiming at the statistical characterisation of the transfer function at fixed frequency. The histogram of the absolute values of the transfer functions computed for the 100 realisations of bundles having a given fractal dimension are shown in Fig. 7, for a frequency of 243 kHz, corresponding to the low frequency band of the transfer function, away from the region affected by line resonances. Figs. 7a, 7b

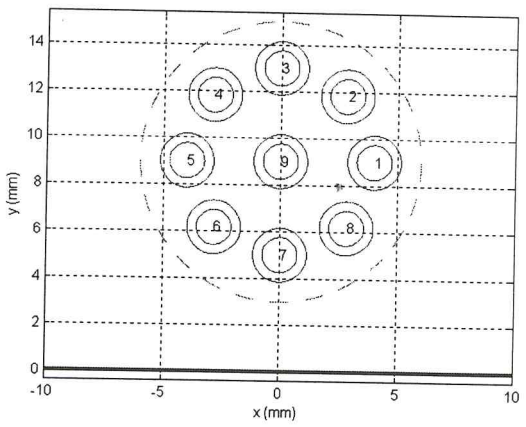

Fig. 5: Cross section of both ends of simulated cables. It corresponds to the geometry of wires on connectors. The external dashed circle indicates the trace of the cylinder that contains the bundle.

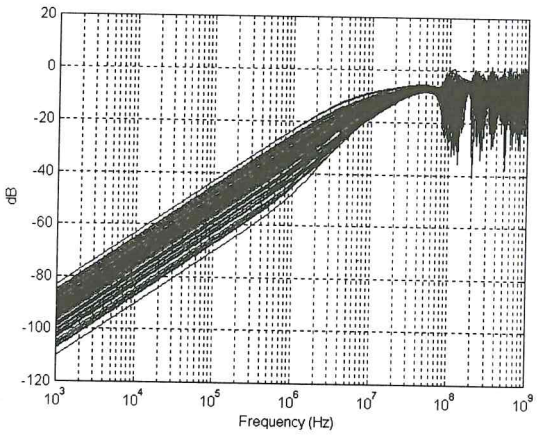

Fig. 6: Amplitude of the coupling transfer function H between wire 1 (excitation on the left end) and wire 9 (left end), evaluated for 100 realisations of bundles of wires having fractal dimension 1.4.

and 7c refer to experiments with bundles having fractal dimensions 1.1, 1.4 and 1.7, respectively. Table 1 reports the means and the standard deviations of the histograms plotted in Fig. 7a, 7b and 7c. For both means and variances, a range is given, corresponding to a 5% confidence level. Finally, in order to investigate the frequency behaviour of statistical characteristics, Fig. 8 is presented, where means and standard deviations of the three sets of simulations are shown *vs.* frequency.

The average value of crosstalk is not influenced by the degree of irregularity of the bundle. This outcome is clearly shown in Fig. 8. Also, Table 1 confirms that the confidence interval of the average has almost the same width for the three simulation sets. On the contrary, the standard deviation of the transfer function depends on the smoothness degree of wire windings. In particular, the higher the fractal dimension of bundles is, the smaller is the variance. This behaviour can be explained with the fact that, if bundles are quite intricate, the distance of the two wires between which the crosstalk is

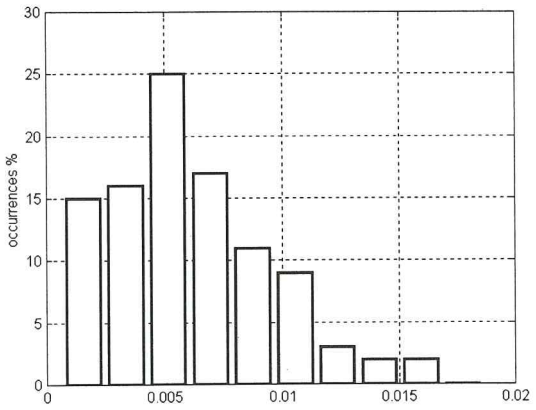

Fig. 7a: Histogram of the amplitude of the transfer function H at 243 kHz, for the case of Fig. 6.

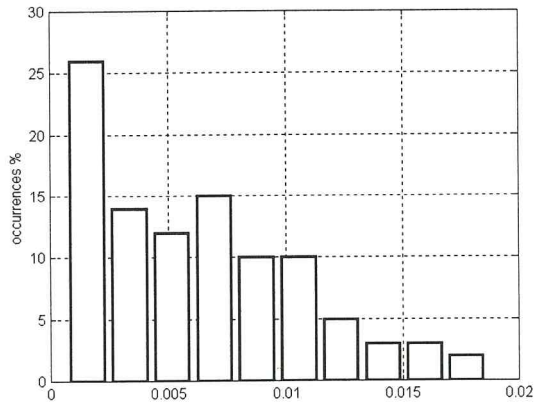

Fig. 7b: Same as Fig. 7a, except that the fractal dimension of simulated cables is 1.1.

measured tends to be equally distributed along the cable length, and the cases in which the two wires are either adjacent or far away all along the line are rare. In this situation, the resulting standard deviation is low because crosstalk tends to assume only intermediate values and the very high or very low values are infrequent.

The outcomes on the average and the standard deviation of crosstalk just described are not valid in general (arbitrary cables and loads characteristics). The investigations aimed at generalising the prediction of crosstalk are still underway.

5. Kriging approach

In this section we introduce Kriging model in order to provide a very efficient tool for the statistical elaboration of the cable transfer functions, that basically allows to avoid the unaffordable simulation times of Monte Carlo approach. The voltage or the current induced on a cable due to crosstalk or to an impinging electromagnetic wave depends on a large number of factors such as the length of the cable, its height above a ground plane, the number of wires, the twisting characteristics, the loads and the frequency. A parametric analysis of the voltage (or its average and maximum values over some frequency band) conducted over the ranges of all or part of the factors leads to a

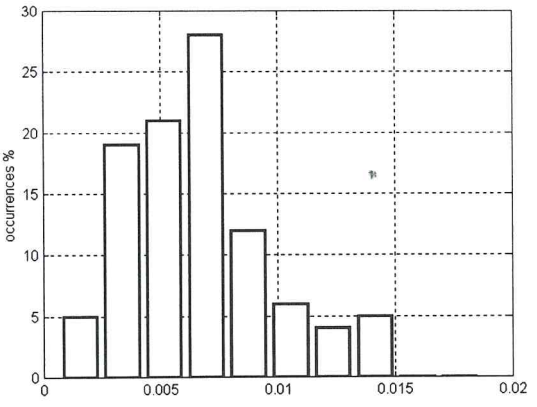

Fig. 7c: Same as Fig. 7a, except that the fractal dimension of simulated cables is 1.7.

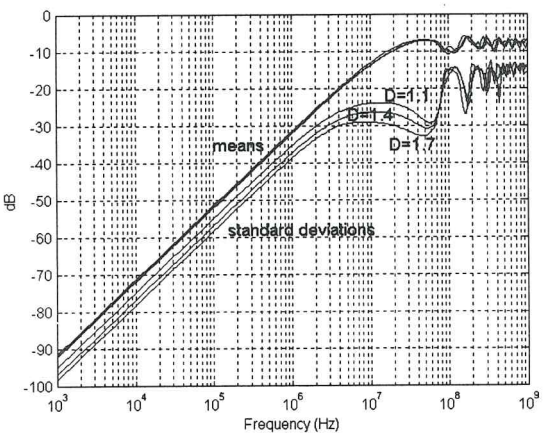

Fig. 8: Means and standard deviations of three sets of simulations having fractal dimensions D=1.1, 1.4 and 1.7.

	D = 1.1 (Fig. 7b)	**D = 1.4** (Fig. 7a)	**D = 1.7** (Fig. 7c)
mean of $\|H\|$ (multiply by 10^{-3})	5.4-**6.3**-7.1	5.4-**6.1**-6.7	6.0-**6.6**-7.2
std of $\|H\|$ (multiply by 10^{-3})	3.8-**4.4**-5.1	3.0-**3.5**-4.0	2.6-**3.0**-3.4

Table 1: Means and standard deviations of the distributions shown in Figs. 7a, 7b and 7c. D is the fractal dimension of the bundles of wires. The intervals of means and variances have been evaluated at 5% confidence level.

high cost in terms of computing time or prototyping. We may then think of building an accurate numerical model that permits prediction of the values of the observable at a lesser computational cost than with the "classical" approach based on browsing the complete set of values of the factors.

This cost-reduction can be looked at in a somewhat unusual manner, at least as far as electromagnetics is concerned, if one takes the following considerations into account:

1. The observable is not equally influenced by all

factors. It is then possible to call on some specific tools in data analysis, such as multivariate regression, to select the main factors and consequently neglect the others. This will lead to a significant reduction of the cost of computation and experimentation.

2. The knowledge of the fine variations of the observable with respect to the factors is not always necessary for the understanding of the phenomenon of interest. Let us consider, for example, the voltage induced at high frequencies on a cable. The density of the resonances in the voltage increases with frequency and it is not generally possible to trace back the origin of these resonances. The determination of the trend of the voltage, or its maximum value over some frequency band, may then be a sufficient information.

In order to achieve the goals stated above, a multi-factor parametric approach to system modeling, called Kriging [5] can be combined with the method of solving the non uniform transmission lines introduced in the previous section. The Kriging model states that the observable, here the transfer function, is the sum of two terms. The first one is an approximate multivariate regression model and the second is an outcome of a stochastic process, the covariance function of which is known. Then a cost function is minimized and optimized values of the parameters of the regression model are obtained. In a final step, we build a predictor for the observable made of a linear combination of a small number of values of this observable obtained by applying a data analysis method named experimental design. Kriging also provides an estimation of the accuracy of the predicted values through the computation of a standard deviation. Kriging is not a substitute for traditional numerical methods, but an efficient tool to bring out trends which, in particular, indicate where exact computation should be performed. Applications of Kriging model on complex systems are available in [7].

6. Conclusions

In this paper, we have provided a statistical model that describes the disturbances due to crosstalk on random non-uniform bundles of wires. This model is based on a very efficient method for generating and solving random non-uniform transmission lines combined with an accurate method for the statistical elaboration of results. In particular, the generation of random bundles of wires that resemble to real industrial cables is performed by means of an algorithm controlled by one parameter, the fractal dimension; also, the fast computation of p.u.l. parameters takes advantage of a moment method based on a natural expansion of charges on circular conductors. Finally, the Kriging model will be adopted as an optimised fitting method for the statistical elaboration of results in order to avoid the lengthy Monte Carlo approach.

References

[1] A. Ciccolella, F. Canavero: "Statistical simulation of crosstalk in bundles," *Proceedings of the 11th International Zurich Symposium on Electromagnetic Compatibility*, ETH, Zurich (Switzerland), 7-9 March 1995, pp. 83-88.

[2] A. Ciccolella, F. Canavero: "Stochastic prediction of wire coupling interference," *Proceedings of the IEEE 1995 International Symposium on Electromagnetic Compatibility*, Atlanta (USA), 14-18 August 1995, pp. 51-56.

[3] C. R. Paul, J. C. Clements, and A. T. Adams, "Two-Dimensional Systems of Dielectric-Coated, Cylindrical Conductors", *IEEE Trans. on Electromagnetic Compatibility*, EMC-17, 238-248, 1975.

[4] C. R. Paul, *Analysis of Multiconductor Transmission Lines*, New York: Wiley, 1994.

[5] Lefèbvre J., Roussel H., Walter E., Lecointe D., Tabbara W., "Prediction from wrong models: the Kriging approach", *IEEE Ant. Prop. Magazine*, vol.38, no. 4, 1996, pp. 35 - 45.

[6] H. O. Peitgen, and Saupe, *The science of fractal images*, Springer Verlag, Berlin (D), 1988.

[7] J. Lefebvre, *Contribution of statistics to the analysis of coupling phenomena in EMC,* PhD Thesis, Univ of Paris VI, Nov. 1997. (In French).