[Proceeding] Investigation on brittle fracture in rounded V-notched structures by Finite Fracture Mechanics

Investigation on brittle fracture in rounded V-notched structures by Finite Fracture Mechanics

A. Sapora, P. Cornetti, A. Carpinteri
Department of Structural Engineering and Geotechnics, Politecnico di Torino, Torino
alberto.sapora@polito.it

ABSTRACT. In the present work, Finite Fracture Mechanics (FFM) is applied to investigate the brittle fracture behaviour of rounded V-notched structures subjected to mode I loading. According to the criterion, fracture does not propagate continuously, but by finite crack extensions, whose value is determined by the contemporaneous fulfilment of a stress requirement and an energy balance. Consequently, the crack advance becomes a structural parameter. By assuming the apparent generalized stress intensity factor as the governing failure parameter, as expected within brittle structural behaviour, the expression for the apparent generalized fracture toughness as a function of the material properties as well as of the notch opening angle and the notch radius is achieved. FFM predictions are compared with those provided by other theoretical approaches based on a critical distance and with experimental data available in the Literature, showing a good agreement.

SOMMARIO. Nel presente lavoro, il criterio *Finite Fracture Mechanics* (FFM) è applicato allo studio del comportamento fragile a rottura delle strutture con intagli a V arrotondati sollecitate in modo I. In base a tale criterio, la frattura non si propaga con continuità, ma attraverso un avanzamento discreto della fessura, il cui valore viene determinato imponendo il contemporaneo soddisfacimento di una condizione tensionale ed un bilancio energetico. L'estensione della fessura risulta essere quindi una proprietà strutturale. Assumendo come parametro governante la frattura il fattore di intensificazione degli sforzi generalizzato apparente, ipotesi legittima per comportamenti strutturali fragili, ne si ricava l'espressione in funzione delle proprietà del materiale, dell'angolo di apertura dell'intaglio e del raggio di curvatura. Le previsioni del criterio FFM sono confrontate con successo con i dati sperimentali presenti in Letteratura e con i risultati forniti da altri approcci teorici basati su una distanza critica.

KEYWORDS. Rounded V-notches; mode I; brittle fracture; Finite Fracture Mechanics.

INTRODUCTION

Different criteria based on a linear-elastic analysis combined with an internal length have been proposed to deal with fracture initiation of brittle notched structures subjected to mode I loading [1-8]. For U-notched samples it has been shown that failure load predictions related to different approaches are generally very close [9]. On the other hand, for what concerns V-notched specimens, the coupled FFM criterion generally provides the most accurate results [7-8]. FFM was introduced to remove some inconsistencies related to the criteria previously introduced [2-5], according to which either stress or energetic considerations result violated. In the present work, it is applied to estimate the failure loads of rounded V-notched structures (Fig.1), subjected to mode I loading [10]. The analysis involves the characterization of the stress field and the stress intensity factor (SIF) functions ahead of the notch tip, which leads to the solution of non-linear equations.

FINITE FRACTURE MECHANICS

The FFM criterion [7-8] is based on the hypothesis of a finite crack advance Δ and assumes a contemporaneous fulfilment of two conditions. The former requires that the average stress $\sigma_y(x)$ upon the crack advance Δ is higher than material tensile strength σ_u

$$\int_0^{\Delta} \sigma_y(x)\,\mathrm{d}x \geq \sigma_u \,\Delta\,, \tag{1}$$

where (*x*, *y*) is the Cartesian coordinate system centered at the notch tip (Fig.1).

The latter one ensures that the energy available for a crack increment Δ is higher than the energy necessary to create the new fracture surface:

$$\int_0^{\Delta} K_I^2(c)\,\mathrm{d}c \geq K_{Ic}^2 \,\Delta\,, \tag{2}$$

$K_I(c)$ and K_{Ic} being the SIF related to a crack of length *c* stemming from the notch root and the fracture toughness, respectively. For positive geometries, in critical conditions, Eqs.(1-2) can be grouped into a system of two equations in two unknowns: the crack advancement and the failure load.

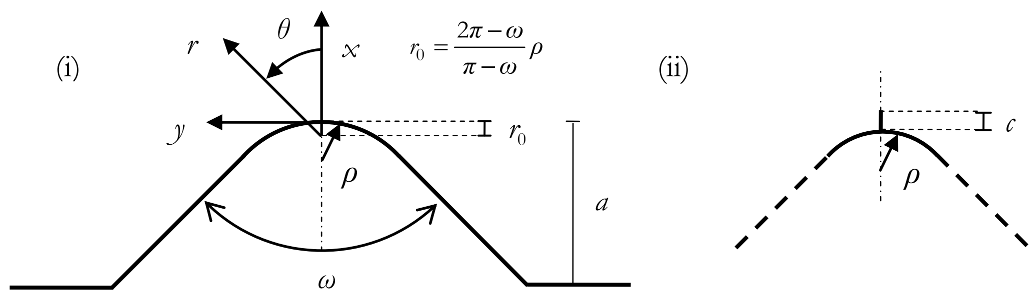

Figure 1: (i) Rounded V-notch with Cartesian and polar coordinate systems. (ii) Crack of length *c* stemming from the notch tip.

Notice that a similar criterion, but based on a point-wise stress requirement, was proposed in [6].

Stress field ahead of the notch tip

By assuming that the notch tip radius ρ is sufficiently small with respect to the notch depth *a* (Fig. 1), the stress field along the notch bisector could be expressed in polar coordinates as [11]:

$$\sigma_\theta(r,0) = \frac{K_I^{V,\rho}}{(2\pi r)^{1-\lambda}}\left[1+\left(r_0/r\right)^{\lambda-\mu}\eta_\theta(0)\right], \tag{3}$$

where $K_I^{V,\rho}$ is the apparent generalized stress intensity factor. The distance between the notch tip and the origin of the polar coordinates system is denoted by r_0: it depends on the notch radius ρ and, as well as the eigenvalues λ and μ, and the function $\eta_\theta(0)$, whose values are tabulated in [11], on the notch opening angle ω:

$$r_0 = \frac{2\pi-\omega}{\pi-\omega}\rho\,. \tag{4}$$

Stress intensity factor function

In order to evaluate the SIF function $K_I(c)$ related to a small crack of length c emanating from the notch root (Fig.1), we have performed some numerical simulations by means of FRANC2D®code [12] on notched structures under tension: different notch opening angles with different ratios c/ρ ranging from 0.01 to 10 have been taken into account, provided that $a/\rho=b/a=20$, b being the overall geometric dimension. On the basis of the obtained results, we propose the following expression for the function $K_I(c)$:

$$K_I(c) = \alpha K_I^{V,\rho}\sqrt{c}\left[\left(\alpha/\beta\right)^{\frac{1}{1-\lambda}} c + r_0\right]^{\lambda-1}, \tag{5}$$

where $\alpha=1.12\sqrt{\pi}\ (1+\eta_\theta(0))\ (2\pi)^{\lambda-1}$ and β can be found tabulated in [8] (according to the present notation, it increases from unity, when $\omega=0°$, up to $1.12\sqrt{\pi}$, when $\omega=180°$). For a very short crack ($c/\rho<<1$), the notch-crack problem can be treated approximately as an edge crack subjected to the local peak stress: Eq.(5) provides consistently $K_I(c)=1.12\ \sigma_{max}\sqrt{\pi c}$ with $\sigma_{max}=\sigma_\theta(r_0, 0)$ (Eq.(2)). On the other hand, for a very long crack ($c/\rho>>1$), the notch radius effect becomes negligible and Eq.(5) gives $K_I(c)=\beta K_I^V c^{\lambda-0.5}$, i.e. the SIF related to a crack stemming from a sharp V-notch. For intermediate values, Eq.(5) generally provides good results: the most significant deviations (below 10%) from the numerical predictions emerge for $0.2<c/\rho<0.8$ and $\omega\leq 90°$. Errors decrease as ω increases.

In order to apply the FFM criterion, Eqs. (3) and (5) must be inserted into Eqs. (1) and (2), respectively (notice that $x=r-r_0$). By supposing that failure takes place when the apparent generalized stress intensity factor reaches its critical conditions $K_I^{V,\rho}= K_{Ic}^{V,\rho}$(as expected within brittle structural behaviour) and integrating, some analytical passages lead to the following expression for the apparent generalized fracture toughness $K_{Ic}^{V,\rho}$:

$$K_{Ic}^{V,\rho} = \sigma_u r_0^{1-\lambda} f(\overline{\Delta}_c), \quad \text{with} \quad r_0 = \left(K_{Ic}/\sigma_u\right)^2 g(\overline{\Delta}_c)/f^2(\overline{\Delta}_c), \text{ and } \overline{\Delta}_c = \Delta_c / r_0 . \tag{6}$$

The two functions f and g are expressed analytically by the following relationships:

$$f(\overline{\Delta}_c) = (2\pi)^{1-\lambda}\,\overline{\Delta}_c\left\{\left[\left(\overline{\Delta}_c+1\right)^{\lambda}-1\right]/\lambda+\eta_\theta(0)\left[\left(\overline{\Delta}_c+1\right)^{\mu}-1\right]/\mu\right\}^{-1},$$

and

$$g(\overline{\Delta}_c) = \frac{\overline{\Delta}_c}{\beta^2}\left\{\left[\left(\overline{\Delta}_c+\left(\frac{\beta}{\alpha}\right)^{\frac{1}{1-\lambda}}\right)^{2\lambda}-\left(\frac{\beta}{\alpha}\right)^{\frac{2\lambda}{1-\lambda}}\right]/2\lambda-\left(\frac{\beta}{\alpha}\right)^{\frac{1}{1-\lambda}}\left[\left(\overline{\Delta}_c+\left(\frac{\beta}{\alpha}\right)^{\frac{1}{1-\lambda}}\right)^{2\lambda-1}-\left(\frac{\beta}{\alpha}\right)^{\frac{2\lambda-1}{1-\lambda}}\right]/(2\lambda-1)\right\}^{-1}.$$

Experimental validation

The theoretical predictions from FFM (Eq.(6)) are compared with some experimental results available in the Literature [13-14] and with those provided by the simple point stress (PS) criterion [1]. Details of the sample geometry and the material properties (PMMA at –60° [13] and Alumina-7%Zirconia [14]) can be found in the quoted references. As can be seen (Fig.2), FFM predictions better agree with experimental data, the maximum percentage error being nearly 10% for the last two data sets related to the 150°-notched PMMA samples. Indeed, in these cases, the notch radius is not so small with respect to the notch depth (ρ/a > 0.15): thus, the stress field according to Eq.(3) results a little overestimated. Summarizing, the good fitting found (Fig.2) confirms the validity of the present approach as well as that related to the expression proposed for function $K_I(c)$ (Eq.(5)).

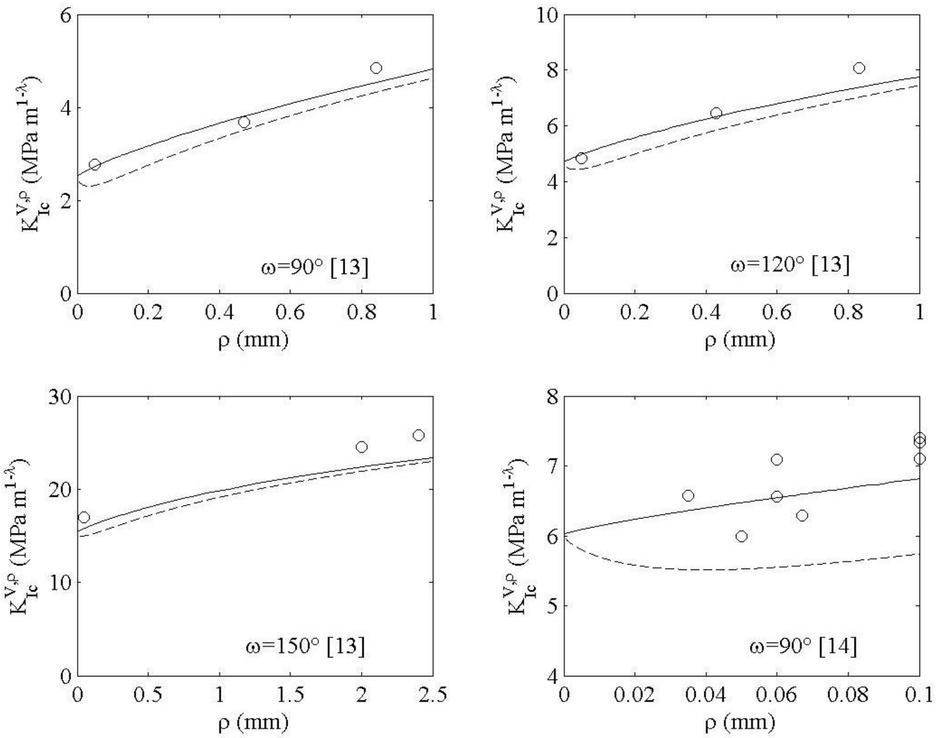

Figure 2: $K_{Ic}^{V,\rho}$ vs. ρ experimental data (circle) and theoretical predictions according to the PS criterion (dashed line) and FFM.

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