Linear generation of multiple time scales by 3D unstable perturbations

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In this paper we present few observations concerning the appearance of different time scales during the transient growth of small three-dimensional perturbations superposed to a sheared flow, the bluff-body wake. The interesting point is that these phenomena are developing in the context of the linear dynamics. Before to comment these results let us shortly describe the method of study.

The early transient and long asymptotic behaviour is studied using the initial-value problem formulation. The base flow is approximated through an analytical expansion solution [1] of the Navier-Stokes equations. The viscous perturbative equations are written in terms of the vorticity and the transversal velocity [2] and then transformed through a Laplace-Fourier decomposition [3] in the plane (x, z) which is normal to the base flow plane (x, y),

$$\frac{\partial^2 \hat{v}}{\partial y^2} - (k^2 - \alpha_i^2 + 2i\alpha_r\alpha_i)\hat{v} = \hat{\Gamma} \tag{1}$$

$$\frac{\partial \hat{\Gamma}}{\partial t} = (i\alpha_r - \alpha_i)(\frac{d^2 U}{dy^2}\hat{v} - U\hat{\Gamma}) + \frac{1}{Re}[\frac{\partial^2 \hat{\Gamma}}{\partial y^2} - (k^2 - \alpha_i^2 + 2i\alpha_r\alpha_i)\hat{\Gamma}] \tag{2}$$

$$\frac{\partial \hat{\omega}_y}{\partial t} = -(i\alpha_r - \alpha_i)U\hat{\omega}_y - i\gamma\frac{dU}{dy}\hat{v} + \frac{1}{Re}[\frac{\partial^2 \hat{\omega}_y}{\partial y^2} - (k^2 - \alpha_i^2 + 2i\alpha_r\alpha_i)\hat{\omega}_y] \tag{3}$$

The transversal velocity and vorticity components are indicated as $\hat{v}$ and $\hat{\omega}_y$ respectively, while $\hat{\Gamma}$ is defined through the kinematic relation $\widetilde{\Gamma} = \partial_x\widetilde{\omega}_z - \partial_z\widetilde{\omega}_x$ that in the physical plane links together the perturbation vorticity components in the x and z directions ($\widetilde{\omega}_x$ and $\widetilde{\omega}_z$) and the perturbed velocity field. Equations (2) and (3) are the Orr-Sommerfeld and Squire equations respectively, from the classical linear stability analysis for three-dimensional disturbances in the phase space. We define k as the polar wavenumber, $\alpha_r = kcos(\phi)$ as the wavenumber in x direction, $\gamma = ksin(\phi)$ as the wavenumber in z direction, ϕ as the angle of obliquity with respect to the physical plane, and α_i as

the spatial damping rate in x direction. We introduce the amplification factor $G(t)$ as the disturbance kinetic energy density $E(t)$, the temporal growth rate $r(t) = log|E(t)|/2t$ and the frequency $\omega(t)$ as the time derivative of the perturbation phase.
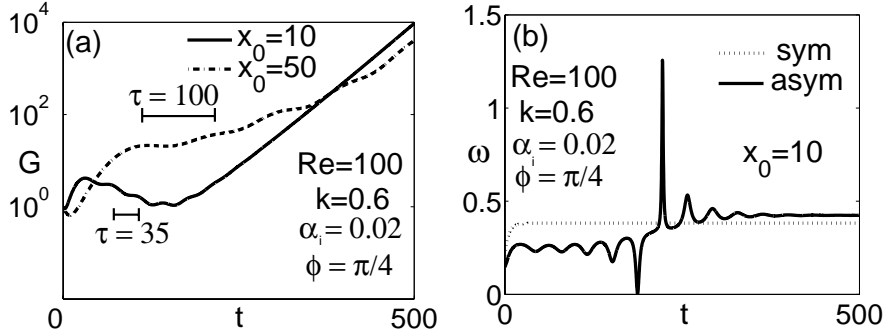

Fig. 1. $Re = 100$, $k = 0.6$, $\alpha_i = 0.02$, $\phi = \pi/4$. (a): Asymmetric case. Amplification factor G(t) for intermediate ($x_0 = 10$) and far ($x_0 = 50$) wake sections. (b): Intermediate section $x_0 = 10$. Pulsation $\omega(t)$ for asymmetric and symmetric cases.

The results on the onset of multiple time scales – obtained by observing the amplification factor $G(t)$ and the pulsation $\omega(t)$ – are presented in Fig. 1 (a, b). In Fig. (1a) the amplification factor $G(t)$ is shown for two typical intermediate ($x_0 = 10$) and far ($x_0 = 50$) wake sections. The perturbations are asymmetric. For $x_0 = 10$ a local maximum, followed by a minimum, is visible in the energy density, then the perturbation is slowly amplifying and the transient is extinguished only after hundreds of time scales. For $x_0 = 50$ these features are less marked. It can be noted that the far field configuration ($x_0 = 50$) has a faster growth than the intermediate field configuration ($x_0 = 10$) up to $t = 400$. Beyond this instant the growth related to the intermediate configuration will prevail on that of the far field configuration. For $x_0 = 10$ the amplification factor $G(t)$ shows a modulation which is very evident in the first part of the transient (see [4]), and which corresponds to a modulation in amplitude of the pulsation of the instability wave depicted in Fig. (1b). Here, the frequency $\omega(t)$ for symmetric and asymmetric perturbations at $x_0 = 10$ is shown. The modulation is only present for the asymmetric wave (for the symmetric case the amplitude is constant after few time scales). This behaviour is in general found for asymmetric longitudinal or oblique waves. In these instances two time scales are simultaneously observed in the transient and long term behaviour: the periodicity associated to the average value of the pulsation in the early transient ($\omega \approx 0.3$) and the asymptotic pulsation ($\omega \approx 0.45$). Moreover, the oscillation of this pulsation $\omega(t)$ in the early transient introduces another time scale τ, which in terms of pulsation is about 0.17.

The frequency determination is validated through the comparison of the temporal asymptotic behaviour ($t \to \infty$) of the initial-value analysis with a recent normal mode analysis [5] and experimental data of nearly supercritical oscillations [6] for different Reynolds numbers ($Re = 50, 70, 100$) (see Fig. 2).
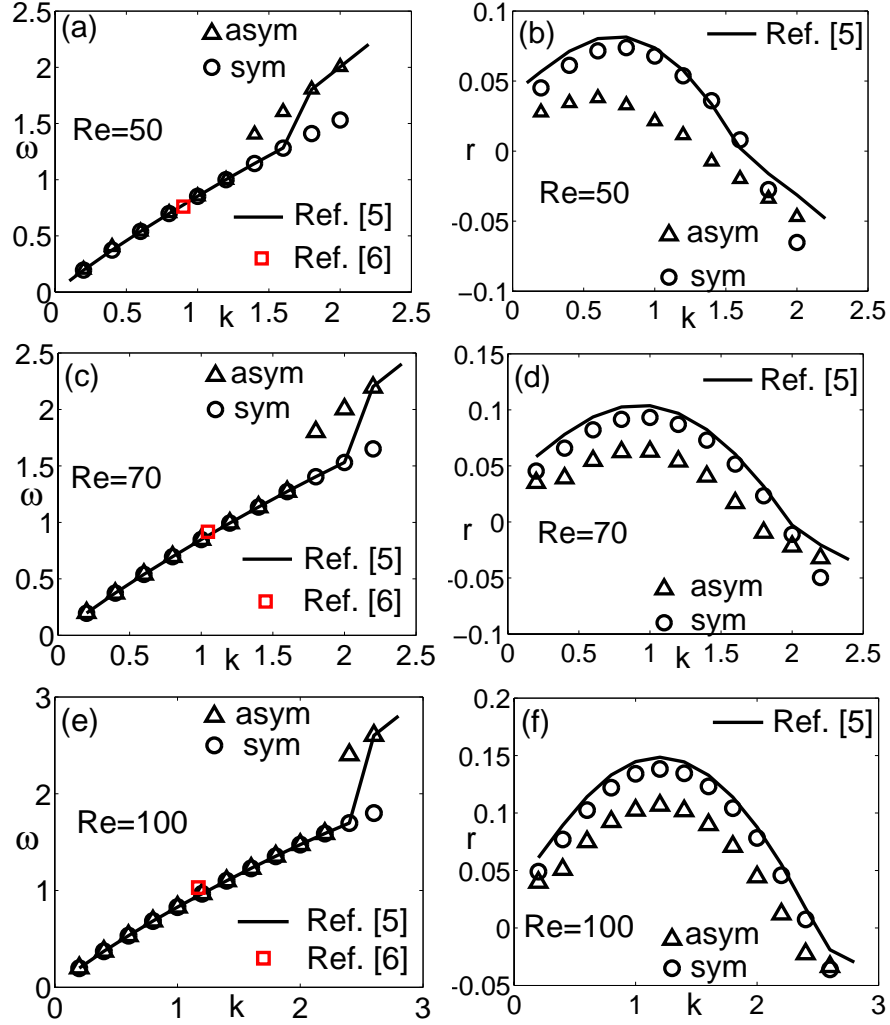

Fig. 2. (a, c, e) pulsation $\omega(t)$ and (b, d, f) temporal growth rate $r(t)$ for present results (asymmetric case: black triangles, symmetric case: black circles), modal analysis [5] (solid curves) and experimental data [6] (red squares). $\alpha_i = 0.05$, $\phi = 0$, $x_0 = 10$, $Re = 50, 70, 100$.

The comparison is quantitatively good for all the Reynolds numbers considered, because it shows that a wavenumber close to the wavenumber that theoretically has the maximum growth rate has a - theoretically deduced - frequency which is very close to the frequency measured in the laboratory.

The noticeable point of this analysis is the variety of temporal scales revealed by the transient, which are associated to a given specific value of the instability wavelength. In particular, if the perturbation is asymmetric and oblique it is possible to count up to five different time scales for the system: (i) the temporal scale $D/U \sim 1$ related to the base flow (where D is the cylinder diameter and U is the free stream velocity), (ii) the length of the transient (200-300 time units), (iii and iv) the scales associated to the instability frequency in the early transient (about 21 time units) and in the asymptotic state (about 14 time units), and (v) the modulation of the pulsation in the early transient (about 35-40 time units). Another interesting point is that these scales are different each other and are also different from the asymptotic value predicted either by the initial-value problem or the modal theory.

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