Power-law decay of the energy spectrum in linearized perturbed systems

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ABSTRACT

The -5/3 power-law scaling of the energy spectrum in the inertial range, confirmed both in laboratory and numerical simulations (see for instance [1]), is a well-known notion in the phenomenology of turbulence (in the sense of Kolmogorov 1941). Here, we study the state that precedes the onset of instability and transition to turbulence to: (i) understand whether the nonlinear interaction among different scales in fully developed turbulence can affect the energy spectrum, and to (ii) quantify the level of generality on the value of the energy decay exponent of the inertial range. In this condition, the system has the same features (e. g. linearized convective transport, linearized vortical stretching, and molecular diffusion) as those characterizing the turbulent state, with the important exception of the nonlinear interaction. The perturbative transient dynamics, ruled by the initial-value problem related to the linearized perturbative Navier-Stokes equations, is very complicated and shows a variety of different behaviours [2]. We ask whether the linearized perturbative system is able to show a power-law scaling for the energy spectrum in an analogous way to the Kolmogorov argument [3].

We determine the decay exponent of the energy spectrum for arbitrary three-dimensional perturbations acting on a typical shear flow, i.e. the bluff-body wake, for different stable and unstable configurations ($Re = 30, 100$). Then, we compare the energy spectrum of the linearized perturbative system - evaluated as the wavenumber distribution of the perturbation kinetic energy density in asymptotic condition - with the well-known -5/3 Kolmogorov power-law scaling. We observe, whether the waves are aligned with the base sheared flow or not, a decay rate of -5/3 in the intermediate range ($2 < k < 100$) for both stable and unstable configurations (see Fig. 1). So far, we can conclude that the nonlinear interaction is not the main factor responsible of the specific value of the -5/3 decay exponent in the energy spectrum and the spectral power-law scaling of inertial waves is a general dynamical property of the Navier-Stokes solutions which encompasses the nonlinear interaction.

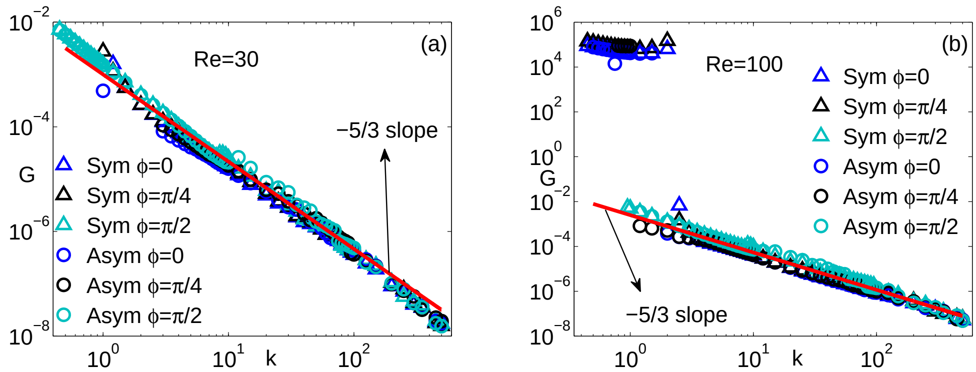

Figure 1: Energy spectrum G of symmetric (triangles) and asymmetric (circles) perturbations (blue: $\phi = 0$, black: $\phi = \pi/4$, light blue: $\phi = \pi/2$). (a) $Re = 30$, (b) $Re = 100$. Red curves: $-5/3$ slope.

REFERENCES

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