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A Simple NWA Calibration Algorithm Based on a Transfer Standard

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Abstract* - A new two-port network analyzer (NWA) calibration technique is here presented. It uses a single two-port transfer standard plus a known reflectance to perform the calibration process. The transfer standard device has to be previously fully characterized with a traceable NWA. The technique here proposed uses less standards than any other up today calibration algorithm, which, on the contrary, requires at least three different devices. The paper presents the calibration algorithm along with some on-wafer experimental results which compare the new solution with a more traditional technique.

1. INTRODUCTION

The accuracy of a NWA is tied to the effectiveness of the technique used to remove the systematic errors. In order to increase the system accuracy, several calibration techniques, based on different physical standards and mathematical algorithms have been proposed. Recently, some authors introduced the concept of using transfer standards, previously characterised by primary laboratories, instead of primary standards (i.e. known by dimensional measurement and EM modelling). By following this concept, an electronic system was developed which allows to calibrate a two port NWA by presenting several hundreds of known load conditions. So far traditional calibration procedures have been extended in order to accept the connection of those transfer standards [1,2].

In this work a new technique is presented, which accomplishes a two port NWA calibration by using a single device as *transfer standard* plus a known one port reflectance. Our transfer standard is a two-port passive network, reciprocal but non-symmetrical, previously fully characterised with a traceable NWA. The reflectance has nó constrain, provided it is fully known. In particular, it could be the input impedance of the transfer standard itself with the output port disconnected.

We solve the calibration problem with a complete measurement of the transfer standard in a forward and reverse configuration, plus a reflectance measurement at one port. To the authors' knowledge this technique, that we called NR (Network Reflection), uses less standards than any other calibration algorithm up today developed, which, on the contrary, requires at least three different devices.

NR calibration technique allows a flexibility to design the transfer standard topology and the connectors to fit the user needs. For example, the classical configuration where two coaxial connectors have the same gender normally requires an adapter removal procedure, based on the insertion of several one and two port standards. With the presented procedure, the problem is solved by means of two connections of the same special designed transfer standard.

2. ERROR MODEL AND DEEMBEDDING FORMULA

With an actual NWA, it is not possible to access directly to the real waves a_i, b_i at the DUT reference planes. According to the error model adopted, the acquired quantities a_{mi}, b_{mi} at each measurement port are linearly related to the corresponding real waves (no leakage assumption):

$$\begin{bmatrix} b_{m1} \\ a_1 \end{bmatrix} = \mathbf{E}_1 \cdot \begin{bmatrix} a_{m1} \\ b_1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} b_{m2} \\ a_2 \end{bmatrix} = \mathbf{E}_2 \cdot \begin{bmatrix} a_{m2} \\ b_2 \end{bmatrix}. \qquad (1)$$

The usual interpretation of the matrices $\mathbf{E}_i$ (i = 1,2) that contain the error coefficients is shown in figure 1: they can be seen as the scattering matrices of two distinct fictitious networks (the *error boxes*) that link the DUT to an ideal, error-free NWA which measures the waves a_{mi}, b_{mi}.

The notation adopted in this paper has been already presented for a generic N-port test-set ([3], [4]) and it is used here in the special case of N equal to two. In particular, the *i*-th error matrix is regarded as:

$$\mathbf{E}_i = \begin{bmatrix} e_i^{00} & e_i^{01} \\ e_i^{10} & e_i^{11} \end{bmatrix} \qquad (2)$$

The aim of the raw data correction process consists in computing the scattering matrix **S** of the DUT, by using the error coefficients found during the calibration and the measured pseudo-scattering matrix $\mathbf{S_m}$, defined as:

$$\begin{bmatrix} b_{m1} \\ b_{m2} \end{bmatrix} = \mathbf{S_m} \cdot \begin{bmatrix} a_{m1} \\ a_{m2} \end{bmatrix} \qquad (3)$$

To obtain this matrix, a proper switch correction procedure is performed, which consists in measuring the DUT under two different excitations. For instance, defining $a_{mi}^{(j)}$ and $b_{mi}^{(j)}$ as the measured waves at the *i*-th port with the RF source connected to the *j*-th port, two equations like (3) can be packed into a single matrix and the measured matrix $\mathbf{S_m}$ can be computed as [5]:

$$\mathbf{S_m} = \begin{bmatrix} b_{m1}^{(1)} & b_{m1}^{(2)} \\ b_{m2}^{(1)} & b_{m2}^{(2)} \end{bmatrix} \cdot \begin{bmatrix} a_{m1}^{(1)} & a_{m1}^{(2)} \\ a_{m2}^{(1)} & a_{m2}^{(2)} \end{bmatrix}^{-1} \qquad (4)$$

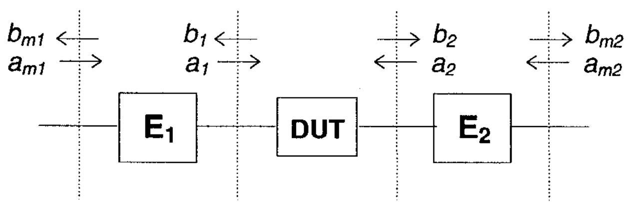

Fig.1. Error model of a two-port NWA.

The deembedding formula can be found re-arranging equations (1) into the following system (in a way similar to [7]):

$$\begin{bmatrix} \mathbf{b} \\ \mathbf{a} \end{bmatrix} = \begin{bmatrix} -\mathbf{H} & \mathbf{L} \\ -\mathbf{M} & \mathbf{K} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{a_m} \\ \mathbf{b_m} \end{bmatrix} \qquad (5)$$

where $\mathbf{a} = [a_1 \;\; a_2]^T$, $\mathbf{b} = [b_1 \;\; b_2]^T$, $\mathbf{a_m} = [a_{m1} \;\; a_{m2}]^T$, $\mathbf{b_m} = [b_{m1} \;\; b_{m2}]^T$.

The error terms are contained in four 2x2 matrices which are diagonal (since no leakage between the test ports has been assumed):

$$\mathbf{K} = \begin{bmatrix} e_1^{01} & 0 \\ 0 & e_2^{01} \end{bmatrix}^{-1} \qquad \mathbf{M} = \mathbf{K} \cdot \begin{bmatrix} e_1^{00} & 0 \\ 0 & e_2^{00} \end{bmatrix} \qquad (6)$$

$$\mathbf{L} = \begin{bmatrix} e_1^{11} & 0 \\ 0 & e_2^{11} \end{bmatrix} \cdot \mathbf{K} \qquad \mathbf{H} = \begin{bmatrix} \Delta_1 & 0 \\ 0 & \Delta_2 \end{bmatrix} \cdot \mathbf{K} \qquad (7)$$

where $\Delta_i = e_i^{00} e_i^{11} - e_i^{01} e_i^{10}$.

Introducing the scattering matrices **S** and $\mathbf{S_m}$, the system (5) with some basic algebra becomes:

$$\mathbf{M} + \mathbf{S} \cdot \mathbf{L} \cdot \mathbf{S_m} - \mathbf{S} \cdot \mathbf{H} - \mathbf{K} \cdot \mathbf{S_m} = \mathbf{0} \qquad (8)$$

and, in the final form:

$$\mathbf{S} = (\mathbf{M} - \mathbf{K} \cdot \mathbf{S_m}) \cdot (\mathbf{H} - \mathbf{L} \cdot \mathbf{S_m})^{-1}. \qquad (9)$$

3. NR CALIBRATION ALGORITHM

For a two-port standard, in the non-leakage case, equation (8) can be seen as four equations of the following type [4]:

$$M_{ii}\delta_{ij} + \sum_{q=1}^{2} S_{iq} L_{qq} S_{mqj} - S_{ij} H_{jj} - K_{ii} S_{mij} = 0 \qquad (10)$$

where i = 1,2, j = 1,2 and δ_{ij} is the Kronecker symbol. For a one-port standard connected to port i, equation (8) becomes:

$$M_{ii} + \Gamma_i \Gamma_{mi} L_{ii} - \Gamma_i H_{ii} - K_{ii} \Gamma_{mi} = 0 \qquad (11)$$

The calibration procedure is performed by collecting a sufficient number of equations like (10) and (11) from a proper set of standard measurements to form the homogeneous linear system in eight unknowns:

$$\mathbf{C} \cdot \mathbf{v} = \mathbf{0} \qquad (12)$$

where **v** is the error coefficient array, expressed as:

$$\mathbf{v} = [M_{11} \quad M_{22} \quad L_{11} \quad L_{22} \quad H_{11} \quad H_{22} \quad K_{11} \quad K_{22}]^T \qquad (13)$$

To avoid the trivial zero solution, system (12) has to be normalised to one of the unknown coefficients, obtaining an equation of the form

$$\mathbf{N} \cdot \mathbf{u} = \mathbf{g}. \qquad (14)$$

Since it can be easily proved that the deembedding equation (9) is invariant for any normalisation of matrix **K** (see the definitions (6) and (7)), we can assume K_{11} equal to one.

The NR calibration procedure is based on the following standard set:

- a fully-known two-port non-symmetrical device (the transfer standard, whose S-matrix is **S**) is first measured in *forward* configuration, obtaining $\mathbf{S}_\mathrm{m}^\mathrm{f}$;
- afterwards, the same transfer standard is measured in *reverse* configuration, i.e. swapping port1 and port2, obtaining $\mathbf{S}_\mathrm{m}^\mathrm{r}$;
- finally, since the previous connections provide only six linearly independent equations to (14), an additional one-port standard has to be measured to complete the calibration procedure. For instance, let Γ_{m1} be the measured reflection coefficient of the standard Γ_1 at port1.

The true scattering matrix of the transfer standard in the reverse configuration is simply the S-matrix of the forward case with both rows and columns swapped (by definition). Thus the final system of nine equation is given by (15). It has a rank of seven and it can be solved with any traditional method (for instance, Gaussian elimination or QR-decomposition algorithm).

It is not useless underlining that the one-port reflectance can be obtained as the input impedance of the transfer standard with the second port disconnected. It can be proved that this measurement adds a new, linearly independent equation. This method obviously requires an additional characterisation of the transfer standard in the open circuit configuration, but it allows to perform an entire calibration process with different measurements of a *single* standard device.

The effectiveness of the NR calibration algorithm strongly depends on the transfer standard parameters. Obviously, the standard can be reciprocal but has to be not symmetrical, in order to provide different equations in the forward and reverse case.

4. VERIFICATION OF THE NR ALGORITHM

In the authors' opinion, the main application of the NR calibration procedure is in coaxial measurements, as it will be clear in the following. However, in order to test the algorithm with different transfer standards, a large set of coplanar (CPW) passive networks with different topologies and parameters have been designed on a thin-film sapphire substrate and measured with 150μm-pitch probes up to 18.5 GHz.

The aim of the experimental investigation is to verify how the NR technique is affected the accuracy of the transfer standard characterisation.

The process follows four distinct steps.

1. First of all, the NWA was calibrated with a traditional LRM procedure [6], using a 1.5 ps line and two dc-measured loads. The so calibrated NWA is assumed as the reference instrument.

$$\begin{bmatrix} 1 & 0 & S_{11}S_{m11}^f & S_{12}S_{m21}^f & -S_{11} & 0 & 0 \\ 0 & 0 & S_{11}S_{m12}^f & S_{12}S_{m22}^f & 0 & -S_{12} & 0 \\ 0 & 0 & S_{21}S_{m11}^f & S_{22}S_{m21}^f & -S_{21} & 0 & -S_{m21}^f \\ 0 & 1 & S_{21}S_{m12}^f & S_{22}S_{m22}^f & 0 & -S_{22} & -S_{m22}^f \\ 1 & 0 & S_{22}S_{m11}^r & S_{21}S_{m21}^r & -S_{22} & 0 & 0 \\ 0 & 0 & S_{22}S_{m12}^r & S_{21}S_{m22}^r & 0 & -S_{21} & 0 \\ 0 & 0 & S_{12}S_{m11}^r & S_{11}S_{m21}^r & -S_{12} & 0 & -S_{m21}^r \\ 0 & 1 & S_{12}S_{m12}^r & S_{11}S_{m22}^r & 0 & -S_{11} & -S_{m22}^r \\ 1 & 0 & \Gamma_1\Gamma_{m1} & 0 & -\Gamma_1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} M_{11} \\ M_{22} \\ L_{11} \\ L_{22} \\ H_{11} \\ H_{22} \\ K_{22} \end{bmatrix} = \begin{bmatrix} S_{m11}^f \\ S_{m12}^f \\ 0 \\ 0 \\ S_{m11}^r \\ S_{m12}^r \\ 0 \\ 0 \\ \Gamma_{m1} \end{bmatrix} \qquad (15)$$

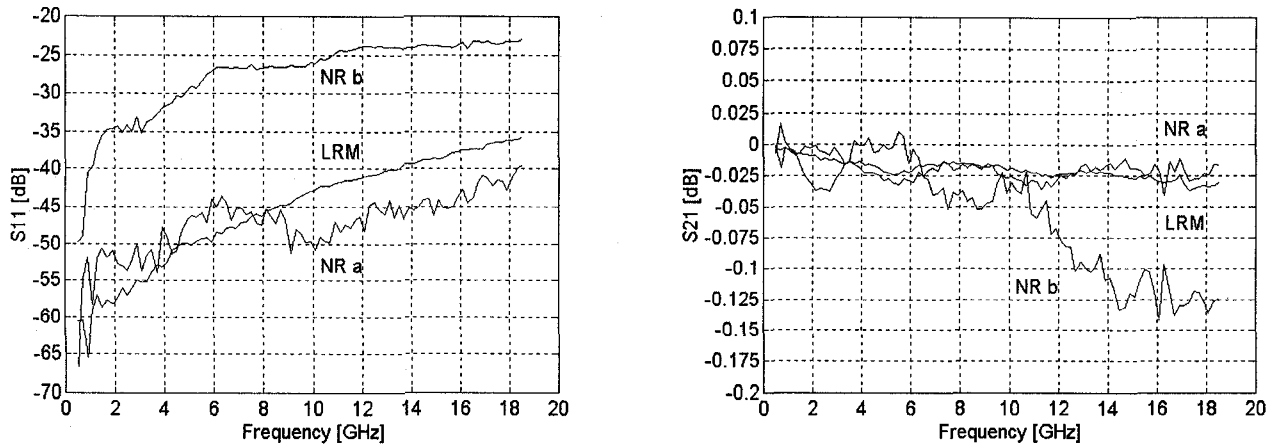

Fig.2. Measurements of a 3 ps verification CPW line with three different calibrations: an LRM reference calibration and two NR calibrations performed with different transfer standards (case a: Rs=200Ω, Rp=50Ω; case b: Rs=25Ω, Rp=25Ω).

2. The LRM calibrated NWA was used to characterise the device adopted as transfer standard.
3. Afterwards, an NR calibration was performed, using the previous transfer standard and with an ideal short (the same used in LRM calibration as reflect) connected at port 1.
4. Finally, some verification devices were measured and corrected with both the LRM and NR method.

Two NR calibrations were performed using different transfer standards with the same topology (shown in fig.3), but different parameters. Fig.2 shows the S-parameters of a 3 ps verification CPW line, obtained by the LRM and the two NR calibrations. It can be noted that the first calibration (NR case a) has a dramatically better performance than the second one (NR case b).

5. CONCLUSION

A new NWA calibration algorithm which uses only one transfer standard has been developed. The technique is particularly suitable for a coaxial test-set with the same gender at both ports. We proved that the accuracy is not significantly reduced with respect to the usual calibration techniques, provided that the standard device has been properly designed.

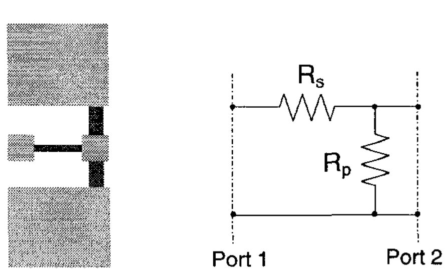

Fig.3. Layout and schematic of the optimum transfer standard used during the measurement.

6. ACKNOWLEDGEMENT

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