# Generation of Passive Macromodels from Transient Port Responses

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**Abstract:** This paper presents a new technique for the generation of linear lumped macromodels from input-output port characterization. A complete set of transient port responses is processed by a new time-domain formulation of the well-known Vector Fitting algorithm. The data processing involves a combination of digital filtering and least squares fitting. Passivity of the obtained macromodel is enforced a posteriori by applying an iterative perturbation technique to the associated Hamiltonian matrix.

## 1 Introduction

This paper is aimed at the construction of lumped macromodels for interconnect structures like junctions, packages, vias, connectors. Such macromodels are essential for accurate system-level signal integrity assessment. A common practice is to use some full-wave electromagnetic solver to characterize the structure in either time or frequency domain. This leads generally to a set of transient port responses to suitable port excitations, or to frequency-domain tables of the structure transfer matrix entries. Alternatively, these port characterization can be obtained via direct measurement. We concentrate here on time-domain characterizations obtained by transient electromagnetic simulation via, e.g., Finite-Difference Time-Domain (FDTD), Finite Integration (FIT), or transient Partial Element Equivalent Circuit (PEEC) methods. The direct derivation of a passive and accurate macromodel from this type of characterization is still a challenging task, especially for highly complex structures. The methodology introduced in this work seems to be very promising for this difficult problem.

The first contribution of this paper is a new macromodeling algorithm which we denote as Time-Domain Vector Fitting (TDVF). This method is basically a time-domain reformulation of the well-known standard Vector Fitting algorithm [6]. It is based on a combination of digital filtering and linear least squares approximations, which are directly applied to process the transient port responses. A first step allows the identification of the dominant poles of the structure. Once these poles are known, the matrices of residues are computed. These three steps are outlined in Section 2. More details can be found in [3]. A remarkable feature of this method is the possibility of using truncated transient responses to generate the macromodel, thus allowing for short full-wave transient simulations.

The second contribution of this paper is a fully automatic algorithm for passivity enforcement. This algorithm is based on spectral perturbations of associated Hamiltonian matrices, and can be applied a posteriori to the macromodel obtained by TDVF in order to check its passivity and possibly compensate it. The procedure for passivity enforcement is only outlined in Section 3. Full derivation, theorem proofs, and details can be found in [4, 5]. The combination of TDVF and Hamiltonian-based passivity tools leads to a powerful macromodeling methodology that has been verified on several applications. Some examples are reported in Section 4.

## 2 Time-Domain Vector Fitting

We consider a multiport structure with an arbitrary number $P$ of ports. Usually, a transient characterization of such a multiport structure is obtained by exciting one port at the time and computing/measuring the responses at all ports. As a result, the raw data set is a matrix of response waveforms $y_{ij}(t)$ at port $(i)$, due to an excitation source $x_j(t)$ located at port $(j)$. We remark that this type of data set is the natural outcome of time-domain full-wave electromagnetic solvers. Transient scattering waveforms are the typical format, although present formulation is applicable also to transient impedance (open-circuit) or admittance (short-circuit) responses. The objective is the derivation of a rational approximation to the matrix transfer function $\boldsymbol{H}(s)$, where $s$ is the Laplace variable. This approximation reads, in terms of poles and residues,

$$\boldsymbol{H}(s) \simeq \boldsymbol{H}_\infty + \sum_{n=1}^{N} \frac{\boldsymbol{R}_n}{s - p_n}. \tag{1}$$

We first introduce, as for standard VF [6], a scalar weight function

$$\sigma(s) = 1 + \sum_{n=1}^{N} \frac{k_n}{s - q_n} = \frac{\prod_{n=1}^{N}(s - z_n)}{\prod_{n=1}^{N}(s - q_n)} \tag{2}$$

with fixed (initial) poles $\{q_n\}$ and unknown residues $\{k_n\}$. This function is used to enforce the following condition,

$$\sigma(s)H(s) \simeq M_\infty + \sum_{n=1}^{N} \frac{M_n}{s-q_n}. \tag{3}$$

Since the right-hand-side has the same poles $\{q_n\}$ as the weight function, a cancellation between the zeros $\{z_n\}$ of $\sigma(s)$ and the poles $\{p_n\}$ of $H(s)$ must occur. This condition provides indeed a way to estimate these poles by solving (3) for the unknown residues $\{k_n\}$, computing the zeros $\{z_n\}$ using standard techniques [6], and by enforcing $p_n = z_n$. This procedure, named *pole relocation*, avoids use of ill-conditioned nonlinear least squares algorithms for the direct approximation of (1). Also, the poles relocation can be iterated using the estimated poles as starting poles for the new iteration.

We reformulate the VF condition (3) in time domain by applying it to the vector of input signals $X(s)$ and by using inverse Laplace transform. We get

$$y_{ij}(t) \simeq M_{\infty,ij}x_j(t) + \sum_{n=1}^{N} M_{n,ij}x_{n,j}(t) - \sum_{n=1}^{N} k_n y_{n,ij}(t), \quad i,j=1,\ldots,P, \tag{4}$$

where the transient waveforms

$$x_{n,j}(t) = \int_0^t e^{q_n(t-\tau)}x_j(\tau)d\tau, \qquad y_{n,ij}(t) = \int_0^t e^{q_n(t-\tau)}y_{ij}(\tau)d\tau \tag{5}$$

are convolutions resulting from inverse Laplace transform of each partial fraction in the expansions (2)-(3). These waveforms are easily obtained by applying a suitable discretization of the convolution integrals. We remark that due to the exponential nature of the convolution kernels, a fast implementation based on recursive convolutions, i.e., digital IIR filtering, is convenient [3].

The condition (4) is enforced in least squares sense using raw and filtered input/output sequences. Note that the residues $\{k_n\}$ of the weight function are common in all the $P^2$ independent equations in (4), which result coupled and not independent. Solution of this linear system returns the residues $\{k_n\}$ of the weight function, which in turn are used as in standard VF to compute its zeros $\{z_n\}$, and consequently the poles $\{p_n\}$ of the sought approximation. Once these poles are known, the residues $R_n$ and the direct coupling matrix $H_\infty$ in (1) are computed componentwise as

$$y_{ij}(t) \simeq H_{\infty,ij}x_j(t) + \sum_{n=1}^{N} R_{n,ij}\widehat{x}_{n,j}(t), \quad i,j=1,\ldots,P, \tag{6}$$

with $\widehat{x}_{n,j}(t)$ defined as in (5) with $\{q_n\}$ replaced by the estimated poles $\{p_n\}$. All residue entries are computed by solving $P$ decoupled least squares problems, one for each transfer matrix column.

The final step in the algorithm is the identification of a state-space realization

$$\begin{cases} \frac{d}{dt}w(t) &= Aw(t) + Bx(t) \\ y(t) &= Cw(t) + Dx(t) \end{cases} \tag{7}$$

from the poles and residues representation in (1). This realization is needed for the application of passivity check and compensation algorithm described in next Section. The construction of the above state-space system is a standard problem and is not detailed here. We adopt the robust procedure developed in [1], which is based on the singular value decomposition.

## 3 Passivity and Hamiltonian matrices

We consider in this paper a multiport characterized via transient scattering waves. Therefore, the transfer matrix $H(s)$ coincides with the scattering matrix of the structure. Passivity is guaranteed if this matrix is unitary bounded at all frequencies, or equivalently, if the set $\sigma(H(j\omega))$ of all its singular values is not larger than one at any frequency. Of course, a frequency sweep can be used to check this condition. However, this procedure may be computationally expensive if the number of ports is large, and it may lead to wrong results if the sampling of the frequency axis is not accurate.

The following theorem, which we report from [2] without proof, motivates the introduction of Hamiltonian matrices for passivity characterization, since it provides the background for a purely algebraic passivity test. If we denote with $\lambda(\cdot)$ the set of eigenvalues of a given matrix, we have

**Theorem 1** *(Scattering representation). Assume $A$ has no imaginary eigenvalues, $\gamma > 0$ is not a singular value of $D$, and $\omega_0 \in I\!R$. Then, $\gamma \in \sigma(H(j\omega_0))$ if and only if $j\omega_0 \in \lambda(M_\gamma)$, where*

$$M_\gamma = \begin{pmatrix} A - BR^{-1}D^TC & -\gamma BR^{-1}B^T \\ \gamma C^TS^{-1}C & -A^T + C^TDR^{-1}B^T \end{pmatrix}, \tag{8}$$

*and $R = (D^TD - \gamma^2 I)$ and $S = (DD^T - \gamma^2 I)$.*

Matrix $M_\gamma$ is said to be Hamiltonian due to its particular block structure. This theorem allows to compute the exact frequencies (if any) at which the singular values of the transfer matrix cross or touch any given threshold $\gamma$. These frequencies are exactly the imaginary eigenvalues of the associated Hamiltonian matrix. The value of interest for the threshold is here the critical level for passivity (i.e., $\gamma = 1$). In case the set of imaginary eigenvalues of $M_{\gamma=1}$ is not empty, a passivity violation is likely. An accurate passivity characterization is provided by the following theorem [4, 5], which we report without proof. The theorem is based on a first-order perturbation of the Hamiltonian matrix

$$M_\gamma = M_1 + (\gamma - 1)M'_1 + \dots \tag{9}$$

induced by small variations of the threshold $\gamma$ around the critical level. We have the following

**Theorem 2** *(Scattering representation) Let the $j\omega_i$ denote the i-th imaginary eigenvalue of the Hamiltonian matrix $M_\gamma$ in (8) at the critical level $\gamma = 1$. Let all these imaginary eigenvalues be simple, sorted in ascending order, and let*

$$\xi_i = \frac{j\, w_i^T v_i}{w_i^T M'_1 v_i}, \tag{10}$$

*where $w_i$, $v_i$ are the left and right eigenvectors of $M_1$ associated to $j\omega_i$. Then, $H(j\omega)$ is locally passive for $\omega \in (\omega_{i-1}, \omega_i)$ if and only if*

$$\Lambda_i = \sum_{k \geq i} \mathrm{sgn}(\xi_k) = 0, \tag{11}$$

*where $\mathrm{sgn}(\cdot)$ extracts the sign of its argument and $\omega_0 = 0$.*

The term $\xi_i$ is related to the first-order perturbation of the corresponding eigenvalue $j\omega_i$. It can be shown [4, 5] that $\xi_i$ is exactly the slope of the singular value curves versus frequency when they cross the passivity threshold $\gamma = 1$. This theorem allows to pinpoint the exact frequency bands where passivity violations occur through an algebraic and numerically stable procedure.

We concentrate now on the passivity compensation in case some violation bands are found. This is equivalent to obtaining a new state-space system having an associated Hamiltonian matrix without imaginary eigenvalues. Therefore, the main algorithm is aimed at displacing these eigenvalues by some controlled amount in order to force them to move off the imaginary axis. This is achieved by a perturbation of the state matrix $C$ by an amount $dC$ to be determined. First-order perturbation is applied again. Each imaginary eigenvalue is displaced into a new location $j\omega_{i,p}$ via

$$j\omega_{i,p} - j\omega_i \simeq \frac{w_i^T dM v_i}{w_i^T v_i}, \tag{12}$$

where the Hamiltonian perturbation term $dM$ is related linearly to the correction $dC$ of the state matrix. Each eigenvalue is displaced towards the direction pointed by its slope $\xi_i$, by an amount that insures applicability of the first-order perturbation assumptions. This procedure leads to another linear least squares problem which gives the corrected state matrix. If passivity is still not satisfied, the procedure is iterated until a passive system is obtained [4, 5].

## 4 Examples

The first example is a single via interconnect crossing a solid metal plane (Fig. 1). The two ports are defined between the vertical conductor and top/bottom ground planes. The structure has been meshed and analyzed via the Partial Element Equivalent Circuit (PEEC) method. A time-domain full-wave formulation has been adopted to obtain the scattering port responses (referenced to 50Ω loads) to a unitary triangle pulse excitation with a 10 ps rise and fall time. These responses have been processed by TDVF using a 5-pole approximation. The resulting state-space realization is non-passive. However, this passivity violation has been corrected by the Hamiltonian-based algorithm as depicted in the middle panel of Fig. 1. The three sets of transient responses (original, non-passive macromodel, passive macromodel) are depicted in the right panel of Fig. 1, showing excellent accuracy.

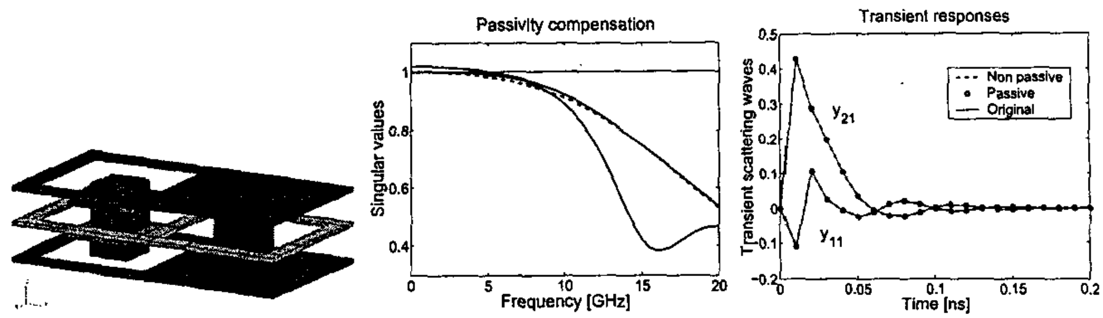


Figure 1: Left: single via structure. Middle: singular values for the non-passive macromodel (solid lines) and for the passive macromodel (dashed lines). Right: transient scattering responses ($y_{11}$: reflected, $y_{21}$: transmitted).

The second example is a commercial 14-pin SOIC package depicted in Fig. 2. The structure has $p = 28$ ports, half being defined between a corresponding pin and the printed circuit board on which the package is mounted, and half being defined between the bonding pad on the included die and the reference plane below the die itself. The structure has been meshed and analyzed with a full-wave electromagnetic solver based on the Finite-Difference Time-Domain (FDTD) method, obtaining a set of $28 \times 28$ transient scattering responses due to Gaussian pulse excitation having a 30 GHz frequency bandwidth. The combination of TDVF and Hamiltonian-based algorithms has been applied to obtain a passive macromodel. Some of the responses are depicted in Fig. 2, together with the frequency-dependent distribution of singular values. The results are excellent compared to the raw FDTD waveforms.

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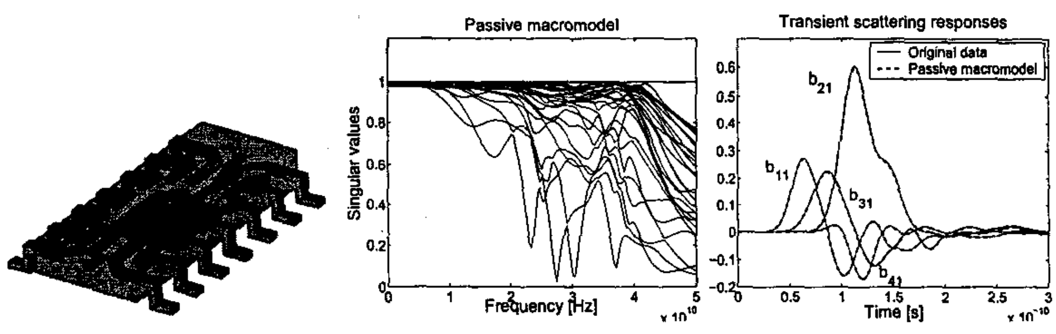


Figure 2: Left: SOIC-14 package. Middle: singular values of the passive macromodel. Right: some responses of the passive macromodel compared to the raw FDTD responses.