Singular High-order Complete Vector Bases For the Finite Element Modeling of Wedge Problems

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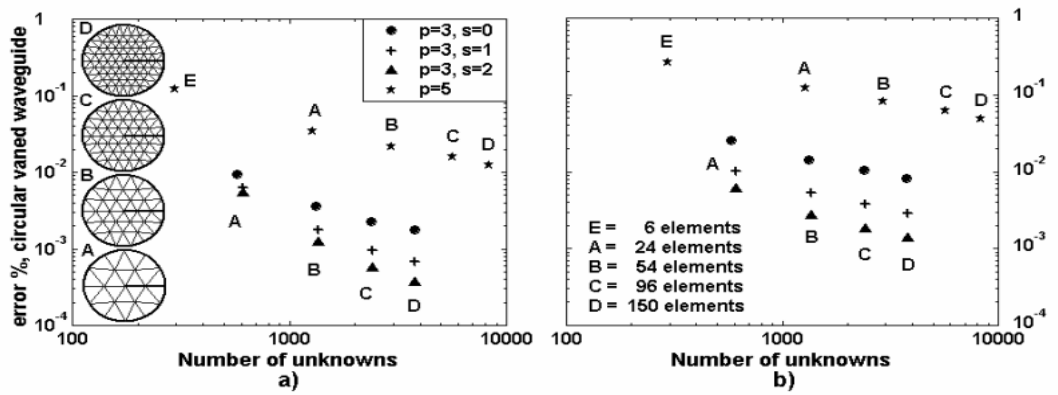

Figure 1.

This paper presents new developments and FEM applications of the singular curl-conforming vector bases given in [1]. These new vector bases incorporate the edge conditions, are complete to arbitrarily high order, and are proved to be fully compatible with the standard, high-order regular vector bases used in adjacent elements [2]. The curl-conforming singular bases guarantee tangential continuity along the edges of the elements allowing for the discontinuity of normal components, adequate modeling of the curl, and removal of spurious modes. Several sample numerical results that confirm the faster convergence of these bases on wedge problems will be presented at the Conference. For example, Fig.1 reports the percentage error in the computed square values k^2_z of the longitudinal wavenumbers versus the number of unknowns for a circular vaned waveguide of radius a. The normalized waveguide dimension is $k_0\ a$, where k_0 is the free-space wavenumber, and the homogeneous waveguide has a zero-thickness radial vane extending to its center. In Fig.1a the error is averaged over the first twenty modes, which involve four singular modes. A mode is defined to be singular when its eigenfield exhibits singular behavior. The modal fields exhibiting a ν=1/2 singularity at the edge of the vane are those of the TE_{1n} and the TM_{1n} modes, and the singular TE_{11} mode is dominant. Fig.1b shows the error averaged over the first four singular modes. These results have been obtained by using five different meshes, with meshes from A to D shown in the inset of Fig.1a. Mesh E (not shown) consists of only six curved triangular elements having as a common vertex the sharp-edge vertex. Notice that all the used meshes have six triangular elements attached to the sharp-edge vertex. The results reported with stars were obtained by using regular (non-singular) vector elements of fifth order [2]. As a matter of fact, Fig.1 shows the effects obtained by trying bases of different singular s-order (but all regular at p=3 order) only on these six sharp-edge elements.

The new numerical technique used to discretize thin or thick layers by means of singular elements having two or more of their corner nodes attached to different edges of the layer wedges will be discussed at the Conference.

REFERENCES

1. R. D. Graglia, G. Lombardi, "Singular higher-order complete vector bases for Finite Methods," due to appear in the *IEEE Trans. Antennas Propagat.*, vol. 52, July 2004.
2. R. D. Graglia, D .R. Wilton and A. F. Peterson, "Higher order interpolatory vector bases for computational electromagnetics," special issue on "Advanced Numerical Techniques in Electromagnetics," *IEEE Trans. Antennas Propagat.*, vol. 45, no. 3, pp. 329-342, Mar. 1997.