Weighting Strategies for Passivity Enforcement Schemes

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Abstract: We present a general class of frequency-selective weighting schemes, allowing for a drastic accuracy improvement when embedded in passivity enforcement algorithms for linear lumped macromodels. The proposed technique minimizes the in-band model perturbation while achieving global passivity, even in the difficult case of large out-of-band passivity violations.

1 Introduction

Passive macromodeling has become a common practice in the design flow of digital, RF, and mixed/signal systems, in several application areas. Linear macromodels usually consist of Laplace-domain rational functions and are commonly derived via curve-fitting from frequency-domain or time-domain tabulated responses. The standard method for this task is the Vector Fitting (VF) algorithm, in one of its many variants [1]-[3]. The resulting macromodels enable fast transient simulations via common circuit solvers, thus enabling effective Signal/Power Integrity assessments.

It is well-known that common macromodeling techniques, including VF, are not able to directly enforce macromodel passivity. This is a fundamental property that should always be enforced, since lack of passivity may be the cause of diverging transient simulations due to a spurious model-induced energy gain. This fact motivated significant reserch efforts over the last few years, aimed at the definition of fast algorithms for model correction and passivity enforcement. Several solutions are now available [4]-[9]. All these techniques are able to perturb the model parameters, using linear or quadratic constraints arising from some formulation of the passivity conditions. We can cite approaches based on a direct enforcement of Bounded or Positive Real Lemma via Linear Matrix Inequalities (LMI) constraints [4, 5], Hamiltonian eigenvalue perturbation schemes [6, 7], and schemes for passivity enforcement at discrete frequency samples [8, 9]. In all cases, a minimal perturbation condition is also used in order to preserve model accuracy while enforcing passivity.

This work illustrates that standard accuracy control methods may be insufficient. We show that out-of-band passivity violations may lead to dramatic in-band accuracy loss during passivity enforcement. We then suggest a general frequency-dependent norm weighting scheme, allowing for systematic improvement over existing techniques. With our proposed method, model accuracy is preserved even in the challenging case of large out-of-band passivity violations.

2 Preliminaries and Notation

We consider linear macromodels in state-space form, described by the following standard shorthand notation

$$\mathbf{H}(s) := \left[\begin{array}{c|c} \mathbf{A} & \mathbf{B} \\ \hline \mathbf{C} & \mathbf{D} \end{array}\right] = \mathbf{D} + \mathbf{C}\,(s\mathbf{I} - \mathbf{A})^{-1}\,\mathbf{B}\,, \tag{1}$$

where s is the Laplace variable, $\mathbf{H}(s)$ is the $p \times p$ transfer matrix of the macromodel, and $\{\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}\}$ are the (real) state-space matrices of some realization associated to $\mathbf{H}(s)$. Throughout this paper we will assume that $\mathbf{H}(s)$ is a scattering matrix, which is the standard representation for Signal Integrity applications. In case (1) is found to be non-passive, some correction is applied to the model. Most passivity enforcement methods try to find a passive model by perturbing the macromodel residue matrices, which are usually stored in state-space matrix $\mathbf{C}$. The perturbed model and the associated perturbation read

$$\mathbf{H}_p(s) = \left[\begin{array}{c|c} \mathbf{A} & \mathbf{B} \\ \hline \mathbf{C} + \delta\mathbf{C} & \mathbf{D} \end{array}\right] \quad \text{and} \quad \delta\mathbf{H}(s) = \mathbf{H}_p(s) - \mathbf{H}(s) = \left[\begin{array}{c|c} \mathbf{A} & \mathbf{B} \\ \hline \delta\mathbf{C} & \mathbf{0} \end{array}\right], \tag{2}$$

respectively. The numerical evaluation of $\delta\mathbf{C}$ is performed in order to keep the induced perturbation in the system response as small as possible. The standard measure that is used to quantify this amount of perturbation is the $\mathcal{L}^2$ (energy) norm, defined as

$$||\delta\mathbf{H}||^2 = \frac{1}{2\pi} \sum_{ik} \int_{-\infty}^{\infty} |\delta H_{ik}(j\omega)|^2 \mathrm{d}\omega = \mathrm{tr}\{\delta\mathbf{C}\,\mathbf{P}\,\delta\mathbf{C}^T\} = \mathrm{tr}\{\mathbf{\Delta}\,\mathbf{\Delta}^T\} = ||\mathrm{vec}(\mathbf{\Delta})||_2^2\,, \tag{3}$$

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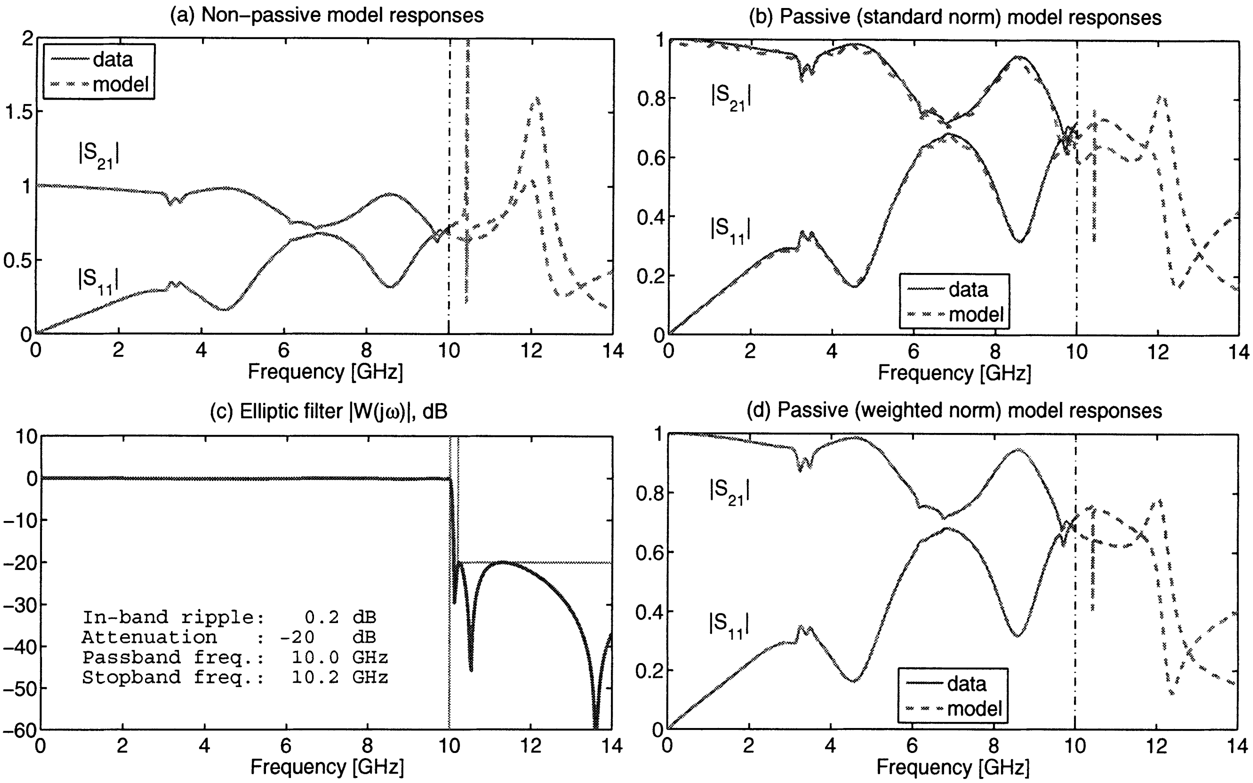

Figure 1: Modeling a VHDM connector. The responses of a non-passive model obtained via Vector Fitting are compared to raw frequency data in (a). Results of a passivity enforcement scheme with standard error control are reported in (b), showing significant in-band accuracy degradation. Use of a norm weighted by an elliptic filter (c) during passivity enforcement leads to a far more accurate passive model (d).

where tr is the matrix trace, operator vec$(\cdot)$ stacks the columns of its matrix argument, $\mathbf{P} = \mathbf{K}^T\mathbf{K}$ is the controllability Gramian of the macromodel (1) computed via the following Lyapunov equation

$$\mathbf{A}\mathbf{P} + \mathbf{P}\mathbf{A}^T + \mathbf{B}\mathbf{B}^T = \mathbf{0}\,, \tag{4}$$

and $\boldsymbol{\Delta} = \delta\mathbf{C}\,\mathbf{K}^T$. Based on this algebraic representation, a generic passivity enforcement scheme can be stated as a minimization of $||\mathrm{vec}(\boldsymbol{\Delta})||$ with passivity constraints. In this work we will consider both schemes based on equality passivity constraints, such as Hamiltonian-based perturbation algorithms [6, 7]

$$\min ||\mathrm{vec}(\boldsymbol{\Delta})|| \quad \text{subject to} \quad \mathbf{Z}\mathrm{vec}(\boldsymbol{\Delta}) = \mathbf{v}\,, \tag{5}$$

and schemes based on linear or linearized inequality constraints [8, 9]

$$\min ||\mathrm{vec}(\boldsymbol{\Delta})|| \quad \text{subject to} \quad \mathbf{Q}\mathrm{vec}(\boldsymbol{\Delta}) \leq \mathbf{u}\,. \tag{6}$$

3 Problem statement

Any raw frequency response is known only over a finite bandwidth $[\omega_{\min}, \omega_{\max}]$. Without loss of generality, we consider the special case with $\omega_{\min} = 0$. Since no raw data are available for $\omega > \omega_{\max}$, no model identification process is able to guarantee model quality beyond $\omega_{\max}$. In principle, this fact has limited practical relevance if the excitation signals have a spectral content within this frequency band. Unfortunately, model behavior for $\omega > \omega_{\max}$ may be very poor in terms of passivity. Indeed, it is well-known that the most severe passivity violations usually occur outside the modeling bandwidth. Figure 1 depicts this typical situation for a VHDM connector. Figure 1(a) compares the frequency responses of a rational model generated via VF to the raw scattering data. Model is very accurate within the modeling bandwidth, in this case up to 10 GHz. However, there are severe passivity violations beyond the last frequency data point, as can be readily noted from the large gain of both return and insertion loss of the model. Therefore, although accurate under bandlimited excitation, the model may lead to instabilities during system-level transient simulations caused by this out-of-band spurious energy gain.

We now apply a passivity enforcement scheme of type (6) based on the minimization of the absolute norm (3). The results are depicted in Figure 1(b). Model is now passive, but significant accuracy degradation occurs inside the modeling bandwidth, despite the explicit accuracy constraint. The reason for this degradation is clear from (3). All energy contributions at all frequencies contribute to the norm with equal weight, both in-band and out-of-band. Most of these contributions come from out-of-band perturbations, since this is the region where the model needs a large correction. However, we are not interested in keeping the out-of-band accuracy under control, since the original non-passive model is wrong for $\omega > \omega_{\max}$. We are only interested in preserving the in-band accuracy while enforcing global passivity. To this end, the ideal approach would be to use a bandlimited norm by considering the energy contributions in (3) only up to $\omega_{\max}$, while neglecting any out-of-band contribution. Next section presents a general approach to meet this goal.

4 Formulation

We start by defining a general $p \times p$ weighting matrix known via its state-space realization,

$$\mathbf{W}(s) = \left[\begin{array}{c|c} \mathbf{A}^w & \mathbf{B}^w \\ \hline \mathbf{C}^w & \mathbf{D}^w \end{array}\right]. \tag{7}$$

Then, we define a weighted model perturbation

$$\delta\mathbf{H}^w(s) = \delta\mathbf{H}(s)\mathbf{W}(s) = \left[\begin{array}{cc|c} \mathbf{A} & \mathbf{B}\mathbf{C}^w & \mathbf{B}\mathbf{D}^w \\ \mathbf{0} & \mathbf{A}^w & \mathbf{B}^w \\ \hline \delta\mathbf{C} & \mathbf{0} & \mathbf{0} \end{array}\right] \tag{8}$$

using standard algebraic manupilations. Finally, we denote as

$$\widehat{\mathbf{P}} = \begin{bmatrix} \mathbf{P}^w & \mathbf{P}_{12} \\ \mathbf{P}_{21} & \mathbf{P}_{22} \end{bmatrix} \tag{9}$$

the controllability Gramian associated to (8). A weighted norm of the model perturbation is readily computed as

$$||\delta\mathbf{H}||_w^2 = ||\delta\mathbf{H}^w||^2 = \mathrm{tr}\{\delta\mathbf{C}\,\mathbf{P}^w\,\delta\mathbf{C}^T\} = \mathrm{tr}\{\boldsymbol{\Delta}_w\,\boldsymbol{\Delta}_w^T\} = ||\mathrm{vec}(\boldsymbol{\Delta}_w)||_2^2\,, \tag{10}$$

where $\boldsymbol{\Delta}_w = \delta\mathbf{C}\,\mathbf{K}_w^T$, with $\mathbf{K}_w$ being the Cholesky factor of $\mathbf{P}^w$. This expression is formally identical to (3), just a different (weighted) controllability Gramian is used. This implies that any existing passivity enforcement scheme, such as (5) or (6), requires minimal modifications in order to incorporate (10) as the accuracy control metric, instead of (3).

A straightforward interpretation of the above weighted norm can be given for the particular case of identical weights applied to each element of $\delta\mathbf{H}(s)$, i.e., $\mathbf{W}(s) = W(s)\mathbf{I}$, where $W(s)$ is a scalar transfer function. In such case, we have

$$||\delta\mathbf{H}||_w^2 = \frac{1}{2\pi}\sum_{ik}\int_{-\infty}^{\infty} |W(j\omega)|^2\,|\delta H_{ik}(j\omega)|^2 \mathrm{d}\omega\,. \tag{11}$$

The scalar weighting factor $W(s)$ can be suitably chosen to enhance or reduce the contribution of individual frequency bands in the overall model perturbation metric. In this work, we concentrate on the reduction of out-of-band passivity violations. Therefore, the natural choice is to define $W(s)$ as a lowpass filter characterized by a cutoff frequency around the upper limit $\omega_{\max}$ of the modeling bandwidth. Among the many alternative choices, we selected for all numerical tests a standard lowpass elliptic filter, parameterized by its in-band admissible ripple r, out-of-band attenuation M, and by the two cutoff frequencies $\omega_{L,H}$ defining the transition between passing and attenuated bands. See Fig. 1(c) for a graphical illustration. This filter provides the desired cutoff performance with the lowest order, hence computational effort in the numerical evaluation of the weighted Gramian $\mathbf{P}^w$.

5 Examples

We apply the proposed weighted norm to enforce the passivity of the VHDM connector model discussed in Section 3. Application of the elliptic filter of Fig. 1(c) leads to the results depicted in Fig. 1(d). Although the filter attenuation is only 20 dB, significant improvements are obtained with respect to a standard error control. The new model responses are now undistinguishable from the raw data, yet global passivity has been enforced.

Our second example is taken from a single-chip RF transceiver structure (courtesy of Infineon). For EDGE/UMTS applications, modern transceivers have separate RF downconversion paths for receiving the different frequency bands. During system operation, only one path is active, with the other paths being switched off in a sleep mode. The example

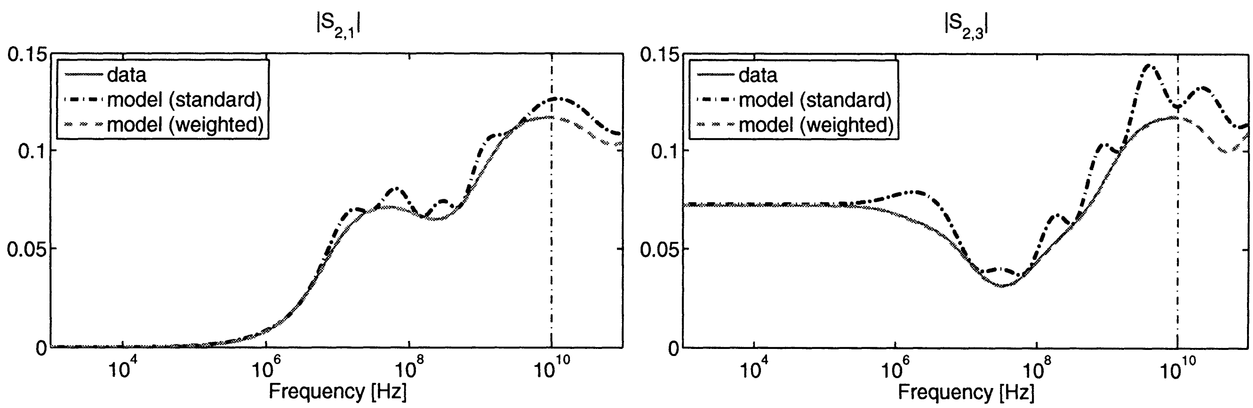

Figure 2: Macromodel of a circuit block in a RF transceiver. Passive models obtained with standard and frequency-weighted error control are compared to raw scattering data for two selected responses. Frequency weighting is here performed via an elliptic filter $W(s)$ with 40 dB attenuation in the stopband.

we consider is a 19-port small-signal rational macromodel of a mixer in switched-off mode, which provides a broadband parasitic block possibly causing Signal Integrity problems to the active RF signal. A first VF-generated model shows major passivity violations up to 8×10^3 GHz, although the modeling bandwidth is only up to 10 GHz. Figure 2 shows the results of standard and frequency-weighted passivity enforcement schemes. The proposed weighted scheme clearly provides superior performance.

In conclusion, the proposed weighting scheme provides a very effective solution to a well-known weak point of state-of-the-art passivity enforcement algorithms. With suitable weighting factors, the model perturbation is reduced to a minimum in the modeling bandwidth, regardless of possibly large out-of-band passivity violations. In addition, this technique can be combined with the relative weighting scheme proposed in [10] to fine-tune the model accuracy. Therefore, this approach represents a significant step towards the quality improvement of broadband passive macromodels.

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