A One-Dimensional Interpretation of the Statistical Behavior of Reverberation Chambers

Ramiro Serra* Flavio Canavero*

Abstract — **This work focuses on building a fairly simple yet physically appropriate 1D model for a Reverberation Chamber which claims to be able to analytically predict the statistical behavior of fields inside such a chamber, without forsaking to the benefits of deterministic models. The statistical properties of the fields are introduced either by varying the size of a 1D stirrer, or (in absence of it) by varying the cavity size itself. A validation analysis was made over 27 different experiments in order to assess the main effects of the frequency and of the stirrer size. The properties derived are in agreement with other theories and with measured results on real Reverberation Chambers.**

1 INTRODUCTION

Reverberation Chambers (RC) are gaining significant confidence in their use for radiated emissions and immunity measurements. RC users need to fully understand its working principles in order to correctly interpret the measurement results and to optimize the performance for various measurement tasks.

The increasing comprehension of RCs has evolved from deterministic to statistical models. Both kind of models provide a reasonable knowledge of the basic principles involved, and help in giving some guidelines in the construction and/or optimization of a RC. Nevertheless, many of the construction suggestions existing in literature were not only derived from applying the mentioned basic physical principles but also in combination with years of practical experience (as in [1] and [2]). There is certainly a widespread use of computer simulations applied to this problem, and they are found of a great usefulness when designing or constructing a RC. However, numerical methods are out of the scope of the present analysis, and we will not discuss their influence here, for the sake of brevity. Since an empirical approach to design is time-consuming and does not guarantee (even if successful) optimal solutions, a thorough understanding of RC mechanisms helps to build more efficient chambers.

Deterministic models (i.e. [3], [4]) very often start with the abstraction of a RC as a simple cavity in order to explain basic, but important concepts such as the number of modes and the modal density. As these models move from an ideal cavity into a lossy one [5], they converge towards a fairly realistic RC, including in the analysis parameters such as the so called quality factor. However, it must be pointed out that deterministic models, mainly treating a RC as if it were a simple cavity resonator, do not succeed in describing the process of mode-stirring, by which the field distribution inside the cavity becomes a stochastic process.

Statistical models (i.e. [6], [7], [8]) are able to derive the probability density functions and the spatial correlation function for each field magnitude, predict antenna or test object responses and some useful expressions for the quality factor. However, they lack of a complete understanding of the chamber, and many important issues are left aside. They frequently start assuming that the modes are "well-stirred" without deepening into the conditions leading to this. Furthermore, they often need to assume special geometrical conditions, not quite realistic and somewhat difficult to apply into a specific RC. For example, the Plane Wave Integral Representation [8] has its rigorous validity only in spherical volumes.

It is not possible to leave one of the mentioned models behind, as they mutually collaborate in the RCs wide-ranging knowledge. Generally speaking, each approach succeeds in areas where the other one fails, and viceversa. There is an obvious gap between deterministic and statistical models which makes us change our methodology depending on what kind of result we seek. Consequently, a call for filling this gap and link the two approximations is needed. This necessity is supported by the aim of having a better understanding, to manage a simpler yet complete model and to reduce up to a reasonable minimum the empirical techniques. Our work focuses on building a fairly simple –yet physically appropriate– 1D model which claims to be able to succeed in describing the statistical behavior of RCs. It is obviously a reduced and somewhat simplistic model, yet it appears to provide sufficient hints, and it is promisingly useful.

In the following, Section 2 presents the basic 1D model and explains the essential functioning of it;

*The authors are with the Dipartimento di Elettronica, Politecnico di Torino, C.so Duca degli Abruzzi 24, 10129 Torino, Italy, tel.: +39 011 5644000, fax: +39 011 5644099. E-mails: ramiro.serra@polito.it – flavio.canavero@polito.it

Section 3 studies the statistical properties of the fields inside the chamber, and Section 4 discusses the effects of some key factors of RCs.

2 THE ONE-DIMENSIONAL CAVITY MODEL

The description of our chamber (see Fig. 1 for a schematic diagram) starts as a 1D cavity including a segment of a dielectric material with relative dielectric constant κ inside the vacuum-filled space and a continuous-wave source located at x_0. The length of the chamber is a.

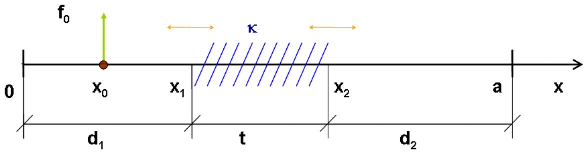

Figure 1: Definition of the one-dimensional cavity under study.

The electromagnetic field inside this chamber obeys the wave equation:

$$\nabla^2 E(x) + \kappa(x) k^2 E(x) = 0 \qquad (1)$$

where $\kappa(x) = \kappa$ for $x_1 \leq x \leq x_2$ and $\kappa(x) = 1$ elsewhere; $k = \omega\sqrt{\mu\epsilon}$ is the free-space wavenumber; μ, ϵ are the free-space permeability and permittivity, respectively. The $e^{-j\omega t}$ time dependence is suppressed. The chamber is divided into three regions: d_1, where the source is; t, the stirrer; and d_2, the Test Volume.

One possible set of eigensolutions ([5], [9]) for each region is:

$$\begin{aligned} E_{n1}(x) &= D_n \sin h_n x \\ E_{n2}(x) &= A_n \sin l_n(x - x_1) + B_n \sin l_n(x - x_2) \\ E_{n3}(x) &= C_n \sin h_n(a - x) \qquad (2) \end{aligned}$$

where subindexes $1, 2, 3$ mean the region of validity of each expression and n is the modal index. The proposed solution automatically satisfy the boundary conditions at the perfectly conducting "walls" of the chamber in $x = 0, a$. The coefficients A_n, B_n, C_n, D_n and the wavenumbers l_n and h_n are determined knowing that at $x = x_1, x_2$, both E and H must be continuous, and that a source is present in $x = x_0$. The method for determining the fields inside our cavity is outlined in [5] and [9], and omitted here due to space limitation. Losses in the walls are introduced according to the method in [5], where the perturbed eigenvalues are given by:

$$l'_n = l_n\left(1 - \frac{j}{2Q}\right) \qquad h'_n = h_n\left(1 - \frac{j}{2Q}\right)$$

where $j = \sqrt{-1}$ and Q is the chamber quality factor (additional discussion about these assumptions, omitted here for the sake for brevity, can be found in [3]).

Figure 2 shows the modification of field distribution inside the chamber, due to a change of the κ value in the dielectric region, assumed to maintain a constant ratio $t/a = 0.1$. From the observation of Fig. 2, where the real part of the electric field inside the chamber for $\kappa = 1$ (i.e., absence of dielectric) and $\kappa = 1.2$, it is evident that the main effect of the dielectric layer inside the chamber is to appreciably change the field distribution inside the "Test Volume" region. Thus, an analogy with the stirrer in real RCs can be established. Additional secondary effects are noticed, such as a reduction in the field magnitude and in the number of modes.

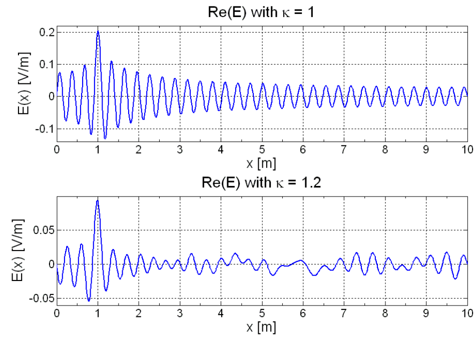

Figure 2: The real part of electric field E inside the 1D cavity model for $\kappa = 1$ (top panel) and $\kappa = 1.2$ (bottom panel).

3 THE ONE-DIMENSIONAL REVERBERATION CHAMBER MODEL

Up to now, we have not been solving a RC but a Cavity Resonator. Here we demonstrate that a random variation of selected parameters can turn the cavity into a RC.

Let us consider the cavity described in the previous Section, and uniformly vary the stirrer length t. Figure 3 shows the field distribution inside the Reverberation Chamber for five values of the stirrer region size $t/a = 0.11, 0.12, 0.13, 0.14, 0.15$.

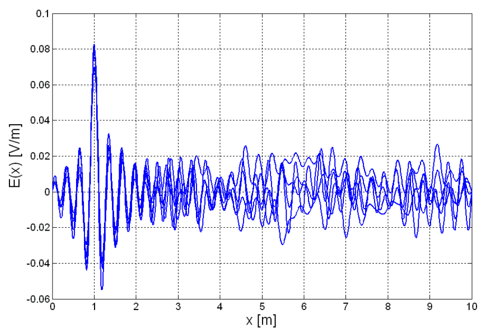

Figure 3: The real part of electric field E inside the 1D cavity model for 5 different values of the stirrer size.

It can be observed that the field is highly coherent in the region where the source is present but,

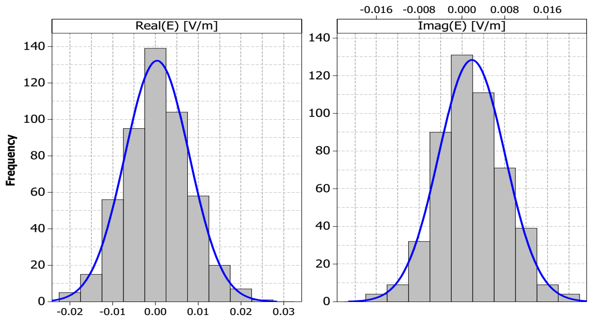

Figure 4: Probability distribution of the real and imaginary parts of the electric field measured at position $x = 8.5$ m after 500 iterations.

on the contrary, a considerably uncorrelated field behavior develops in our "Test Volume".

The results of 500 independent calculations of the electromagnetic field at a fixed measurement position inside the test volume are shown in Fig. 4, that presents the histograms of the real and imaginary parts of the electric field with their fitted Normal Distributions.

The Anderson-Darling Normality Test (A-D) [10] was applied to these values to determine whether the data of the sample is nonnormal. The resulting p-values were 0.762 and 0.503 for the real and imaginary part, respectively, thus largely justifying the hypothesis that they follow the Normal Distribution. Figure 5 shows the empirical cumulative distributions of the data with almost all the measurements laying inside the 95% Confidence Interval. These results reproduce the literature findings, i.e., that the field-components distributions match the Probability Density Functions ([6], [8]).

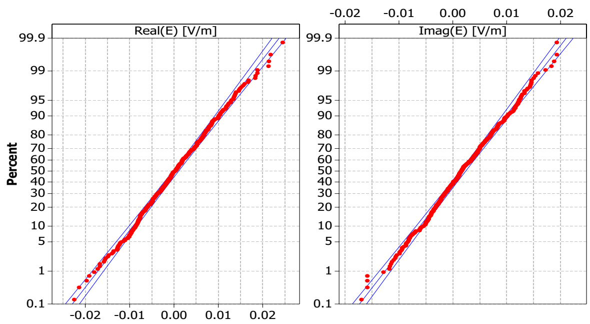

Figure 5: Cumulative distribution of the real and imaginary parts of the electric field measured at position $x = 8.5$ m after 500 iterations with their 95% Confidence Intervals.

Alternatively, if we solve the cavity without the stirrer region, but we make the chamber length a to randomly vary, we are able to reproduce the behavior of a vibrating-wall chamber [11]. The A-D test was applied resulting in p-values of 0.434 and 0.387,

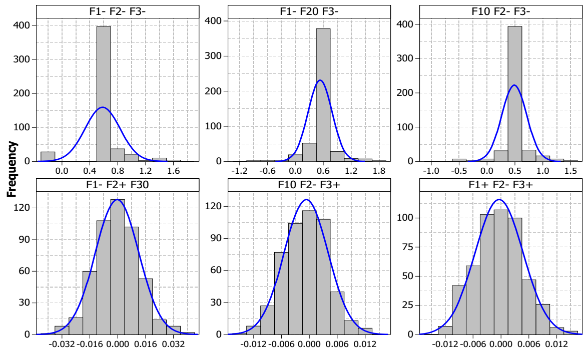

Figure 6: Histograms and their fitted Normal Distributions of the worst three and the best three performances out of the 27 experiments of the Validation Analysis. Parameters values are coded in F-symbols, as explained in the text.

largely justifying again the hypothesis of normality. Many other stirring processes can be studied by means of the proposed model, but are omitted here for brevity.

4 VALIDATION ANALYSIS

Several parameters (or "factors") can influence the distribution of the electromagnetic field inside the chamber. In this section, we study the effects of the following geometrical factors:

$$F_1 = \frac{t_0}{a}; \qquad F_2 = \frac{\Delta t}{a} \qquad and \qquad F_3 = \frac{a}{\lambda} \qquad (3)$$

The actual length t of each stirrer was randomly obtained by means of $t = t_0 + 2U(0, \Delta t)$, where $U(a, b)$ stands for the Uniform Distribution within the interval (a, b). The t_0 value is the fixed part of our 1D stirrer, while the Δt value is related to the varying part.

These parameters are defined as dimensionless quantities in order to gain generality. A factorial design was defined as indicated in Table 1, outlining three levels of variation for every factor, and each level was chosen according to empirical experience.

Factors	Low	Medium	High
F_1	0.05	0.1	0.15
F_2	0.03	0.06	0.09
F_3	3	30	60

Table 1: Factorial Plan.

As in Section 3, the A-D test was repeated for the resulting 27 experiments. For each configuration of the factors levels, we calculated the real part of the electric field for 500 different stirrer sizes t as explained above.

The A-D test was run for all experiments, and Fig. 6 presents the worst three and the best three performances of all, for brevity. A code was added for clarity attaching a $-, 0$ or $+$ symbol whether a factor receives a *Low, Medium* or *High* level, respectively. For the worst cases, the p-values are lower than 0.005, while the best three cases show p-values equal to 0.825, 0.724 and 0.569.

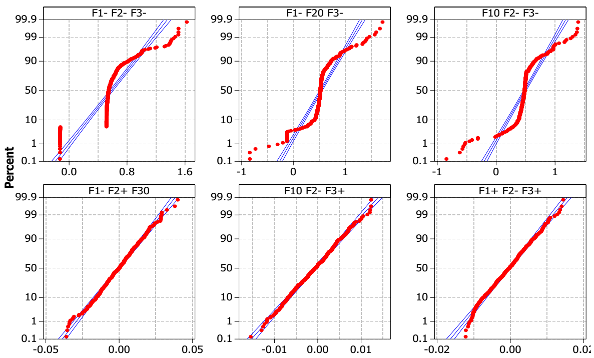

Figure 7: Cumulative distributions of the worst three and the best three performances out of the 27 experiments of the Validation Analysis, with their 95% Confidence Intervals. Parameters values are coded in F-symbols, as explained in the text.

Figure 7 shows the empirical cumulative distributions corresponding to the Normal Distributions of Fig. 6; the limits of the 95% Confidence Interval are also shown for all cases.

A complete analysis of the factors indicates a total agreement with the behavior found in practice for RCs and with what is reported in literature. The following considerations represent a summary of our observations:

1) all factors have a main effect on the response;

2) the effect of F_3 (indirectly corresponding to the operation frequency f_0) results comparably superior to the rest;

3) the effect of every single factor on the response is significantly influenced by the other two factors; thus, a strong interaction is working between them;

4) when the frequency is low, no matter how large the change of the stirrer size or variations could be, the performance is not acceptable.

The above properties are in agreement with the published RC theories and with measured results on real RCs. Hence, we can conclude that our 1D model (although simplistic) provides a good representation of reality.

5 CONCLUSION

This paper describes a 1D RC model that presents a strong behavioral analogy with 3D RCs. It simulates the electromagnetic field distribution inside a theoretical vacuum-filled 1D segment with the presence of a 1D "stirrer" and of losses in the walls. In this model, the statistically uniform field can be obtained in two different ways: either by varying the size of the stirrer, or (in absence of it) by varying the cavity size. Both processes show reliable normality conditions. The effects of the stirrer size and the frequency are in agreement with theory and measurements. The main convenience of this model consists in providing a complete understanding of RCs, without leaving a gap in the theoretical development. Further work (currently under way) involves both the development of a correlation between the real stirrer and its 1D parameters, and a 3D extension of this model.

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