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Computation of Potentials on Curvilinear Elements.

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Abstract

This paper presents new techniques to compute with an extremely high accuracy the moment integrals of high order vector basis functions on curved patches.

1 Introduction.

Fully interpolatory higher-order divergence-conforming vector basis functions on surface patches of triangular and quadrilateral shape were defined in [1]. These basis functions become very effective in the numerical solution of the electric or the magnetic field integral equation provided one is able to compute with a good accuracy the moment integrals. The kernel of the moment integrals is the free space Green's function $\exp(-jkR)/R$, and these integrals are quite difficult to compute when the integration patch contains or is close to the observation point.

A feature of the bases derived in [1], is that they contain a factor $1/\mathcal{J}$, where $\mathcal{J}$ is the Jacobian of the curved patch, which cancels with a similar factor in the differential surface area, allowing evaluation of the moment integrals in parent dependent coordinates (ξ_1, ξ_2, ξ_3 for triangular patches; $\xi_1, \xi_2, \xi_3, \xi_4$ for quadrilateral patches). Furthermore, in parent coordinates, the bases are a product of a polynomial P of degree p times a low order vector function, a feature which facilitates determining a Taylor's series expansion of each basis function near the observation point for arbitrarily high polynomial order p.

We present two integration schemes, both based on integration in the plane tangent to the surface of the patch at the observation point, or its projection onto the patch surface. The first scheme, summarized in Section 2, is used to deal with observation points not lying on the integration surface. This integration scheme is based on a systematic (recursive) procedure able to express the integrand as the divergence of a finite vector sum, so that surface integration is easily reduced to an integration along the patch contour. The second scheme, discussed in Section 3, considers the case of observation points located on the patch surface. In this case we define a new quadrature function $q_m(n, \alpha)$ used to analytically integrate in one direction $[Q\, R^m \exp(-jkR)]$, for $m \geq -1$, where Q is a monomial of n-th degree (for a plane patch it suffices to consider $m = -1$); the moment integrals are then reduced to one-dimensional integrals along the patch contour. The functions $q_m(n, \alpha)$ satisfy recursive formulas which permit their evaluation from knowledge of $q_{-1}(0, \alpha)$.

2 Integration for observation point located outside of the surface of the patch or of the plane of the tangent patch

For sake of brevity we can consider in this Section only the case of a plane patch. We call R_t the distance from the observation point $\boldsymbol{r}'$ to the (integration) point $\boldsymbol{r}_t$ located on the plane of the patch. The subscript t is used to clarify that here we are talking of a distance from the observation point to a

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point on a plane patch (subscript t stays for "tangent" plane when dealing with curved patches). We call ρ the length of the projection of $\boldsymbol{r}_t - \boldsymbol{r}'$ onto the plane of the patch and $\hat{\boldsymbol{\rho}}$ the unit vector directed toward $\boldsymbol{r}_t$ along the projection of $\boldsymbol{r}_t - \boldsymbol{r}'$ onto the patch plane.

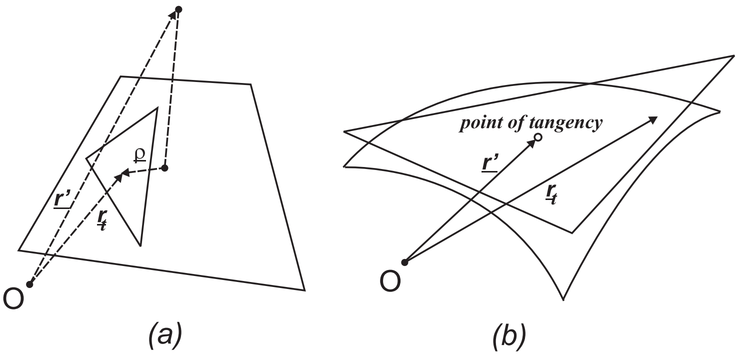

Figure 1: a) Planar triangular patch with observation point $\boldsymbol{r}'$ located outside the plane of the patch; b) planar triangular patch tangent to a curved triangular element at the observation point $\boldsymbol{r}'$.

Our derivation has been inspired after recognizing that it is very convenient to work in *parent* independent coordinates; in fact, the polynomial $P^p(\xi_1, \xi_2)$ is supposed to be known as a function of the two independent parent coordinates ξ_1, ξ_2. The normal projection of $\boldsymbol{r}'$ onto the plane patch specifies the point with parent coordinates (ξ_1', ξ_2'). Hence, the vector $\boldsymbol{\rho}$ (in the observation domain, not in parent domain) can be expressed in terms of the two unitary basis vectors $\boldsymbol{\ell}^1$, $\boldsymbol{\ell}^2$ as follows:

$$\boldsymbol{\rho} \;=\; (\xi_1 - \xi_1')\,\boldsymbol{\ell}^1 + (\xi_2 - \xi_2')\,\boldsymbol{\ell}^2$$

where $\boldsymbol{\ell}^1$, $\boldsymbol{\ell}^2$ are constant vectors which are known for a given patch. The knowledge of the unit vector $\hat{\boldsymbol{n}}$ normal to the plane patch and the introduction of the reciprocal basis vectors $\boldsymbol{h}_1 = \boldsymbol{\ell}^2 \times \hat{\boldsymbol{n}}$, $\boldsymbol{h}_2 = \hat{\boldsymbol{n}} \times \boldsymbol{\ell}^1$ allows one to write:

$$\frac{\boldsymbol{h}_1}{\mathcal{J}_t} \cdot \boldsymbol{\rho} \;=\; \xi_1 - \xi_1' \,, \qquad \frac{\boldsymbol{h}_2}{\mathcal{J}_t} \cdot \boldsymbol{\rho} \;=\; \xi_2 - \xi_2' \tag{1}$$

Use of the above dot-product expressions is the key point of our derivation.

For both the triangular and quadrilateral patch $\boldsymbol{h}_1$ ($\boldsymbol{h}_2$) is the inward vector in the plane of P normal to side 1 (2), and it is constant; $\mathcal{J}_t$ is the Jacobian relative to the plane patch P. By assuming, for a moment, that the parent-domain point (ξ_1', ξ_2') is located outside of the parent patch we are able to demonstrate the following result:

$$P^p(\xi_1, \xi_2)\,\frac{\exp(-jkR_t)}{R_t} \;=\; -\sum_{\substack{m=2 \\ m \text{ even}}}^{p+2} \boldsymbol{\nabla}_s \cdot \left[g_m \boldsymbol{F}^{p+2-m}\right] \tag{2}$$

where the differential operator $\boldsymbol{\nabla}_s$ operates on the unprimed (integration) coordinates and it is a surface operator relative to the surface of the plane patch. This result will then be estended to consider also the cases where the point (ξ_1', ξ_2') is inside or on the border of the parent patch. The nice features of expression (2) are:

1. the scalar functions g_m are computed recursively for increasing m, on the basis of the knowledge of the function g_0;
2. the vector functions $\boldsymbol{F}^{p+2-m}$ are computed recursively for increasing m (decreasing $p+2-m$), on the basis of the knowledge of the function $\boldsymbol{F}^p$.

The scalar functions g_{2n} are defined by the recursive formulas (the minus sign is important):

$$g_0(R_t) \;=\; \frac{\exp(-jkR_t)}{R_t}$$

$$g_{2n}(R_t) \;=\; -\int_{R_t} R_t\, g_{2(n-1)}\,\mathrm{d}R_t \implies \boldsymbol{\nabla}_s\left[g_{2n}\right] \;=\; -\boldsymbol{\rho}\, g_{2(n-1)} \quad \text{for } n \geq 1 \tag{3}$$

The integration constants defining each of these functions are chosen so to make the limit for $k \to 0$ finite. The technique and all the details to obtain the vector functions $\boldsymbol{F}^{p+2-m}$ will be presented at the Conference. Knowledge of all the functions g_{2n} up to a given n permits one to express $P^p \exp(-jkR_t)/R_t$ as the divergence of a known vector, for all values of p up to $p = 2(n-1)$ (for example, one can easily handle cases with $p = 6$ knowing g_{2n} up to $2n = 8$). Use of (2) yields:

$$\int_{\mathrm{P}} \frac{P^p\,(\xi_1, \xi_2)}{\mathcal{J}_t} \frac{\exp(-jkR_t)}{R_t} \,\mathrm{d}S \;=\; -\frac{1}{\mathcal{J}_t} \sum_i \hat{\boldsymbol{u}}_i \cdot \int_{\partial_i \mathrm{P}} \sum_{\substack{m=2 \\ m \text{ even}}}^{p+2} g_m \,\boldsymbol{F}^{p+2-m} \,\mathrm{d}\ell \tag{4}$$

where ∂P is the boundary of the plane patch P and quantities with subscript i are associated with the i-th edge, ∂_iP, of ∂P. The surface divergence theorem has been used to convert the surface integral to a line integral, and $\hat{\boldsymbol{u}}$ denotes the outgoing unit normal to the boundary in the plane of P.

If the parent-domain point (ξ_1', ξ_2') is located inside or on the border of the parent patch one has to break P into the two regions P_ϵ and (P-P_ϵ). P_ϵ is the intersection of P and a small disk of radius ϵ centered at (ξ_1', ξ_2'). In the limit for $\epsilon \to 0$ (principal value integral) the following result is obtained:

$$\int_{\mathrm{P}} \frac{P^p\,(\xi_1, \xi_2)}{\mathcal{J}_t} \frac{\exp(-jkR_t)}{R_t} \,\mathrm{d}S \;=\; \int_{\mathrm{P}_\epsilon} \frac{T^p_{0,0}}{\mathcal{J}_t} \frac{\exp(-jkR_t)}{R_t} \,\mathrm{d}S_\epsilon - \frac{1}{\mathcal{J}_t} \sum_i \hat{\boldsymbol{u}}_i \cdot \int_{\partial_i \mathrm{P}} \sum_{\substack{m=2 \\ m \text{ even}}}^{p+2} g_m \,\boldsymbol{F}^{p+2-m} \,\mathrm{d}\ell \tag{5}$$

where $T^p_{0,0} = P^p\,(\xi_1', \xi_2')$, and the integral over P_ϵ is intended in the limit for $\epsilon \to 0$; expressions of the integral over P_ϵ are available in the literature. The technique used to integrate on a curved patch for observation point located outside of the tangent patch will be discussed at the Conference.

3 Integration for observation point located on the surface of the patch or on the plane of the tangent patch

In this section we consider the problem of integrating on a two-dimensional patch P a polynomial function P^p of p-th degree times a scalar function of the form

$$\exp(-jkR_t)\,R_t^m\,, \quad m = -1, 0, 1, 2, \ldots \quad \text{for a plane patch}$$
$$\exp(-jkR)/R\,, \qquad \text{for a curved patch}$$

where R_t is the distance from the observation point $\boldsymbol{r}'$ to the (integration) point $\boldsymbol{r}_t$ located on the plane of the tangent patch; while R is the distance from the observation point $\boldsymbol{r}'$ to the (integration) point $\boldsymbol{r}$, both located on the patch surface.

Indeed, the subscript t is used to clarify that here we are talking of a distance from the observation point to a point on a plane patch (subscript t stays for "tangent" plane when dealing with curved patches). The reason why to consider kernels of the form $\exp(-jkR_t)\,R_t^m$ rather than just the 3D scalar Green's kernel is due to the fact that we want to derive formulas to be used to integrate on curved patches, where the curvature of the patch plays a very important role when the observation point *belongs* to the curved patch.

P^p is a polynomial function of two independent parent coordinates ξ_1, ξ_2 as well as of some dependent parent coordinates (ξ_3 for a triangular patch and ξ_3, ξ_4 for a quadrilateral patch).

Our derivation is presented here by considering, for simplicity, the case of a triangular patch; however, the results are readily estended to quadrilateral patches.

3.1 Integration on a plane patch for observation point located on the plane of the patch

In case of a triangular patch, the parent coordinates of the observation point (point of tangency) are (ξ_1', ξ_2', ξ_3'). *In the parent domain*, the point of tangency (pot) is used to subdivide the triangular patch into three subtriangles, by joining with straight lines the vertices of the triangle to the point of tangency. *In object space* we call R_1, R_2, R_3 the distances from observation point to vertex 1, 2 and 3, respectively; while the length of side 1, 2, 3 is ℓ_1, ℓ_2 and ℓ_3, respectively. Each subtriangle of the parent domain is mapped into a square domain $\{0 \leq s_i \leq 1; \quad 0 \leq \lambda_i \leq 1\}$. For each subtriangle, the point of tangency (ξ_1', ξ_2', ξ_3') is reached at $\lambda_i = 1$ (where s_i is undetermined). The Jacobian $\mathcal{J}_i$ of the i-th nonlinear transformation ($i = 1, 2, 3$) has a first order zero at $\lambda_i = 1$.

In object space, the distance R_t of a point $\boldsymbol{r}_t$ of the plane (tangent) triangle from $\boldsymbol{r}' = \boldsymbol{r}(\xi_1', \xi_2', \xi_3')$ is a function of λ and s of the form $R_t = (1-\lambda_i)\sqrt{A_i - s_i B_i + s_i^2 C_i}$, where A_i, B_i, C_i ($i = 1, 2, 3$) are known positive constants

We will see that when dealing with curved patches one need also to evaluate some surface integral by means of some quadrature rule which specifies the sampling points. When a sampling point is given, it is important to be able to evaluate the λ and s coordinates. For a given sampling point $(\xi_1^s, \xi_2^s, \xi_3^s)$ *inside* one subtriangle (as said, the sampling points on the subtriangle are chosen, for example, by the surface integration routine used to integrate the "regular" part), one can easily evaluate s_i (for i=1, 2, 3) and then the parent coordinates $(\xi_1^c, \xi_2^c, \xi_3^c)$ of any point close to (ξ_1', ξ_2', ξ_3') having the same value of s_i.

By use of the above specified nonlinear transformations, on each subtriangle of the plane (tangent) patch, it is then possible to integrate polynomial functions of (ξ_1, ξ_2, ξ_3) times some radial functions:

$$\iint P(\xi_1, \xi_2, \xi_3)\ \exp(-jkR_t)\ R_t^m\ \mathrm{d}\xi_1\ \mathrm{d}\xi_2 \qquad \text{for}\ \ m = -1, 0, 1, 2, \ldots$$

For $m = -1$, the singularity $(1/R_t)$ is in fact eliminated by the first order zero of the Jacobian of the NL-transformation. The first integration along λ is performed analytically, the remaining integration along s is performed numerically. This technique *"de facto"* reduces the surface integration to a contour integration along the oriented contour of the patch.

To perform this integration the polynomial function P^p must be represented in terms of the λ and s variables. Since P^p is of p-th degree and the parent variable vary linearly in λ, the expression of P^p as a function of λ will be a polynomial in λ of p-th degree:

$$P^p \quad = \quad a_p(s)\,\lambda^p \ + \ a_{p-1}(s)\,\lambda^{p-1} \ + \ a_{p-2}(s)\,\lambda^{p-2} \ + \ \cdots \ + \ a_0(s)$$

The algorithm to obtain the coefficients $a_r(s)$ (r =0, 1, ..., p) on the basis of the expression of ξ_i (i=1, 2, 3) in terms of λ and s variables and of $P^p(\xi_1, \xi_2, \xi_3)$ is so simple that does not need to be discussed.

3.2 The quadrature function $q_m(p, \alpha)$

The potential integrals we are interested in have the following form

$$\iint P(\xi_1, \xi_2, \xi_3)\ \exp(-jkR_t)\ R_t^m\ \mathrm{d}\xi_1\ \mathrm{d}\xi_2 \ =$$

$$= \sum_i \xi_i' \int_0^1 \mathrm{d}s_i \left[\left(\frac{\alpha}{jk}\right)^m \sum_{p=0}^{p_{max}} a_p(s_i) \int_0^1 \mathrm{d}\lambda\, \lambda^p\ \exp\{-\alpha\,(1-\lambda)\}\,(1-\lambda)^{m+1} \right]$$

with $\alpha = jk\sqrt{A_i - s_i B_i + s_i^2 C_i}$

Thus, to integrate along λ we need to define and use the following function:

$$q_m(p, \alpha) \ = \ \int_0^1 (1-\lambda)^m \lambda^p \exp\{-\alpha\,(1-\lambda)\}\,(1-\lambda)\ \mathrm{d}\lambda$$

$$m \ \text{and}\ p\ \text{integer;}\ \ m \geq -1\,,\ p \geq 0 \tag{6}$$

For $m = -1$ and $p = 0$ this function has a simple analytic expression whereas, for other values of m and p, this function can be evaluated by recursive formulas. In fact, for $m = -1$, one has:

$$q_{-1}(0, \alpha) \quad = \quad \frac{1\ -\ \exp(-\alpha)}{\alpha} \tag{7}$$

$$q_{-1}(p, \alpha) \quad = \quad \frac{1\ -\ p\, q_{-1}(p-1, \alpha)}{\alpha}\,, \quad p \geq 1 \tag{8}$$

(7), (8) permit one to recursively compute $q_{-1}(p,\alpha)$, for $p=0$ up to $p=p_{max}$. This knowledge is then used to compute the functions $q_0(p,\alpha)$ for $0\le p\le p_{max}$ which, in turns, allows computation of $q_1(p,\alpha)$ up to $p=p_{max}$, etc.. In fact, for $m\ge 0$ one has:

$$q_m(0,\alpha) = q_{m-1}(0,\alpha) - q_{m-1}(1,\alpha) \tag{9}$$

$$q_m(p,\alpha) = \frac{(1+m)\, q_{m-1}(p,\alpha) \;-\; p\, q_m(p-1,\alpha)}{\alpha}\ , \quad (p\ge 1) \tag{10}$$

The recursive evaluation of $q_m(p,\alpha)$ becomes critical for $|\alpha|\approx 0$. In this case one can use Taylor series approximations which are always convergent and, for α real positive, take the form of alternating series. For $|\alpha|$ small, a general and more effective (under a computational point of view) expression can be given in terms of infinite continued products.

3.3 Integration on the tangent plane or on a plane patch

For a plane patch P_t, the results of the previous two subsections yield:

$$\int_{\mathrm{P}_t} \frac{P^p(\xi_1,\xi_2,\xi_3)}{\mathcal{J}} \frac{\exp(-jkR_t)}{R_t}\, \mathrm{d}S = \int_{\mathrm{P}_t} P^p(\xi_1,\xi_2,\xi_3)\, \frac{\exp(-jkR_t)}{R_t}\, \mathrm{d}\xi_1\, \mathrm{d}\xi_2$$

$$= \sum_i \xi_i' \int_0^1 \frac{\mathrm{d}s_i}{\sqrt{A_i - s_i B_i + s_i^2 C_i}} \left[\sum_{p=0}^{p_{max}} a_p(s_i)\, q_{-1}(p,\alpha)\right]$$

$$\text{with } \alpha = jk\sqrt{A_i - s_i B_i + s_i^2 C_i} \tag{11}$$

In order to derive the result valid for a curved patch we first need to study an approximation of the 3D Green's kernel on the tangent plane. This is done in the following subsection.

3.4 Approximation of the kernel on the tangent plane along a line s = constant

Let us call R the distance from a point $\boldsymbol{r}(\xi_1,\,\xi_2,\,\xi_3)$ of the curved patch and the point of tangency $\boldsymbol{r}' = \boldsymbol{r}(\xi_1',\,\xi_2',\,\xi_3')$ (observation point). In the neighborhood of the point of tangency, the integral kernel $\exp(-jkR)/R$ can be approximated by a function of the distance R_t from a point $\boldsymbol{r}_t(\xi_1,\,\xi_2,\,\xi_3)$ on the tangent patch and the point of tangency. The point $\boldsymbol{r}_t$ of the tangent patch is reached for the same values of dependent coordinates relative to the point $\boldsymbol{r}$ of the curved patch. In the neighborhood of the point of tangency, and along a line $s=$ constant, one has:

$$\frac{\exp(-jkR)}{R} \approx \frac{\exp(-jkR_t)}{R_a} = \exp(-jkR_t)\left[\frac{1}{R_t} - \kappa_1(s) + \kappa_2^2(s) R_t + \kappa_3^3(s) R_t^2\right] \tag{12}$$

The argument of the exponential function on the left-hand side of eq. (12) is approximated on the right-hand side to first order in R_t, whereas the factor inside the square brackets on the right-hand side of (12) is obtained by assuming the following representation for small R ($\approx R_a$) and R_t:

$$R_a = \frac{R_t}{1 - \kappa_1(s)\, R_t + \kappa_2^2(s)\, R_t^2 + \kappa_3^3(s)\, R_t^3}$$

which yields

$$\frac{1}{R_a} = \frac{1}{R_t} - \kappa_1(s) + \kappa_2^2(s)\, R_t + \kappa_3^3(s)\, R_t^2$$

and

$$R_a \approx R_t \left\{1 + \kappa_1(s)\, R_t + \left[\kappa_1^2(s) - \kappa_2^2(s)\right] R_t^2 + \left[\kappa_1^3(s) - 2\,\kappa_2^2(s)\,\kappa_1(s) - \kappa_3^3(s)\right] R_t^3 + \cdots \right\}$$

In the latter equation $\kappa_1(s)$ approximates the curvature of the patch at the point of tangency (where $R_t \approx 0$), but it is not the true curvature. For $\kappa_1(s)$ to become the curvature one should represent R in the neighbourhood of the point of tangency as it follows (infinite series):

$$R = \frac{R_t}{1 - \kappa_1(s)\, R_t + \kappa_2^2(s)\, R_t^2 + \cdots + \kappa_{n+1}^{n+1}(s)\, R_t^n + \cdots}$$

Therefore, the function coefficients $\kappa_1(s)$, $\kappa_2^2(s)$ and $\kappa_3^3(s)$ must be evaluated numerically by enforcing (12) at three points located very close to the point of tangency. Because of the presence of the complex exponential functions, these coefficients are complex. The three points used to evaluate the coefficients have the same value of s-coordinate (s is known when one integrates along s, while it can be evaluated when integrating the regular part, since the sampling quadrature-point is always given); whereas the λ-values are $(1-\Delta)$, $(1-2\Delta)$ and $(1-3\Delta)$ (with $\Delta = 10^{-2}$, for example). By calling (R_{t1}, R_1), (R_{t2}, R_2), (R_{t3}, R_3) the distance couples (R_t, R) relative to the three interpolation points, the numerical-curvature coefficients are easily expressed in terms of matrix product.

The numerically obtained values for κ_1, κ_2^2 and κ_3^3 yield a regular part which, numerically, is zero at $\boldsymbol{r} = \boldsymbol{r}'$ (i.e., at the point of tangency, where $R = R_t = 0$) and with zero first and second derivatives along the direction s=constant.

Since R and R_t are continuous functions of s, also $\kappa_1(s)$, $\kappa_2^2(s)$ and $\kappa_3^3(s)$ are continuous functions of s.

3.5 Integration on a curved patch

We either recall or introduce the following functions:

$$\frac{\exp(-jkR_t)}{R_a} = \exp(-jkR_t)\left[\frac{1}{R_t} - \kappa_1(s) + \kappa_2^2(s)R_t + \kappa_3^3(s)R_t^2\right] \tag{13}$$

$$\alpha = jk\sqrt{A_i - s_iB_i + s_i^2C_i} \tag{14}$$

$$P^p = a_p(s)\,\lambda^p + a_{p-1}(s)\,\lambda^{p-1} + a_{p-2}(s)\,\lambda^{p-2} + \cdots + a_0(s) \tag{15}$$

$$\begin{aligned} Q\,(s_i) = {} & \frac{q_{-1}(p,\alpha)}{\sqrt{A_i - s_iB_i + s_i^2C_i}} - \kappa_1(s_i)\,q_0(p,\alpha) \\ & + \kappa_2^2(s_i)\,q_1(p,\alpha)\sqrt{A_i - s_iB_i + s_i^2C_i} + \kappa_3^3(s_i)\,q_2(p,\alpha)\,\left(A_i - s_iB_i + s_i^2C_i\right) \end{aligned} \tag{16}$$

For a curved patch one has:

$$\begin{aligned} & \int_{\mathrm{P}} \frac{P^p\,(\xi_1,\xi_2,\xi_3)}{\mathcal{J}}\,\frac{\exp(-jkR)}{R}\,\mathrm{d}S \\ & = \int_{\mathrm{P}_t} P^p\,(\xi_1,\xi_2,\xi_3)\left[\frac{\exp(-jkR)}{R} - \frac{\exp(-jkR_a)}{R_a}\right]\mathrm{d}\xi_1\,\mathrm{d}\xi_2 \\ & \quad + \sum_i \xi_i' \int_0^1 \left[\sum_{p=0}^{p_{max}} a_p(s_i)\,Q\,(s_i)\right]\mathrm{d}s_i \end{aligned} \tag{17}$$

The surface integral on the right-hand side is evaluated by first subdividing the parent patch into subtriangles. The quadrature rule used to integrate on each subtriangle defines some sampling points in parent space. To evaluate R_a one needs first to evaluate the s_i coordinate of the sampling point (and then $\kappa_1(s_i)$, $\kappa_2^2(s_i)$, $\kappa_3^3(s_i)$), as explained previously.

References

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