# DETERMINATION OF DENSITY AND CONCENTRATION FROM FLUORESCENT IMAGES OF A GAS FLOW

## Applications to laboratory hypersonic jets

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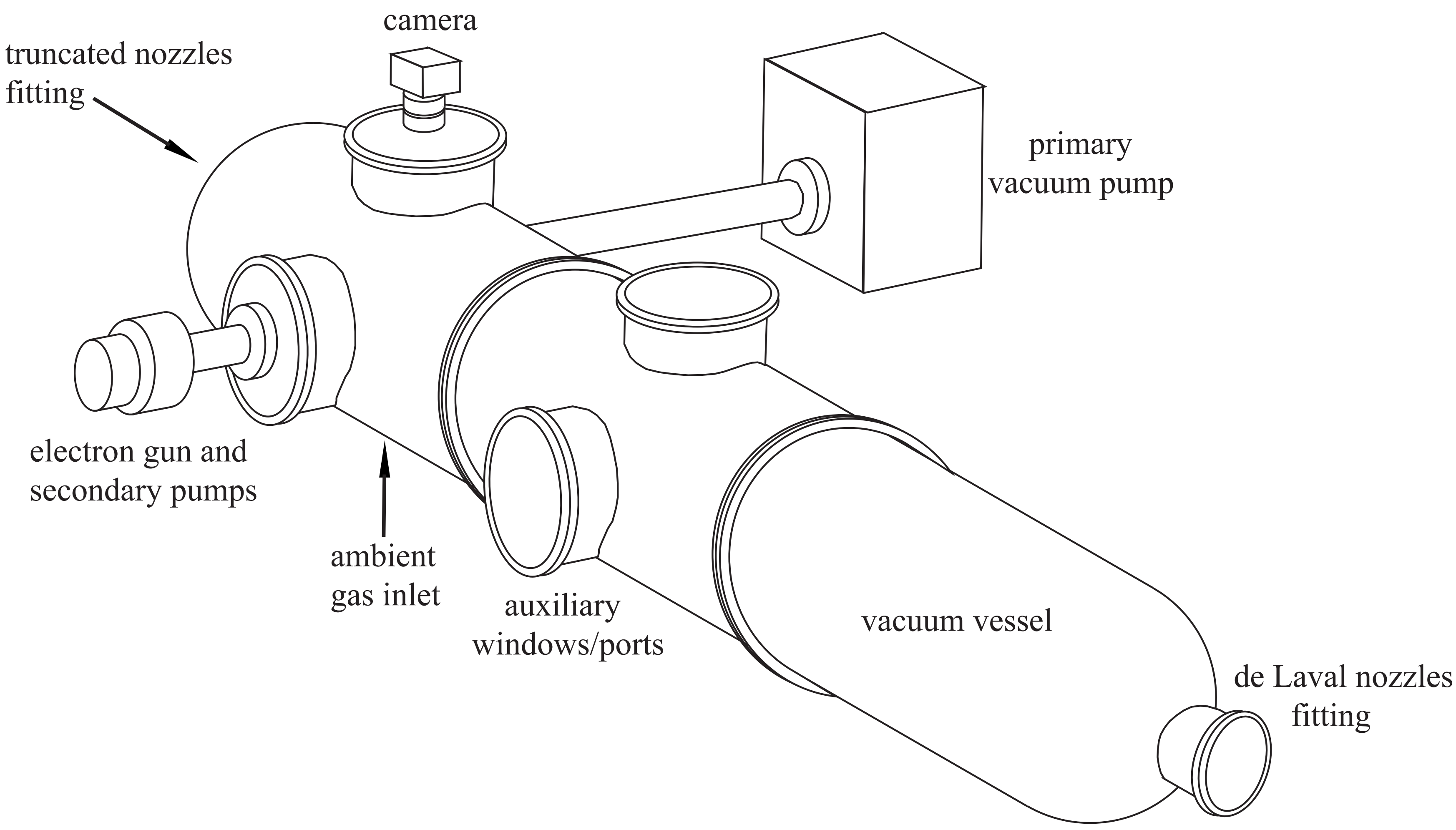
# 1 Facilities




•Vacuum vessel •Nozzles •Electron gun •High-sensitivity camera

## 2 Capability of the system / tested flows

(ICIASF meeting 2001, Astrophysics and Space Science 2004)

- Long-scale jet visualization (up to 100 initial diameters)
- 2D-visualizations of 3D-flows (slices)
- Density measurements
- Concentration measurements, jet gas $\neq$ environment gas
- Mixing layer thickness, shock thickness measurements

## 3 Physical conditions

- Rarefied gases ($n < 10^{22}\,\mathrm{m}^{-3}$, $p < 100\,\mathrm{Pa}$ and $\rho < 1\,\mathrm{g/m}^3$ for air)
- Stagnation/ambient pressure ratio: $p_0/p_{\mathrm{amb}}$ up to $10^5$
- Mach number $M$ up to 50 (or more, $v \sim v_{limit}$).
- Density ratio $0.04 < \rho_{\mathrm{jet}}/\rho_{\mathrm{amb}} < 45$ (0.01 to 100 is possible).
- $Re_D$ up to 2000 (diameter), $Re_x > 10^5$ (axial length)

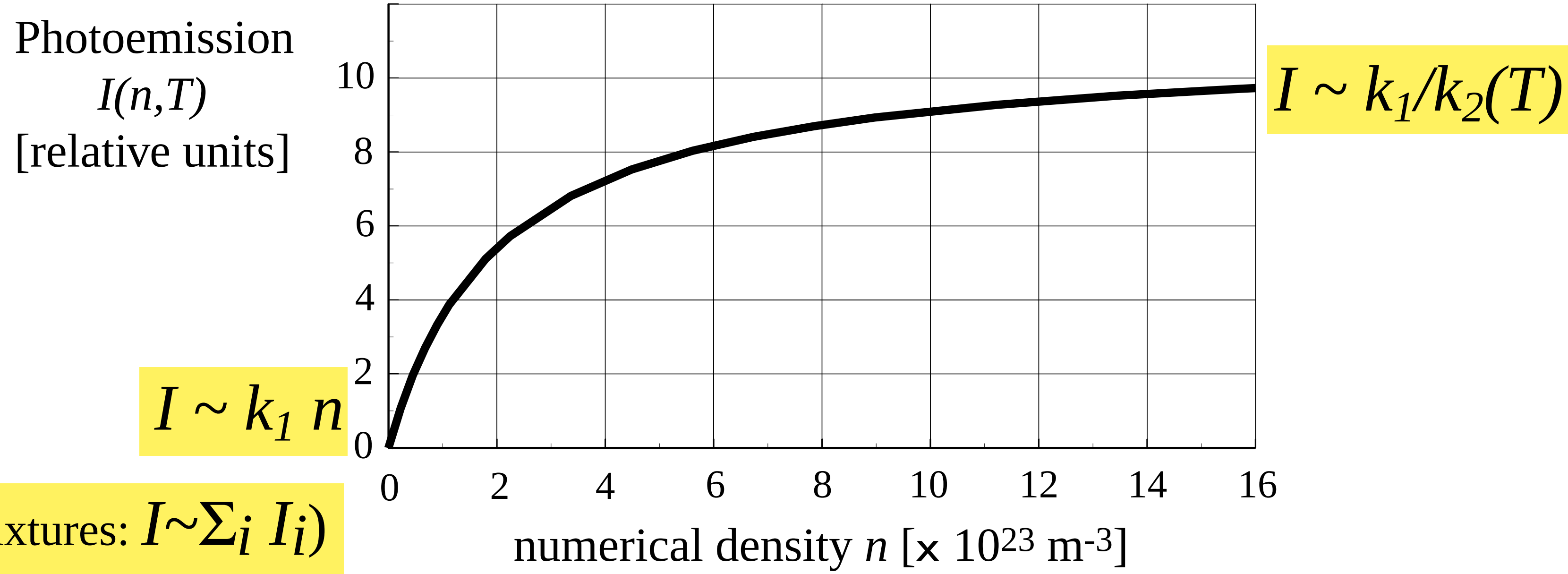
# 4 Density and concentration (mixing) measurements

## 4.1 *Fluorescent emission of rarefied gases*

Radiant intensity $I$ vs numerical density $n$ (Grün, 1954):

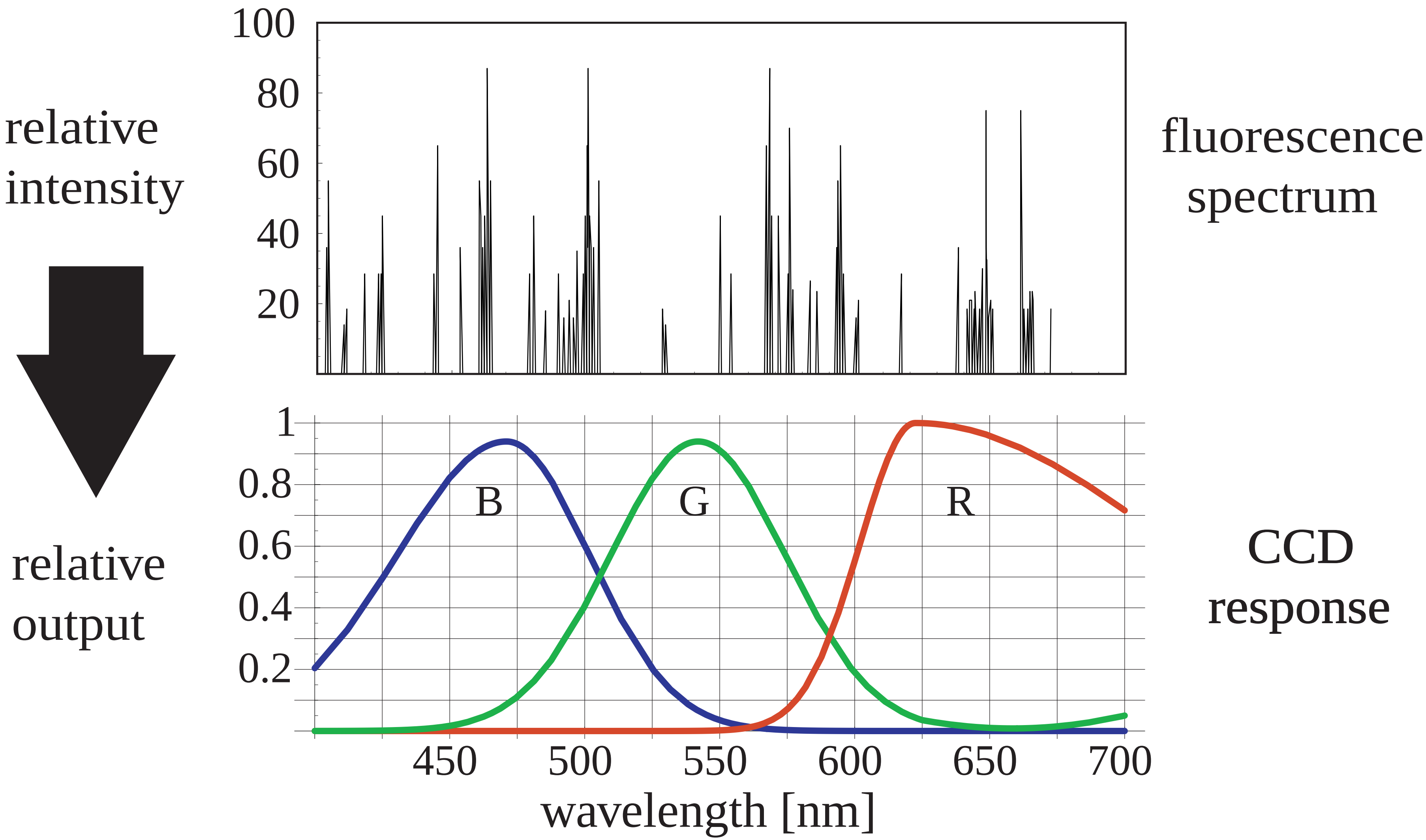
$$I = \frac{k_1\, n}{1 + \left(2\, n\, \sigma^2 P_{ij}{}^{-1} \sqrt{\frac{4\pi}{m} kT}\right)} = \frac{k_1\, n}{1 + k_2(T)\, n}$$

Typical fluorescent emission at constant T:




(gas mixtures: $I \sim \Sigma_i I_i$)

## *Image analysis: concentration*



Algorithm: (Exp Fluids 2008)

$I = k\,n$ total emission $\propto$ numerical density $n$

$$\begin{cases} R = k_R\, n \\ G = k_G\, n \\ B = k_B\, n \end{cases}$$ CCD output colors (partial emission) $\propto n$

$C = aR + bG + cB = k_C\, n$ weighted spectral superposition $\propto n$

For different gases:

jet gas $\begin{cases} R_{\text{jet}} = k_{Rj} n_{\text{jet}} \\ G_{\text{jet}} = k_{Gj} n_{\text{jet}} \\ B_{\text{jet}} = k_{Bj} n_{\text{jet}} \end{cases}$ ambient gas $\begin{cases} R_{\text{amb}} = k_{Ra} n_{\text{amb}} \\ G_{\text{amb}} = k_{Ga} n_{\text{amb}} \\ B_{\text{amb}} = k_{Ba} n_{\text{amb}} \end{cases}$

Example: He jet in Ar ambient $(I_{He} < I_{Ar})$.

Decoupled emission gives $R = R_{He} + R_{Ar}$ , $G = G_{He} + G_{Ar}$

$$\Longrightarrow r_{\text{pure Ar}} \leq \frac{G}{R} \leq r_{\text{pure He}}$$

In general, for a mixture of 2 gases,

$$r_{\text{min}} \leq \frac{C_1}{C_2} \equiv \frac{a_1 R + b_1 G + c_1 B}{a_2 R + b_2 G + c_2 B} \leq r_{\text{max}}$$

Decoupled emission for a given color combination $C$:

$$C_{\text{mix}} = C_{\text{amb}} + C_{\text{jet}}$$
$$k_C\, n = k_{\text{a}} n_{\text{amb}} + k_{\text{j}} n_{\text{jet}}$$

Introducing $z_{\text{gas}} = n_{\text{gas}}/n$:

$$k_C = k_{\text{a}} z_{\text{amb}} + k_{\text{j}} z_{\text{jet}} = k_{\text{a}} (1 - z_{\text{jet}}) + k_{\text{jet}} z_{\text{jet}}$$

Color combinations setup (choice of $a_1, b_1, c_1,\ a_2, b_2, c_2$):

$$C_1 = a_1 R + b_1 G + c_1 B \quad ; \quad C_2 = a_2 R + b_2 G + c_2 B$$

Definition of the color ratio:

$$r \equiv \frac{C_1}{C_2} = \frac{k_{C1}}{k_{C2}} = \frac{k_{\text{a1}} z_{\text{amb}} + k_{\text{j1}} z_{\text{jet}}}{k_{\text{a2}} z_{\text{amb}} + k_{\text{j2}} z_{\text{jet}}}$$

Concentration (solving for $z_{\text{jet}}$):

$$z_{\text{jet}}(r) = \frac{k_{\text{a1}} - k_{\text{a2}}\, r}{(k_{\text{a1}} - k_{\text{j1}}) + (k_{\text{j2}} - k_{\text{a2}})\, r}$$

($k$-constants are obtained by experimental calibration)

## 4.3 *Image analysis: density*

Radiant intensity of gases (decoupled emissions):

$$I_{\mathrm{jet}} = k_{\mathrm{j}}\, n_{\mathrm{jet}}$$
$$I_{\mathrm{amb}} = k_{\mathrm{a}}\, n_{\mathrm{amb}}$$

Radiant intensity of the gas mixture gives the total numerical density:

$$I = I_{\mathrm{jet}} + I_{\mathrm{amb}} = (k_{\mathrm{j}}\, z_{\mathrm{jet}} + k_{\mathrm{a}}\, z_{\mathrm{amb}})n$$
$$\Longrightarrow n = I/(k_{\mathrm{j}}\, z_{\mathrm{jet}} + k_{\mathrm{a}}\, z_{\mathrm{amb}})$$

Density ($m$: molar masses):

$$\rho = n(z_{\mathrm{amb}} m_{\mathrm{amb}} + z_{\mathrm{jet}} m_{\mathrm{jet}})$$

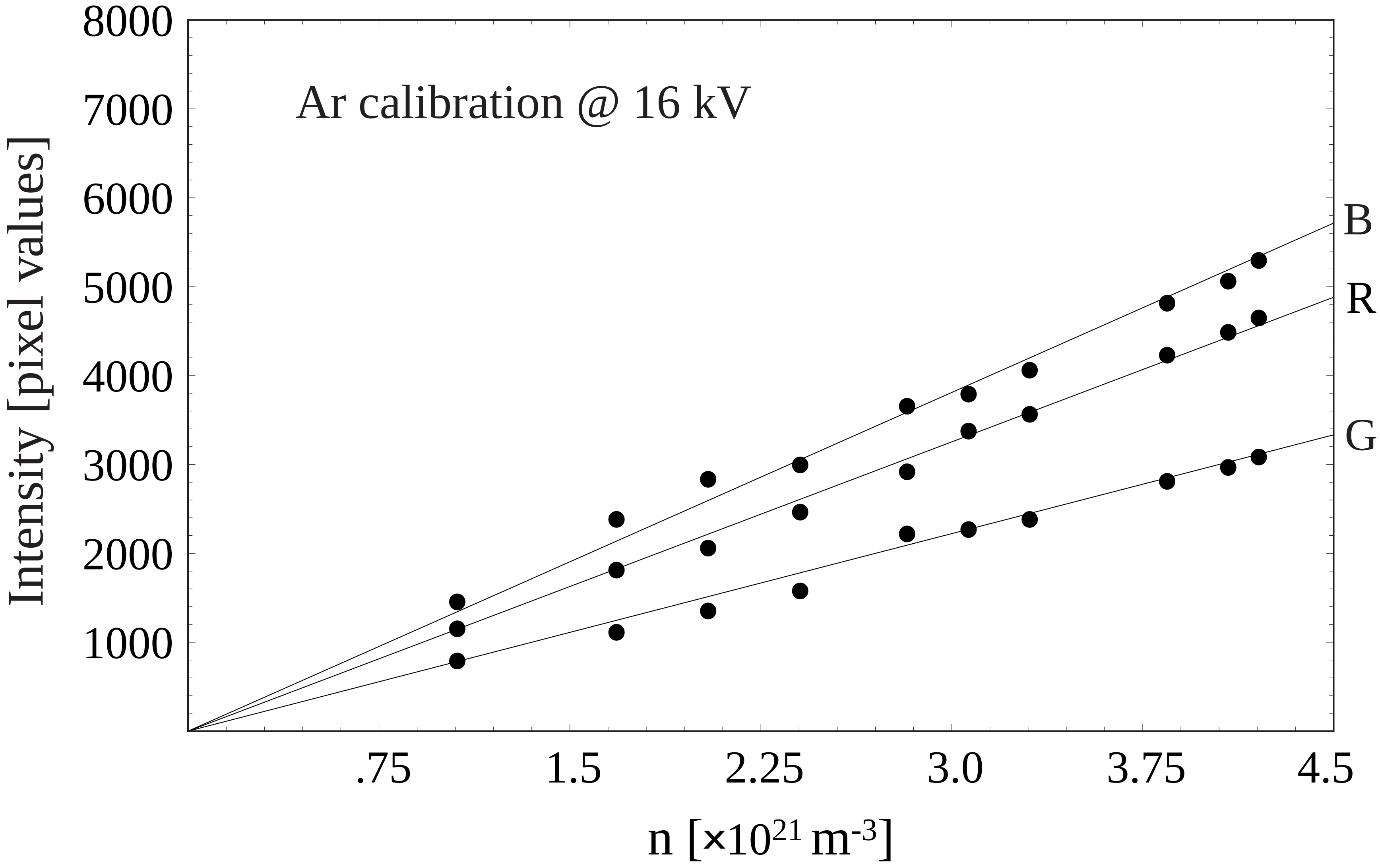
Remarks:

On the image domain $r(x, y) \mapsto z_{\mathrm{jet}}(x, y), z_{\mathrm{amb}}(x, y) \mapsto \rho_{\mathrm{mix}}(x, y)$

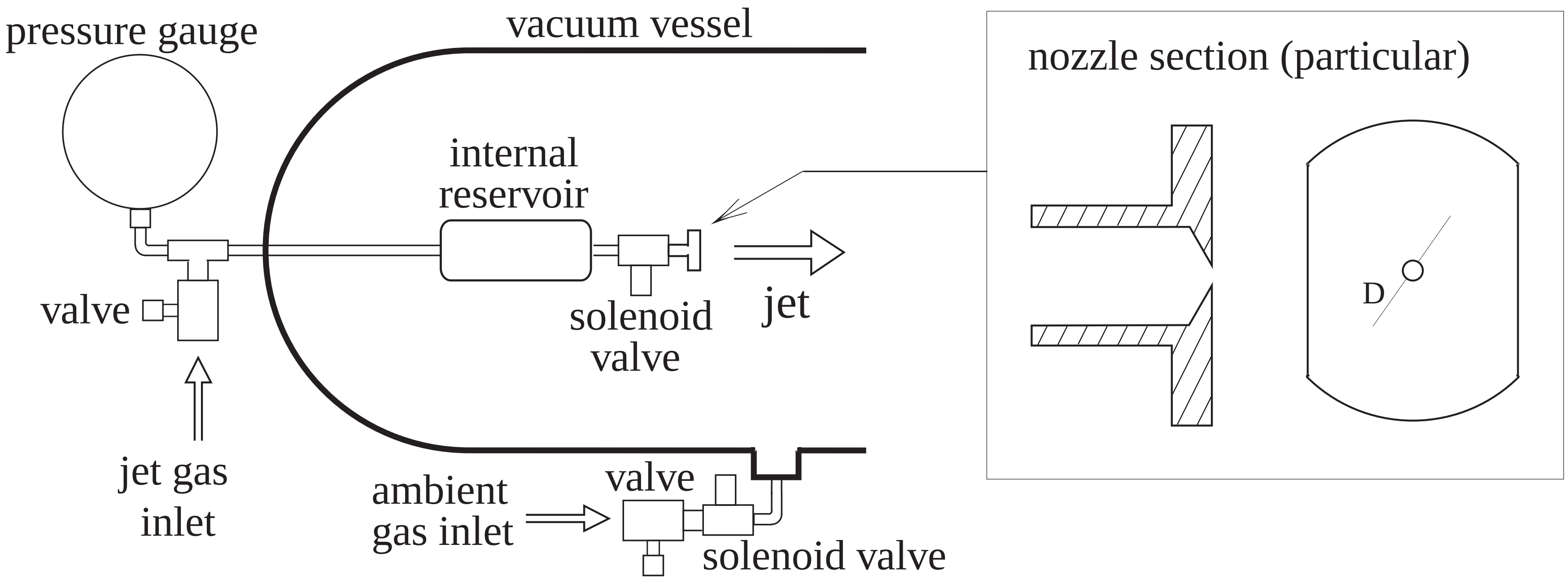
- $C_1, C_2$ should be chosen to get the widest range for $r = C_1/C_2$
- results may be strongly noise-sensitive
- if (jet gas)=(ambient gas) the density is much easier to calculate

## 4.4 *Calibration (example)*

(* 2008)



## 5 Underexpanded jets

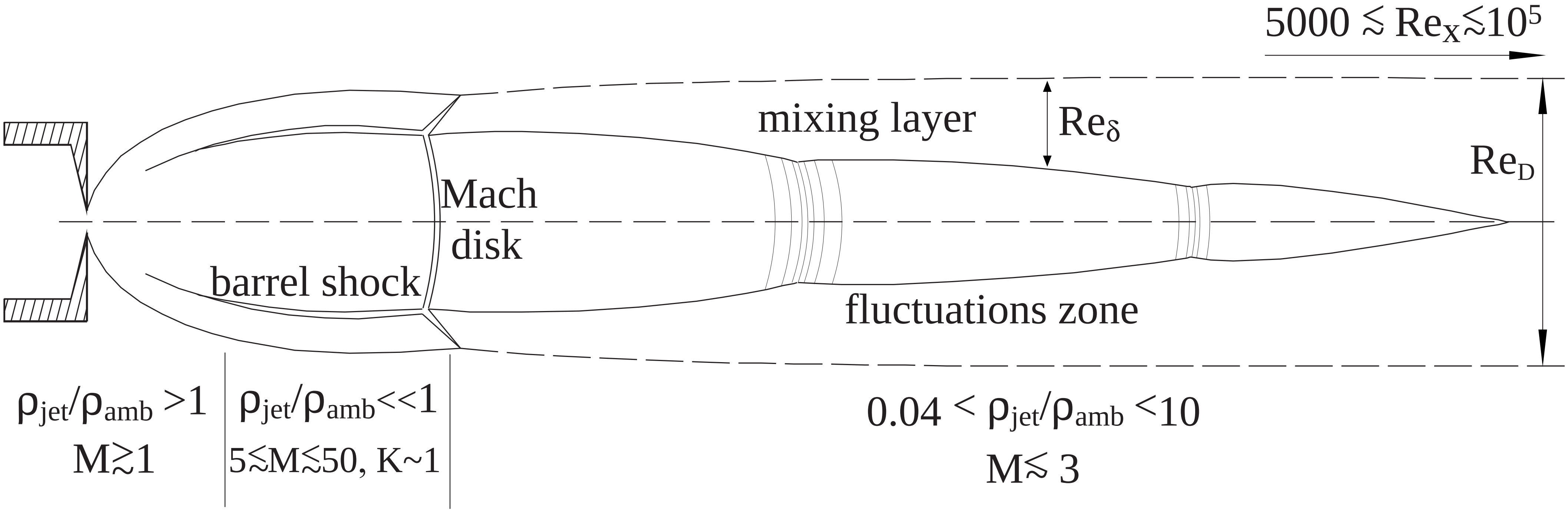



- Small jet radius: $r_{jet} < 0.1\, r_{vessel}$
- Quasi-stationary jet: $\Delta t_{valve} \gg t_{jet}$

Adjustable parameters:

- (stagnation/ambient) pressure ratio $p_0/p_{\mathrm{amb}}$: $M_{\max} = f(p_0/p_{\mathrm{amb}})$
- density ratio $\rho_{\mathrm{jet}}/\rho_{\mathrm{amb}}$ (selection of light or heavy gases)

Jet structure:




Jets similarity:

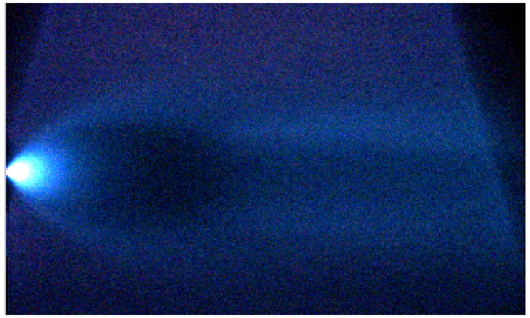
- Short range (barrel zone): $\rho_{\text{jet}}/\rho_{\text{amb}}$ similarity, (the jet properties depend on the pressure ratio $p_0/p_{\text{amb}}$)
- Long range: Mach similarity, (the jet properties depend on the density ratio $\rho_{\text{jet}}/\rho_{\text{amb}}$)

# 6 Results

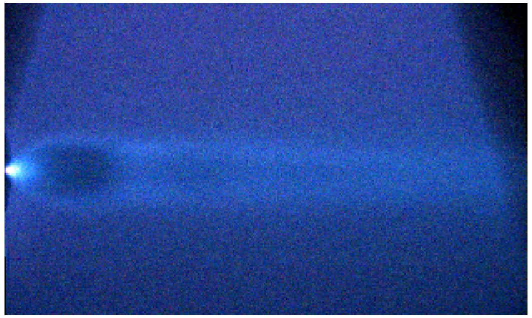
Example: Argon jet in Helium medium

## 7 Results

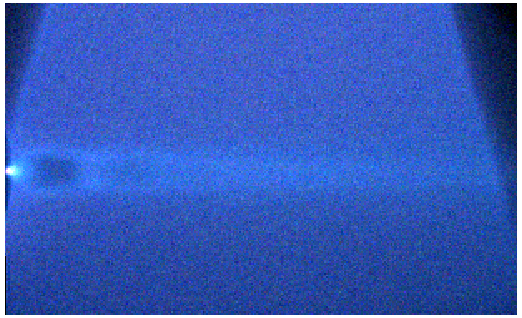
Helium jets in Argon medium: jets at several pressure ratios $p_0/p_{\,\mathrm{amb}}$



$M_{\mathrm{max}} \simeq 36$

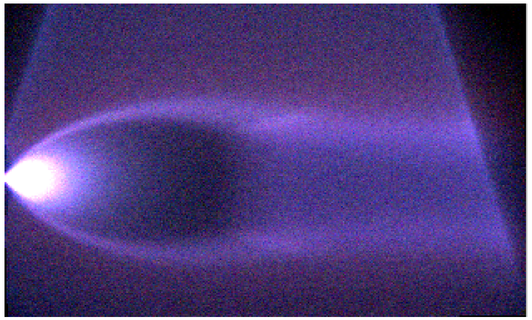


$M_{\mathrm{max}} \simeq 25$

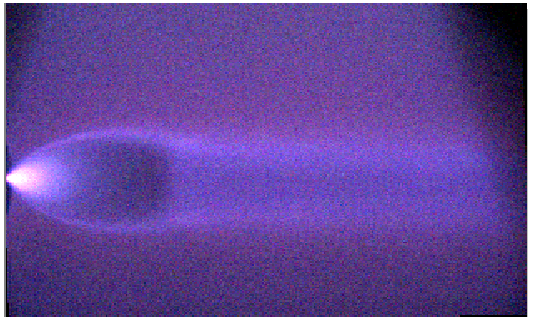


$M_{\mathrm{max}} \simeq 17$

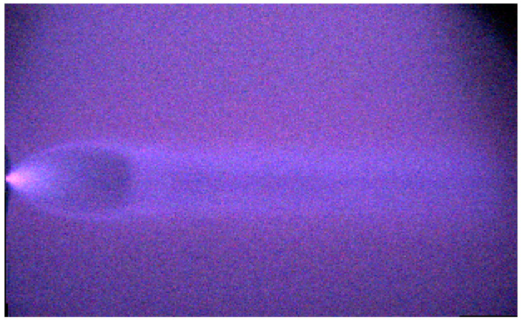
Argon jets in Helium medium: jets at several pressure ratios $p_0/p_{\,\mathrm{amb}}$



$M_{\mathrm{max}} \simeq 36$



$M_{\mathrm{max}} \simeq 29$

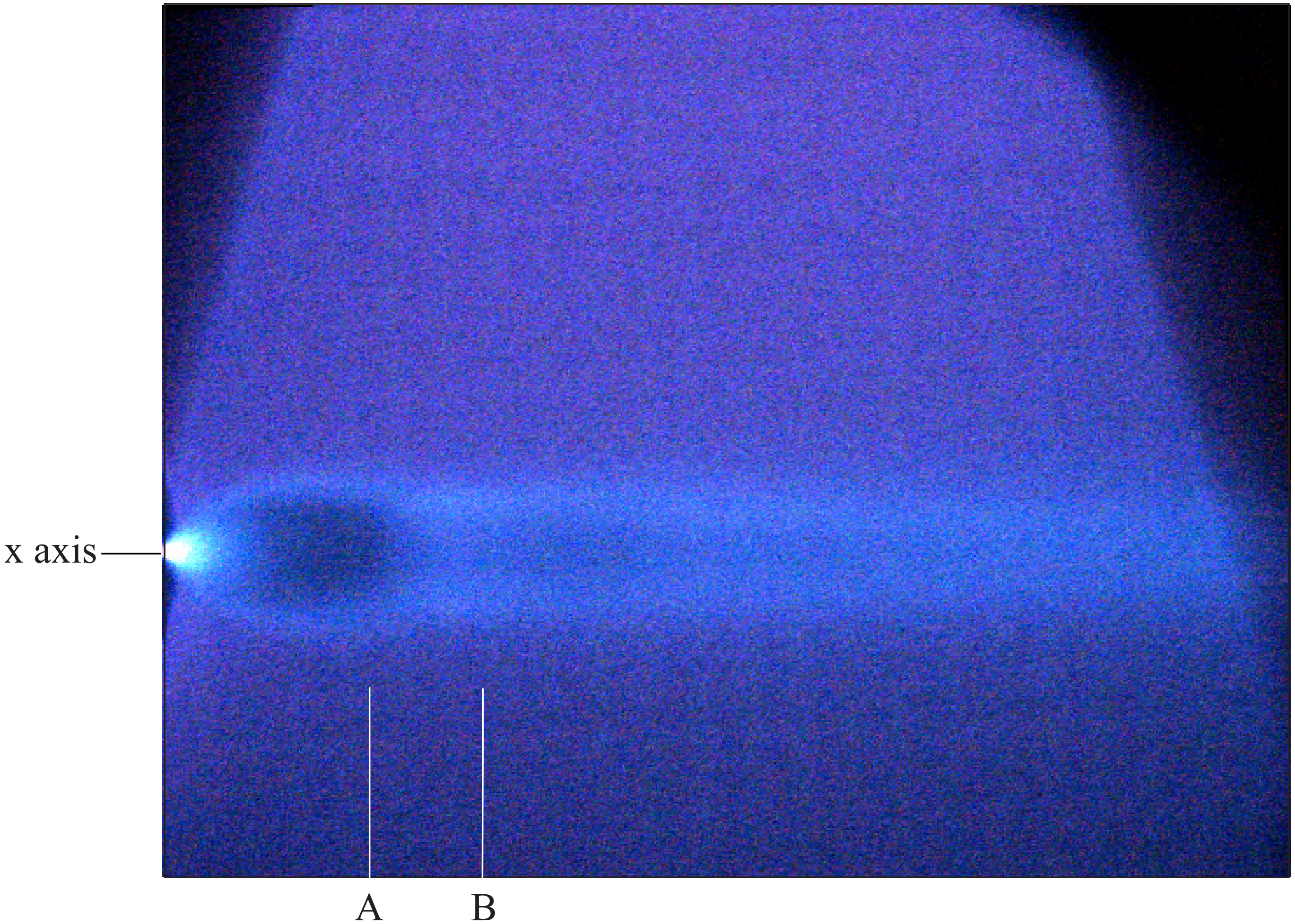


$M_{\mathrm{max}} \simeq 25$

(other gases in use: air, xenon)

## 7.1 *Light jet in heavy medium, same $\gamma$ (monoatomic gases)*

Example: Helium jet in Argon medium.

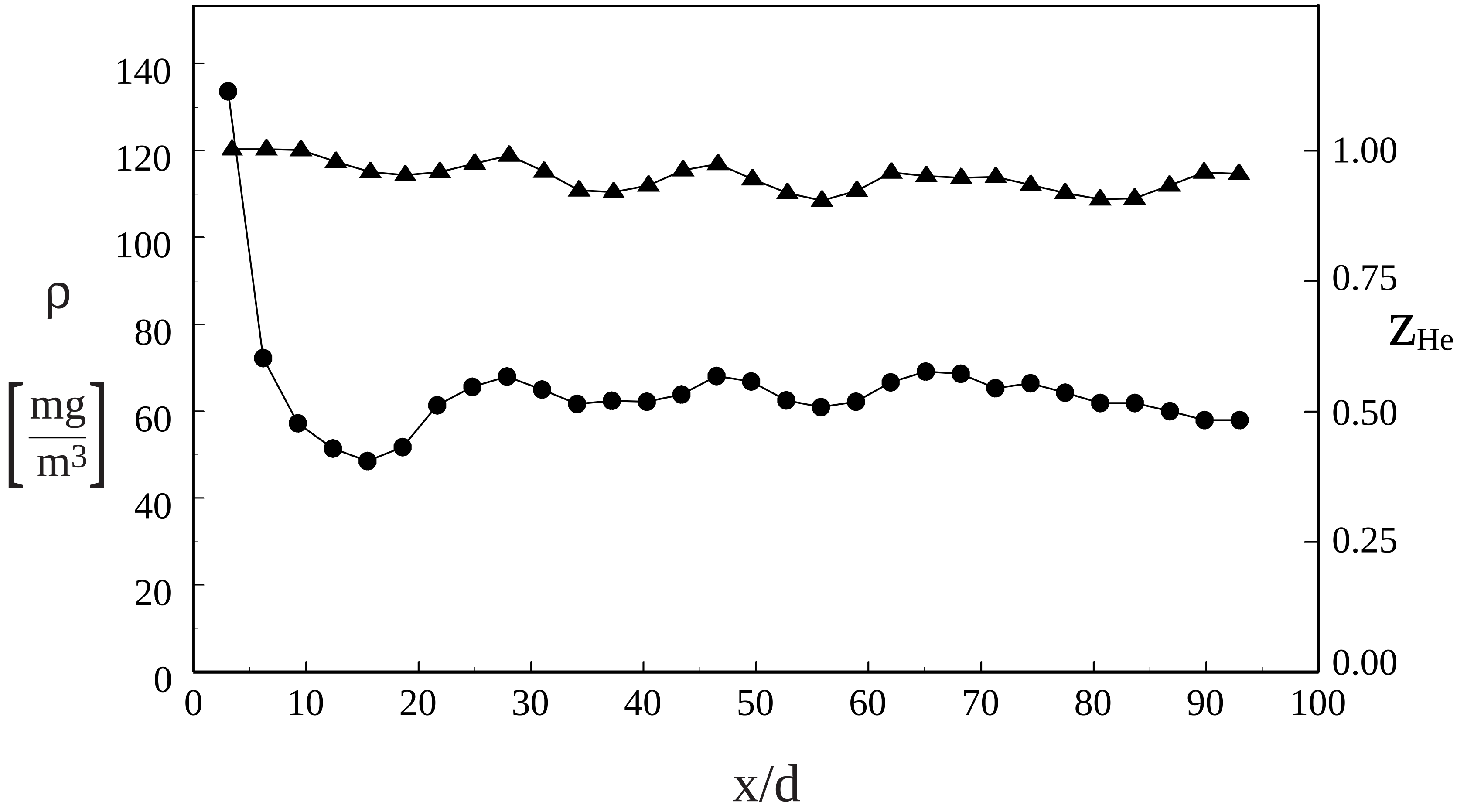



unbalanced RGB image, long exposure time (80ms $\gg$ time scale)

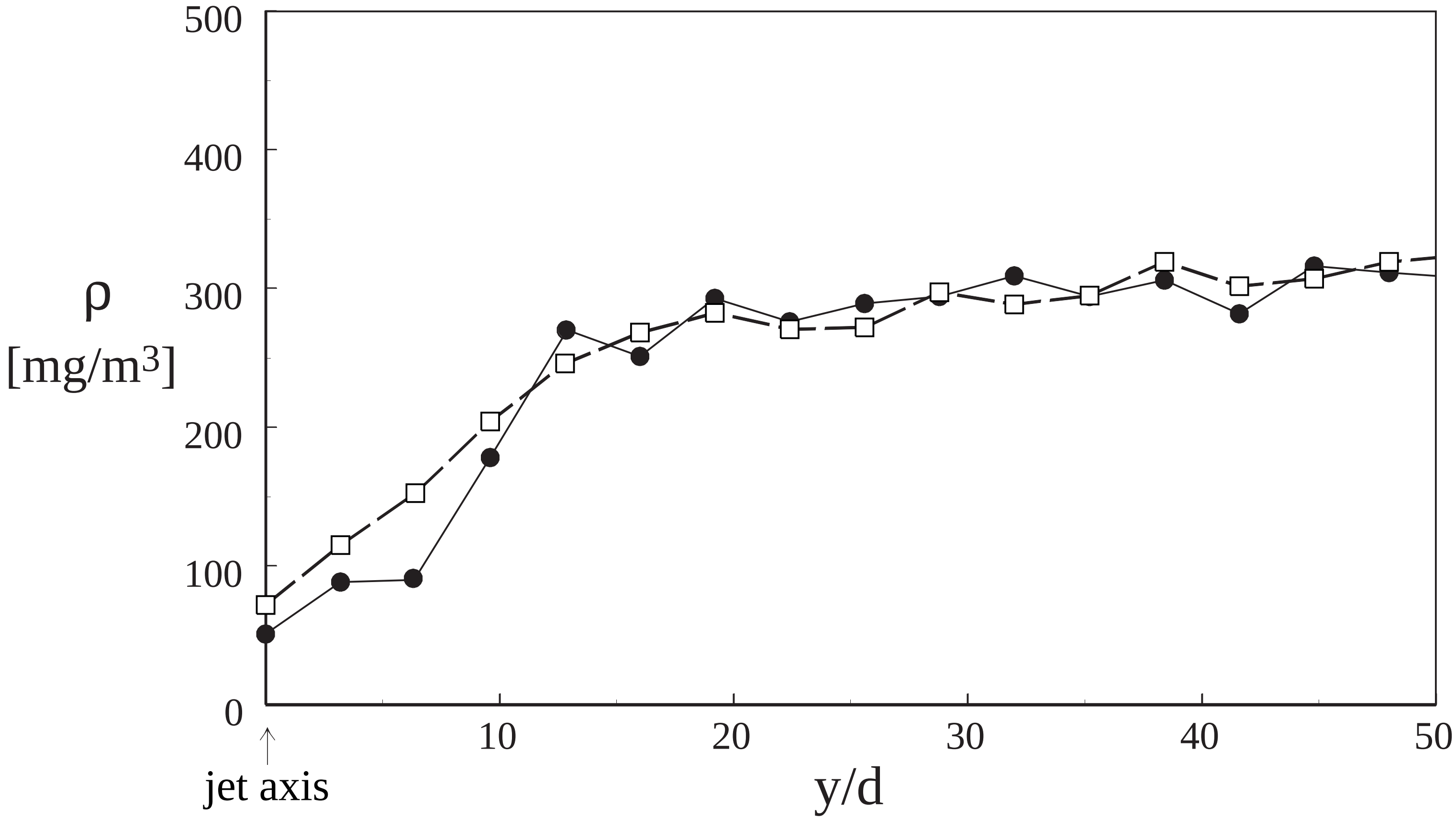
$$p_0/p_{\text{amb}} \simeq 840 \; ; \; M_{\text{max}} \simeq 26 \; ; \; \rho_{\text{jet}}/\rho_{\text{amb}} \simeq 0.14$$

weak gradients, density/pressure fluctuations after the shock

# Helium jet in argon medium: axial density and concentration

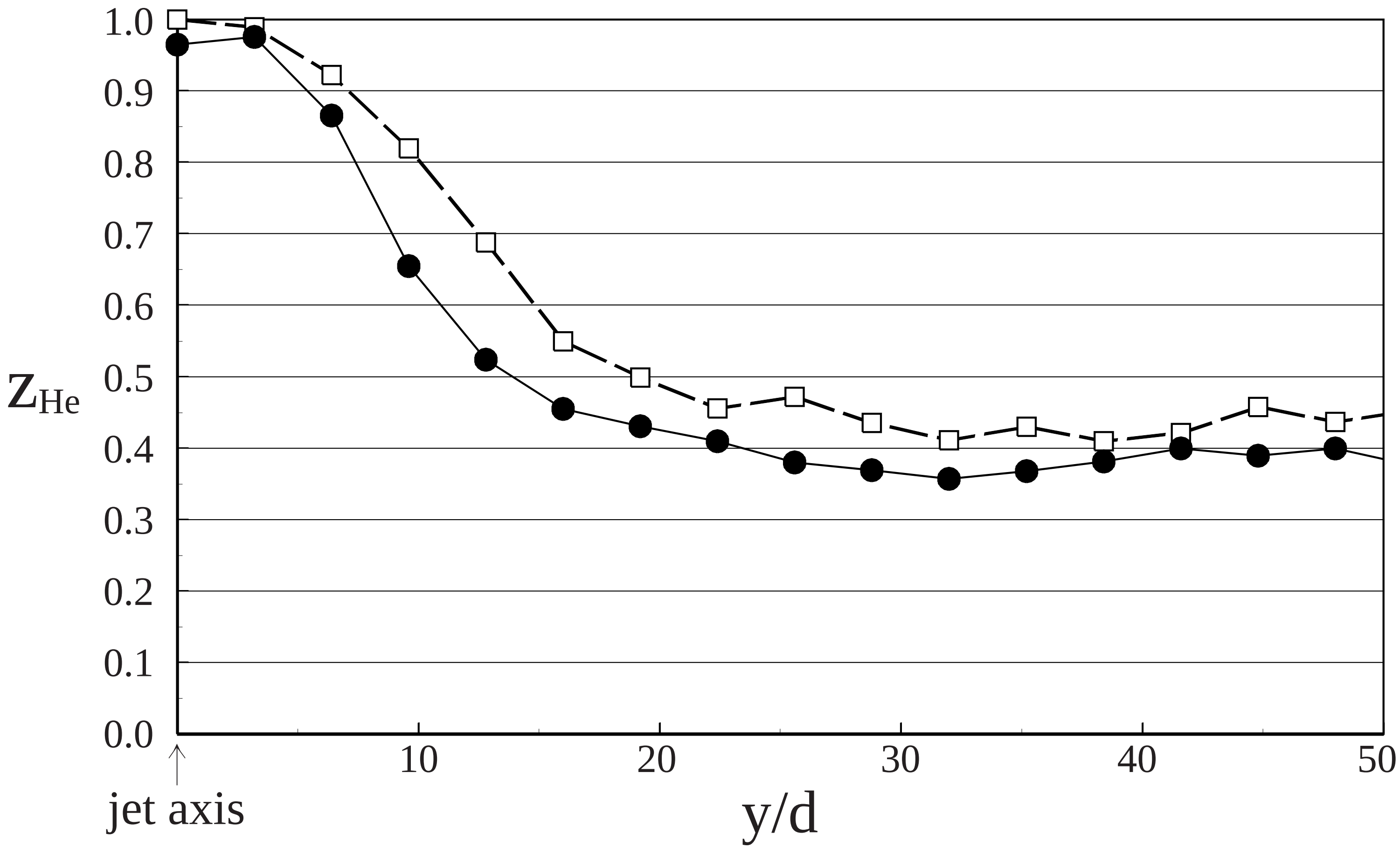


# Helium jet in argon medium: radial density




●: section (A), before the Mach disk. □: section (B), after the Mach disk.

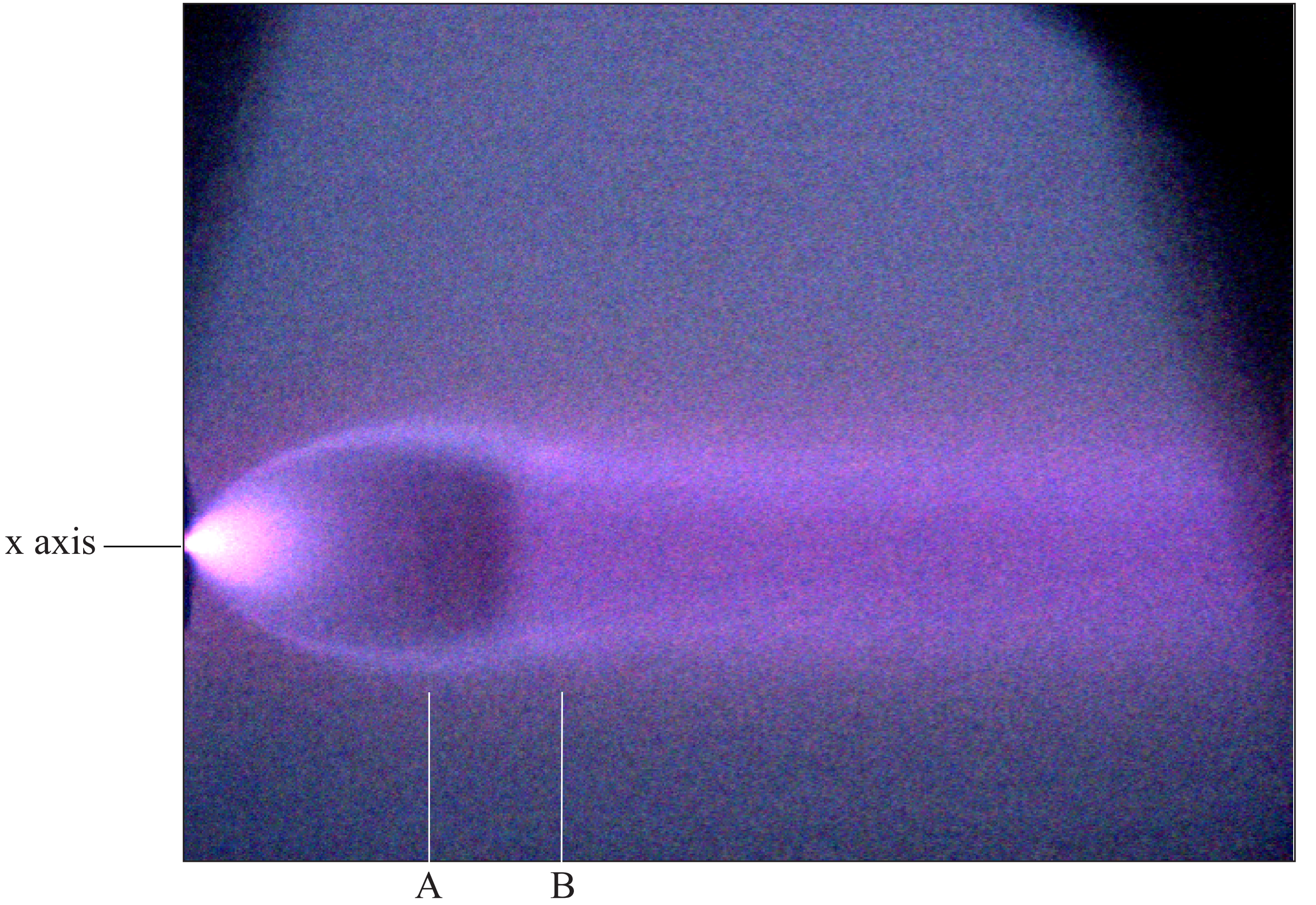
Helium jet in argon medium: radial concentration




●: section (A), before the Mach disk. □: section (B), after the Mach disk.

## 7.2 *Heavy jets in light mediums, same $\gamma$ (monoatomic gases)*

Example: Argon jet in Helium medium.

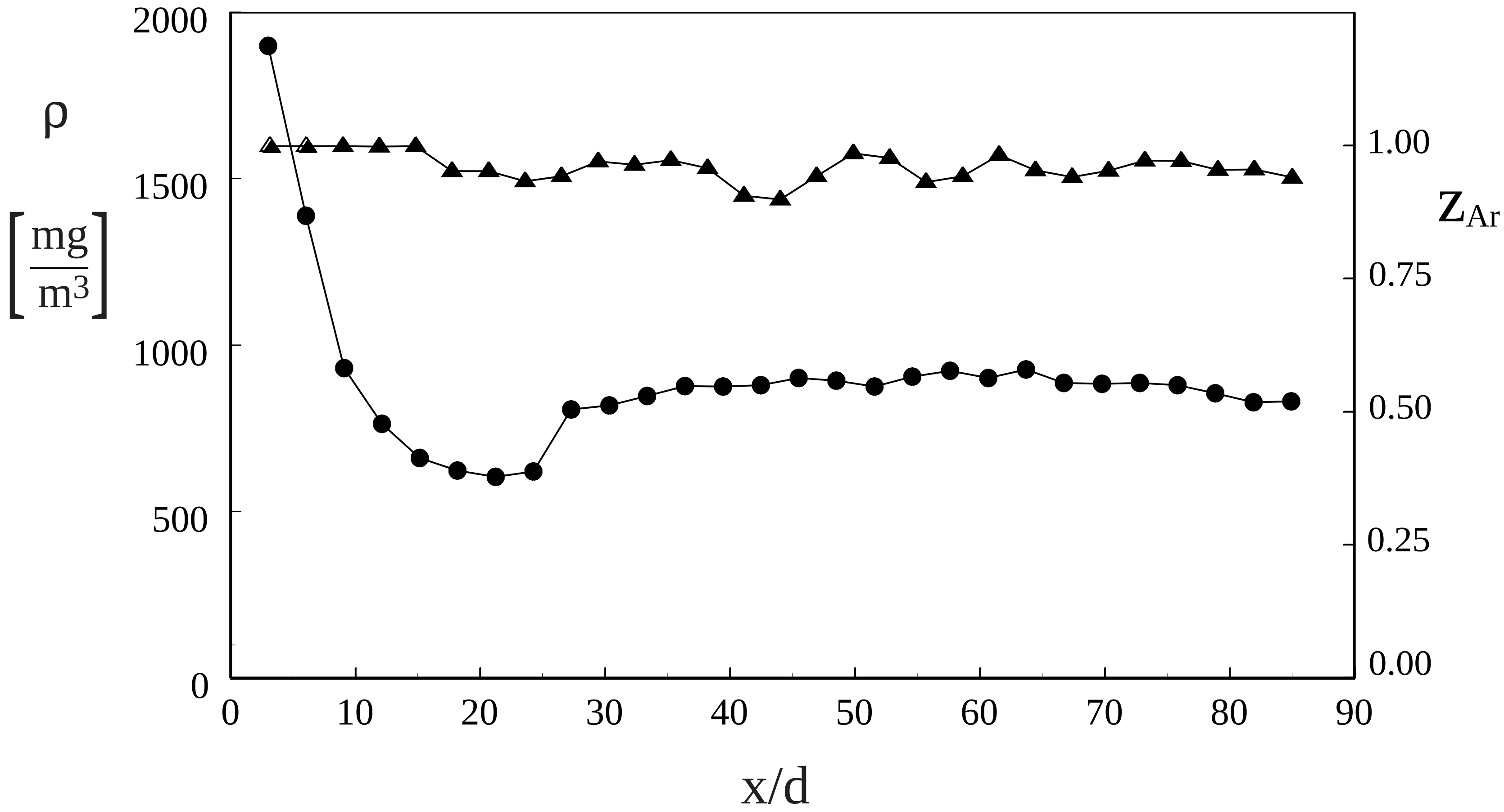



unbalanced RGB image, long exposure time (80ms $\gg$ time scale)

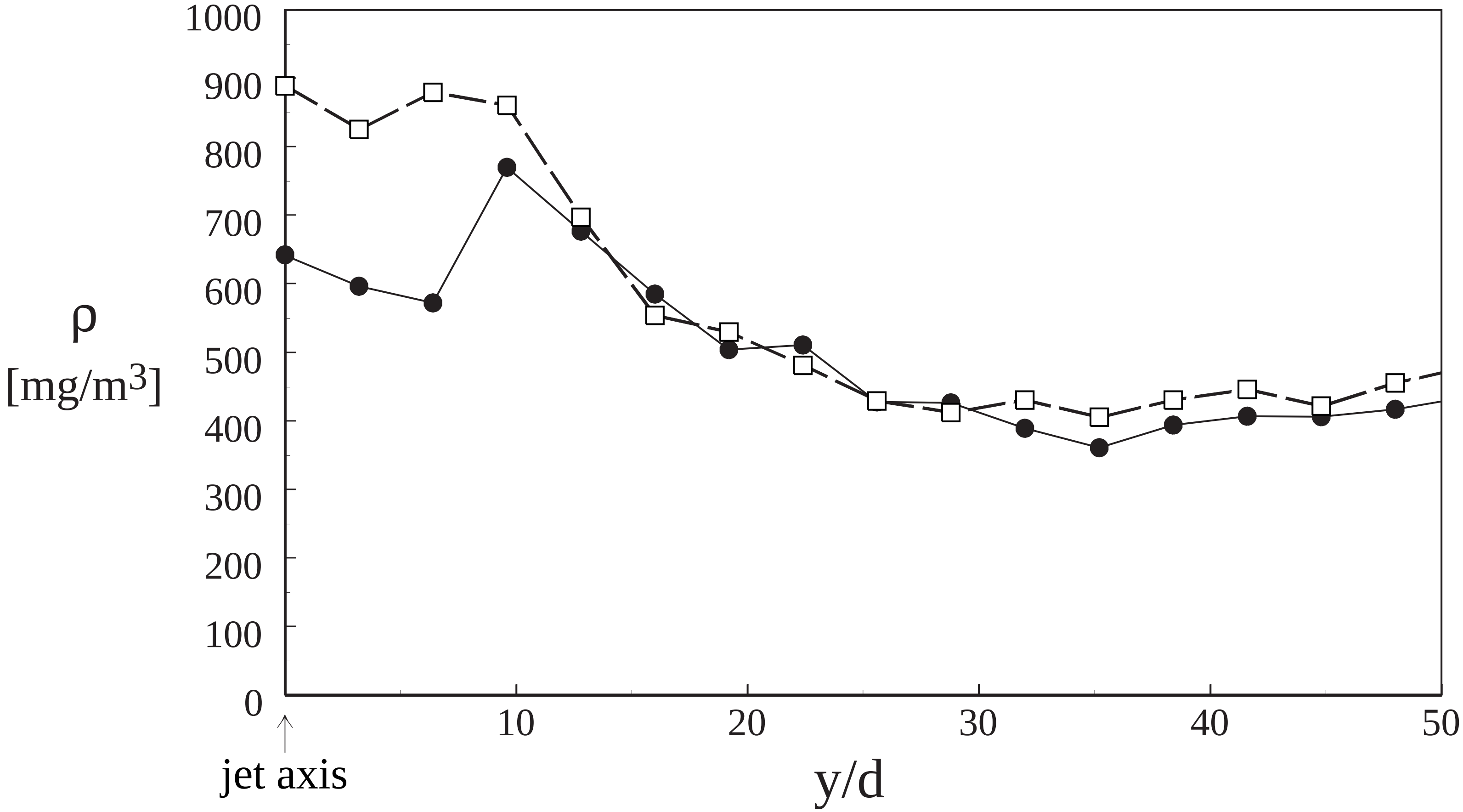
$$p_0/p_{\text{amb}} \simeq 1200 \ ; \ M_{\text{max}} = 29 \ ; \ \rho_{\text{jet}}/\rho_{\text{amb}} \simeq 11$$

sharp gradients, no fluctuations after the normal shock

# Argon jet in helium medium: axial density and concentration

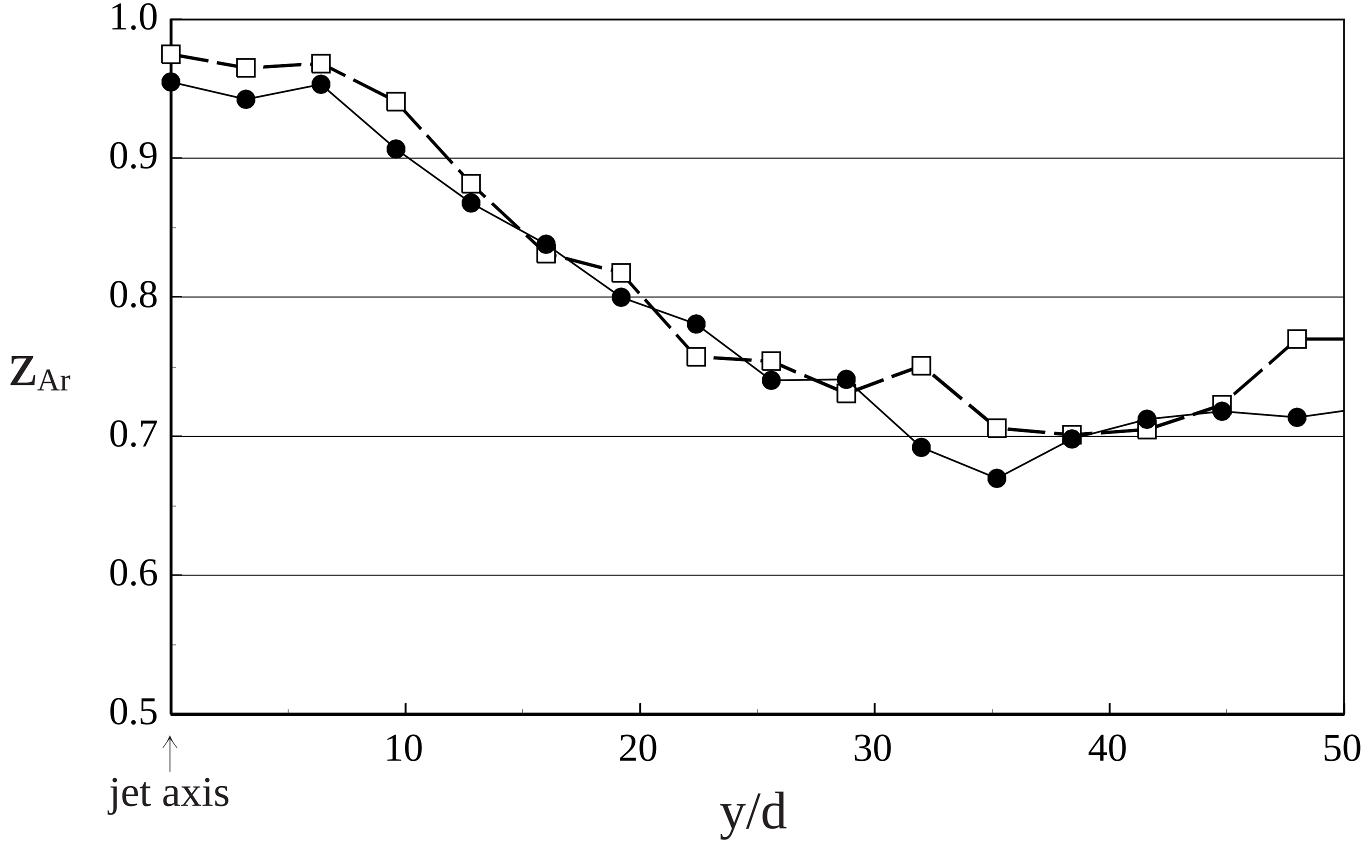


# Argon jet in helium medium: radial density

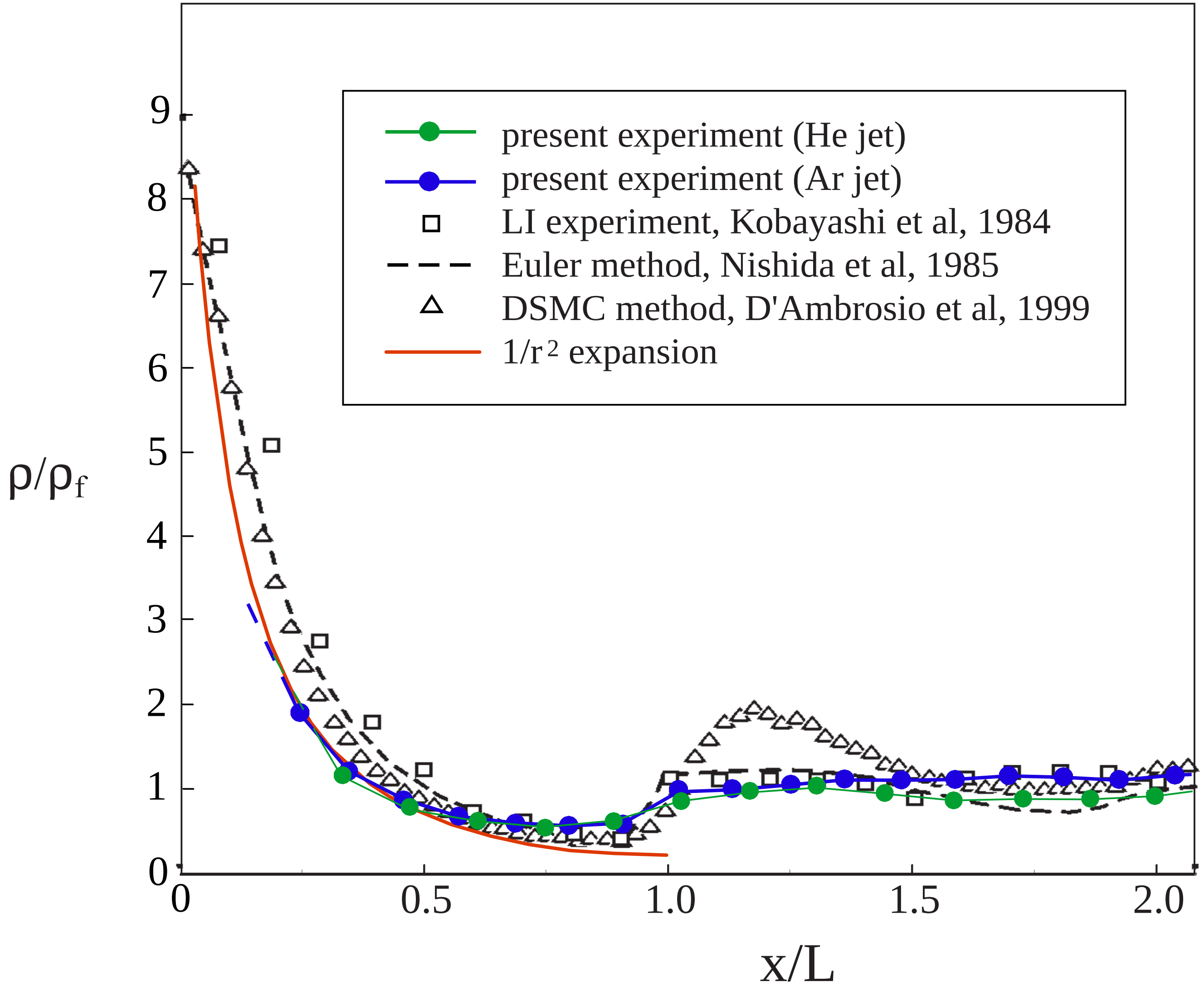



●: section (A), before the Mach disk. □: section (B), after the Mach disk.

# Argon jet in helium medium: radial concentration

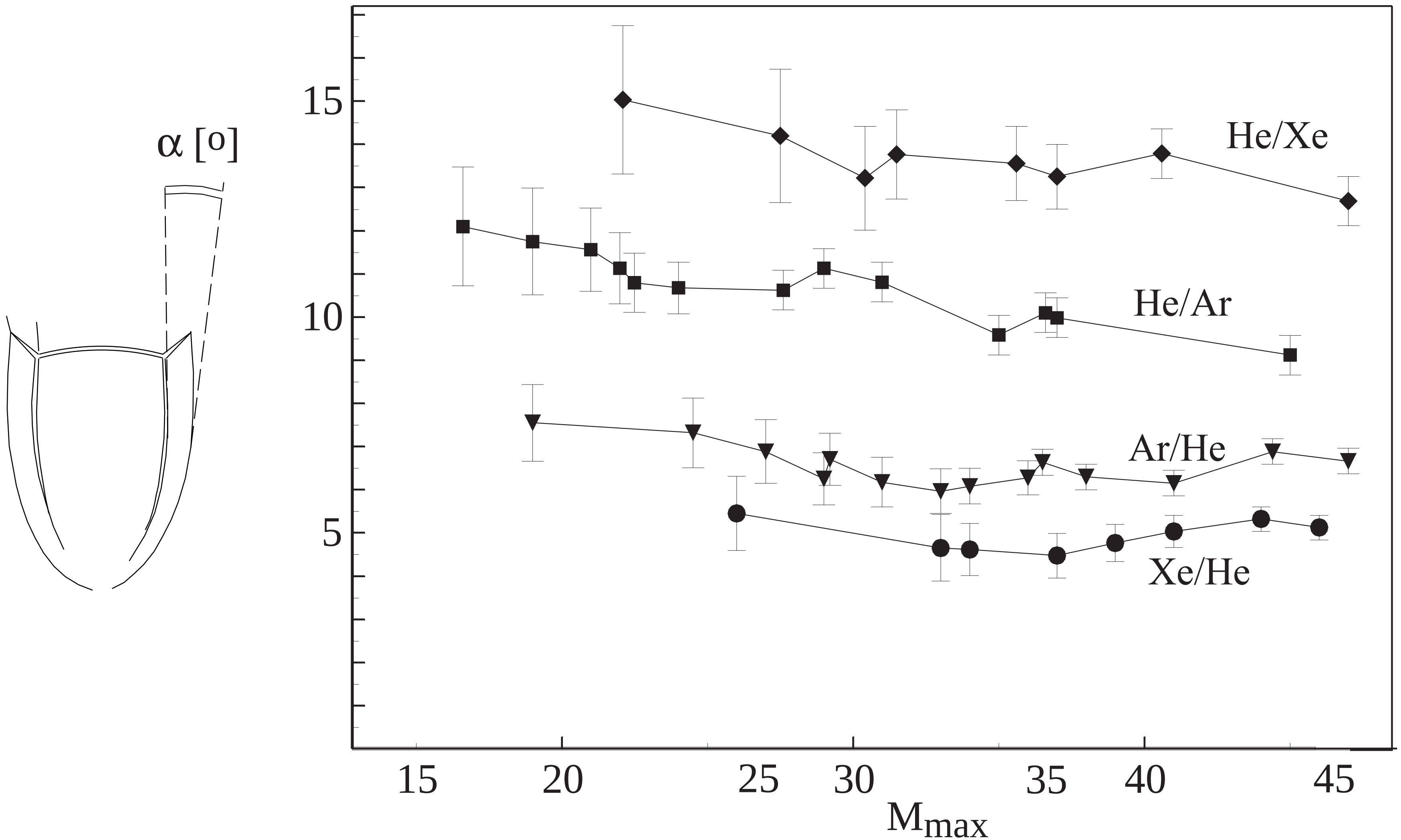



●: section (A), before the Mach disk. □: section (B), after the Mach disk.



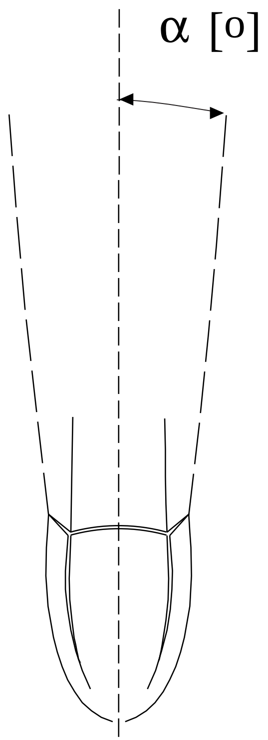
present experiment (He jet)
present experiment (Ar jet)
LI experiment, Kobayashi et al, 1984
Euler method, Nishida et al, 1985
DSMC method, D'Ambrosio et al, 1999
1/r 2 expansion
ρ/ρf
x/L
0
1
2
3
4
5
6
7
8
9
0
0.5
1.0
1.5
2.0

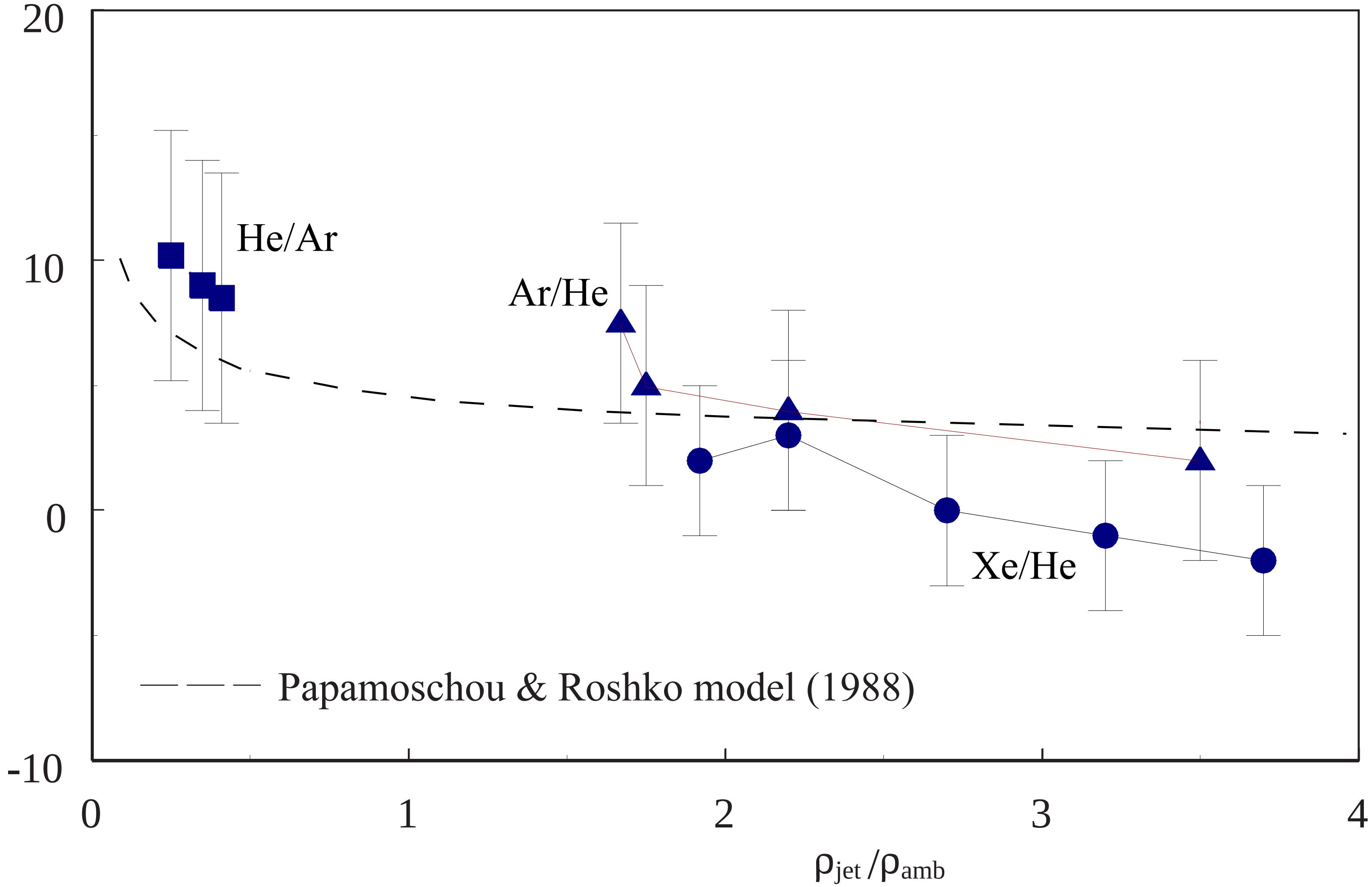
Near jet: Mixing layer (plume) spreading angle $\alpha$ vs $M_{max}$. (* 2008)




Barrel zone, before the Mach disk. Cross-section at $x = 0.8$ barrel lengths

Far jet: jet spreading angle $\alpha$ vs $\rho_{jet}/\rho_{amb}$ (* 2007, unpublished [Msc thesis])

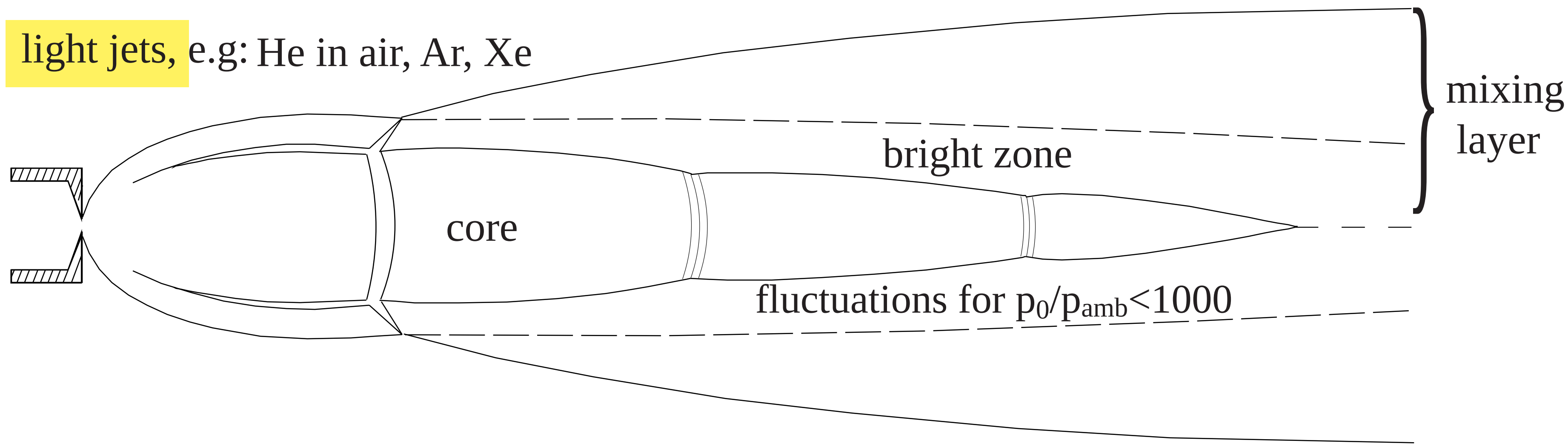


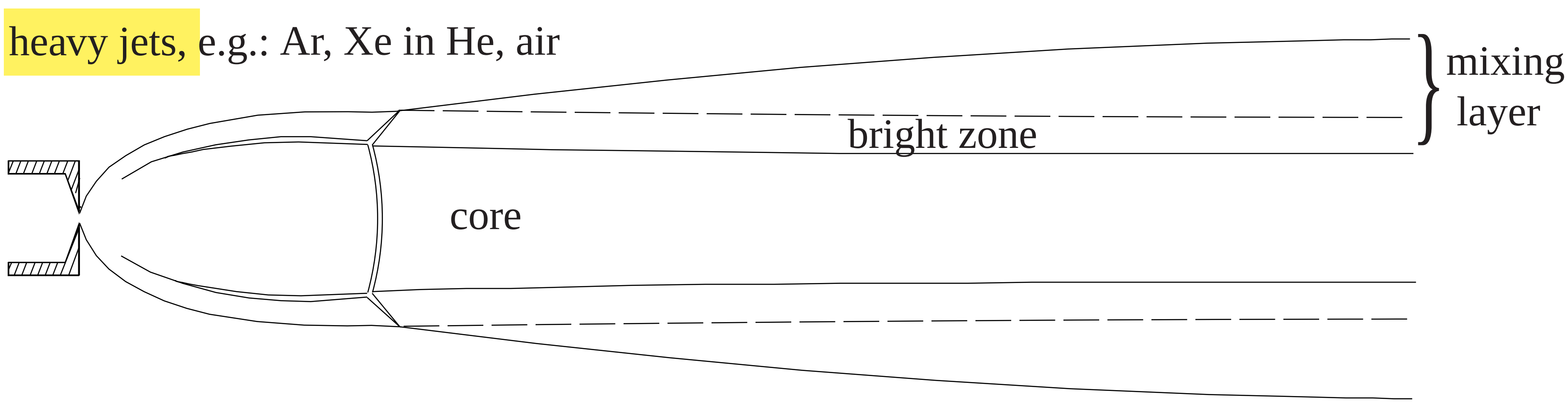




long range, Cross-section at 2.5 barrel lengths

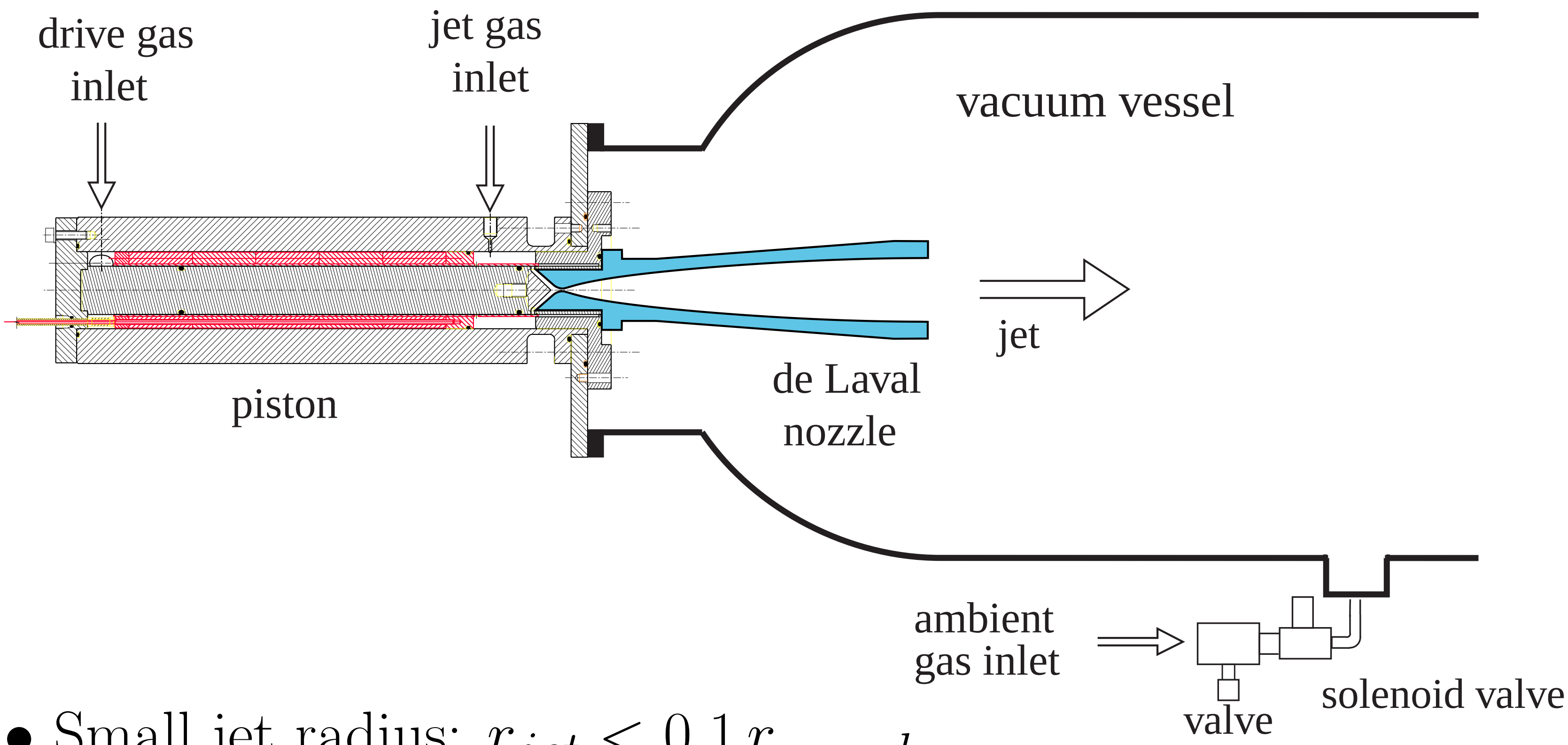
# 8 Underexpanded jet: results outline

Jet morphology in the range $500 < p_0/p_{\mathrm{amb}} < 10^5$, $0.1 < \rho_{\mathrm{jet}}/\rho_{\mathrm{amb}} < 15$



# Isoentropic jets

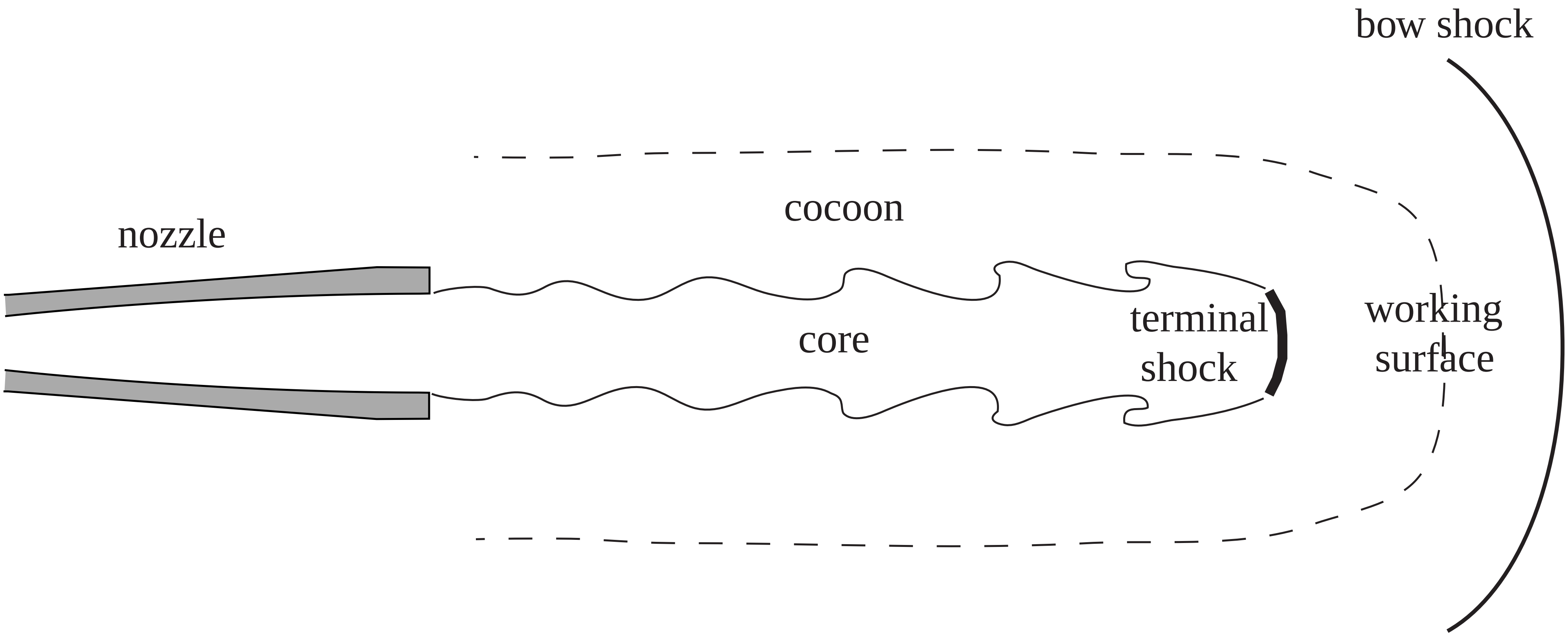



- Small jet radius: $r_{jet} < 0.1\, r_{vessel}$
- Fast (single pulse) jets: $\Delta t_{piston} \simeq t_{jet}$

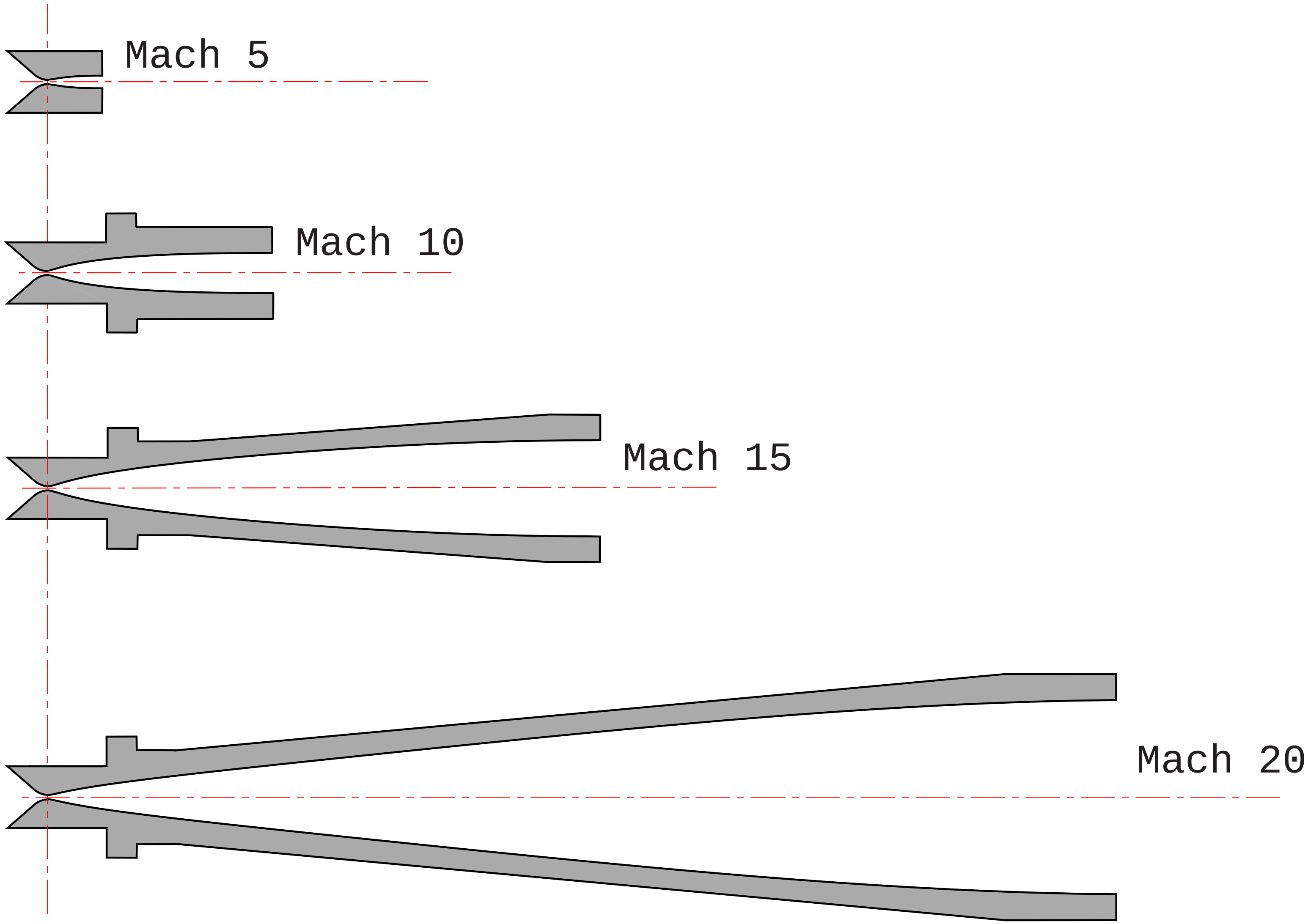
Adjustable parameters:

- jet Mach number $M_{\rm jet}$
- density ratio $\rho_{\rm jet}/\rho_{\rm amb}$ (selection of light or heavy gases)

Expected jet structure:

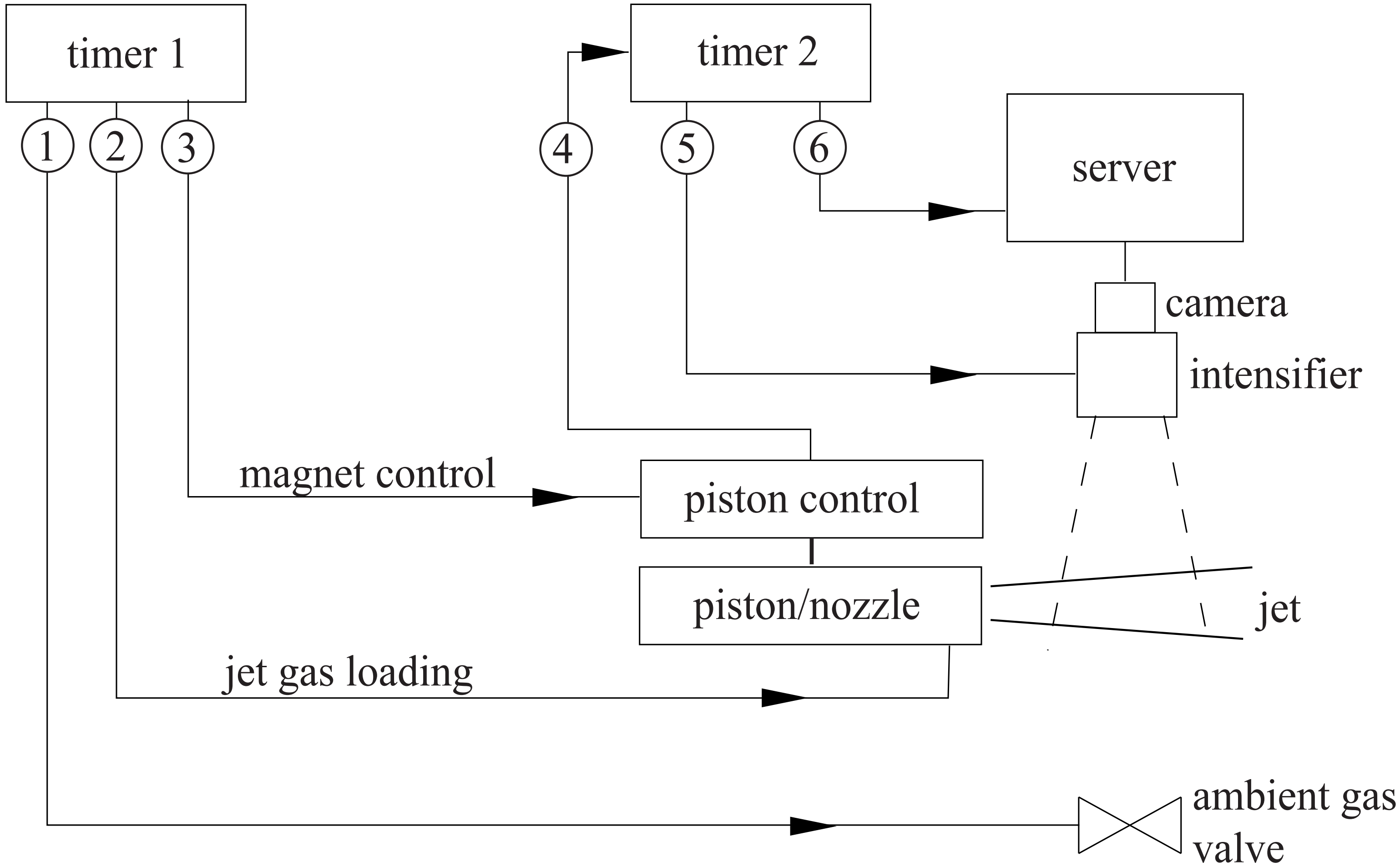


# Interchangeable de Laval nozzles

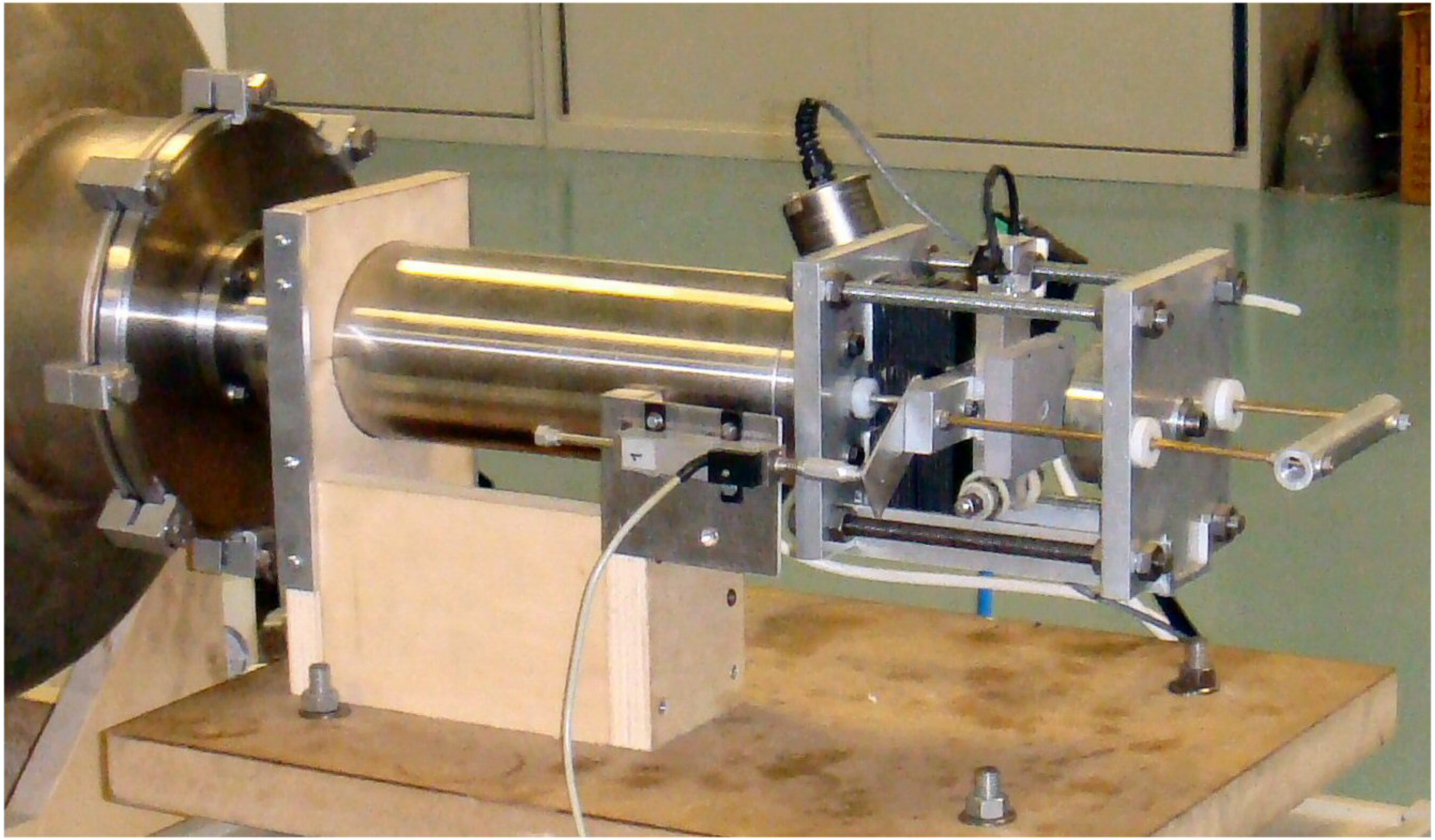


Working principle of the annular piston:

System timing

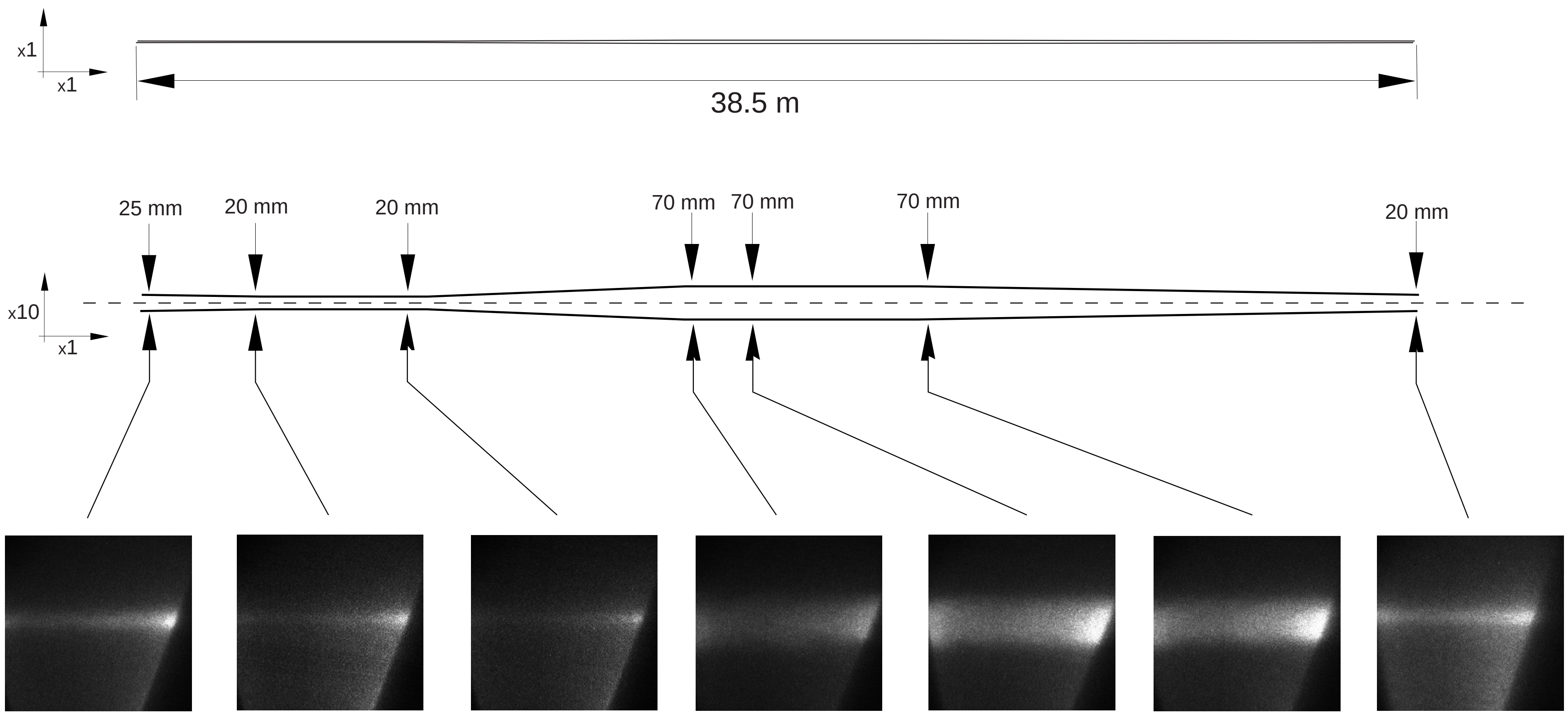


Piston view (connected to the vacuum vessel):



# Camera / triggered intensifier view

# Working tests: a M=15 Argon jet



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