Accuracy of Propagation Modeling on Transmission Lines

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Abstract: This paper investigates the accuracy of the Nonuniform Multiconductor Transmission Lines (NMTL) equations when they are used to reproduce voltages and currents on structures made of nonparallel conductors. A commercial software based on the NEC2 kernel is used as a virtual measurement tool to validate the outcome of the NMTL model. Three factors that affect the accuracy of the NMTL equations are evidenced, namely radiation, nonuniformity of the cross-section, and asimmetry between different conductors. The results show that the NMTL equations constitute indeed a quite robust model for a wide range of structures. Therefore, it is confirmed that the NMTL equations can be used to simulate the electrical behavior of many interconnects of practical interest.

INTRODUCTION

The Multiconductor Transmission Lines (MTL) model is widely used to represent structures made of parallel conductors with a translation-invariant cross-section. The accuracy of this model is well trusted, because many numerical and experimental validations can be found in the literature. Also, the solution of the MTL equations is a simple task both in the frequency and in the time domain (see [1] and references therein).

An extension of the MTL model is provided by the Nonuniform Multiconductor Transmission Lines (NMTL) equations. These equations introduce a longitudinal variation in the per-unit-length parameters, thus allowing to represent structures of conductors with a nonuniform cross-section. The cross-sectional parameters are evaluated at fixed locations around the line, by assuming that the dominant mode of propagation is the quasi-TEM mode. Implicitely, this imposes a bound on the longitudinal variations of the cross-section, which must be smooth and "not too fast". However, it is not yet clear how fast can be these variations in order to insure the validity of the transmission line formalism. The purpose of this paper is to investigate which are the factors that affect the accuracy of the NMTL model.

The validity of a model can only be assessed through direct measurement or with a more refined model. In this paper, we employ the latter solution, and we use a NEC2-based software [2] as a virtual measurement tool. This well-known code solves the integral equations for the currents induced on systems of wires by incident fields or by lumped sources. This approach provides a highly accurate and versatile tool for electromagnetic analysis. The possibility of including lumped loads makes it an ideal tool for our purpose. Some considerations on the low-frequency limitations of the NEC code can be found in the forthcoming sections.

In this paper, we consider structures made of two nonparallel straight wires in free space placed above a ground plane. We chose these simple structures for a number of reasons. First, we are mainly interested in the effect of longitudinal nonuniformities rather than on the effect of nonhomogeneous dielectrics. Second, the NEC kernel is ideally suited for structures made of wires. Finally, the simplicity of the considered structures will allow a straightforward interpretation of the deviations in the NMTL solution with respect to the NEC solution, in terms of electromagnetic (radiation) and geometric (divergence angle and asymmetry) factors. It will be shown that the geometric factors become critical only when the deviations from the uniform case are quite large.

GEOMETRY DESCRIPTION

We will analyze in this paper two different classes of structures, which will be identified as Type I and Type II, respectively. They are depicted in Fig. 1. In both cases the wires are parallel to a perfectly conducting ground plane. The distance from the center of each wire and the ground plane is $h = 0.6$ mm, while the radius is $r = 0.1$ mm. The separation of the two wires at the left section $z = 0$ will always be equal to $s_0 = 0.6$ mm, while the separation s_1 at the right section $z = \mathcal{L}$ will be determined by the divergence angle ϑ. The latter is taken as a parameter and will be allowed to vary in a wide range, from 0 up to 45 degrees. The length of the two structures is defined in the figure and is equal to $\mathcal{L} = 12.8$ cm. It should be noted that $\mathcal{L}$ is not equal to the length of each wire except when the divergence angle is vanishing.

The frequencies at which the structures will be analyzed range from 50 MHz up to 5 GHz. This range includes four

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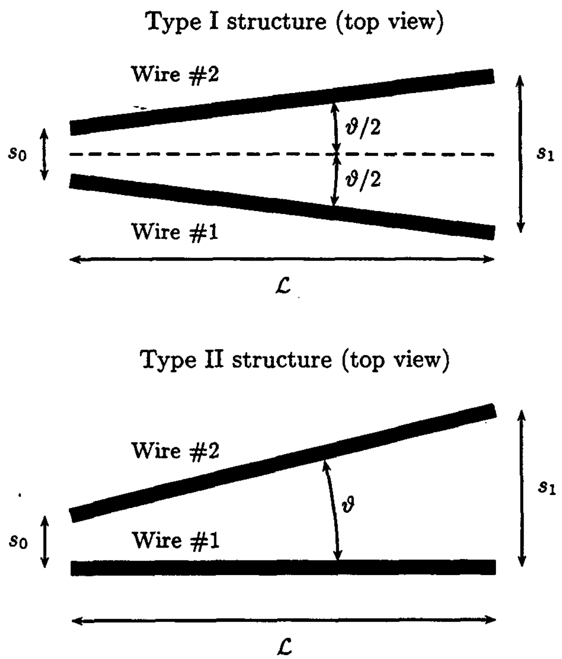

Figure 1: The structures analyzed in this paper are made of nonparallel straight wires above a perfectly conducting ground plane. They can be simmetric (top panel) or asimmetric (bottom panel).

resonances, as the frequency corresponding to $\mathcal{L} = 2\lambda$ is equal to $f_{2\lambda} = 4.69$ GHz. It should be noted that in this range of frequencies the NMTL equations are clearly not applicable when the divergence angle is large. For instance, when $\vartheta = 45°$, the separation s_1 for the Type II structure is equal to the length $\mathcal{L}$ of the line, and consequently the cross section is far from being small with respect to the wavelength. Indeed, one of the purposes of this paper is to determine a bound on the divergence angle ϑ that, when respected, will guarantee the validity of the NMTL equations.

Each wire is terminated by equal series resistances $R = 50\ \Omega$. A complete set of simulations were performed also by setting the termination resistances to $R = 5\ \Omega$ and $R = 5\ k\Omega$, but the results did not show significant deviations and are therefore not reported here. As the diagonal entries of the characteristic impedance matrix are approximately equal to $Z_{ii} = 149\ \Omega$, this load condition can be regarded as an intermediate impedance level with a moderate mismatch. The excitation of the structures is provided by two additional series voltage sources V_{S1} and V_{S2} at the left termination of each wire. The subscripts refer to wire number 1 or 2, with reference to the labeling shown in Fig. 1. The values of the generators will be selected to excite a common mode ($V_{S1} = V_{S2} = 1$ V), a differential mode ($V_{S1} = -V_{S2} = 1$ V), or a single conductor ($V_{S1} = 1$ V, $V_{S2} = 0$ V and vice-versa). It should be kept in mind that for the Type II structure, due to the asimmetry in the cross-section, the common mode and the differential mode are not independent. Therefore, even if the generators are set up to excite only one of these two modes, the current distributions will not be those of a pure common and differential mode.

The NMTL equations representing the structures under investigation are

$$\frac{\partial}{\partial z}\mathbf{v}(z,t) + \mathbf{L}(z)\frac{\partial}{\partial t}\mathbf{i}(z,t) = 0, \quad (1)$$

$$\frac{\partial}{\partial z}\mathbf{i}(z,t) + \mathbf{C}(z)\frac{\partial}{\partial t}\mathbf{v}(z,t) = 0. \quad (2)$$

The per-unit-length matrices $\mathbf{L}(z)$ and $\mathbf{C}(z)$ are determined et each fixed z by solving a static field problem in the cross-sectional plane [1]. It should be noted that the definition of the cross-sectional plane is different for Type I and Type II structures, as the longitudinal coordinate coincides with the simmetry axis in the first case and with one of the conductors in the second case. However, if we neglect the fact that the cross-section of each wire is not exactely round due to the skew angle, it can be easily shown that the per-unit-length parameters depend only on the separation between the wires at a fixed z. Consequently, if the divergence angle is chosen so that the separation s_1 is the same for Type I and Type II structures, their representation with the NMTL equations will be exactely the same. Therefore, also the electrical solution will be the same. We can conclude that the NMTL equations cannot account for a non simmetric placement of the conductors. We will show that this is indeed the most critical issue for the validity of the NMTL equations.

Once the per-unit-length parameters are evaluated for a given structure, the resulting NMTL equations are solved in the frequency-domain through standard piecewise uniform discretization [1]. The line is first subdivided into a large number N of sections, each of length $\Delta z = \mathcal{L}/N$. Then, each subsection i is approximated by a uniform MTL characterized by the per-unit-length parameters evaluated in its middle cross-section. The chain matrix $\mathbf{\Phi}_i$ is evaluated for each subsection, and all the subsections are finally connected through cascading. The number of sections is set to $N = 100$ for all the simulations shown in the paper. This quite fine subdivision insures a negligible discretization error in the overall solution.

VALIDATIONS

This section illustrates which are the main factors that affect the accuracy of the NMTL model. Three aspects will be covered, namely radiation, divergence angle, and asimmetry. We will report in all cases the currents at the terminations of the wires, by indicating

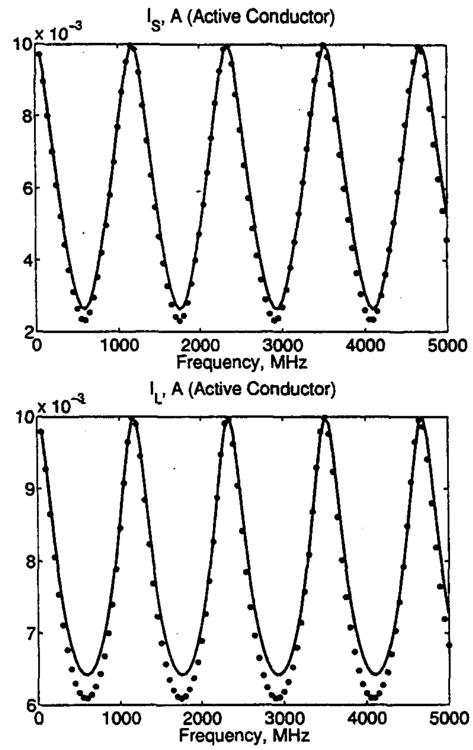

Figure 2: Uniform case $\vartheta = 0^o$ with single conductor excitation. The continuous line is the NMTL output, while the circles are obtained with NEC.

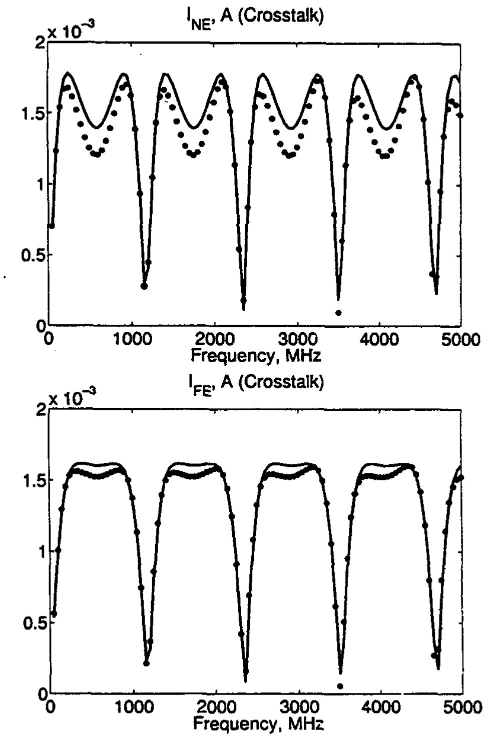

Figure 2: continued

- I_S: Current in Wire #1 at $z = 0$,
- I_L: Current in Wire #1 at $z = \mathcal{L}$,
- I_{NE}: Current in Wire #2 at $z = 0$,
- I_{FE}: Current in Wire #2 at $z = \mathcal{L}$,

for all types of excitations.

Radiation

The standard NMTL equations (1)-(2) represent a lossless structure, as the surrounding medium is supposed to be free space and the conductors are perfect. Therefore, power dissipation can only occur through ohmic losses at the line terminations. However, since each of the conductors carries a nonvanishing current, the overall structure radiates a nonvanishing field, behaving like an antenna. Clearly, the amount of radiated power is small when compared to the power dissipated at the line terminations, since the currents in the conductors return through the ground plane. Nonetheless, this assumption may not be verified when the separation between conductors becomes large and transversal propagation effects start to become important.

In order to quantify the "bias" in the NMTL solution due to radiation, we consider first the uniform configuration obtained from either Type I or II structure by setting $\vartheta = 0$. The sources are set to $V_{S1} = 1$ V and $V_{S2} = 0$ V. Figure 2 compares the currents at the terminations obtained with the NMTL equations and with NEC. The location of the resonances, which are corresponding to the integer multiples of the frequency at which the line is half wavelength long, compare well. There is also a good agreement for the magnitude of the currents, especially at the frequencies corresponding to $\mathcal{L} = n\lambda/2$. On the other hand, the NMTL and NEC output show evident discrepancies at those frequencies corresponding to $\mathcal{L} = (2n+1)\lambda/4$. This behavior is indeed typical of radiation effects. To further explain this we report in Fig. 3 the current distribution along the active conductor (Wire #1) at the frequencies corresponding to $\lambda/4$ and $\lambda/2$. From the right panel, referring to the case $\lambda/2$, we can note that the current distribution suffers a sign change along the line. Consequently, the overall radiation is formed by a superposition of contributions that tend to cancel. This does not occur in the $\lambda/4$ case, where no cancellation takes place.

In conclusion, the radiation effects can be significant also in the uniform case. The differences between a full-wave simulation, which includes radiation losses, and a NMTL simulation can be easily explained by considering the cur-

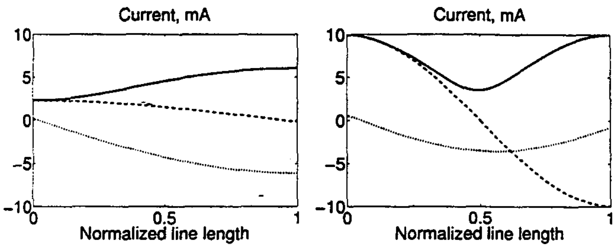

Figure 3: Current distribution along the active conductor at frequencies corresponding to $\mathcal{L} = \lambda/4$ (left panel) and $\mathcal{L} = \lambda/2$ (right panel). Magnitude (continuous line), Real part (dashed line), and Imaginary part (dotted line).

rent distribution along the wires. Unfortunately, this distribution can only be determined a posteriori once the line has been solved, being heavily dependent on the load conditions. Some attempts to include radiation effects in the transmission line equations have already been done [3], but they are limited to pairs of straight wires in free space. The Authors think that simplified models allowing an approximate inclusion of radiation losses are much needed to further extend the validity of the Transmission Lines equations.

Divergence angle

This section will focus on the effects due to a nonvanishing divergence angle ϑ. We will concentrate therefore on the Type I structure, because the effects due to asimmetry will be dealt with in the next section. Due to simmetry considerations, it is convenient to excite the Type I structure with common mode or differential mode sources. We will show that, in this situation, the NMTL equations can be used for divergence angles much larger than allowed by the usual "small cross-section" limitation, provided that a frequency axis rescaling is performed.

Figure 4 shows the currents obtained with a differential mode excitation for three different divergence angles. The plots referring to common mode excitation show the same features and are not reported here. The figure shows that the main discrepancies between the NMTL model and NEC, introduced by an increasing divergence angle, consist of a frequency shift. The radiation effects were already analyzed in the preceding section and will not be further discussed here. This frequency shift is due to the fact that the currents travel along each conductor, which is longer than the actual length $\mathcal{L}$ considered for the transmission line. More precisely, the exact length of each wire is

$$\mathcal{L}_w = \mathcal{L} \sec(\vartheta/2). \tag{3}$$

This is the characteristic length leading to the resonances in the load currents. To prove this we calculate the fre-

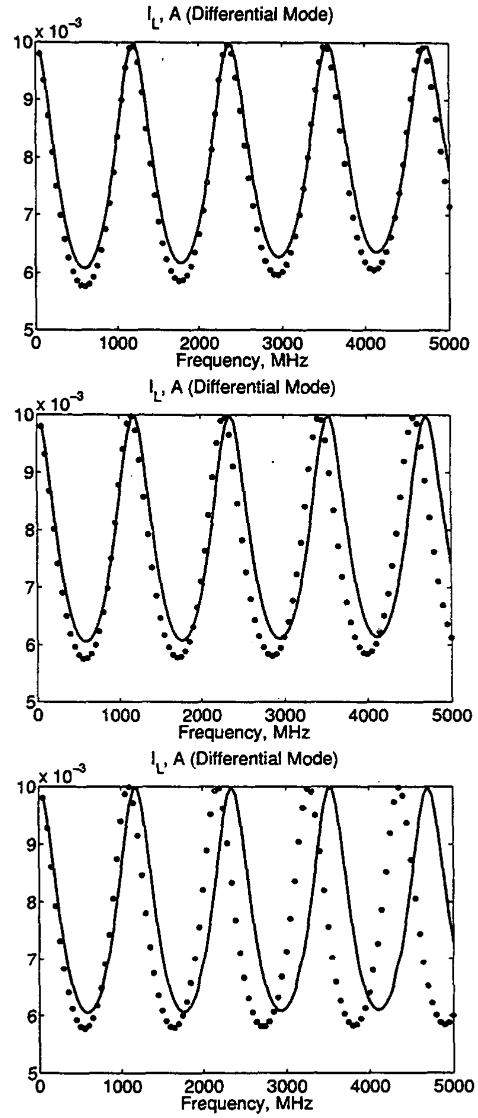

Figure 4: Type I structure with differential mode excitation. Load current obtained with the NMTL model (continuous line) and NEC (circles) for different divergence angles: $\vartheta = 9^o$ (top panel), $\vartheta = 27^o$ (middle panel), $\vartheta = 43^o$ (bottom panel).

quency $f_{2\lambda}$ corresponding to $\mathcal{L}_w = 2\lambda$ for the three different configurations reported in the plots,

$$\vartheta = 9^o \rightarrow \mathcal{L}_w = 12.84\text{cm} \rightarrow f_{2\lambda} = 4.67\text{GHz}$$
$$\vartheta = 9^o \rightarrow \mathcal{L}_w = 13.16\text{cm} \rightarrow f_{2\lambda} = 4.56\text{GHz}$$
$$\vartheta = 9^o \rightarrow \mathcal{L}_w = 13.76\text{cm} \rightarrow f_{2\lambda} = 4.36\text{GHz}$$

These values compare well with the locations of the 2λ-resonances in the NEC output of Fig. 4. Figure 5 reports the frequency shift from the uniform case in the 2λ res-

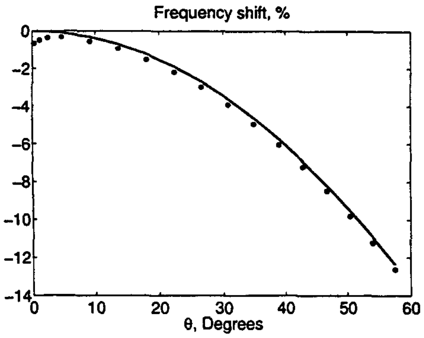

Figure 5: Percentage frequency shift for the 2λ resonance in the differential mode excitation of Type I structure. NEC output (circles) and predicted frequency shift according to the wire lengths (continuous line).

onance for a wide range of angles. The continuous line reports the predicted frequency shift according to the actual wire length in Eq. (3), while the circles are the NEC results. The match is within an acceptable tolerance even for very large divergence angles.

To further assess the validity of these considerations, we show in Fig. 6 the Near-End crosstalk currents excited by a single voltage source V_{S1}. The frequency shift is evident also from these plots. The crosstalk levels are slightly larger in the NEC output with respect to the NMTL simulation, but only for very large divergence angles. Therefore, these results lead to the conclusion that even when the cross-section suffers significant variations and is not much smaller than the wavelength, quite accurate predictions for crosstalk can be obtained by applying the NMTL model with an eventual frequency rescaling.

Simmetry Breaking

This last part is devoted to investigate the effects of simmetry breaking of the structure. To this end, we will first consider the Type II structure of Fig. 1 and excite it with a common-mode configuration. The near-end currents on the two wires for a divergence angle $\vartheta = 25^o$ are shown in Fig. 7. Excellent agreement is found on the current I_S, which refers to Wire #1. Indeed, the length of this wire is equal to the length $\mathcal{L}$ of the transmission line. Moreover, for this relatively large separation we can assume a weak coupling situation, and consider the perturbations on Wire #1 due to the currents on Wire #2 to be small. Conversely, the current I_{NE} on Wire #2 suffers a significant frequency shift due to an effective length of the wire equal to

$$\mathcal{L}_{w2} = \mathcal{L} \sec \vartheta = 1.412\,\text{cm}. \tag{4}$$

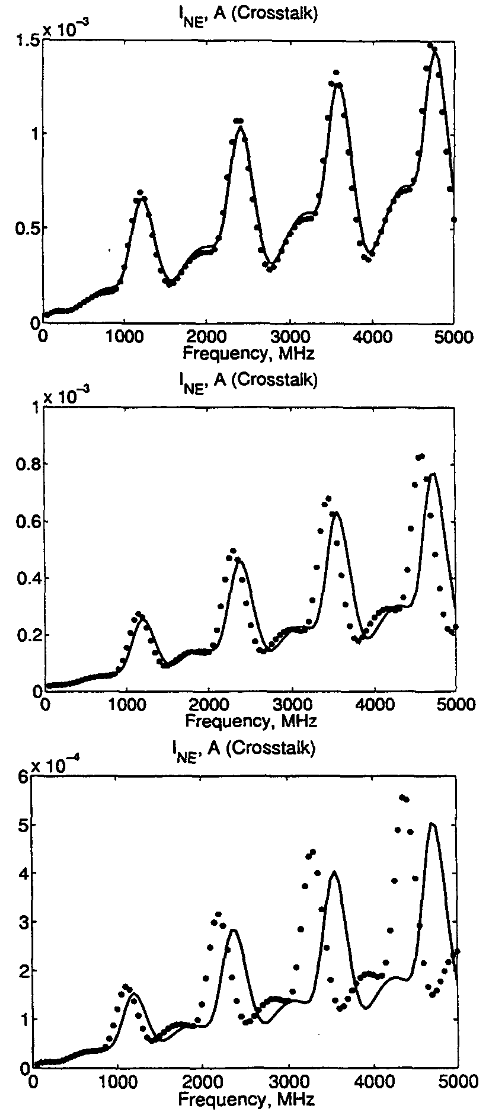

Figure 6: Type I structure with single conductor excitation. Near-End Crosstalk obtained with the NMTL model (continuous line) and NEC (circles) for different divergence angles: $\vartheta = 9^o$ (top panel), $\vartheta = 27^o$ (middle panel), $\vartheta = 43^o$ (bottom panel).

In fact, the resonance frequency corresponding to $\mathcal{L}_{w2} = 2\lambda$ is equal to $f_{2\lambda} = 4.25$ GHz. As this estimate compares well to the NEC results, it can be argued that a frequency rescaling could be performed also in this case. However, this is not possible in a general setting, when the coupling is stronger and the resonances due to different lengths of the conductors interact more efficiently (on the other hand, a stronger coupling would require the wires to be more closely spaced, thus reducing the relative difference in their

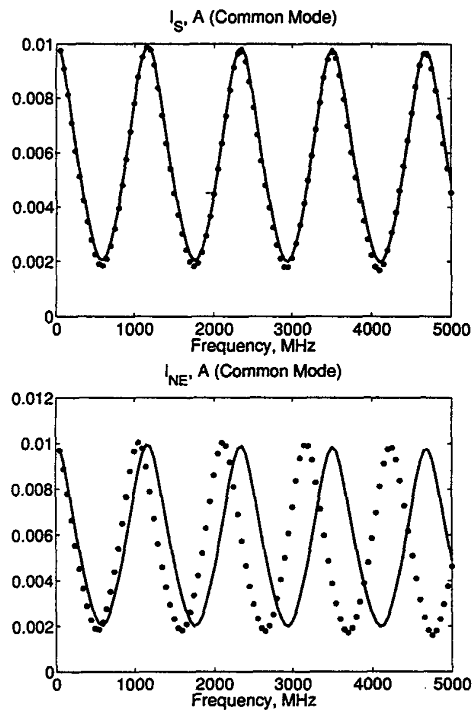

Figure 7: Type II structure with common mode excitation. Near-End currents obtained with the NMTL model (continuous line) and NEC (circles) for $\vartheta = 25^o$.

length).

Finally, we show in Fig. 8 how the crosstalk currents are significantly distorted and cannot be predicted through the NMTL model when the skew angle is too large. It should be noted that the discrepancies for very low frequencies are due to the approximations in the NEC kernel rather than to the NMTL model. It can be concluded that even when the simmetry of the structure is broken, divergence angles up to 10–15 degrees can be accepted to predict crosstalk voltages and currents through the NMTL model.

CONCLUSIONS

In this work the critical aspects of the NMTL equations for modeling structures of nonparallel conductors have been investigated. The employed validation tool is a full-wave electromagnetic code based on the NEC2 kernel. It was shown for simple test cases which is the influence of radiation effects, divergence angle of the conductors, and simmetry breaking. The general conclusion is that even when the longitudinal variations in the cross-section are significant, the validity of the NMTL equations can be assumed. Consequently, quite accurate predictions of crosstalk can be obtained with the transmission line formalism.

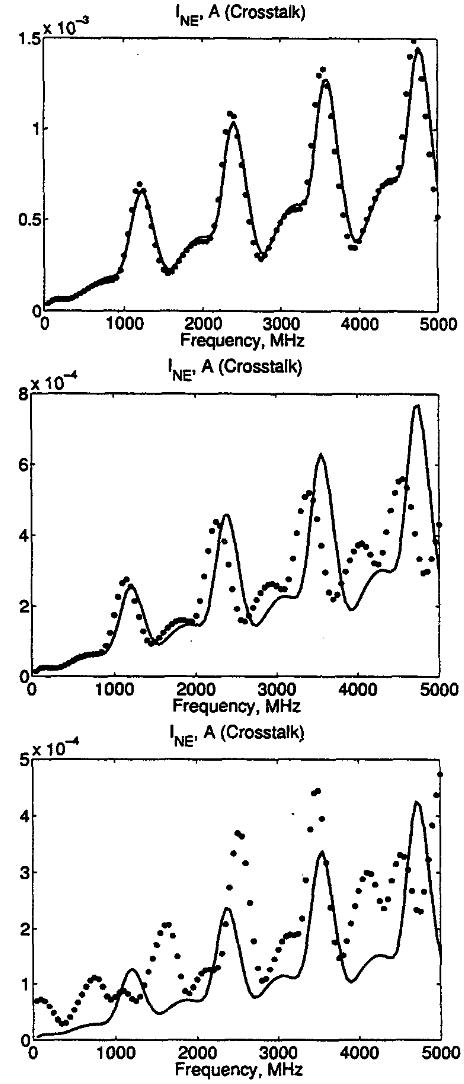

Figure 8: Type II structure with single conductor excitation. Near-End Crosstalk obtained with the NMTL model (continuous line) and NEC (circles) for different divergence angles: $\vartheta = 9^o$ (top panel), $\vartheta = 25^o$ (middle panel), and $\vartheta = 43^o$ (bottom panel).

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