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REFINED SHELL ELEMENTS FOR THE ANALYSIS OF FUNCTIONALLY GRADED STRUCTURES

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ABSTRACT

Functionally graded materials (FGMs), proposed in 1980s by the Japanese school of material science as thermal barrier materials [1], present a number of advantages that make them attractive in potential future applications, such a reduction of in-plane and transverse through-the-thickness stresses, an improved residual stress distribution, enhanced thermal properties, higher fracture toughness, and reduced stress intensity factors. For these reasons, an accurate evaluation of displacements, strains and stresses can be fundamental in the design of such structures. During the years theoretical formulations and finite element models have been presented to deal with the analysis of FGM plates and shells for which reference is made to [2] among the others.

In this paper, we propose a shell finite element based on the refined bi-dimensional theories contained in the Carrera’s Unified Formulation (CUF) [3] to perform the mechanical

analysis of FGM structures. According to CUF, the governing equations are written in terms of few fundamental nuclei which do not formally depend on the order of expansion *N* used in the thickness direction *z*. Both cases of equivalent single layer (the multilayered structure is seen as an equivalent single-layered one) and layer-wise (each layer is considered as an independent shell) variable descriptions are accounted for. The classical models, such as Koiter and and Naghdi models, can be also obtained from an equivalent single layer theory with linear expansion of displacements along the thickness by simply applying penalty techniques.

In this work, nine-nodes finite elements and the exact geometry of cylindrical shells are considered. In the wake of Chinosi et al. [4], the Mixed Interpolation of Tensorial Components (MITC) method, extended to CUF, is used in order to overcome membrane and shear locking phenomenon that affect shell elements. According to this technique, the shell elements are formulated by using – instead of the strain components directly computed from the displacements – an interpolation of these strain components within each element using a specific interpolation strategy for each component. The feasibility of such extension to higher-order plate and shell elements has been already proved in [5] and [6], respectively, and it shows to be numerically efficient.

Usually in FGMs to vary the elastic properties in the thickness direction, a gradually changing of the volume fraction of the constituents is considered. So the key point is an accurate description of the variables and the material properties in the thickness direction, to perform a satisfactory analysis of the mechanical behavior of FGM structures. The variation of material characteristics are usually given in terms of exponential and/or polynomial functions applied directly to the engineering constants such as Young's modulus *E* and/or Poisson's ratio v or to the elastic material constants C_{ij} $(i,j=1,\dots,6)$. Actually, since in each point of the shell a relation between the engineering constants and the elastic material constants holds, only the second case can be treated.

A number of examples are carried out in this work to show the efficiency and robustness of CUF shell finite element. In particular, comparisons with classical approaches and analytical results provided in [7] are made to highlight the accuracy and computational cost of the present formulation.

REFERENCES

[1] M. Koizumi, "The concept of FGM", *Ceram. Trans. Funct. Graded Mater.*, 34:3-10 (1993).

[2] J.N. Reddy and C.D. Chin, "Thermomechanical analysis of functionally graded cylinders and plates", *Journal of Thermal Stresses*, 26(1):93-126 (1998).

[3] E. Carrera, "Theories and finite elements for multilayered plates and shells: a unified

compact formulation with numerical assessment and benchmarking”, *Arch. Comput. Methods Eng*, 10: 215–297 (2003).

[4] C. Chinosi and L. Della Croce, “Mixed-interpolated elements for thin shells”, *Communications in Numerical Methods in Engineering*, 14:1155-1179 (1998).

[5] E. Carrera, M. Cinefra and P. Nali, “MITC technique extended to variable kinematic multilayered plate elements”, *Composite Structures*, 92:1888-1895 (2010).

[6] M. Cinefra, E. Carrera, C. Chinosi, L. Della Croce and S. Belouettar, “Mixed-interpolated shell finite elements based on refined theories”, *Computers and Structures*, submitted (2010).

[7] M. Cinefra, S. Brischetto, E. Carrera and S. Belouettar, “Thermo-mechanical analysis of functionally graded shells”, *Journal of Thermal Stresses*, in press (2009).