

Transient Analysis of PCB Lines with the Inclusion of Parameters Uncertainties

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Abstract—This paper presents an effective solution for the transient analysis of long bus-like interconnects with the inclusion of geometrical and material uncertainties of the structure. The proposed approach is based on the expansion of the well-known frequency-domain telegraph equations in terms of orthogonal polynomials and on the back conversion to time domain via Fourier superposition. The method is validated by means of a systematic comparison with the results of Monte Carlo simulations, for an application example involving a PCB coupled-microstrip interconnect with uncertainties in the relative dielectric permittivity and trace separation.

Index Terms—Stochastic analysis, Tolerance analysis, Uncertainty, Circuit modeling, Circuit simulation, Transmission lines.

I. INTRODUCTION

The manufacturing process of electronic devices introduces sources of uncertainty that may cause significant differences between simulated and measured responses, like higher crosstalk levels, thus possibly causing violations of noise margins. Simulation and verification of such systems is a fundamental need for discovering and correcting problems and avoiding very expensive refabrication. Recently, methods and tools for the stochastic analysis of circuits have become available, and their importance for the simulation of high-performance interconnected electronic equipments with the inclusion of parameters uncertainties has grown. A relevant example is the process-induced variability that unavoidably impacts on the performance of PCB planar structures [1].

The typical resource allowing to collect quantitative information on the statistical behavior of the circuit response is based on the application of the brute-force Monte Carlo (MC) method, or possible complementary methods based on the optimal selection of the subset of model parameters in the whole design space. Such methods, however, are computationally expensive, and this fact prevents us from their application to the analysis of complex realistic structures.

Recently, an effective solution that overcomes the previous limitation has been proposed. This methodology is based on the polynomial chaos (PC) theory and on the representation of the stochastic solution of a dynamical circuit in terms of orthogonal polynomials. For a comprehensive and formal discussion of PC theory, the reader is referred to [2], [3] and references therein. PC technique enjoys applications in several domains of Physics; we limit ourselves to mention recent results on the extension of the classical modified nodal analysis

(MNA) approach to the prediction of the stochastic behavior of circuits with uncertain parameters [4]. Also, the authors of this contribution have recently proposed an extension of PC theory to distributed structures described by transmission-line equations [5].

This paper demonstrates the feasibility and strength of the PC approach, that is validated for a PCB coupled microstrip structure, for which the stochastic behavior of time-domain crosstalk is analyzed.

II. POLYNOMIAL CHAOS PRIMER

This section outlines a brief overview of the PC method. The idea underlying this technique is the spectral expansion of a stochastic function (intended as a given function of a random variable) in terms of a truncated series of orthogonal polynomials. Within this framework, any function H , carrying the effects of parameter variability (in this paper, it will be the per-unit-length parameters and the time- and frequency-domain response of a transmission line), can be approximated by means of the following truncated series

$$H(\xi) = \sum_{k=0}^P H_k \cdot \phi_k(\xi), \quad (1)$$

where $\{\phi_k\}$ are suitable orthogonal polynomials expressed in terms of the random variable ξ . The above expression is defined by the class of the orthogonal bases, by the number of terms P (limited to the range $2 \div 5$ for practical applications) and by the expansion coefficients H_k . The choice of the orthogonal basis relies on the distribution of the random variables being considered. The uncertainties arising from fabrication tolerances turn out to be properly characterized in terms of gaussian variability. Therefore, in this case, the most appropriate orthogonal functions for the expansion (1) are the Hermite polynomials, the first three being $\phi_0 = 1$, $\phi_1 = \xi$ and $\phi_2 = (\xi^2 - 1)$, where ξ is the standard normal random variable, with zero mean and unity standard deviation. It is relevant to remark that any random parameter in the system, e.g., the substrate permittivity ε_r , can be related to ξ as follows

$$\varepsilon_r = \bar{\varepsilon}_r + \Delta\varepsilon_r \xi, \quad (2)$$

where $\bar{\varepsilon}_r$ and $\Delta\varepsilon_r$ are the parameter mean and standard deviation, respectively. The orthogonality property of Hermite polynomials is expressed by

$$\langle \phi_k, \phi_j \rangle = \langle \phi_k, \phi_k \rangle \delta_{kj}, \quad (3)$$

where δ_{kj} is the Kronecker delta and $\langle \cdot, \cdot \rangle$ denotes the inner product in the Hilbert space of the variable ξ with Gaussian weighting function, i.e.,

$$\begin{cases} \langle \phi_k, \phi_j \rangle = \int_{-\infty}^{+\infty} \phi_k(\xi) \phi_j(\xi) W(\xi) d\xi \\ W(\xi) = \exp(-\xi^2/2)/(\sqrt{2\pi}). \end{cases} \quad (4)$$

With the above definitions, the expansion coefficients H_k of (1) are computed via the projection of H onto the orthogonal components ϕ_k . It is worth noting that relation (1), which is a known nonlinear function of the random variable ξ , can be used to predict the probability density function (PDF) of $H(\xi)$ via numerical simulation or analytical formulae [6]. For the sake of brevity, the formal development of PC theory for multiple variables is omitted here.

III. PC APPLICATION TO STOCHASTIC TRANSMISSION-LINE EQUATIONS

This section discusses the modification of the classical transmission-line equations, as needed for incorporating the effects of the statistical variation of the per-unit-length (p.u.l.) parameters via the PC theory. The problem is addressed in frequency domain first, and then extended to time domain via Fourier superposition.

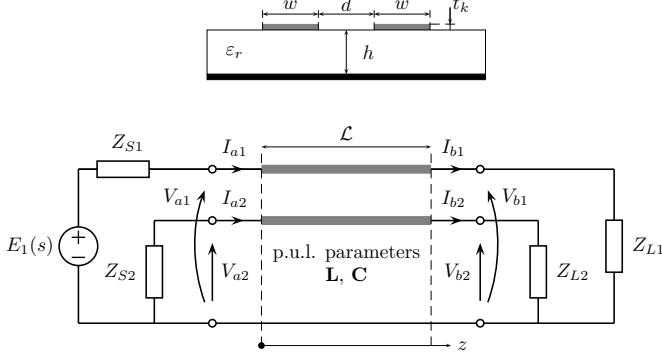


Fig. 1. Test structure considered to demonstrate the proposed approach. Top panel: transmission-line cross-section; bottom panel: simulation test case.

A. Classical Frequency-Domain Transmission-Line Model

For the sake of simplicity, the discussion is based on a lossless three-conductor line, as the coupled microstrip structure shown in Fig. 1, in presence of a single random parameter. The wave propagation on the structure is governed by the telegraphers equation in the Laplace domain [7]

$$\frac{d}{dz} \begin{bmatrix} \mathbf{V}(z, s) \\ \mathbf{I}(z, s) \end{bmatrix} = -s \begin{bmatrix} \mathbf{0} & \mathbf{L} \\ \mathbf{C} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{V}(z, s) \\ \mathbf{I}(z, s) \end{bmatrix}. \quad (5)$$

In the above equation, s is the Laplace variable, $\mathbf{V} = [V_1(z, s), V_2(z, s)]^T$ and $\mathbf{I} = [I_1(z, s), I_2(z, s)]^T$ are vectors collecting the voltage and current variables along the

multiconductor line (z coordinate) and \mathbf{C} and \mathbf{L} are the p.u.l. capacitance and inductance matrices, depending on the geometrical and material properties of the structure [7].

In order to account for the uncertainties affecting the guiding structure, we must consider the p.u.l. parameters as random quantities, with entries depending on the random variable ξ . Hence, (5) becomes a stochastic differential equation, leading to randomly-varying voltages and currents along the line.

B. Stochastic Frequency-Domain Transmission-Line Model

The application of expansion (1) in terms of Hermite polynomials to the p.u.l. parameters and to the unknown voltage and current variables yields a modified version of (5), whose second row becomes

$$\begin{aligned} \frac{d}{dz} (\mathbf{I}_0(z, s) \phi_0(\xi) + \mathbf{I}_1(z, s) \phi_1(\xi) + \mathbf{I}_2(z, s) \phi_2(\xi)) = \\ -s (\mathbf{C}_0 \phi_0(\xi) + \mathbf{C}_1 \phi_1(\xi) + \mathbf{C}_2 \phi_2(\xi)) (\mathbf{V}_0(z, s) \phi_0(\xi) + \\ + \mathbf{V}_1(z, s) \phi_1(\xi) + \mathbf{V}_2(z, s) \phi_2(\xi)), \end{aligned} \quad (6)$$

where a second-order expansion (i.e., $P = 2$) is assumed; the expansion coefficients of electrical variables and of p.u.l. parameters are readily identifiable in the above equation.

Projection of (6) and of the companion relation arising from the first row of (5) on the first three Hermite polynomials leads to the following augmented system, where the random variable ξ does not appear explicitly, due to the integral projection form given in (4):

$$\frac{d}{dz} \begin{bmatrix} \tilde{\mathbf{V}}(z, s) \\ \tilde{\mathbf{I}}(z, s) \end{bmatrix} = -s \begin{bmatrix} \mathbf{0} & \tilde{\mathbf{L}} \\ \tilde{\mathbf{C}} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{V}}(z, s) \\ \tilde{\mathbf{I}}(z, s) \end{bmatrix}. \quad (7)$$

In the previous equation, vectors $\tilde{\mathbf{V}} = [\mathbf{V}_0, \mathbf{V}_1, \mathbf{V}_2]^T$ and $\tilde{\mathbf{I}} = [\mathbf{I}_0, \mathbf{I}_1, \mathbf{I}_2]^T$ collect the different coefficients of the polynomial chaos expansion of the voltage and current variables. The new p.u.l. matrix $\tilde{\mathbf{C}}$ turns out to be

$$\tilde{\mathbf{C}} = \begin{bmatrix} \mathbf{C}_0 & \mathbf{C}_1 & 2\mathbf{C}_2 \\ \mathbf{C}_1 & \mathbf{C}_0 + 2\mathbf{C}_2 & 2\mathbf{C}_1 \\ \mathbf{C}_2 & \mathbf{C}_1 & \mathbf{C}_0 + 4\mathbf{C}_2 \end{bmatrix} \quad (8)$$

and a similar relation holds for matrix $\tilde{\mathbf{L}}$.

It is worth noting that (7) is analogous to (5) and plays the role of the set of equations of a multiconductor transmission line with a number of conductors that is $(P + 1)$ times larger than those of the original line. It should be noted that the increment of the equation number is not detrimental for the method, since for small values of P (as typically occurs in practice), the additional overhead in handling the augmented equations is much less than the time required to run a large number of MC simulations.

Extension of the procedure to the general case of lossy transmission lines with multiple random parameters is straightforward, and amounts to including the resistance and conductance matrices in (5) and the corresponding augmented matrices in (7). Moreover, the univariate polynomials in (1) must be replaced by proper multivariate polynomials, built as

the product combinations of the univariate ones. Similarly, the inner product (4) needs to be suitably modified according to the formulae summarized in [2].

C. Boundary Conditions and Simulation

The standard procedure for the solution of a loaded transmission line like the one of Fig. 1 amounts to combining the port electrical relations of the terminal elements defining the source and load with the transmission-line equation, and solving the resulting system (cfr Ch.s 4 and 5 of [7]).

Similarly, when the problem becomes stochastic, the augmented transmission-line equation (7) is used in place of (5) together with the projection of the source and the load equations on the first $P + 1$ Hermite polynomials. For the example of Fig. 1, the augmented port equations of the line terminations become

$$\begin{cases} \tilde{\mathbf{V}}_a(s) &= [E_1(s), 0 \dots 0]^T - \tilde{\mathbf{Z}}_S(s)\tilde{\mathbf{I}}_a(s) \\ \tilde{\mathbf{V}}_b(s) &= \tilde{\mathbf{Z}}_L(s)\tilde{\mathbf{I}}_b(s), \end{cases} \quad (9)$$

where the port voltages and currents need to match the solutions of the differential equation (7) at line ends (e.g., $\tilde{\mathbf{V}}_a(s) = \tilde{\mathbf{V}}(z=0, s)$, $\tilde{\mathbf{V}}_b(s) = \tilde{\mathbf{V}}(z=\mathcal{L}, s)$). It is worth noting that in this specific example, no variability is included in the terminations, hence the augmented characteristics of the source and load turn out to have a diagonal structure.

Once the unknown voltages and currents are computed, the quantitative information on the spreading of circuit responses can be readily obtained from the analytical expression of the unknowns. As an example, the frequency-domain solution of the terminal voltage V_{b2} , arising from (9) and (7) with $P = 2$, is

$$V_{b2}(j\omega) = V_{b2,0}(j\omega) + V_{b2,1}(j\omega)\xi + V_{b2,2}(j\omega)(\xi^2 - 1), \quad (10)$$

where the first numerical index denotes the conductor and the second one denotes the expansion term. The above relation can be used to compute the PDF of the output quantity (e.g., the magnitude $|V_{b2}(j\omega)|$) using the rules of random variable transformations given in [6].

D. Time-Domain Solution

The time-domain response can be readily obtained from the frequency-domain solution by considering a periodic input source and expressing it in terms of a truncated Fourier series

$$e_1(t) \simeq c_0 + \sum_{n=1}^N c_n e^{j2\pi f_n t} + c_n^* e^{-j2\pi f_n t}, \quad (11)$$

where c_0 is the signal average over one period and c_n is the complex Fourier coefficient for the n -th harmonic at frequency f_n . Being the system of Fig. 1 linear, its time-domain behavior is in principle obtainable by the superposition of the analyses carried out for all signal harmonics. For the individual solution at frequency f_n , the voltage source of Fig. 1, appearing also in (9), is replaced by its n -th harmonic component, i.e., $E(s_n) = E(j2\pi f_n) = c_n$. The time-domain expression of the

output voltage $v_{b2}(t)$ is then obtained as a linear superposition of harmonics:

$$v_{b2}(t) = \sum_{n=1}^N V_{b2}(j2\pi f_n) e^{j2\pi f_n t} + V_{b2}^*(j2\pi f_n) e^{-j2\pi f_n t}, \quad (12)$$

and the output coefficients $V_{b2}(j2\pi f_n)$ are computed according to (10). The linearity of Fourier decomposition assures that the PC structure is preserved also for the time-domain expression of the output:

$$v_{b2}(t) = v_{b2,0}(t) + v_{b2,1}(t)\xi + v_{b2,2}(t)(\xi^2 - 1), \quad (13)$$

where

$$v_{b2,j}(t) = \sum_{n=1}^N V_{b2,j}(j2\pi f_n) e^{j2\pi f_n t} + V_{b2,j}^*(j2\pi f_n) e^{-j2\pi f_n t} \quad (14)$$

with $j = 0, 1, 2$. This is the expression that is used in practice for the numerical validation in the next section.

In the above analysis, the DC term is neglected due to the differential nature of crosstalk, though in general it can be obtained from a DC calculation. However, in our specific application, DC terms are not affected by variability, since transmission line is irrelevant (except for losses) at DC.

IV. NUMERICAL RESULTS

In this section, the proposed technique is applied to the time-domain analysis of a coupled microstrip, with an active line fed by a Gaussian voltage pulse. Referring to Fig. 1, the nominal parameters are $w = 100 \mu\text{m}$, $d = 80 \mu\text{m}$, $h = 60 \mu\text{m}$, $t_k = 35 \mu\text{m}$, $\epsilon_r = 3.7$ and $\mathcal{L} = 5 \text{ cm}$. The source and load elements are defined according to the notation in (9) with $Z_{S1} = Z_{S2} = 50 \Omega$ and $Z_{L1} = Z_{L2} = 1/(sC_L + G_L)$, being $C_L = 10 \text{ pF}$, $G_L = 1/(10 \text{ k}\Omega)$. The input pulse waveform has a peak of 1 V and a width of approximately 0.35 ns at half amplitude. To compute its Fourier series (11), a period of 6 ns and $N = 30$ harmonics are considered. The variability is provided by the relative permittivity ϵ_r and the trace separation d , that are assumed to behave as two independent Gaussian random variables with 3.7 and $80 \mu\text{m}$ mean values, respectively, and identical 10% relative standard deviations. The approximate relations given in [8] were used to compute the PC expansion of the p.u.l. parameters.

Figure 2 shows the time-domain voltage source as well as the near-end and far-end crosstalk on the quiet line. Referring to the lower panels, the black thick line represents the response of the structure for the nominal values of its parameters, while the thinner black lines indicate the limits of the 3σ bound, determined from the results of the proposed technique. Finally, a qualitative set of 100 MC simulations is plotted using gray lines. Clearly, the parameter variations lead to a spread in the crosstalk levels that is well predicted by the estimated 3σ limits.

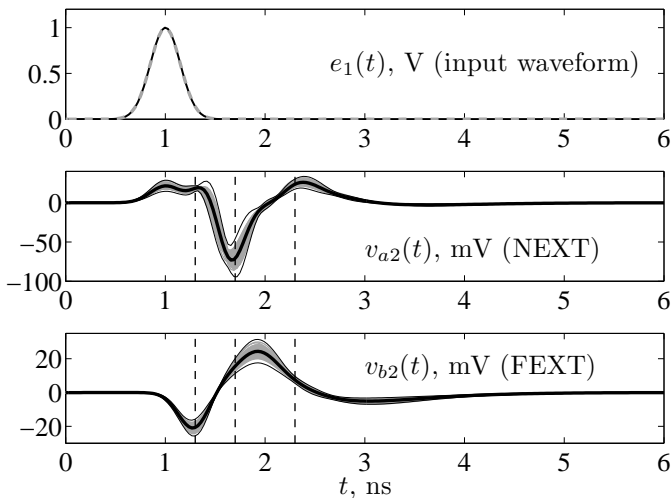


Fig. 2. Top panel: input waveform $e_1(t)$; solid black line: analytical expression; dashed gray line: Fourier series approximation with 30 terms. Lower panels: near-end crosstalk $v_{a2}(t)$ (central panel) and far-end crosstalk $v_{b2}(t)$ (bottom panel); solid black thick line: deterministic response; solid black thin line: 3σ limits of the second-order PC expansion; gray lines: a sample of responses obtained by means of the MC method (limited to 100 curves, for graph readability).

A better quantitative prediction is possible from the knowledge of the actual PDF of the network response. To this end, Figures 3 and 4 show the PDFs of $v_{a2}(t)$ and $v_{b2}(t)$, respectively, obtained from the analytical PC expansions at different times, and compare them with the distributions generated by 20,000 MC simulations. The time points selected for this comparison correspond to the dashed lines shown in Fig. 2.

The good agreement between the PDFs obtained from the PC model and the corresponding set of MC simulations confirms the potential of the proposed method. It is also clear from this example that a PC expansion with two terms is already accurate enough to capture the dominant statistical information of the system response, while leading to a speedup factor of $100 \div 600\times$, depending on the number of time points considered.

V. CONCLUSIONS

In this paper, a methodology for the stochastic simulation of PCB interconnects is presented. The proposed model is based on a frequency-domain extended set of telegraph equations, obtained by means of the expansion of the voltage and current variables into a sum of a limited number of orthogonal basis functions. The transient simulation is then performed via the back-conversion of the extended frequency-domain characteristics into the time domain. The advocated method, while providing accurate results, turns out to be more efficient than the classical Monte Carlo technique in determining the transmission-line response sensitivity to parameters variability. The strength of the approach is verified on a PCB coupled microstrip for which the impact of fabrication tolerances like the variability of the relative dielectric permittivity and trace separation on the crosstalk is predicted.

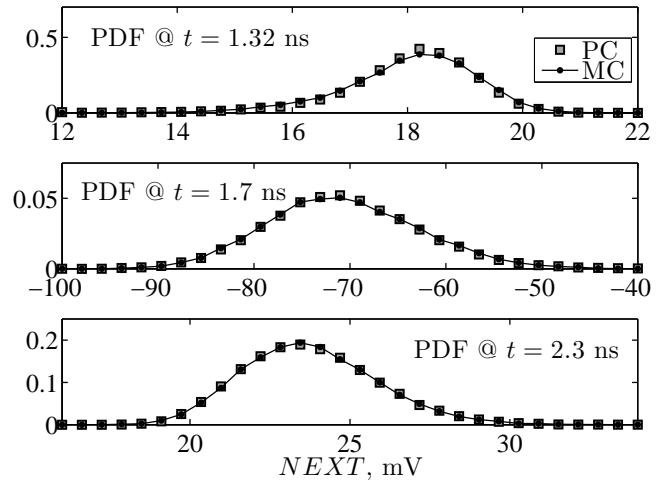


Fig. 3. Probability density function of the near-end crosstalk $v_{a2}(t)$ computed at different times. Of the two distributions, the one marked MC refers to 20,000 MC simulations, while the one marked PC refers to the response obtained via a second-order PC expansion.

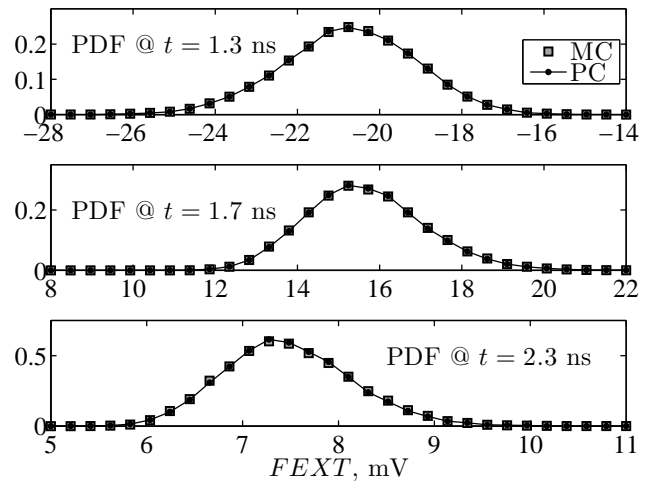


Fig. 4. Probability density function of the far-end crosstalk $v_{b2}(t)$ computed at different times. Same comments of Fig. 3 apply here

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