

# Higher-order vector bases for singular fields on curved 2D-elements.

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Curl-conforming functions are useful in the FEM solution of the transverse vector Helmholtz equation, whereas divergence-conforming functions are used in the Moments Method solution of surface integral equations. This work improves on the results of a previous work (R.D. Graglia, G. Lombardi, *2002 IEEE AP-S Symp. Digest*, vol. I, pp. 62-65) and presents higher-order singular curl- and divergence-conforming bases on curved triangular and quadrilateral elements, directly defined in their parent space. The method used to construct such bases is simple and general, and can be used for any order. Several issues are addressed including completeness of the bases and number of degrees of freedom. Our bases incorporate the edge condition and are able to approximate the unknown field components in the neighborhood of the edge of a wedge for any order of the singularity coefficient  $\nu$ , that is supposed given and known *a priori*. The wedge can be penetrable in the curl-conforming case, while it is supposed metallic in the divergence conforming case. For metal wedges of aperture angle  $\alpha$ , one has  $\nu = \pi/(2\pi - \alpha)$ . Our curl (divergence) conforming singular functions are compatible with standard  $p$ -th order vector functions in adjacent elements (R.D. Graglia, D.R. Wilton and A.F. Peterson, *IEEE Trans. Antennas Propagat.*, vol. 45, no. 3, pp. 329–342, 1997), and guarantee tangential (normal) continuity along the edges of the elements allowing for the discontinuity of normal (tangential) components, adequate modelling of the curl (divergence), and removal of spurious modes (solutions). The Galerkin form of FEM to study with curl-conforming elements a homogeneous circular waveguide of unit radius with a baffle extending to its center has been implemented. The figure reports the relative errors on  $k_z^2$  for the lowest order TE mode, that at the edge vertex supports singular fields with  $\nu = 1/2$ .

The mesh used for these simulations required to increase only by 20 the number of degrees of freedom in order to switch from regular to singular bases. The results obtained with regular vector functions of order  $p = 2, 3$  are less precise than those obtained with singular vector bases of order  $[p = 2, 3; s = 0]$ . More results will be presented at the Conference.

