A guaranteed-convergence framework for passivity enforcement of linear macromodels

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Abstract

Passivity enforcement is a key step in the extraction of linear macromodels of electrical interconnects and packages for Signal and Power Integrity applications. Most state-of-the-art techniques for passivity enforcement are based on suboptimal or approximate formulations that do not guarantee convergence. We introduce in this paper a new rigorous framework that casts passivity enforcement as a convex non-smooth optimization problem. Thanks to convexity, we are able to prove convergence to the optimal solution within a finite number of steps. The effectiveness of this approach is demonstrated through various numerical examples.

1 Introduction

Macromodeling of electrical interconnects and packages is a standard practice in Signal and Power Integrity verification flows. This approach involves a first step based on full-wave simulations or direct measurements, in order to capture all possible signal and power degradation effects due to electromagnetic propagation and local/global coupling. This process results in the characterization of the interconnect network as a multiport, known through frequency-domain samples of its scattering matrix. This dataset is then fed to a rational fitting scheme [1], in order to produce a closed-form state-space macromodel. The latter is finally synthesized as an equivalent circuit and used in system-level transient simulations.

Since any electrical interconnect is unable to generate energy, the corresponding macromodels should satisfy passivity constraints [2]. Passivity may be lost due to numerical approximations during the model identification stage and must be corrected, otherwise the system-level transient simulation may become unstable [3].

Passivity conditions on scattering input-output representations require that the transfer matrix of the macromodel is bounded real [2]. Since bounded realness conditions involve checking the model transfer matrix over infinite (continuous) frequency sets, various alternative formulations have been presented, in forms that are more convenient for numerical passivity enforcement. The most rigorous approach for state-space macromodels is the so-called Bounded Real Lemma (BRL), which leads to a convex [4] formulation of passivity enforcement based on Linear Matrix Inequality (LMI) constraints [5]. This approach is unfortunately too expensive in terms of CPU and especially memory consumption for practical applications. Other formulations resort to a spectral perturbation of suitably defined Hamiltonian matrices [3, 6, 7]. Although very popular and very effective in some cases, the corresponding passivity enforcement schemes are not guaranteed to converge. A last class of methods is based on iterative perturbations of the frequency-dependent energy gain of the system at a finite set of frequency points, cast as an iterative solution of linear or quadratic programs [7, 8]. Also these schemes do not guarantee convergence since an approximate form of the bounded realness constraints is used. There exist heuristic schemes that always converge to some passive macromodel, e.g. the pole perturbation approach in [9], but such methods do not guarantee optimal accuracy preservation.

This paper presents a new and rigorous approach. The passivity enforcement problem is formally cast as a minimization of the $\mathcal{H}_\infty$ norm of the model. This problem can be cast as a convex optimization [4], thus guaranteeing global optimality and convergence. The objective function to be optimized is verified to be convex but non-smooth and non-differentiable as a function of the decision variables. Therefore, we introduce a dedicated projection algorithm based on descent directions computed from subgradients and subdifferentials of the objective function. Numerical examples show that the obtained scheme is able to always guarantee convergence within any prescribed tolerance in a finite number of iteration steps.

2 Notation and problem statement

Our starting point is a nominal macromodel $\boldsymbol{H}(\boldsymbol{0})$, whose $p \times p$ transfer (scattering) matrix is expressed as

$$\boldsymbol{H}(\boldsymbol{0}, s) = \boldsymbol{C}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{B} + \boldsymbol{D}, \tag{1}$$

where the argument $\boldsymbol{0}$ will be used later to denote suitably defined perturbation variables, s is the Laplace variable, and $\boldsymbol{A} \in \mathbb{R}^{n,n}, \boldsymbol{B} \in \mathbb{R}^{n,p}, \boldsymbol{C} \in \mathbb{R}^{p,n}, \boldsymbol{D} \in \mathbb{R}^{p,p}$ are the state-space matrices. It is assumed that the eigenvalues λ_i of matrix $\boldsymbol{A}$ have strictly negative real part, as easily enforced by most rational macromodeling schemes, so that the nominal macromodel (1) is asymptotically stable. Under these assumptions, the model is passive if and only if the following condition holds

$$\|\boldsymbol{H}(\boldsymbol{0})\|_{\mathcal{H}_\infty} = \sup_{\omega \in \mathbb{R}} \sigma_1(\boldsymbol{H}(\boldsymbol{0}, \mathrm{j}\omega)) \leq 1, \tag{2}$$

where σ_1 denotes the maximum singular value. Passivity thus implies the unitary boundedness of the so-called $\mathcal{H}_\infty$ norm, defined in (2), which provides a quantitative measure of the maximum energy gain of the model throughout the frequency axis.

Let as assume that condition (2) does not hold, so that the nominal model is not passive. We want to perturb the state-space matrices such that the resulting perturbed macromodel is passive. As typical in most published passivity enforcement schemes, we choose to perturb only the state-to-output mapping matrix $\boldsymbol{C}$, which is usually constructed by collecting the residue matrices of a partial fraction expansion of $\boldsymbol{H}(\boldsymbol{0}, s)$. This is only feasible if $\sigma_1(\boldsymbol{D}) \leq 1$, a condition that is easily enforced during the model identification stage. We define a perturbation matrix $\boldsymbol{X} \in \mathbb{R}^{p,n}$ and its corresponding "vectorized" form $\boldsymbol{x} \in \mathbb{R}^{pn,1}$,

which are related through

$$\boldsymbol{x} = \mathrm{vec}(\boldsymbol{X}), \quad \boldsymbol{X} = \mathrm{mat}(\boldsymbol{x}), \qquad (3)$$

where the "vec" operator stacks the columns of its matrix argument in a single column vector, and the "mat" operator performs the inverse operation. We then define the perturbed macromodel $\boldsymbol{H}(\boldsymbol{x})$ through its transfer matrix

$$\boldsymbol{H}(\boldsymbol{x}, s) = (\boldsymbol{C} + \boldsymbol{X})(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{B} + \boldsymbol{D}. \qquad (4)$$

The $\mathcal{H}_\infty$ norm of the perturbed macromodel is

$$h(\boldsymbol{x}) = \|\boldsymbol{H}(\boldsymbol{x})\|_{\mathcal{H}_\infty} = \sup_{\omega \in \mathbb{R}} h_\omega(\boldsymbol{x}), \qquad (5)$$

where $h_\omega(\boldsymbol{x}) = \sigma_1(\boldsymbol{H}(\boldsymbol{x}, \mathrm{j}\omega))$ denotes the maximum singular value of the transfer matrix (4) at a fixed frequency ω. The perturbed macromodel (4) is passive if and only if $h(\boldsymbol{x}) \leq 1$.

A second minimal perturbation condition will be also needed for our passivity enforcement scheme, in order to guarantee that the model accuracy will be preserved. In this work, we will measure the amount of perturbation as

$$f(\boldsymbol{x}) = \|\boldsymbol{x}\|_2 = \|\boldsymbol{X}\|_F, \qquad (6)$$

where $_2$ and $_F$ denote the Euclidean and the Frobenius norm, respectively. We will then formulate our optimal passivity enforcement scheme as the following optimization problem

$$\min_{\boldsymbol{x}} f(\boldsymbol{x}), \quad \text{s.t. } h(\boldsymbol{x}) \leq 1, \qquad (7)$$

where the minimal perturbation condition is set as an objective function and the passivity condition appears as an inequality constraint. Other equivalent or weighted perturbation norms [6] can be used as well with suitable modifications of (6).

3 Convexity and smoothness

We recall that a set $\mathcal{S}$ is convex if

$$\vartheta \boldsymbol{x}_1 + (1 - \vartheta)\boldsymbol{x}_2 \in \mathcal{S}, \quad 0 \leq \vartheta \leq 1.$$

A function $\phi : \mathbb{R}^n \to \mathbb{R}$ is convex if its domain $\mathcal{D}$ is a convex set and if for any $\boldsymbol{x}_1, \boldsymbol{x}_2 \in \mathcal{D}$ we have

$$\phi(\vartheta \boldsymbol{x}_1 + (1 - \vartheta)\boldsymbol{x}_2) \leq \vartheta \phi(\boldsymbol{x}_1) + (1 - \vartheta)\phi(\boldsymbol{x}_2)$$

with $0 \leq \vartheta \leq 1$. It follows from the triangle inequality that any norm is convex, therefore both $f(\boldsymbol{x})$ in (6) and $h(\boldsymbol{x})$ in (5) are convex functions. Therefore, the optimization problem (7) minimizes a convex function over a convex set. It is well known that these properties guarantee that there exist a unique global optimum $\boldsymbol{x}^*$.

A vector $\boldsymbol{g} \in \mathbb{R}^n$ is called subgradient of a convex function ϕ at $\boldsymbol{x}$, if for all $\boldsymbol{z}$ in the domain of ϕ, it holds that

$$\phi(\boldsymbol{z}) \geq \phi(\boldsymbol{x}) + \boldsymbol{g}^\top(\boldsymbol{z} - \boldsymbol{x}). \qquad (8)$$

If ϕ is differentiable, then $\boldsymbol{g} = \nabla\phi(\boldsymbol{x})$ is the unique subgradient. However, subgradients exist also at points where ϕ is non differentiable. The set $\partial\phi(\boldsymbol{x})$ collecting all such subgradients is called subdifferential and is always closed and convex.

4 Subdifferential of the $\mathcal{H}_\infty$ norm

Let us consider the perturbed macromodel (4) and compute its singular value decomposition at frequency $\bar{\omega}_i$

$$\boldsymbol{H}(\boldsymbol{x}, \mathrm{j}\bar{\omega}_i) = \boldsymbol{U}^{(i)}\boldsymbol{\Sigma}^{(i)}[\boldsymbol{V}^{(i)}]^H \qquad (9)$$

We denote with ℓ_i the multiplicity of the largest singular value $\sigma_1^{(i)}$, and the first ℓ_i columns of $\boldsymbol{U}^{(i)}$ and $\boldsymbol{V}^{(i)}$ as $\boldsymbol{U}_1^{(i)}$ and $\boldsymbol{V}_1^{(i)}$, respectively. Since the $\mathcal{H}_\infty$ norm is defined (2) as the supremum of the largest singular values among all frequencies, let us collect all frequencies $\{\bar{\omega}_i, 1 \leq i \leq q\}$ at which this supremum is attained, i.e., such that

$$\sigma_1^{(i)} = \sup_{\omega \in \mathbb{R}} h_\omega(\boldsymbol{x}) = h(\boldsymbol{x}), \quad 1 \leq i \leq q. \qquad (10)$$

The subdifferential of the $\mathcal{H}_\infty$ norm can be expressed as

$$\partial h(\boldsymbol{x}) = \left\{ \mathrm{vec}\left(\sum_{i=1}^q \Re\{\boldsymbol{\Psi}(\mathrm{j}\bar{\omega}_i)\boldsymbol{V}^{(i)}\boldsymbol{Y}_i\boldsymbol{U}^{(i)H}\}^\top \right) \right\}, \quad (11)$$

where $\boldsymbol{\Psi}(\mathrm{j}\omega) = (\mathrm{j}\omega\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{B}$ and where the q matrices $\boldsymbol{Y}_i \in \mathbb{R}^{\ell_i, \ell_i}$ are such that $\boldsymbol{Y}_i = \boldsymbol{Y}_i^\top \geq 0$ and $\sum_{i=1}^q \mathrm{Tr}\,\boldsymbol{Y}_i = 1$. The above result is reported here without proof. For additional details, see [11, 12].

We remark that when that there are multiple extremal frequencies ($q > 1$) or multiple singular values ($\ell_i > 1$ for some i), the $\mathcal{H}_\infty$ norm $h(\boldsymbol{x})$ results non-differentiable at $\boldsymbol{x}$. Conversely, if $q = 1$ and $\ell_1 = 1$, a much simpler characterization is possible, since in this case $h(\boldsymbol{x})$ results differentiable, and the subdifferential includes a single element $\nabla h(\boldsymbol{x})$, which can be expressed as

$$\nabla h(\boldsymbol{x}) = \mathrm{vec}\left(\Re\{\boldsymbol{\Phi}(\mathrm{j}\bar{\omega})\boldsymbol{v}_1\boldsymbol{u}_1^H\}^\top \right). \qquad (12)$$

It is worth noting that most existing passivity enforcement schemes based on singular value perturbation employ expression (12) as the basis for their algorithm setup. The lack of smoothness is typically not addressed, leading to inevitably approximate, non-convex, and possibly incorrect formulations.

5 Smooth and non-smooth descent schemes

Let ϕ be a convex function. If ϕ is differentiable, the direction pointed by the negative gradient $-\nabla\phi(\boldsymbol{x})$ is the steepest descent direction. Gradient-based descent schemes for minimization of ϕ find the solution by applying iterative update rules of type

$$\boldsymbol{x}^{(k+1)} = \boldsymbol{x}^{(k)} - \alpha_k \nabla\phi(\boldsymbol{x}^{(k)}),$$

where α_k is a suitable step size. In case ϕ is convex but non-smooth, as for the $\mathcal{H}_\infty$ norm $h(\boldsymbol{x})$, a similar approach for minimization can be adopted, by generalizing the update rule as

$$\boldsymbol{x}^{(k+1)} = \boldsymbol{x}^{(k)} - \alpha_k \boldsymbol{g}^{(k)}, \qquad (13)$$

where $\boldsymbol{g}^{(k)} \in \partial\phi(\boldsymbol{x}^{(k)})$ is a subgradient. Although it is not guaranteed that a generic subgradient is a descent direction for ϕ, it is possible to select a particular subgradient $\boldsymbol{g}^*$ that minimizes the corresponding directional derivative of ϕ over all directions. To this end, it suffices to solve the following additional convex optimization problem

$$\boldsymbol{g}^* = \arg \min_{\boldsymbol{g} \in \partial\phi(\boldsymbol{x})} \|\boldsymbol{g}\|, \qquad (14)$$
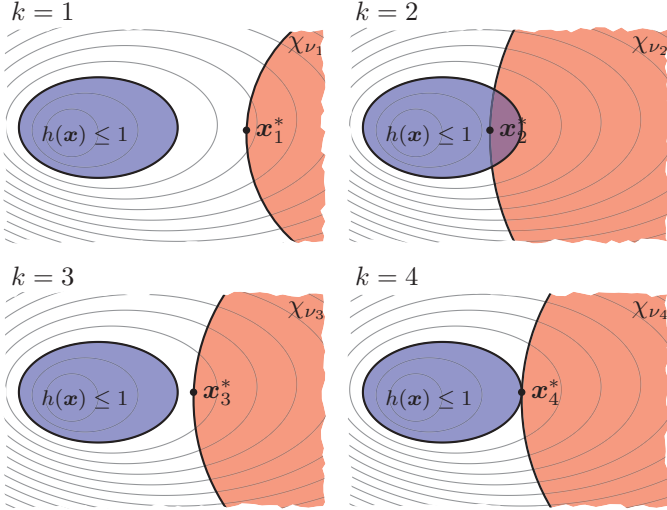
as proved in [10].

Figure 1: First three iterations of the outer bisection loop on the macromodel perturbation amount ν. For each iteration, $\boldsymbol{x}^*$ denotes the optimal solution of problem (15). Bottom right panel depicts the situation at convergence.

6 The main algorithm

Here, we restate the direct formulation (7) in a form that will be easily solved numerically. First, we define the (convex) set

$$\mathcal{X}_\nu = \{\boldsymbol{x} : f(\boldsymbol{x}) \leq \nu\},$$

including all parameter configurations defining perturbed models that differ from the nominal system less than ν. For instance, ν may be some prescribed desired accuracy level that is set by the designer in the problem setup phase. Among all such models, we seek the one with minimal $\mathcal{H}_\infty$ norm by solving problem

$$\min_{\boldsymbol{x}} h(\boldsymbol{x}), \quad \text{s.t. } \boldsymbol{x} \in \mathcal{X}_\nu. \tag{15}$$

Denoting the optimal solution as $\boldsymbol{x}^*$, the following two cases may apply

1. if $h(\boldsymbol{x}^*) \leq 1$, we have found a passive macromodel with controlled accuracy with respect to nominal macromodel; in other words, problem (15) with the additional passivity constraint $h(\boldsymbol{x}) \leq 1$ is feasible;

2. if instead $h(\boldsymbol{x}^*) > 1$, we can conclude that there exist no passive macromodel which deviates less than ν from the original model.

We then argue that there exists an optimal accuracy $\nu = \nu^*$ such that problem

$$\min_{\boldsymbol{x}} h(\boldsymbol{x}), \quad \text{s.t. } \boldsymbol{x} \in \mathcal{X}_\nu, \quad h(\boldsymbol{x}) \leq 1 \tag{16}$$

is feasible. We will look for the optimal accuracy ν^* by an outer bisection loop, as illustrated in Fig. 1 and described below.

Let us assume that at the first iteration $k = 1$ (top left panel) problem (16) is not feasible. Therefore, the accuracy ν_1 is too stringent and the set $\mathcal{X}_{\nu_1}$ is too small. We then need to relax the accuracy to a larger value $\nu_2 > \nu_1$ which makes problem (16)
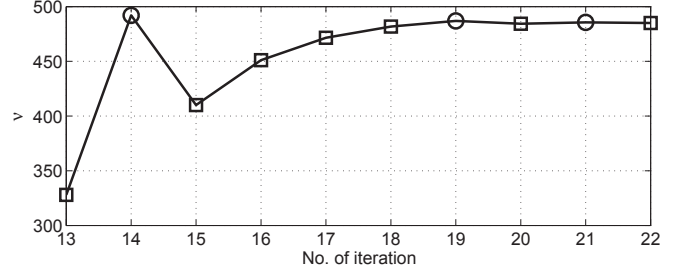

Figure 2: PCB interconnect: values attained by ν during the last few bisection iterations. Squares and circles denote iterations where $h(\boldsymbol{x}^*) > 1$ and $h(\boldsymbol{x}^*) < 1$, respectively.

feasible. The top right panel in Fig. 1 illustrates this situation, highlighting that the intersection of sets $\mathcal{X}_{\nu_2}$ and $\{\boldsymbol{x} : h(\boldsymbol{x}) \leq 1\}$ is nonempty. The optimal accuracy is such that $\nu^* \in [\nu_1, \nu_2]$. We then define $\nu_3 = (\nu_1 + \nu_2)/2$ and solve problem (16) again (bottom left panel). This bisection process on ν is repeated until convergence (bottom right panel). We remark that we do not need to obtain the optimal solution $\boldsymbol{x}^*$ of problem (16) at each iteration. Rather, we need to determine only the feasibility of this problem. If the problem is feasible, we decrease ν. If the problem is infeasible, we increase ν.

7 A projected subgradient scheme

The feasibility of problem (16) is addressed via an iterative projected subgradient algorithm. We pick a generic initial point $\boldsymbol{x}^{(0)}$, and we generate the next point by performing a step in the direction $-\boldsymbol{g}^{(0)}$

$$\tilde{\boldsymbol{x}}^{(0)} = \boldsymbol{x}^{(0)} - \alpha_0 \boldsymbol{g}^{(0)}$$

where $\boldsymbol{g}^{(0)}$ is a subgradient of the function $h(\boldsymbol{x})$ in $\boldsymbol{x}_0$ and α_0 is a suitable step size. Generally, $\tilde{\boldsymbol{x}}^{(0)}$ does not belong to the feasible set $\mathcal{X}_\nu$, therefore we project the $\tilde{\boldsymbol{x}}^{(0)}$ on the set $\mathcal{X}_\nu$ obtaining the new candidate solution

$$\boldsymbol{x}^{(1)} = [\tilde{\boldsymbol{x}}^{(0)}]_{\mathcal{X}_\nu} = [\boldsymbol{x}^{(0)} - \alpha_0 \boldsymbol{g}^{(0)}]_{\mathcal{X}_\nu}$$

where $[\cdot]_{\mathcal{X}}$ is the operator that performs an orthogonal Euclidean projection of its argument onto $\mathcal{X}$. The above process is repeated following the iterative scheme

$$\boldsymbol{x}^{(k+1)} = [\boldsymbol{x}^{(k)} - \alpha_k \boldsymbol{g}^{(k)}]_{\mathcal{X}} \tag{17}$$

until convergence. See [13] for technical details on fundamental assumptions of this method and convergence proofs.

8 Numerical examples

We demonstrate the proposed passivity enforcement algorithm through two practical cases. The first example is a 4-port coupled PCB interconnect. Its scattering matrix has been measured from DC up to 20 GHz with resolution 10 MHz. These samples have been processed by the well known Vector Fitting (VF) algorithm [1] to obtain an initial macromodel (1), with $n = 272$ states. The projected subgradient method has been applied to the model in order to enforce its passivity. The algorithm required 22 outer bisection iterations before reaching a relative accuracy 3.33×10^{-7} on the optimal perturbation
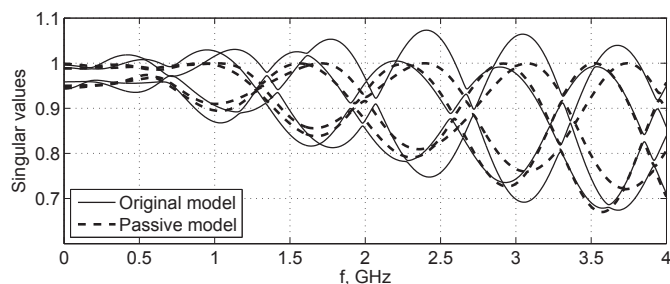
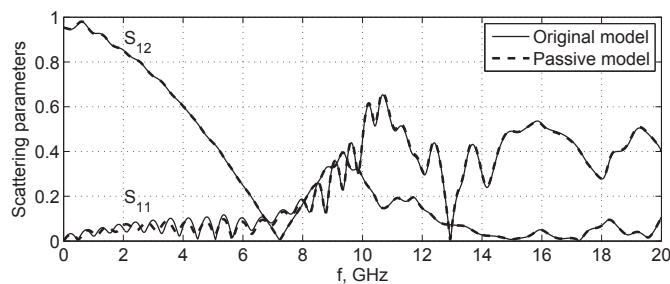Figure 3: PCB interconnect: singular values of original non passive and compensated models.

Figure 4: PCB interconnect: scattering responses of original and compensated models.
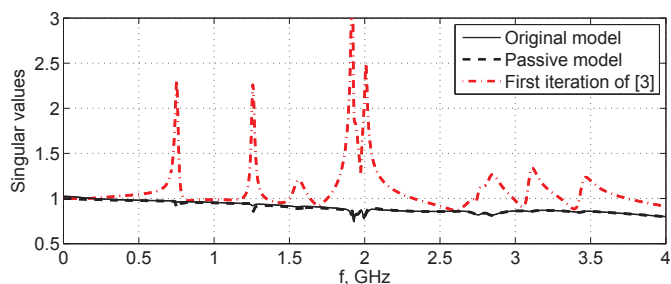

Figure 5: Filter: frequency-dependent largest singular value of original non-passive and final compensated models. The red dash-dot line depicts the largest singular value obtained after the first iteration of the passivity enforcement scheme [3].
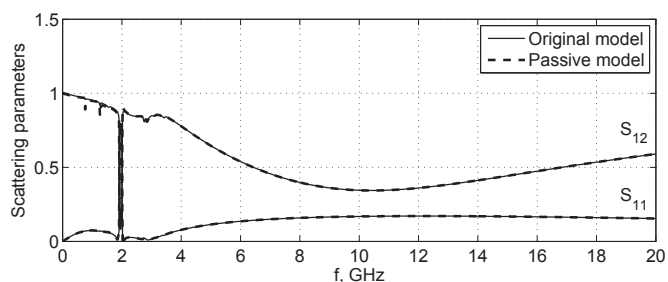

Figure 6: Filter: scattering responses of original and compensated models.

ν^* . The values attained by ν at the last few iterations are reported in Fig. 2. The singular values of the starting non passive model and of the final compensated model are depicted in the Fig. 3. A comparison between few scattering responses before and after passivity enforcement is depicted in Fig. 4, showing an excellent accuracy preservation.

We demonstrate through a second example the reliability of the proposed scheme, by processing a model for which the standard methods [3, 6] fail. A nominal macromodel ($n = 60$ poles) has been obtained by applying the VF algorithm to the scattering responses of a sharp 2-port filter. This model was first subject to the iterative passivity enforcement scheme [3], which is based on passivity constraints derived from a linearized expression of the Hamiltonian eigenvalues as a function of residues. The optimal algorithm settings discussed in [6] were used. Figure 5 shows that very large perturbations of the singular values and model responses are induced throughout the frequency axis. This method diverges in very few iterations. We then applied our proposed projected subgradient algorithm, easily obtaining a passive model with a very good accuracy, as demonstrated in Fig. 6, where few scattering responses of original and passive models are compared.

This last example shows that, thanks to the convex formulation, the proposed algorithm is able to manage cases where other state-of-the-art methodologies fail to converge.

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