

# Spectral Properties of Wedge Problems

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**Abstract**— This paper presents our recent results on the study of the scattering and diffraction of an incident plane wave by wedge structures. A review about the impenetrable wedge problem at skew incidence and about the penetrable wedge at normal incidence is discussed. In particular we focus the attention on the spectral properties of the solution in the angular domain. These studies seem to provide a new tool to enhance the fast computation of the solution in terms of fields via a quasi-heuristic approach.

## I. INTRODUCTION

The behavior of electromagnetic fields scattered by wedge structures has been the subject of numerous important paper in the past using different techniques.

The impenetrable wedge problems modeled via impedance surfaces are treated since the mid-sixties [1] while the penetrable wedge has been one of the most important canonical diffraction problem for engineering, mathematical and physical communities since the mid-nineties [2].

Different mathematical approaches have been used to deal these diffraction problems in particular we mention the Sommerfeld-Malyuzhinets method, the method of separation of variable (Mc Donald), the Kontorovich-Lebedev transform method and the method of Wiener-Hopf (W-H). All these technique have been demonstrated their effectiveness, see references of [3] and [4] for a list of papers.

During last decade our effort has been focused on the Wiener-Hopf technique applied to angular region problems [3-9].

The first full paper on the application of the Wiener-Hopf technique was the one produced by Daniele in 2003 [5] for impenetrable wedge problems. These papers provided the mathematical formulation to obtain the results published in [3] and [4] respectively for the impenetrable wedge and penetrable wedge. Approximate techniques of factorization via Fredholm integral equations have been the mathematical tool to obtain effective approximate solutions [7].

This paper is devoted to review our solutions and to investigate the spectral properties of the spectral solution in the angular domain  $w$ . The analysis of these properties will provide a new tool to enhance the fast computation of the solution in terms of fields via a quasi-heuristic approach.

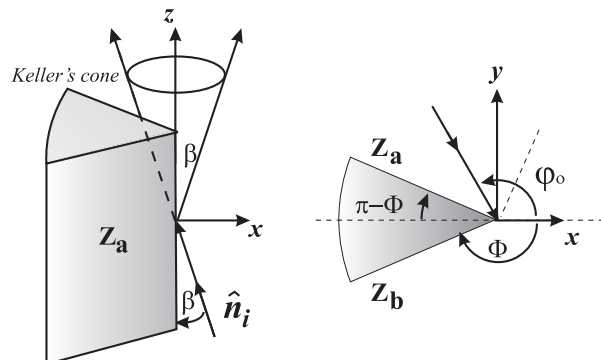


Fig. 1: The impenetrable wedge.

## II. IMPENETRABLE WEDGE

Following the procedure proposed in [3] we obtain that the problem of a plane wave incident on an impenetrable wedge with the Leontovich conditions can be modeled by the following generalized matrix Wiener-Hopf equation:

$$G(\eta)X_+(\eta) = X_-(m) \quad (1)$$

where the plus and the minus unknowns are defined in two different complex planes  $\eta$  and  $m$  as follow:

$$\begin{aligned} X_+(\eta) &= |V_{z+}(\eta, 0), V_{\rho+}(\eta, 0), Z_o I_{z+}(\eta, 0), Z_o I_{\rho+}(\eta, 0)|^t \\ X_-(m) &= |Z_o I_{\rho+}(-m, \Phi), -I_{z+}(-m, \Phi), -I_{\rho+}(-m, -\Phi), I_{z+}(-m, -\Phi)|^t \end{aligned} \quad (2)$$

with

$$\begin{aligned} V_{z+}(\eta, \varphi) &= \int_0^\infty E_z(\rho, \varphi) e^{jn\rho} d\rho \\ I_{z+}(\eta, \varphi) &= \int_0^\infty H_z(\rho, \varphi) e^{jn\rho} d\rho \\ V_{\rho+}(\eta, \varphi) &= \int_0^\infty E_\rho(\rho, \varphi) e^{jn\rho} d\rho \\ I_{\rho+}(\eta, \varphi) &= \int_0^\infty H_\rho(\rho, \varphi) e^{jn\rho} d\rho \end{aligned} \quad (3)$$

Classical W-H equations defined in the complex plane  $\bar{\eta}$  are obtained using a special mappings see [3]. The solution of the problem is obtained through the Fredholm factorization of the problem [7] and by extracting the source term based on the GO solution with source coefficient  $R$  and pole  $\bar{\eta}_o$  [3].

$$\bar{G}(\bar{\eta})\bar{X}_{i+}(\bar{\eta}) + \frac{1}{2\pi j} \int_{-\infty}^{+\infty} \frac{(\bar{G}(x) - \bar{G}(\bar{\eta}))\bar{X}_{i+}(\bar{\eta})}{x - \bar{\eta}} dx = \frac{R_i}{\bar{\eta} - \bar{\eta}_o} \quad (4)$$

$i=1..4$

The numerical evaluation of (4) through a simple quadrature is performed in the classical angular  $w$ -plane along a contour deformation to enhance the convergence.

The numerical procedure provides only an analytical element of the spectral solution in the  $w$  plane. We define the starting spectra as the spectra in the regularity strip  $-\Phi < \text{Re}[w] < 0$  that belongs to the proper sheet  $P_w$  defined in [3].

In the angular complex plane  $w$  (1) becomes

$$\hat{G}(w)\hat{X}_+(w) = \hat{Y}_+(-w - \Phi) \quad (5)$$

with  $Y_+(m) = X_-(-m)$  and by mathematical elaboration we obtain the recursive equation useful to extend the validity of the numerical solution in the whole complex plane  $w$ :

$$\hat{X}_+(w) = \hat{G}^{-1}(-w)\hat{G}^{-1}(w - 2\Phi)\hat{X}_+(w - 2\Phi) \quad (6)$$

In particular the far field solution is obtained asymptotically through the evaluation of  $\hat{X}_+(w)$  in the real range  $-\pi - \Phi < w < -\pi + \Phi$  (diffraction component).

By studying the properties of the starting spectra we observe that it can be approximated by using the starting spectra of the GO solution. The solution in the real range  $-\pi - \Phi < w < -\pi + \Phi$  is obtained through (6).

We observe that only the incident pole is located in the regularity strips and if the other GO poles are far from the regularity strip we can approximate the GO spectra just with the incident plane wave spectra.

This strong approximation has several potential advantages:

- 1) the solution in  $-\pi - \Phi < w < -\pi + \Phi$  is obtained recursively and improved by the use of the recursion eq. (6),
- 2) the spectral solution assumes a closed form that does not require any numerical approximation,
- 3) the solution contains all the GO singularities and the structural singularities (poles and branch points)

These properties are useful to efficient calculate the field.

Implementation and validation will be provided at the conference.

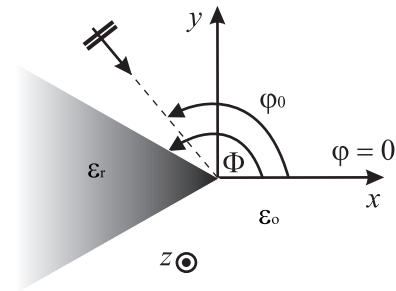


Fig. 1: The dielectric wedge

### III. PENETRABLE WEDGE

Following the procedure proposed in [4], [8-9] we obtain that the problem of a plane wave incident on a penetrable wedge can be modeled by a system of generalized matrix Wiener-Hopf equation defined in three complex planes that are related together.

In this problem the equations are more cumbersome but we expect that the same properties introduced for the impenetrable wedge problems are extendable to efficient calculate the solution in terms of field.

Further discussion, implementation and validation will be provided at the conference

### ACKNOWLEDGEMENT

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