# Introduction of Randomness in Deterministic Descriptions of Reverberation Chambers

Ramiro Serra* Flavio Canavero*

*Abstract* — **The main purpose of this work is to present some simple yet complete models that are able to describe the statistical behavior of Reverberation Chambers (RC) as well as the influence of their main factors and their overall functioning. We start by building a simplified two-dimensional model which is able to reproduce the statistical behavior of real RCs without renouncing the advantages of deterministic descriptions. The statistical properties of the fields inside the cavity are introduced either by varying the width or the relative dielectric constant of a perturbing lossless layer inside a 2D cavity. The field statistics derived with this process are shown to correspond to the statistics found in properly functioning RCs in the overmoded regime. With the use of a suitable coordinate transformation, we assume our non-homogeneous rectangular cavity to be a virtual mathematical domain, and find an analogous homogeneous non-rectangular cavity as our physical domain. With this simple process we are able to link our 2D "dielectric" stirrer with actual geometrical discontinuities. By means of this procedure, a novel design for RCs appears as feasible.**

## 1 INTRODUCTION

Reverberation Chambers (RC) are gaining significant confidence in use for radiated emissions and immunity measurements. RC users need to fully understand its working principles in order to correctly interpret the measurement results and to optimize the performance for various measurement tasks.

Reverberation chamber knowledge to date results from a partial juxtaposition of four different approaches: the deterministic models, the statistical models, the empirical techniques, and the computer/numerical methods. It is not possible to leave one of these approaches behind, as each one of them behaves as a non-exhaustive, non-excluding part of a RC description. Furthermore, they mutually collaborate to give fairly successful answers in fields where the other one fails, and viceversa. Therefore, there is an obvious gap which makes us change our methodology depending on what kind of result we seek. In this sense it is to point out that a statistical description is meaningful only if the chamber is working in an overmoded regime and only on special chamber geometries. On the other hand, a deterministic model's success is intimately linked to the specificity of the chamber geometry and it compels us to pay no attention to the mode stirring process, which is an essential constituent of the RC performance. Consequently, a call for filling this gap and linking the two approximations is needed. This necessity is supported by the aim of having a better understanding, to manage a simpler yet complete model, and to reduce up to a reasonable minimum the empirical techniques.

*The authors are with the Dipartimento di Elettronica, Politecnico di Torino, C.so Duca degli Abruzzi 24, 10129 Torino, Italy, tel.: +39 011 5644000, fax: +39 011 5644099. E-mails: ramiro.serra@polito.it - flavio.canavero@polito.it

## 2 THE 1D RC MODEL

An attempt at filling this gap was presented in [1], where a one-dimensional RC model was shown to have a statistical behavior equal to real RCs. It simulates the electromagnetic field distribution inside a theoretical vacuum-filled 1D segment with the presence of a 1D "stirrer" (a perturbing lossless dielectric layer) and of losses in the walls. In this model, the statistically uniform field can be obtained by means of different stirring processes, each one of them finding a strong analogy with real RCs.

### 2.1 Size stirring and dielectric stirring results

For reference, we show the results of 500 independent calculations of different stirrer sizes (e.g., "size stirring") and different relative dielectric constants (e.g., "dielectric stirring"). The electromagnetic field was calculated at a fixed position inside the test volume. Histograms of the real and imaginary parts of the electric field with their fitted normal distributions for every stirring process are shown in Fig. 1.

The Anderson-Darling Normality Test (A-D) [2] was applied to these values to determine whether the data of the sample are nonnormal. The results largely justify the hypothesis that they follow the normal distribution as reported in [1], thus reproducing the literature findings, i.e., that the field-components distributions match the probability density functions ([3], [4]).

## 3 THE 2D RC MODEL

We will next start the description of the 2D RC inspired by [1] and conceived as a 2D simple extension of the basic configuration of its analogous 1D

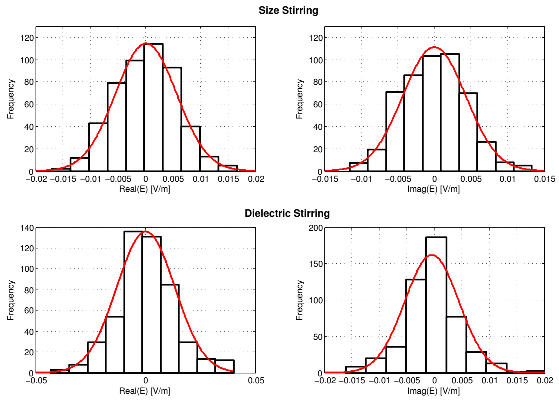


Figure 1: Probability distribution of the real and imaginary parts of the electric field measured at a given position after 500 iterations of the stirrer size and of the relative dielectric constant with their fitted normal distributions.

model, and afterwards some statistical results will be shown.

### 3.1 The 2D Cavity Model

The description of our chamber (see Fig. 2 for a schematic diagram) starts as a 2D cavity including a layer of a perturbing medium with relative dielectric constant $\kappa$ inside the vacuum-filled space and a continuous-wave source located at $(x_0, y_0)$. The dimensions of the chamber are $a$ and $b$.

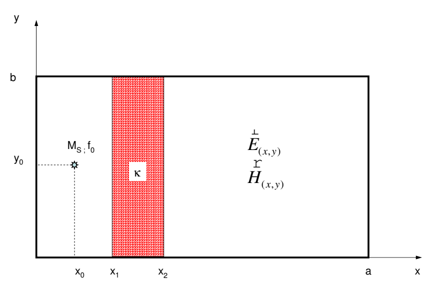


Figure 2: Definition of the two-dimensional cavity under study.

The electromagnetic field inside this chamber obeys the wave equation:

$$\nabla^2 \vec{E}(x,y) + \kappa(x,y) k^2 \vec{E}(x,y) = g(x,y)\,, \qquad (1)$$

where

$$\kappa(x,y) = \begin{cases} 1 & 0 \le x < x_1 \text{ and } x_2 \le x \le a \\ \kappa & x_1 \le x < x_2 \end{cases}$$

and $k = \omega\sqrt{\mu\epsilon}$ is the free-space wavenumber; $\mu, \epsilon$ are the free-space permeability and permittivity, respectively. The $e^{-j\omega t}$ time dependence is suppressed. One possible set of eigensolutions ([7], [8]) is:

$$\vec{E}_{mn}(x,y) = A_{mn}\psi_m(x) sin\left(\frac{n\pi}{b}y\right)\,, \qquad (2)$$

where $A_{mn}$ and $\psi_m(x)$ are found by means of the process described in [1] and subindexes $m, n$ are the modal indexes. The proposed solution automatically satisfies the boundary conditions at the walls of the chamber at $x = 0, a$ and $y = 0, b$.

Losses in the walls are introduced according to the method described in [7] and shown in [1].

Figure 3 shows the modification of the field distribution inside the chamber, due to a change of the $\kappa$ value in the dielectric region, assumed to maintain a constant ratio $t/a = 0.1$, being $t = x_2 - x_1$ the width of the stirrer. From the observation of Fig. 3, where the real part of the electric field inside the chamber for $\kappa = 1$ (i.e., absence of dielectric) and $\kappa = 1.2$, it is evident that the main effect of the dielectric layer inside the chamber is to appreciably change the field distribution inside the "Test Volume" region. Thus, an analogy with the stirrer in real RCs can be established. This result matches what was shown in [1].

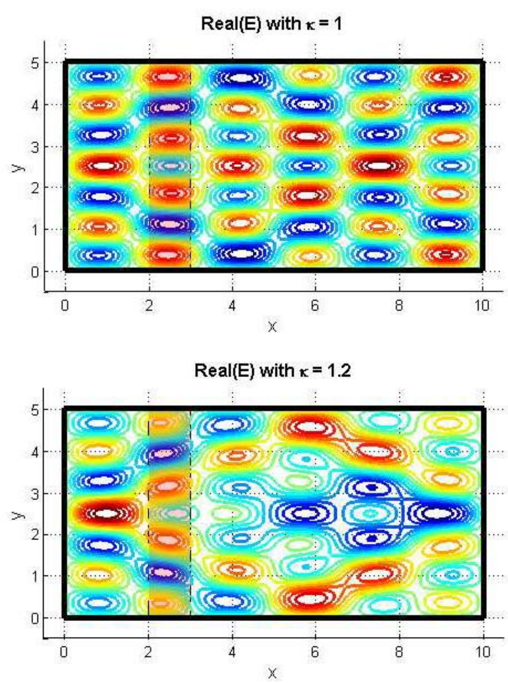


Figure 3: The real part of electric field $E$ inside the 2D cavity model for $\kappa = 1$ (top panel) and $\kappa = 1.2$ (bottom panel).

### 3.2 The 2D Reverberation Chamber Model

Up to now, we have not been solving an RC but a cavity resonator. As shown previously, and in coherence with ergodicity, we can reproduce the statistical behavior of an RC if we randomly vary some selected parameters (in this work, the stirrer size and the stirrer relative dielectric constant).

The results of 500 independent calculations of the electromagnetic field at a fixed measurement position inside the test volume are shown in Fig. 4, that presents the histograms of the real and imaginary parts of the electric field with their fitted normal distributions for every stirring process.

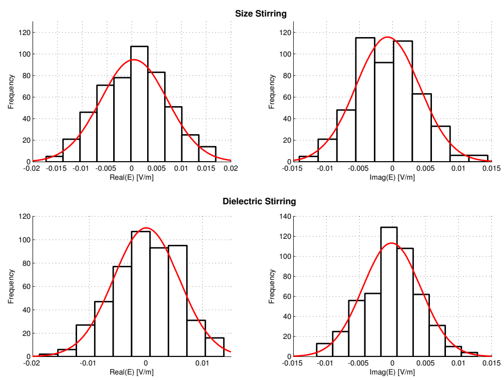


Figure 4: Probability distribution of the real and imaginary parts of the electric field calculated at position $(x, y) = (8.5, 2.5)$ m after 500 iterations for size stirring and dielectric stirring with their fitted normal distributions.

The A-D Test again justified the hypothesis of normality giving p-values of 0.462 and 0.092 for the real and imaginary part of the electric field for size stirring and 0.069 and 0.058 for dielectric stirring, respectively. Even though the p-value of 0.462 is largely better than the other three, all of them are above the commonly accepted level of significance of 0.05.

### 3.3 Validity of the Physical Analogy of the Dielectric Stirrers

A question arises: what is the link and the correlation between real stirrers and our virtual "dielectric" stirrers? Different interpretations could be given to explain the influence of our dielectric stirrer inside the chamber:

The effect of placing a material with a considerable dielectric constant inside an electric field as in our cavity is mainly to change the configuration of the eigenvalues. It is analogous to adding a capacitance to a circuit, or a reactive load into a transmission line. As we are assuming that no losses are into the dielectric, what happens is that this new material inserted has stored some energy from the system. This would result in a particular field configuration. It is then intuitively easy to imagine that a little change in size or in its relative dielectric constant, could generate a fairly different distribution of the field. The present analogy could be justified by the fact that usually, "complex-shaped" stirrers outperforms simpler ones (i.e., see [5], [6]).

Nevertheless, it is true that real stirrers do not "store" energy and so the problem remains unanswered in a closed manner. A conformal mapping technique will be presented in the next section to solve this problem.

## 4 CONFORMAL MAPPING

The technique of conformal mapping will be applied to (1), resulting in a successful strategy for the study of our two-dimensional RC model. This process will help us identify one irregularly-shaped homogeneously-filled cavity, that maps into the 2D cavity of Section 3.1.

### 4.1 Basic Theory

Let us consider the inhomogeneous Helmholtz equation in two dimensions:

$$\left[\nabla^2_{xy} + k^2\right] f(x, y) = g(x, y)\,. \qquad (3)$$

Let us apply the conformal mapping $w = w(z)$ between the complex variable $z = x + jy$ and $w = u + jv$ to (3) as was shown in [9] to obtain a slightly more complicated equation in the transformed domain:

$$\left[\nabla^2_{uv} + k^2/J_{uvxy}\right] F(u, v) = G(u, v)\,, \qquad (4)$$

where $\nabla^2_{uv} = \partial^2/\partial u^2 + \partial^2/\partial v^2$ and $J_{uvxy}$ is the Jacobian of the transformation $(x, y) \Leftrightarrow (u, v)$ associated with the conformal mapping

$$J_{uvxy} = \begin{vmatrix} \partial u/\partial x & \partial u/\partial y \\ \partial v/\partial x & \partial v/\partial y \end{vmatrix}\,. \qquad (5)$$

Equation (4) corresponds to an inhomogeneous medium situation, where the wavenumber $k$ would be a function of the coordinates.

### 4.2 Changing Domains

The cavity described in Section 3.1 is indeed of the same class of (4) if we assume that $\kappa(u, v) = 1/J_{uvxy}$. Let us now suppose it as if it were a certain cavity in the transformed domain and find out a geometry in the physical domain that maps into it. In other words, we would like to find out a conformal mapping

$$w = w(z) \Leftrightarrow \{u = u(x, y); v = v(x, y)\}\ , \qquad (6)$$

whose Jacobian corresponds with the inverse of the $\kappa(u, v)$ function. One possible solution was found to use the following transformation,

$$\begin{aligned} u(x, y) &= x \\ v(x, y) &= \begin{cases} y + b/2 & 0 \le x < x_1 \\ y/\kappa + b/2 & x_1 \le x < x_2 \\ y + b/2 & x_2 \le x \le a \end{cases} \end{aligned} \qquad (7)$$

in the cavity depicted in Figure 5. It is quite straightforward to see that the conformal mapping in 7 will have a Jacobian as the one we search for.

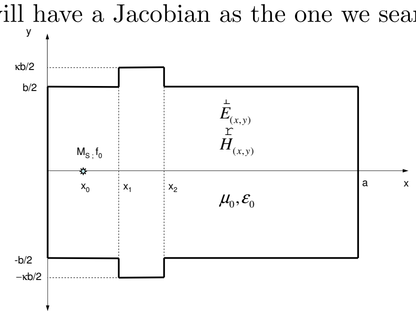


Figure 5: The cavity of Fig. 2 in the assumed physical domain.

### 4.3 Highlighting the Analogy

The cavity geometry conceived as our new physical domain has two discontinuous steps in the horizontal walls of the cavity. The difference of height of the steps and the walls is $(\kappa - 1)b/2$. Pushing our problem to some simple limits, we can make clearer the effect of our transformation and correlate the dielectric stirrer with a geometrical discontinuity.

Let us, as a first example, make $\kappa = 1$. In the physical domain, this means that no discontinuity steps are present. On the transformed domain, this means that the perturbing layer is made of vacuum. So the conformal mapping will transform a rectangular cavity into itself.

As a second example, we change instead of $\kappa$, the width of the discontinuity steps so as to make $x_1 = 0$ and $x_2 = a$. In this case, the physical domain turns again into a rectangular geometry, without discontinuities. The transformed domain results to be homogeneously filled, because the layer occupies all the space. Both of them are regular rectangles, but with a difference: their sizes. The solutions inside these two cavities are equal, but one of them is smaller than the other one and filled with a dielectric material. This is in perfect coherence of what is known in waveguides theory.

It is again clear to see that all the statistical results pointed out in Section 3.2 will be reproduced in our new physical domain.

## 5 CONCLUSIONS

This paper describes a 2D RC model inspired by [1] that presents a strong behavioral analogy with 3D RCs. In this model, the statistically uniform field can be obtained in two different ways: either by varying the size of the stirrer, or its relative dielectric constant. Both processes show reliable normality conditions. The main convenience of this model consists in providing a complete understanding of RCs, without leaving a gap in the theoretical development. With the use of the conformal mapping technique, we can interpret the role of a dielectric stirrer as a geometrical discontinuity. Changing the width of the stirrer is equivalent to changing the width of the discontinuity steps. Changing the relative dielectric constant is equal to changing the height of the discontinuity steps. From this analysis, a novel RC design arises. A chamber with a profile as Fig. 5 appears to be feasibly constructible. Further work (currently under way) should involve both the development of a correlation between the real stirrer and its 1D parameters and a 3D extension of this model.

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