The Wiener-Hopf method applied to dielectric angular regions: the wedge

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Abstract – **This paper reports the state of the art on the study of diffraction by a dielectric wedge and it proposes a new method to compute the diffracted field. In particular the paper presents the application of the Wiener-Hopf method to the problem of diffraction of a plane wave by a dielectric wedge immersed in free space. The formulation and the equations are proposed and discussed in the spectral domain.**

1 INTRODUCTION

This paper examines the problem of the diffraction by a plane wave on a penetrable wedge immersed in free space.

Several attempts to find the solution has been reported in literature but a general and complete solution of this problem is still not available.

Exact solutions have been obtained only for isorefractive and the ideal double negative (DNG) wedges. The solution of isorefractive wedges has been accomplished in the past by using the Kontorovich-Lebedev transform [1-2] in the frequency domain and the Green function in the time domain [3]. More recently solutions have been obtained [5-10].

The more interesting attempt to solve the dielectric wedge problem was the one proposed by Radlow [11] for the diffraction by the right-angled dielectric wedge. This method was based on multidimensional Wiener-Hopf (W-H) equations, but unfortunately it has been ascertained that this solution is wrong Kraut & Lehaman [12]. The interest to the Radlow method is due to the fact that he introduced multidimensional Wiener-Hopf equations to model the problem. However the factorization of multidimensional W-H equations needs function-theoretic techniques employing two complex variables that are cumbersome to handle.

At present the more interesting results obtained for the penetrable wedge geometries arise from the reduction of the problems to integral equations both one-dimensional and two-dimensional. These equations have been formulated both in the space domain and in the spectral domain [13]-[24]. Several techniques were used for their solution and many of them are based on regularization approaches.

According to our opinion, the Wiener-Hopf (W-H) technique is the most powerful method for solving field problems in presence of geometrical discontinuities [25-30]. Nevertheless only recently [26, 29] this technique has been successfully applied to wedges with arbitrary aperture angle. In general, the W-H formulation of the wedge problems yields generalized W-H equations (GWHE). GWHE can be reduced to classical W-H equations (CWHE) only for impenetrable wedges; therefore the direct approximation of GWHE is required for the dielectric wedge. In particular similarly to the CWHE also the GWHE can be reduced to Fredholm equations of second kind. The main aim of this work is the application of this technique to solve the dielectric wedge problem.

This technique based on the approximate solution of the GWHE can be extended to solve wedge problems involving anisotropic or bianisotropic media [25]. Apparently this extension is not possible for approximate solution obtained in the framework of the Sommerfeld-Malyuzhinets method [17, 23] since their applicability seems limited only to media where the Helmholtz wave equation holds.

2 THE DIELECTRIC WEDGE

Let us consider a dielectric wedge where we have identified four angular regions, see Fig. 1:

$0<\varphi<\Phi$, $-\Phi<\varphi<0$, $\Phi<\varphi<\pi$, $-\pi<\varphi<-\Phi$.

The wedge is illuminated by plane wave at skew incidence β and azimuthal incident angle φ_o.

The Wiener-Hopf technique [26] for angular problems is based on the introduction of the following Laplace transforms (1)-(2), where the subscript + indicates plus functions, *i.e.* functions having regular half-planes of convergence that are upper half-planes in the η-plane.

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$$V_{z+}(\eta,\varphi)=\int_0^{\infty}E_z(\rho,\varphi)e^{j\eta\rho}d\rho\,,\; I_{\rho+}(\eta,\varphi)=\int_0^{\infty}H_\rho(\rho,\varphi)e^{j\eta\rho}d\rho \qquad (1)$$

$$V_{\rho+}(\eta,\varphi)=\int_0^{\infty}E_\rho(\rho,\varphi)e^{j\eta\rho}d\rho\,,\; I_{z+}(\eta,\varphi)=\int_0^{\infty}H_z(\rho,\varphi)e^{j\eta\rho}d\rho \qquad (2)$$

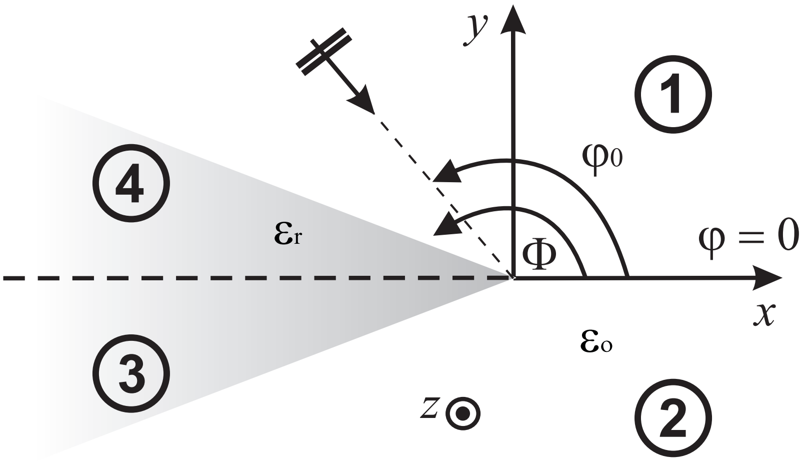

Fig. 1: the dielectric wedge and the four angular regions.

$$\begin{cases}\xi\, V_{z+}(\eta,0)-\dfrac{\tau_o^2}{\omega\varepsilon}I_{\rho+}(\eta,0)-\dfrac{\alpha_o\eta}{\omega\varepsilon}I_{z+}(\eta,0)=-n\; V_{z+}(-m,\Phi)-\dfrac{\tau_o^2}{\omega\varepsilon}I_{\rho+}(-m,\Phi)+\dfrac{\alpha_o m}{\omega\varepsilon}I_{z+}(-m,\Phi)\\ \xi\, I_{z+}(\eta,0)+\dfrac{\tau_o^2}{\omega\mu}V_{\rho+}(\eta,0)+\dfrac{\alpha_o\eta}{\omega\mu}V_{z+}(\eta,0)=-n\,I_{z+}(-m,\Phi)+\dfrac{\tau_o^2}{\omega\mu}V_{\rho+}(-m,\Phi)-\dfrac{\alpha\, m}{\omega\mu}V_{z+}(-m,\Phi)\end{cases} \qquad (3)$$

$$\begin{cases}-\xi\, V_{z+}(-\eta,-\pi)+\dfrac{\tau_o^2}{\omega\varepsilon}I_{\rho+}(-\eta,-\pi)-\dfrac{\alpha_o\eta}{\omega\varepsilon}I_{z+}(-\eta,-\pi)=-nV_{z+}(-m,-\Phi)+\dfrac{\tau_o^2}{\omega\varepsilon}I_{\rho+}(-m,-\Phi)-\dfrac{\alpha_o m}{\omega\varepsilon}I_{z+}(-m,-\Phi)\\ -\xi\, I_{z+}(-\eta,-\pi)-\dfrac{\tau_o^2}{\omega\mu}V_{\rho+}(-\eta,-\pi)+\dfrac{\alpha_o\eta}{\omega\mu}V_{z+}(-\eta,-\pi)=-n\,I_{z+}(-m,-\Phi)-\dfrac{\tau_o^2}{\omega\mu}V_{\rho+}(-m,-\Phi)+\dfrac{\alpha_o m}{\omega\mu}V_{z+}(-m,-\Phi)\end{cases} \qquad (4)$$

For region 1 we obtain the functional equations (3), where: $\alpha_o=k\cos\beta$, $\tau_o=\sqrt{k^2-\alpha_o^2}$, $\mathrm{Im}[\tau_o]\le 0\,,\xi=\xi(\eta)=\sqrt{\tau_o^2-\eta^2}$.

Using symmetry and variable substitutions we obtain similar functional equations for the other regions. For example in region 3 the equations (4) hold.

However we need to notice that the quantities involved in equations (3) and (4) depend on the constitutive parameters of the angular region (aperture angle and material), therefore:

for region 1

$\xi=\xi_1=\sqrt{\tau_1^2-\eta^2}$, $\tau=\tau_1=\sqrt{k_1^2-\alpha_o^2}$,

$\varepsilon=\varepsilon_1,\ \mu=\mu_1,\ k=k_1=\omega\sqrt{\varepsilon_1\mu_1}$

$m=m_1=-\eta\cos\Phi+\xi_1\sin\Phi$,

$n=n_1=-\xi_1\cos\Phi-\eta\sin\Phi$

for region 3

$\xi=\xi_3=\sqrt{\tau_3^2-\eta^2}$, $\tau=\tau_3=\sqrt{k_3^2-\alpha_o^2}$,

$\varepsilon=\varepsilon_3,\ \mu=\mu_3,\ k=k_3=\omega\sqrt{\varepsilon_3\mu_3}$

$m=m_3=-\eta\cos(-\Phi)+\xi_3\sin(-\Phi)$,

$n=n_3=-\xi_3\cos(-\Phi)-\eta\sin(-\Phi)$

With reference to Fig. 1, let us consider an E-polarized plane wave at normal incidence [31-32]: $\beta=\pi/2$ and $\alpha_o=0$, $\tau_j=k_j$.

The W-H functional equations assume the following form (5). Because of the symmetry we can rewrite the equations only using two angular regions and therefore by using only two kind of constitutive parameters and functions *m, n*…

From (5), after some mathematical manipulation we obtain two uncoupled system of GWHE functional equations of the form presented in (6).

$$\begin{cases} \xi \; V_{z+}(\eta,0) - \omega\,\mu I_{\rho+}(\eta,0) = -n \quad V_{z+}(-m \;,\Phi) - \omega\,\mu I_{\rho+}(-m \;,\Phi) \\ \xi \quad V_{z+}(\eta,0) + \omega\,\mu I_{\rho+}(\eta,0) = -n V_{z+}(-m,-\Phi) + \omega\,\mu I_{\rho+}(-m,-\Phi) \\ \xi_1 \; V_{z+}(-\eta,\pi) + \omega\,\mu I_{\rho+}(-\eta,\pi) = n_r \quad V_{z+}(-m_r,\Phi) + \omega\,\mu I_{\rho+}(-m_r,\Phi) \\ \xi_1 V_{z+}(-\eta,-\pi) - \omega\,\mu I_{\rho+}(-\eta,-\pi) = n_r \quad V_{z+}(-m_r,-\Phi) - \omega\,\mu I_{\rho+}(-m_r,-\Phi) \end{cases} \tag{5}$$

$$\begin{cases} Y_{i+}(\eta) = X_{i+}(-m_1) - \dfrac{\xi_{1-}}{n_{1+}} X_{j+}(-m_1) \\ Y_{j+}(\eta) = X_{i+}(-m_2) - \dfrac{\xi_{2-}}{n_{2+}} X_{j+}(-m_2) \end{cases} \tag{6}$$

where the unknowns are related to the physical quantities (1)-(2). Notice that the unknown are defined into three different complex planes: η, m_1, m_2. As reported in [26, 29] we can apply a special transformation to map unknowns defined in η, m_i into a new unique complex plane $\overline{\eta}_i$, therefore we obtain CWHE from a GWHE. Each (η, m_i) requires the definition of a new $\overline{\eta}_i$ plane. The dielectric wedge is modeled after the transformations by two uncoupled systems of two functional equations defined into two different complex planes $\overline{\eta}_1, \overline{\eta}_2$:

$$\begin{cases} Y_{i+}(\overline{\eta}_1) = X_{i+}(\overline{\eta}_1) - \sqrt{\dfrac{k_1 + \overline{\eta}_1}{k_1 - \overline{\eta}_1}} X_{j+}(\overline{\eta}_1) \\ Y_{j+}(\overline{\eta}_2) = X_{i+}(\overline{\eta}_2) - \sqrt{\dfrac{k_2 + \overline{\eta}_2}{k_2 - \overline{\eta}_2}} X_{j+}(\overline{\eta}_2) \end{cases} \tag{7}$$

In order to solve (6) we can apply the general procedure described in [28-30]: the Fredholm technique.

Since the unknowns are defined into two complex planes, we use the Cauchy formula to relate them:

$$X_{i+}(m_2) = \frac{1}{2\pi j} \oint_{\gamma 1} \frac{X_{i+}(m_1)}{m_1 - m_2} dm_1 \tag{8}$$

The use of the angular plane w and w_1 and of special warping improves the convergence of the numerical discretization of the equations (7)-(8).

Further details on the procedure to get the solution and numerical results in terms of diffraction coefficients of a dielectric wedge will be discussed and presented at the conference.

The solution is given in terms of the diffracted components of the unknown (1)-(2). A complete study of the field will show the GO, GTD, UTD components.

We note that for different physical parameters the field components will show different spectral properties.

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