Characterization and Macromodeling of 3D Interconnects

S. Grivet-Talocia
Politecnico di Torino, Italy
grivet@polito.it
http://www.eln.polito.it/research/emc

Introduction
Introduction

S. Grivet-Talocia, SPI tutorial, 9 May 2004
High-speed Data transmission requires integrity of the signals thru lines, bends, vias, connectors, …
An example

Complete propagation path from driver to receiver
An example

Transmission eye @ 1Gb/s (700 bits)

Introduction

Signal Integrity issues in high-speed digital systems

Crosstalk, couplings, reflections, losses, dispersion, attenuation, resonances, ground noise, nonlinear effects, radiation, EMI, …
Introduction

Drivers/Receivers

Linear junctions (connectors, vias, packages,...)

Bus

Macromodel (SPICE,...)

“Lego” approach

Drivers/Receivers

Discontinuities

Transmission Lines

SPICE solver

v(t)

t
3D Interconnects

S. Grivet-Talocia, SPI tutorial, 9 May 2004
Outline

- Introduction
- Macromodeling approaches for 3D Interconnects
- Model Order Reduction methods
  - PRIMA
- Model Identification methods
  - Frequency-Domain Vector Fitting
  - Time-Domain Vector Fitting
  - Passivity characterization and enforcement
- SPICE synthesis
Macromodeling approaches

Macromodeling of 3D interconnects for Signal Integrity

1. Capture physical effects leading to signal degradation
   - Must take into account 3D electromagnetic fields
   - Simulation or measurement
   - Many different characterizations are possible!

2. Use this information to build a macromodel
   - Many macromodeling approaches available!

Characterization via equations

Discretization of Maxwell full-wave equations

Model Order Reduction methods: build a simplified model from an existing (large) one

Characterization via port responses (Black Box)

Time or frequency domain

Simulated or measured

Reduced-Order Model Identification methods: build a model from samples of the port responses
Macromodeling approaches

Main goal of all (lumped) macromodeling methods:

produce a rational approximation

\[ H_q(s) = H_\infty + \sum_{n} \frac{R_n}{s - p_n} \]

Lumped circuits

• have rational transfer functions
• are governed by Ordinary Differential Equations

Outline

• Introduction
• Macromodeling approaches for 3D Interconnects
  • Model Order Reduction methods
    • PRIMA
• Model Identification methods
  • Frequency-Domain Vector Fitting
  • Time-Domain Vector Fitting
  • Passivity characterization and enforcement
• SPICE synthesis
**Moder Order Reduction**

\[ H(s) \approx H_q(s) \]

Possible scenarios

PEEC (Partial Element Equivalent Circuit) discretization

Large circuit
Possible scenarios

Spatial discretization of Maxwell equations (FDTD, FEM, MoM, …)

Set of ODEs (Ordinary Differential Equations)

Large system

Set of ODEs (Ordinary Differential Equations)

\[
\begin{align*}
Gx + C\ddot{x} &= Bu \\
y &= L^T x \\
H(s) &= L^T (G + sC)^{-1} B
\end{align*}
\]
Approximation via moment matching

\[ H(s) = M_0 + M_1 s + M_2 s^2 + \cdots + M_N s^N + \cdots \]

\[ H_q(s) = M_0 + M_1 s + M_2 s^2 + \cdots + M_q s^q + \cdots \]

Moment matching: an example

Original system: 2530
Reduced system: 135

23 GHz
Moments

\[
\begin{align*}
\begin{cases}
Gx + Cx &= Bu \\
y &= L^T x
\end{cases}
\quad \begin{cases}
x &= A\dot{x} + Ru \\
y &= L^T x
\end{cases}
A &= -G^{-1}C \\
R &= G^{-1}B
\end{align*}
\]

\[
H(s) = L^T (I - sA)^{-1} Bu
\]

\[
H(s) = M_0 + M_1 s + M_2 s^2 + \cdots
\]

Moments
\[
\begin{align*}
L^T R \\
L^T AR \\
L^T A^2 R
\end{align*}
\]

Moment matching techniques

**Explicit**

\[
M_i = L^T A^i R \quad \rightarrow H_q(s)
\]

**Asymptotic Waveform Evaluation (AWE)**

**Pade` Approximations**

**Complex Frequency Hopping (CFH)**

- Good theoretical properties, convergence
- Bad numerical properties, intrinsic ill-conditioning due to
  - Moment generation
  - Moment matching
Moment matching techniques

**Implicit**

**Krylov subspace projection methods**
- Same information stored in moments
- Much better numerical performance, robustness
- Several versions
  - Arnoldi, PRIMA, Lanczos, …
- Possibility of preserving stability and passivity by construction!

Krylov subspaces

\[ H(s) = M_0 + M_1s + M_2s^2 + \cdots \]

Moments
\[ L^T R \quad L^T AR \quad L^T A^2 R \]

\[ Kr(A, R, q) = \text{span}\{R, AR, A^2 R, \ldots, A^{q-1} R\} \]

\[ V_q = \text{basis of } Kr(A, R, q) \]

Constructed via iterative (stable) algorithms
**Arnoldi (basic) algorithm**

\[
\begin{align*}
\mathbf{x} &= \mathbf{A}\dot{\mathbf{x}} + \mathbf{R}\mathbf{u} \\
\mathbf{y} &= \mathbf{L}^T\mathbf{x}
\end{align*}
\]

\[
\mathbf{V}_q^T \mathbf{V}_q \mathbf{A} \mathbf{V}_q = \mathbf{A}_q
\]

\[
\mathbf{x} \approxeq \mathbf{V}_q \mathbf{x}_q
\]

\[
\begin{align*}
\mathbf{x}_q &= \mathbf{A}_q \mathbf{x}_q + \mathbf{R}_q \mathbf{u} \\
\mathbf{y} &= \mathbf{L}_q^T \mathbf{x}_q
\end{align*}
\]

**PRIMA algorithm**

\[
\begin{align*}
\mathbf{G}\mathbf{x} + \mathbf{C}\dot{\mathbf{x}} &= \mathbf{B}\mathbf{u} \\
\mathbf{y} &= \mathbf{L}^T \mathbf{x}
\end{align*}
\]

\[
\mathbf{V}_q^T \mathbf{V}_q \mathbf{C} \mathbf{V}_q = \mathbf{C}_q
\]

\[
\mathbf{V}_q^T \mathbf{G} \mathbf{V}_q = \mathbf{G}_q
\]

\[
\mathbf{x} \approxeq \mathbf{V}_q \mathbf{x}_q
\]

\[
\begin{align*}
\mathbf{G}_q \mathbf{x}_q + \mathbf{C}_q \dot{\mathbf{x}}_q &= \mathbf{B}_q \mathbf{u} \\
\mathbf{y} &= \mathbf{L}_q^T \mathbf{x}_q
\end{align*}
\]
Passivity conditions (PRIMA)

\[
\begin{cases}
Gx + Cx = Bu \\
y = L^T x
\end{cases}
\]

Can often be enforced by construction building the original system

\[
\begin{cases}
G_{qq}x_q + C_{qq}x_q = B_u \\
y = L_{qq}^T x_q
\end{cases}
\]

\[G \geq 0 \quad C \geq 0\]

\[C\text{ symmetric}\]

\[L = \pm B\]

\[V_q\text{ must be full rank}\]

An example: RLC tree circuit

Circuit order: 2530
Ports: 9
RLC tree circuit: order reduction

\[ |Y_{11}| \]

- Original circuit 2530
- PRIMA order 36
- Arnoldi order 36

- 1 GHz

- 2 GHz

S. Grivet-Talocia, SPI tutorial, 9 May 2004
RLC tree circuit: order reduction

7 GHz

10 GHz
RLC tree circuit: order reduction

![Graph showing |Y_{11}| vs. frequency for different orders of reduction.]

- **Original circuit**: 2530
- **PRIMA order 135**
- **Arnoldi order 135**

At 15 GHz:
- Red line: 15 GHz

At 23 GHz:
- Red line: 23 GHz
**RLC tree circuit: efficiency**

![Bar chart showing relative CPU time for different reduced systems compared to the original circuit.](image1)

**RLC tree circuit: transient analysis**

![Graph showing transient analysis with input signal and original circuit.](image2)
RLC tree circuit: transient analysis

- Input signal
- Original circuit
- PRIMA order=18
- Arnoldi order=18

RLC tree circuit: transient analysis

- Input signal
- Original circuit
- PRIMA order=27
- Arnoldi order=27
RLC tree circuit: transient analysis

![Graph of RLC tree circuit transient analysis](image)

**Comparison of Voltage (V) over Time (s)**

- **Input signal**
- **Original circuit**
- **PRIMA order=36**
- **Arnoldi order=36**

**Axes:**
- **Time (s)**: 0 to 3 x 10^-3
- **Voltage (V)**: -0.4 to 1.4

---

**Graph Details:**
- The graph illustrates the transient response of a RLC circuit over time.
- Various orders of approximation are compared to the original circuit.
- Key parameters include the input signal and different orders of the PRIMA and Arnoldi methods.

---

S. Grivet-Talocia, SPI tutorial, 9 May 2004
Example: MNA, 22 ports, order 4863

(www.win.tue.nl/niconet/NIC2/benchmodred.html)

3 GHz

Example: MNA, 22 ports, order 4863

6 GHz
Example: MNA, 22 ports, order 4863

Example: MNA, 22 ports, order 4863
Key references


…and references therein


…and references therein

Outline

• Introduction
• Macromodeling approaches for 3D Interconnects
• Model Order Reduction methods
  • PRIMA
• Model Identification methods
  • Frequency-Domain Vector Fitting
  • Time-Domain Vector Fitting
  • Passivity characterization and enforcement
• SPICE synthesis
Model Identification

From samples to model: identification process

Reduced-order identification: approximation process

Several identification methods exist

Characterized by use of different:

- Input data
- Modeling criteria
- Model parameter estimation
Identification methods

Block Complex Frequency Hopping (BCFH)

Rational Padé approximation of network functions

Convergence property in a neighborhood of the expansion point

Hopping along frequency axis to cover the modeling bandwidth

May lead to ill-conditioned numerical systems when used for identification from sampled responses

Identification methods

Global Rational Approximation
[J.Morsey, A.C.Cangellaris, Proc. EPEP, 2001]
[… many, many, many others…]

A matrix of rational functions is fitted to the samples of a network function matrix (e.g. the Y matrix)
Identification methods

Pencil of Functions


Time-domain data

Estimates model poles by fitting a sum of exponential functions to the samples of transient port responses

Poles obtained as eigenvalues of a generalized eigenvalue problem

Automatic order estimation

Identification methods

Subspace-based State-Space System Identification methods (4SID)

[M. Viberg, Automatica, 12/1995]


Based on projections of data onto orthogonal subspaces, leading to direct state-space estimation

Built-in automatic order estimation (based on SVD)

Available in both time and frequency domain

Equivalent to Pencil of Functions methods
Identification methods

Nevanlinna-Pick Interpolation


Interpolation of samples of the scattering matrix with a (unitary bounded) matrix rational function

Nice theoretical properties

Very complex

Leads to models with large dynamical order

Identification methods

Vector Fitting


Performs data fitting with rational functions avoiding nonlinear optimization

Iterative process converging to the dominant poles

Available for both time and frequency domain
Identification methods

Identification methods are not expected to work for every possible problem

Any method performs well for a certain class of identification problems

Vector Fitting is selected here as one of the most promising methods for a wide range of applications

Outline

• Introduction
• Macromodeling approaches for 3D Interconnects
• Model Order Reduction methods
  • PRIMA
• Model Identification methods
  • Frequency-Domain Vector Fitting
  • Time-Domain Vector Fitting
  • Passivity characterization and enforcement
• SPICE synthesis
Frequency-domain macromodeling

Model identification from frequency-domain responses

Possible scenarios

Frequency-Domain full-wave simulation (MoM, FEM, …)

Frequency tables of transfer matrix (S, Y, Z, …)
Possible scenarios

Time-Domain full-wave simulation (FIT, FDTD)
FFT postprocessing

Frequency tables of transfer matrix
(S, Y, Z, ...)

Possible scenarios

Direct VNA measurement

Frequency tables of transfer matrix (S)
Frequency-Domain Macromodeling

Input data
\[ \{ \hat{H}(j\omega_k), \ k = 1, \ldots, K \} \]

Approximation
\[ H(s) = \sum_{n=1}^{N} \frac{R_n}{s - p_n} + H_\infty \]

Fitting condition
\[ H(j\omega_k) \approx \hat{H}(j\omega_k), \ \forall k \]

Unknowns:
- Poles \( p_n \)
- Residues \( R_n \)
- Constant \( H_\infty \)

Frequency-Domain Macromodeling

Direct fitting condition: nonlinear!
\[ \sum_{n=1}^{N} \frac{R_n}{j\omega_k - p_n} + H_\infty \approx \hat{H}(j\omega_k), \ \forall k \]

- Nonlinear dependence on poles
- Requires nonlinear optimization (e.g. nonlinear least squares)
- Convergence problems (local minima, etc…)
Frequency-Domain Macromodeling

Direct fitting condition: nonlinear!
\[ \sum_{n=1}^{N} \frac{R_n}{j \omega_k - p_n} + H_\infty \approx \hat{H}(j \omega_k), \quad \forall k \]

Vector Fitting avoids nonlinear optimization


Frequency-Domain Vector Fitting

Input data
\[ \{ \hat{H}(j \omega_k), \quad k = 1, \ldots, K \} \]

Weight function
\[ w(s) = \sum_{n=1}^{N} \frac{c_n}{s - q_n} + 1 \]
- \( w(s) \) is unitary for \( s \to \infty \)
- poles \( q_n \) are fixed a priori
- residues \( c_n \) are unknown

Approximation
\[ H(s) = \sum_{n=1}^{N} \frac{R_n}{s - p_n} + H_\infty \]

Vector Fitting condition
\[ w(s) H(s) \approx \sum_{n=1}^{N} \frac{\tilde{c}_n}{s - q_n} + \hat{H}_\infty \]

The poles of \( w(s) H(s) \) are \( \{ q_n \} \) only!
Frequency-Domain Vector Fitting

**Input data**
\[ \{ \hat{H}(j\omega_k), \quad k = 1, \ldots, K \} \]

**Weight function**
\[ w(s) = \sum_{n=1}^{N} \frac{c_n}{s - q_n} + 1 \]
\[ w(s) = \prod_{n=1}^{N} \frac{(s - z_n)}{(s - q_n)} \]
There are \( N \) zeros \( \{ z_n \} \)

**Approximation**
\[ H(s) = \sum_{n=1}^{N} \frac{R_n}{s - p_n} + H_\infty \]

**Vector Fitting condition**
\[ w(s) \, H(s) \cong \sum_{n=1}^{N} \frac{\tilde{c}_n}{s - q_n} + \tilde{H}_\infty \]
\[ \{ p_n \} \cong \{ z_n \} \]
### Frequency-Domain Vector Fitting

#### Unknowns

\[ \{ c_n, \hat{c}_n, \hat{H}_\infty \} \]

#### Linear least squares problem: easy to solve!

\[
\left\{ \sum_{n=1}^{N} \frac{c_n}{j\omega_k - q_n} + 1 \right\} \hat{H}(j\omega_k) \approx \sum_{n=1}^{N} \frac{\hat{c}_n}{j\omega_k - q_n} + \hat{H}_\infty
\]

\[
w(s) = \sum_{n=1}^{N} \frac{c_n}{s - q_n} + 1 = \prod_{n=1}^{N} \frac{(s - \hat{z}_n)}{(s - q_n)}
\]

#### Theorem: the zeros \( \{ \hat{z}_n \} \) are the eigenvalues of

\[ Q = A - b \, c^T \]

where

\[ A = \text{diag} \{ q_n \} \]

\[ b = (1 \quad 1 \quad \cdots \quad 1)^T \]

\[ c = (c_1 \quad c_2 \quad \cdots \quad c_N)^T \]
Pole relocation

Starting poles \( \{ q_n \} \) of \( w(s) \)

New poles \( \{ p_n \} \) of \( H(s) \)

Iteration for pole convergence

Choice of starting poles

Uniform coverage of approximation bandwidth

Real poles

Complex poles

\( \Omega \)

\( j \Omega \)

\( -\Omega \)

\( -j \Omega \)
Vector Fitting: residues

Input data
\[ \{ \hat{H}(j \omega_k), \quad k = 1, \ldots, K \} \]

Approximation
\[ H(s) = \sum_{n=1}^{N} \frac{R_n}{s - P_n} + H_\infty \]

Fitting condition
\[ H(j \omega_k) \approx \hat{H}(j \omega_k), \quad \forall k \]

Another linear least squares problem!

Example 1: 18 random poles/residues

![Graph showing poles distribution](image-url)
Example 1

$|H(j\omega)|$

$\omega$

Example 1

$\Re(s)$

$\Im(s)$

Poles

True

Initial poles
Example 1

![Graph showing poles](image1)

Example 1

![Graph showing magnitude](image2)
Example 1

\[ H(j\omega), \text{ phase} \]

\[ \begin{array}{cccc}
10^2 & 10^3 & 10^4 & 10^5 \\
-200 & -150 & -100 & 0 \\
-50 & 0 & 50 & 100 \\
\end{array} \]

\[ \omega \]

Exact

Approx

Example 2

Same 18-pole rational function

Reduced-order fitting (12th order)
Example 2

Poles

- True
- Approx

Example 2

Poles: iteration 1

- True
- Approx
Example 2

Poles: iteration 2

Real(s) vs. Imag(s)

- True
- Approx

Example 2

Poles: iteration 3

Real(s) vs. Imag(s)

- True
- Approx
Example 2

Poles: iteration 4

**Example 2**

Poles: iteration 5

S. Grivet-Talocia, SPI tutorial, 9 May 2004
Example 2

Same 18-pole rational function

Reduced-order fitting (12th order)

Different starting poles (real poles)
Example 3

Uniform coverage of bandwidth

Example 3

Poles: iteration 1

S. Grivet-Talocia, SPI tutorial, 9 May 2004
Example 3

Poles: iteration 2

Example 3

Poles: iteration 3
Example 3

\[ |H(j\omega)| \]

![Graph showing the magnitude of \( |H(j\omega)| \) with two lines representing 'Exact' and 'Approx.'](image)

Example 4

\[ H(j\omega) \]

![Graph showing the real and imaginary parts of \( H(j\omega) \). The graph also shows a non-rational smooth function with the equation \( H(s) = \sqrt{\frac{s + 0.001}{s + 100}} \).](image)
Example 4

S. Grivet-Talocia, SPI tutorial, 9 May 2004
Example 4

$H(j\omega)$

- Real, Exact
- Imag, Exact
- Real, Approx
- Imag, Approx

3 poles

Example 4

$H(j\omega)$

- Real, Exact
- Imag, Exact
- Real, Approx
- Imag, Approx

4 poles
Example 4

**H(\omega)**

- **Real, Exact**
- **Imag, Exact**
- **Real, Approx**
- **Imag, Approx**

5 poles

---

Example 5: MCM-board connector

[Diagram of MCM-board connector]
Example 5: MCM-board connector

Data: 10-port structure, frequency-domain S-matrix
Example 5: MCM-board connector

Macromodel: 4-poles

Error: 0.1%

Scattering matrix entries, magnitude (dB)

Scattering matrix entries, phase (degrees)

Frequency [GHz]
Example 6: stripline+launches

Data: measured S-parameters

Scattering matrix entries, magnitude

Scattering matrix entries, phase

Frequency [GHz]

S(1,1), data
S(1,1), model
S(2,1), data
S(2,1), model

Example 6: stripline+launches

Macromodel: 60 poles

Scattering matrix entries, magnitude

Scattering matrix entries, phase

Frequency [GHz]
Example 7: PCB path, measured
VF with frequency-selective weighting

Scattering matrix entries, magnitude

Scattering matrix entries, phase

S(3,1), data
S(3,1), model

S(4,1), data
S(4,1), model

VF with frequency-selective weighting
Example 8: Connector, measured
VF with frequency-selective weighting

Vector Fitting: summary

- Tool for frequency-domain rational approximation
  - rational transfer functions (system identification)
  - rational transfer functions (reduced-order modeling)
  - non-rational transfer functions
- Data from full-wave simulations
- Direct frequency-domain measurements
Vector Fitting: summary

- Very accurate and robust
- Only linear least squares + eigenvalues required
- Stability is not guaranteed
  - can be fixed by flipping real part during relocation
- Passivity is not guaranteed
  - can be fixed a posteriori (see later)

Outline

- Introduction
- Macromodeling approaches for 3D Interconnects
- Model Order Reduction methods
  - PRIMA
- Model Identification methods
  - Frequency-Domain Vector Fitting
  - Time-Domain Vector Fitting
  - Passivity characterization and enforcement
- SPICE synthesis
Time-domain macromodeling

Model identification from time-domain responses

Possible scenarios

Time-Domain full-wave simulation (FIT, FDTD)

Port responses to transient excitations (usually gaussian)
Possible scenarios

- Time-Domain full-wave simulation (FIT, FDTD)
- Truncated waveforms from short FDTD runs
- Port responses to transient excitations (usually gaussian)

Possible scenarios

- Time-domain measurements (work in progress)
- Port responses to transient excitations (usually gaussian)
Time-Domain Macromodeling

- Input pulse: \( x(t) \) \( t \)-domain
- Output responses: \( y(t) \) \( t \)-domain
- Transfer function:
  \[
  Y(s) = H(s)X(s)
  \]

Rational approximation:

\[
H(s) \approx H_\infty + \sum_n \frac{R_n}{s - p_n}
\]

Unknowns:
- Poles \( p_n \)
- Residues \( R_n \)
- Constant \( H_\infty \)

---

Time-Domain Vector Fitting

Step 1. Find the dominant poles via "relocation"

Guess poles \( \{ q_n \} \) \nNew poles \( \{ p_n \} \)

Iterative refinement

How to do it using time-domain data?
How to insure convergence to the right poles?
Time-Domain Vector Fitting

1a. Start with initial poles: \( \{ q_n \} \)

1b. Define weight function: unknown \( \{ k_n \} \)

\[
w(s) = 1 + \sum_n \frac{k_n}{s - q_n}
\]

1c. Assume the following condition

\[
w(s)H(s) = a + \sum_n \frac{b_n}{s - q_n}
\]

Poles of \( H(s) \) = Zeros of \( w(s) \)

Time-Domain Vector Fitting

\[
w(s)H(s) = a + \sum_n \frac{b_n}{s - q_n}
\]

Apply the input pulse \( X(s) \)

\[
w(s)Y(s) = \left( a + \sum_n \frac{b_n}{s - q_n} \right) X(s)
\]

Compute inverse Laplace transform

\[
y(t) + \sum_n k_n y_n(t) = a x(t) + \sum_n b_n x_n(t)
\]

\[
x_n(t) = \int_0^t e^{q_n(t-t')} x(t') \, dt'
\]

\[
y_n(t) = \int_0^t e^{q_n(t-t')} y(t') \, dt'
\]

Low-pass filtered input and output signals

S. Grivet-Talocia, SPI tutorial, 9 May 2004
Time-Domain Vector Fitting

1d. Solve a linear least squares system for $k_n$, $a$, $b_n$

$$y(t) + \sum_{n} k_n y_n(t) = a x(t) + \sum_{n} b_n x_n(t)$$

1e. Compute the zeros $\{p_n\}$ of the weight function

$$w(s) = 1 + \sum_{n} \frac{k_n}{s - q_n} = \prod_{n} (s - p_n) \prod_{n} (s - q_n)$$

These are the dominant poles!


---

Time-Domain Vector Fitting

Step 2. Compute the residues

2a. Low-pass filter input signals with new poles

$$\tilde{x}_n(t) = \int_{0}^{1} e^{p_n(t-\tau)} x(\tau)d\tau$$

2b. Solve a linear least squares system for $R_n$ and $H_\infty$

$$y(t) = H_\infty x(t) + \sum_{n} R_n \tilde{x}_n(t)$$
Subsampling

Transient waveform

Spectrum

Nyquist frequency of resampled waveform = Effective bandwidth

Example 1: single via

Raw data:
Triangle Impulse Responses obtained by a transient PEEC solver (by Dr. Ruehli, IBM)

Transient responses

S. Grivet-Talocia, SPI tutorial, 9 May 2004
Example 2: segmented power bus

- 2-port structure
- Time-Domain solution
- CST Microwave Studio
- Bandwidth: 3 GHz
- 50 Ω port terminations

80-poles model (Time-Domain Vector Fitting)
Example 2: segmented power bus

Comparison vs. frequency-domain scattering data

Full-wave simulation time (CST) to compute…

… frequency scattering data: 60 hours

(wait until transients are finished for reliable FFT)

… macromodel: 6 hours

(can use truncated waveforms for TD-VF)
Example 3: 42-pin connector

3x14 pins, 84 ports
Characterized via FIT
(CST Microwave Studio 4)
(Courtesy: Erni - AdMOS)

Handling many ports

Frequency-Domain Vector Fitting

\[
(1 + \sum \frac{k_n}{s - q_n}) H(s) = a + \sum \frac{b_n}{s - q_n}
\]

Time-Domain Vector Fitting

\[
y(t) + \sum k_n y_n(t) = a x(t) + \sum b_n x_n(t)
\]

Processing all responses may lead to a large system!
Handling many ports

1. Split port responses into subsets

Transfer matrix $H(s)$  
Subsets $\{h_k(s)\}$

2. Macromodel each subset via FD-VF or TD-VF

$$h_k(s) \approx h_{k,\infty} + \sum_n \frac{r_{k,n}}{s - p_{k,n}}$$

Partial state-space representation

$$\begin{aligned}
\dot{w}_k &= A_k w_k + B_k x_k \\
y_k &= C_k w_k + D_k x_k
\end{aligned}$$
Handling many ports

4. Assemble all partial models into a global model

\[
\begin{align*}
\dot{w} &= Aw + Bx \\
y &= Cw + Dx
\end{align*}
\]

All matrices can be constructed as sparse!
Example 3: 42-pin connector

3x14 pins, 84 ports
Characterized via FIT
(CST Microwave Studio 4)
(Courtesy: Erni - AdMOS)

Example 3: model order selection

Automatic (iterative) order selection on each of the 84 subsets of port responses (reduced model complexity)
Example 3: macromodel accuracy

Transient scattering responses - transmitted

Transient scattering responses - Xtalk
Example 4: 14-pin package

14-pin SOIC package
Simplified CAD for FDTD
Bandwidth: 40 GHz
50 Ω port terminations

FDTD transient scattering responses

Example 4: macromodel responses

Transient scattering responses

No visible difference between data and model
Example 4: macromodel responses

Lines: raw data             Dots: TDVF macromodel

No visible difference between data and model

Example 4: macromodel accuracy

Maximum deviation between model and data for all 28x28 responses

TD-VF produces highly accurate macromodels
Macromodel properties

😊 Accuracy
Good initial data ⇒ small approximation errors

😊 Stability
All poles with negative real part

😊 Passivity
The macromodel may not be passive

Example 4: change terminations

Port terminations:
\[ R = 50 \, \text{m}\Omega \div 50 \, \Omega \]
\[ L = 1 \, \text{nH} \]
\[ C = 1 \, \text{pF} \]

Effects of passivity violation

Macromodel may become unstable!
Outline

• Introduction
• Macromodeling approaches for 3D Interconnects
• Model Order Reduction methods
  • PRIMA
• Model Identification methods
  • Frequency-Domain Vector Fitting
  • Time-Domain Vector Fitting
  • Passivity characterization and enforcement
• SPICE synthesis

Passivity conditions
Scattering representation

\[ \begin{bmatrix} a_j \\ b_j \end{bmatrix} \xrightarrow{A \text{ strictly stable}} \begin{bmatrix} x \\ y \end{bmatrix} \xrightarrow{A} \begin{bmatrix} a_k \\ b_k \end{bmatrix} \]

Scattering matrix: must be bounded real

\[ H(s) = D + C(sI - A)^{-1}B \]

\[ \{\text{singular values of } H(j\omega)\} \leq 1, \quad \forall \omega \]
Passivity conditions

Scattering representation

\[ v_j = A w + B x \]
\[ y = C w + D x \]

Admittance representation

\[ H(s) = D + C(sI - A)^{-1} B \]

\[ \{ \text{eigenvalues of } \left( H(j\omega) + H^H(j\omega) \right) \geq 0, \quad \forall \omega \} \]
Passivity conditions
Admittance representation

Checking passivity
Scattering representation

\[ \{ \text{singular values of } H(j\omega) \leq 1, \quad \forall \omega \} \]

Several techniques can be used

**Frequency sweep test**: most straightforward

- Choose a set of frequency samples
- Compute \( H \) and its singular values, and check
- **Time-consuming** for large models
- May give wrong answers due to poor sampling
Checking passivity
Scattering representation

\{ \text{singular values of } H(j\omega) \leq 1, \ \forall \omega \}

Equivalent purely algebraic conditions:

- Linear Matrix Inequalities (LMI)
- Algebraic Riccati Equations (ARE)
- Eigenvalues of Hamiltonian matrices


Linear Matrix Inequality (LMI)

\begin{pmatrix}
    A^T P + PA + C^T C & PB + C^T D \\
    B^T P + D^T C & D^2 - I
\end{pmatrix} \leq 0 \quad P = P^T, \ P > 0

Real matrix P is the variable
Checking passivity
Scattering representation

\[ \{ \text{singular values of } H(j\omega) \leq 1, \quad \forall \omega \} \]

Algebraic Riccati Equation (ARE)

\[
A^T P + PA + C^T C + \left( PB + C^T D \right) \left( I - D^T D \right)^{-1} \left( PB + C^T D \right)^T = 0
\]

\[ P = P^T \]

Real matrix \( P \) is the variable

Checking passivity
Scattering representation

\[ \{ \text{singular values of } H(j\omega) \leq 1, \quad \forall \omega \} \]

Eigenvalues of Hamiltonian matrix

\[
M = \begin{pmatrix}
A - B \left( D^T D - I \right)^{-1} D^T C & - B \left( D^T D - I \right)^{-1} B^T \\
C^T \left( D D^T - I \right)^{-1} C & - A^T + C^T D \left( D^T D - I \right)^{-1} B^T
\end{pmatrix}
\]

Real matrix \( M \) must have no imaginary eigenvalues
Theorem [Boyd, Balakrishnan, Kabamba, 1989]

\( j\omega_0 \) is an eigenvalue of \( M \Leftrightarrow \sigma = 1 \) is a singular value of \( H(j\omega_0) \)

The slope allows to count the number of singular values exceeding the threshold in each frequency band.
Checking passivity
Scattering representation

First-order perturbation of Hamiltonian eigenvalues

\[
Slope = \text{Im} \left\{ \frac{w^T v}{w^T M' v} \right\}
\]

\( w, v \): Left and right eigenvectors of \( M \) associated to \( \omega_0 \)

\( M' \): Another Hamiltonian matrix (computed via \( A, B, C, D \))

Example 4: change terminations

Port terminations:
- \( R = 50 \ \text{m\Omega} \div 50 \ \Omega \)
- \( L = 1 \ \text{nH} \)
- \( C = 1 \ \text{pF} \)

Effects of passivity violation

Macromodel may become unstable!
Example 4: passivity characterization

Passivity characterization

Singular values

Frequency [Hz]

Example 4: passivity characterization

Passivity characterization

Singular values

Frequency [Hz]
Example 4: passivity characterization

Passivity characterization

Singular values

Frequency [Hz]
Passivity enforcement

- Generate a new passive macromodel
- Apply small correction to preserve accuracy
  - original dataset should be passive
  - original macromodel should be accurate
  - (usually) preserve poles

\[
\begin{align*}
\dot{w} &= Aw + Bx \\
y &= Cw + Dx
\end{align*}
\]

\[
\begin{align*}
\dot{w} &= A w + B x \\
y &= (C + dC)w + D x
\end{align*}
\]

Several different approaches are possible. Examples are:

**Quadratic/convex optimization**

**Trace parameterization/Semi-Definite Programming**

**Perturbation of residues**

**Perturbation of Hamiltonian eigenvalues**

Many others… Hot research topic!
Perturbation of Hamiltonian Eigs

Singular values of $H$

Eigenvalues of $M$

S. Grivet-Talocia, SPI tutorial, 9 May 2004
Perturbation of Hamiltonian Eigs

First-order perturbation of eigenvalues (again)

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx + Du
\end{align*}
\]

Perturb state matrix \( C \)

\( \tilde{C} = C + dC \)

\( \tilde{M} \approx M + dM \) (first-order: \( dM \) is linear in \( dC \))

\[
\begin{align*}
w_m^T dM v_m &\approx j(\tilde{\omega}_m - \omega_m) w_m^T v_m
\end{align*}
\]

Linear constraint on the correction matrix \( dC \)
Perturbation of Hamiltonian Eigs

Iterate

\[ Z c - r \]

Until passive

\[ \min c^T Q c \]

Minimizes the perturbation on the original responses

Linear coefficients

Entries in matrix \( dC \)

Perturbations of eigenvalues

Related to controllability Gramian

Preserve accuracy of macromodel

Minimize this norm!

\[ \sum_{i,j} \int_0^\infty (\tilde{h}_{i,j}(t) - h_{i,j}(t))^2 dt = \| dC W dC^T \|_F^2 \]

Induced perturbation in the impulse responses

Weighted norm of state matrix perturbation

\( W \): controllability Gramian

\[ AW + WA^T = -BB^T \]
Example 4: passivity compensation

Port terminations:
- $R = 50 \, \text{m}\Omega \div 50 \, \Omega$
- $L = 1 \, \text{nH}$
- $C = 1 \, \text{pF}$
Example 4: passive macromodel

Raw data ➔ Macromodel

Passive macromodel

Example 1: single via, nonpassive

Raw data:
Triangle Impulse Responses obtained by a transient PEEC solver (by Dr. Ruehli, IBM)
Example 1: single via, passive

Raw data:
Triangle Impulse Responses obtained by a transient PEEC solver (by Dr. Ruehli, IBM)

Example 2: segmented power bus

• 2-port structure
• Time-Domain solution
• CST Microwave Studio
• Bandwidth: 3 GHz
• 50 Ω port terminations
Example 2: segmented power bus

Passivity characterization

Passivity violation intervals according to Hamiltonian test

Singular values of Scattering matrix

Frequency [Hz]

Passivity compensation

Passivity violation intervals according to Hamiltonian test

Singular values of Scattering matrix

Frequency [Hz]
Example 2: segmented power bus

80-poles **passive** model

Transient scattering responses

---

Example 2: segmented power bus

Comparison vs. frequency-domain scattering data
More examples...

41 poles, 2 ports
- Compensation
- Accuracy

110 poles, 5 ports
- Compensation
- Accuracy

308 poles, 11 ports
- Compensation
- Accuracy

Outline

- Introduction
- Macromodeling approaches for 3D Interconnects
- Model Order Reduction methods
  - PRIMA
- Model Identification methods
  - Frequency-Domain Vector Fitting
  - Time-Domain Vector Fitting
  - Passivity characterization and enforcement
- SPICE synthesis
Macromodel implementation

Main approaches

1. Synthesize an equivalent circuit in SPICE format
   - No access to SPICE kernel
   - Must use standard circuit elements
2. Direct SPICE implementation via recursive convolution
   - Laplace element, most efficient
3. Other languages for mixed-signal analyses
   - Verilog-AMS, VHDL-AMS, …
   - Equation-based

SPICE synthesis

Admittance representation
One-port, one-pole

\[
\begin{aligned}
\dot{w} &= a \, w + b \, v \\
i &= c \, w + d \, v
\end{aligned}
\]
SPICE synthesis

Scattering representation
One-port, one-pole
\[ x = G_0 v + i, \quad y = G_0 v - i \]
\[ \dot{w} = a w + b x \]
\[ y = c w + d x \]

Admittance representation
One-port, two-poles (complex)
\[ p_{1,2} = \alpha \pm j\beta \]
\[ \dot{w} = A w + b v \]
\[ i = c w + d v \]
SPICE synthesis

Admittance representation

General state-space synthesis

\[
\begin{align*}
\dot{w} &= A w + B v \\
i &= C w + D v
\end{align*}
\]

Recursive convolutions

\[
H(s) = D + C(sI - A)^{-1} B = D + \sum_n \frac{R_n}{s - p_n}
\]

\[
h(t) = D\delta(t) + \sum_n R_n e^{p_n t} u(t)
\]

\[
y(t) = Dx(t) + \sum_n R_n \int_0^t e^{p_n(t-\tau)} x(\tau) d\tau
\]

S. Grivet-Talocia, SPI tutorial, 9 May 2004

EMC GROUP
Recursive convolutions

\[ \tilde{y}(t_k) = \int_{0}^{t_k} e^{p(t_k-\tau)} x(\tau) \, d\tau \]

Discrete time \( t_k = t_{k-1} + \Delta t_k \)

\[ = \int_{0}^{t_{k-1}} e^{p(t_k-\tau)} x(\tau) \, d\tau + \int_{t_{k-1}}^{t_k} e^{p(t_k-\tau)} x(\tau) \, d\tau \]

\[ = e^{p\Delta t_k} \int_{0}^{t_{k-1}} e^{p(t_k-\tau)} x(\tau) \, d\tau + \int_{t_{k-1}}^{t_k} e^{p(t_k-\tau)} x(\tau) \, d\tau \]

\[ \approx e^{p\Delta t_k} \tilde{y}(t_{k-1}) + \frac{1 - e^{p\Delta t_k}}{p} x(t_k) \]

Approximation!

The macromodeling dream…

Arbitrary characterization of the structure

• Equation-based or Black-Box

• Time or frequency, simulation or measurement

Generation of a broadband macromodel

• Any order, any number of ports

• Any prescribed accuracy

• Stable and passive by construction

• Efficient (reduced-order and low-complexity)

• Fully automatic