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Self-similarity of the turbulences mixing with a constant in time macroscale gradient

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Abstract

In the absence of kinetic energy production, we consider that the influence of the initial conditions is characterized by the presence of an energy gradient or by the concurrency of an energy and a macroscale gradient on turbulent transport. Here, we present a similarity analysis that interprets two new results on the subject recently obtained by means of numerical experiments on shearless mixing (Tordella & Iovieno, 2005). In short, the two results are: i – The absence of the macroscale gradient is not a sufficient condition for the setting of the asymptotic Gaussian state hypothesized by Veeravalli and Warhaft (1989), where, regardless of the existence of velocity variance distributions, turbulent transport is mainly diffusive and the intermittency is nearly zero up to moments of order four. In fact, it was observed that the intermittency increases with the energy gradient, with a scaling exponent of about 0.29; ii – If the macroscale gradient is present, referring to the situation where the macroscale gradient is zero but the energy gradient is not, the intermittency is higher if the energy and scale gradients are concordant and is lower if they are opposite. The similarity analysis, which is in fair agreement with the previous experiments, is based on the use of the kinetic energy and the two-point correlation equations, which contain information on the second and third order moments of the velocity fluctuations. The analysis is based on two main hypotheses: first, the decays of the turbulences being mixed are nearly equal (as suggested by the experiments), second, the pressure-velocity correlation is almost proportional to the convective transport associated to fluctuations (Yoshizawa, 2002).

The dependence of turbulence mixings on the initial conditions has been considered and documented through single-point statistics, obtained by means of direct and large eddy numerical simulations (Tordella & Iovieno, 2005, Iovieno & Tordella, 2002). The simulations were carried out using of a new technique for the parallel dealised pseudospectral integration of the Navier-Stokes equations (Iovieno et al., 2001). In all the shearless mixing experiments a self-similar state appears to exist. The statistical distributions of orders higher than the second maintain features that depend on the initial values of the ratio of energy, $\mathcal{E} = E_1/E_2$, of the ratio of macroscale, $\mathcal{L} = \ell_1/\ell_2$, and on the sign of $\nabla\ell$. Here and in the following subscript 1 and 2 refer to the high/low energy regions respectively. Independently of the values of the control parameters and the concurrency, or lack of it, of the energy and scale gradients, a set of common properties exists for all the studied mixings. First, the statistical distributions become self-similar after nearly a decay of three time units. Second, in the self-similarity region of the decay, the lateral spreading rate is on averege close to 0.15. Third, the kinetic energy distribution has a common shape (see, (12)). Fourth, all the mixings – including the mixing with $\mathcal{L} = 1$ – are very intermittent, as the skewnes S and kurtosis K distributions show, see fig.s 1b and 2.

To carry out the similarity analysis,we considered the second moment equations for the velocity fluctuations (u, in the inhomogeneous direction x, v_1, v_2 in the plane normal to x),

$$\partial_t\overline{u^2} + \partial_x\overline{u^3} = -2\rho^{-1}\partial_x\overline{pu} + 2\rho^{-1}\overline{p\partial_x u} - 2\varepsilon_u + \nu\partial_x^2\overline{u^2} \quad (1)$$

$$\partial_t\overline{v_1^2} + \partial_x\overline{v_1^2 u} = 2\rho^{-1}\overline{p\partial_{y_1}v_1} - 2\varepsilon_{v_1} + \nu\partial_x^2\overline{v_1^2} \quad (2)$$

$$\partial_t\overline{v_2^2} + \partial_x\overline{v_2^2 u} = 2\rho^{-1}\overline{p\partial_{y_2}v_2} - 2\varepsilon_{v_2} + \nu\partial_x^2\overline{v_2^2} \quad (3)$$

The two mixed turbulences decay in a similar way, as shown by the rical simulations (Tordella & Iovieno, 2005). Thus, in the decay laws:

$$E_1(t) = A_1(t+t_0)^{-n_1}, \quad E_2(t) = A_2(t+t_0)^{-n_2}$$

the exponents n_1, n_2 are close each other, which assures the constancy of $\mathcal{E}$ with respect to the time variable. Here, we suppose $n_1 = n_2 = n = 1$, a value which corresponds to $R_\lambda \gg 1$ (Batchelor & Townsend, 1948).

In the absence of energy production, the pressure-velocity correlation has been shown to be approximately proportional to the convective fluctuation transport (Yoshizawa, 1982, 2002)

$$-\overline{pu} = a\rho\frac{\overline{u^3} + 2\overline{v_1^2 u}}{2}, \quad a \approx 0.10,$$

moreover all experiments show no appreciable difference in the second order moments in the mixing, i.e. $\overline{u^2} \simeq \overline{v_i^2}$, so that $\overline{u^3} - \overline{v_1^2 u} \simeq 2\rho^{-1}\overline{p\partial_x u}$ and consequently

$$-\rho^{-1}\overline{pu} = \alpha\overline{u^3}, \quad \alpha = \frac{3a}{1+2a} \approx 0.25. \quad (4)$$

In this initial value problem, the moment distributions are determined by the coordinates x, t, and by the energy E and the macroscale ℓ of the two mixing turbulences. Thus, through dimensional analysis

$$\overline{u^k} = E_1^{\frac{k}{2}} \varphi_{u^k}(\eta; R_{\ell_1}, \vartheta_1, \mathcal{E}, \mathcal{L}) \;\; \forall k, \;\; \varepsilon_u = E_1^{\frac{3}{2}} \ell_1^{-1} \varphi_{\varepsilon_u}(\eta; R_{\ell_1}, \vartheta_1, \mathcal{E}, \mathcal{L}), \quad (5)$$

where $\eta = x/\Delta(t)$, $\Delta(t)$ is the mixing layer thickness, $R_{\ell_1} = E_1^{\frac{1}{2}}(t)\ell_1(t)/\nu$ is the Reynolds number relevant to the high energy turbulence, $\vartheta_1 = tE_1^{\frac{1}{2}}(t)/\ell_1(t)$ is the dimensionless time scale of the flow and $\mathcal{E} = E_1(t)/E_2(t)$, $\mathcal{L} = \ell_1(t)/\ell_2(t)$. It should be noticed that, if $n = 1$, $\mathcal{E}$, $\mathcal{L}$, $\vartheta_1 = n/f(R_{\lambda_1})$ and $R_{\ell_1} \propto t^{1-n}$ are constantin time. The mixing is then driven by constant scale and energy gradients. By inserting relation (5) in (1), it is possible to deduce that $\Delta(t) \propto \ell_1(t)$. By putting $\Delta(t) = \ell(t)$, one obtains:

$$-\frac{1}{2}\eta\frac{\partial \varphi_{uu}}{\partial \eta} + \frac{1}{f(R_{\lambda_1})}(1-2\alpha)\frac{\partial \varphi_{uuu}}{\partial \eta} - \frac{\nu}{Af(R_{\lambda_1})^2}\frac{\partial^2 \varphi_{uu}}{\partial \eta^2} =$$
$$= \varphi_{uu} - \frac{2}{f(R_{\lambda_1})}\varphi_{\varepsilon_u} \quad (6)$$

The right hand side of equation (6) is zero in homogeneous turbulence where decay balances dissipation. As a consequence, this rhs vanishes when $\eta \to \pm\infty$. To get indication of its behaviour within the mixing layer, we can consider two-point correlation equations. In the absence of mean velocity, the two-point correlation equations can be derived from Navier-Stokes equations as

$$\frac{\partial}{\partial t}B_{ij} + \frac{\partial}{\partial x_k}B_{ik|j} + \frac{\partial}{\partial r_k}\left(B_{i|kj} - B_{ik|j}\right) =$$
$$= -\frac{\partial}{\partial x_i}B_{pj} + \frac{\partial}{\partial r_i}B_{pj} - \frac{\partial}{\partial r_j}B_{ip} +$$
$$+\nu\left[\frac{\partial^2}{\partial x_k \partial x_k}B_{ij} + 2\frac{\partial^2}{\partial r_k \partial r_k}B_{ij} - 2\frac{\partial^2}{\partial x_k \partial r_k}B_{ij}\right] \quad (7)$$

where $B_{ij}(\mathbf{x}, \mathbf{r}, t) = \overline{u_i(\mathbf{x}, t)u_j(\mathbf{x} + \mathbf{r}, t)}$, $B_{pi}(\mathbf{x}, \mathbf{r}, t) = \overline{p(\mathbf{x}, t)u_i(\mathbf{x} + \mathbf{r}, t)}$ and $B_{ij|k}(\mathbf{x}, \mathbf{r}, t) = \overline{u_i(\mathbf{x}, t)u_j(\mathbf{x}, t)u_k(\mathbf{x} + \mathbf{r}, t)}$. Equations (1-3) can be deduced from (7) with the limit $\mathbf{r} \to 0$. In particular, when the cylindrical symmetry for the correlation distance vector $\mathbf{r}$ is used, ε_u in (1) is given by the following limit

$$\varepsilon_u = -\lim_{r_0 \to 0}\lim_{r_x \to 0} \nu \left(\frac{\partial^2}{\partial r_0^2} + \frac{1}{r_0}\frac{\partial}{\partial r_0} + \frac{\partial^2}{\partial r_x^2}\right) B_{uu}(x, r_0, r_x, t)$$

where $B_{uu}(x, r_0, r_x, t)$ is the two-point correlation of velocity component u while r_x, r_0 are the distance between two points along the mixing direction and normal to it. When the correlation distance $\mathbf{r}$ is zero, B_{uu} is equal to $\overline{u^2}$. We note that B_{uu} is a transversal correlation when $r_x \to 0$, so that the following definitions

$$\frac{1}{\lambda^2(x,t)} = -[\frac{1}{2B_{uu}}\frac{\partial^2 B_{uu}}{\partial^2 r_0}]_{(x,0,0,t)} \quad (8)$$

$$\ell(x,t) = 2\int_0^\infty \frac{B_{uu}(x, r_0, 0, t)}{B_{uu}(x, 0, 0, t)}\mathrm{d}r_0$$

for Taylor's and integral scales hold. The similarity form of B_{uu} equivalent to (5) is

$$B_{uu} = E_1^{\frac{1}{2}} \hat{\varphi}_{uu}(\eta, \xi_0, \xi_x)$$

with $\xi_0 = r_0/\ell(x,t)$, $\xi_x = r_x/\ell(x,t)$. For $\xi_x = \xi_0 = 0$ we have $\hat{\varphi}_{uu} = \varphi_{uu}$. Relation (8) leads to

$$\frac{1}{\lambda^2(x,t)} = -\frac{1}{\ell(x,t)} \frac{1}{\hat{\varphi}_{uu}(\eta,0,0)} \partial^2_{\xi_0} \hat{\varphi}_{uu}(\eta,0,0)$$

The dissipation ε_u can then be expressed as

$$\varepsilon_u = 3\nu \frac{E_1}{\ell^2} \partial^2_{\xi_0} \hat{\varphi}_{uu}(\eta,0,0) = E_1 \left(\frac{\ell}{\lambda}\right)^2 \varphi_{uu}.$$

When this equation is introduced into (5) and similarity expressions for the scales distributions are introduced, i.e. $\ell(x,t) = \ell_1(t)\lambda_I(\eta)$, $\lambda(x,t) = \ell_1(t)\lambda_T(\eta)$, we get

$$\varphi_{\varepsilon_u} = \frac{3}{R_{\ell_1}} \frac{\lambda_I^2(\eta)}{\lambda_T^2(\eta)} \varphi_{uu}$$

This allows to write the right-hand side of equation (6) as

$$\varphi_{uu} - \frac{2}{f(R_{\lambda_1})} \varphi_{\varepsilon_u} = \varphi_{uu} \left[1 - \frac{3}{f(R_{\lambda_1})} \frac{1}{R_{\ell_1}} \frac{\lambda_I^2(\eta)}{\lambda_T^2(\eta)} \right] \qquad (9)$$

In conclusion, the right-hand side of (6) dependes on the ratio between the Taylor and integral scales through the mixing, that is on the shape of the two-point correlations. When the shape does not change through the mixing, ratio λ_T/λ_I remains constant. Consequently, it remains equal to the value it assumes when $\eta \to \pm\infty$ where it vanishes. A larger value of λ_I/λ_T reduces the skewness, while a lower value tend to increase the intermittency. The comparison of the third moment distribution with experimental data could then give the ratio λ_T/λ_I through the mixing. When mixings without a gradient of interal scale are considered ($\mathcal{L} = 1$), the previously mentioned experiments (Tordella-Iovieno, 2005) suggest that the **rhs** of (6) could be modelled by means of a diffusive term, so that

$$\frac{2}{f(R_{\lambda_1})} \varphi_{\varepsilon_u} - \varphi_{uu} = \beta \frac{\partial^2 \varphi_{uu}}{\partial \eta^2} \qquad (10)$$

where β is a constant of proportionality; $\beta = 0$ corresponds to an hypothesis of local equilibrium.

In the following, by simply writing f instead of $f(R_{\lambda_1})$, integration of (6) leads to the following expression for the skewness, $S = \varphi_{uuu}/\varphi_{uu}^{3/2}$

$$S = \frac{\varphi_{uu}^{-\frac{3}{2}}}{(1-2\alpha)} \left[\frac{f}{2} \int_{-\infty}^{\eta} \eta \frac{\partial \varphi_{uu}}{\partial \eta} \mathrm{d}\eta + \left(\frac{\nu}{A_1 f} - \beta f \right) \frac{\partial \varphi_{uu}}{\partial \eta} \right] \qquad (11)$$

By representing the second moments with the fitting curve given by the experimental distributions (Veeravalli-Warhaft, 1989 and Tordella-Iovieno, 2005)

$$\varphi_{uu} = \frac{1+\mathcal{E}^{-1}}{2} - \frac{1-\mathcal{E}^{-1}}{2} \mathrm{erf}(\eta) \qquad (12)$$

one obtains

$$S = \frac{1-\mathcal{E}^{-1}}{\sqrt{\pi}} \frac{f}{4(1-2\alpha)} \left(1 - \frac{4\nu}{A_1 f^2} + 4\beta\right) \mathrm{e}^{-\eta^2} \times \left[\frac{1+\mathcal{E}^{-1}}{2} - \frac{1-\mathcal{E}^{-1}}{2}\mathrm{erf}(\eta)\right]^{-\frac{3}{2}} \tag{13}$$

Figure 1 shows the good agreement of the modelled variance and skewness distributions (relations 12 and 13) with the experimental data. The intermittency parameter associated to the lateral penetration of the mixing is compared in fig.2 with the values given by the present similarity law. It can be observed that the scaling exponent deduced from the experiment (Tordella & Iovieno, 2005), which is approximately equal to 0.29, is qualitatively represented. It should be noticed that such scaling is independent from the energy-dissipation model (10), because the model coefficient β does not influence the shape of the skewness distribution (13) and does not modify the position of the skewness maximum, which appears to be a function of the energy ratio $\mathcal{E}$ only. However, β determines the value of the maximum of the skewness distribution, and $\beta \approx 0.08$ gives the best fit with experimental data by Tordella-Iovieno (2005). The other parameters that appear in figures 1 and 2 are $\alpha = 0.25$ (see equation 4) and $f(R_{\lambda_1}) = 0.65$. This last has been obtained for $Re_{\lambda_1} = 45$ from experimental and direct numerical simulation data collected by Sreenivasan (1998). The different penetration observed when $\mathcal{L} \neq 1$ could allow to evaluate the right-hand side of equation (6) and then, through (9), the distribution through the mixing of the ratio between the integral scale and the Taylor microscale, which defines the shape of the two-point correlations.

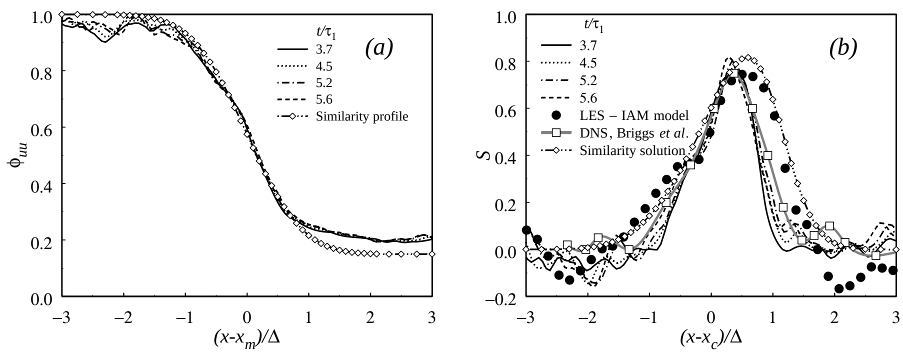

Figure 1: Normalized energy and skewness distributions; $\mathcal{E} = 6.7$ and $\mathcal{L} = 1$.

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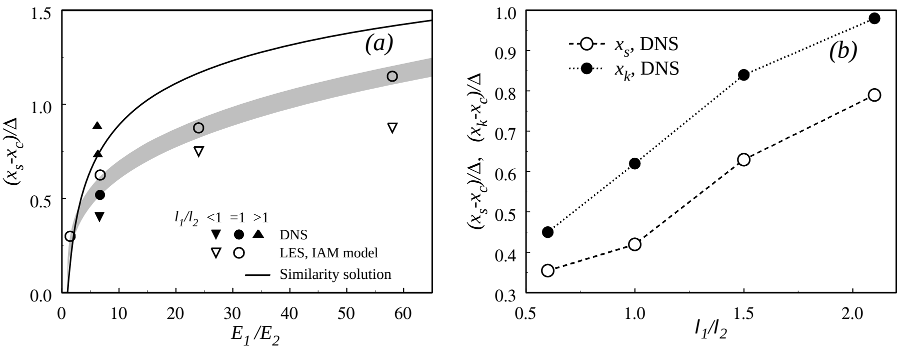

Figure 2: Position of the maximum of the skewness S and kurtosis K distributions as a function of the initial ratio of energy, part (a), and the initial ratio of integral scale, part (b): x_s and x_k are the positions of the maximum of $S(x)$ and $K(x)$, respectively.

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