Angular timing error of a gear set

A. Vigliani

Dipartimento di Meccanica - Politecnico di Torino
C.so Duca degli Abruzzi, 24 - 10129 Torino - ITALY
E-mail: *alessandro.vigliani@polito.it*

Keywords

Abstract

Introduction

A simple solution to the problem of measuring the position of actuators in servomechanisms is achieved by means of a rotary pick-off that transforms an angular position into an electrical voltage. Depending on the application, there are two possible layouts: the pick-off can be directly installed on the shaft of the actuator, or it can be mounted on an auxiliary shaft that derives the motion from the actuator by means of a gear set. A usual system, for instance, is represented by a linear actuator connected to a pinion and a rack.
Despite their simplicity, these systems present problems related to the way the mechanical link is achieved. In fact, if the measure chain consists of two meshing gears, an error in the measure of the actuator position arises if the distance between the axes of the two gears changes after the initial rigging. A change of the distance between the axes may be the result of a temperature change and a different thermal expansion of the gears and housing materials. A change in the distance between the centerlines of two involute gears doesn't prevent the system from working, for gears can still operate properly, but with a different pressure line and two different pitch circles. The geometry of involute gears is such that the gear ratio is still the same as before, but the relative angular position of the two gears is shifted by a small amount. This is clearly shown in FIG.1. Suppose the initial contact point between the teeth was P. Then an increase x of the distance between the two axes shifts the contact point to P'. As a result, gear 2 rotates of an angle α with respect to gear 1. In the following analysis it is assumed that gear 1 is fixed and gear 2 rotates, but the same results are obtained if the opposite condition is considered.
In engineering practice, it is customary to compute this angular error with the assumption that the pressure angle does not change. This is actually true only for the case of a pinion meshing with a rack: in these conditions a simple relationship can be obtained, as shown later in the course of the analysis. In general, however, a more complex relationship applies, as it will be derived in the following analysis.

Analysis

To obtain the relative angular rotation α as a function of the increase x of the distance between the two axes, the geometrical properties of involute gears must be considered [1]. Let ρ_1 and ρ_2 be the radii of the base circles of the two gears (FIG.1); furthermore, in the initial condition let θ be the pressure angle and H_1H_2 the pressure line that crosses in P the line O_1O_2 connecting the gears centers.

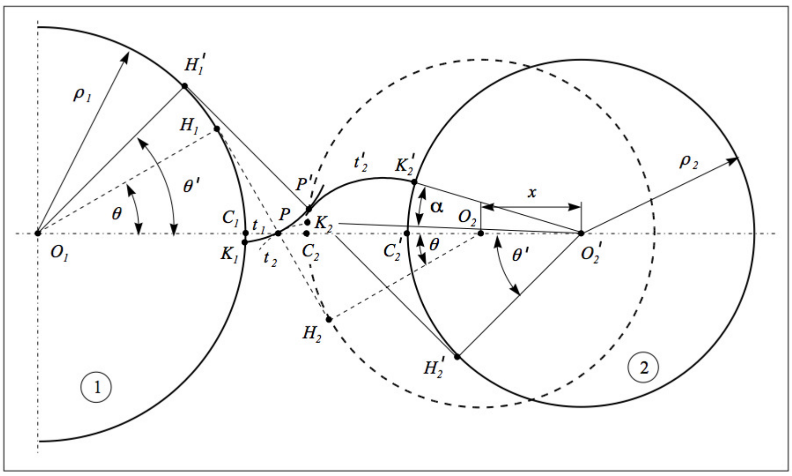

Figure 1: Geometry of two meshing gears

Consider the involute t_1 and t_2 of the two teeth that are in contact at point P : because of the well known properties of involutes [2], it is possible to write

$$\overline{H_1H_2} = \overset{\frown}{H_1K_1} + \overset{\frown}{H_2K_2} = (\rho_1 + \rho_2)\tan\theta,$$

where K_1 and K_2 are the roots of the two teeth t_1 and t_2. The arc $\overset{\frown}{K_2C_2}$ is given by

$$\overset{\frown}{K_2C_2}=\overset{\frown}{H_2K_2} -\rho_2\theta = \rho_2(\tan\theta - \theta). \tag{1}$$

If the center O_2 of gear 2 is moved of the distance x to O_2', the new pressure line is $H_1'H_2'$ and the new pressure angle is θ'. If gear 1 is supposed to be fixed, gear 2 must rotate of an angle α to keep its teeth in contact with the teeth of gear 1. In the new position, the tooth surface is now t_2', while the contact point between t_1 and t_2 is P'.
The arc $\overset{\frown}{K_2'C_2'}$ is given by

$$\overset{\frown}{K_2'C_2'}=\overset{\frown}{H_2'K_2'} -\rho_2\theta'$$

where

$$\overset{\frown}{H_2'K_2'}=\overline{H_1'H_2'}-\overline{H_1'P'}=\overline{H_1'H_2'}-\overline{H_1'K_1'}=(\rho_1+\rho_2)\tan\theta' - \{\rho_1\tan\theta + \rho_1\tan(\theta'-\theta)\}.$$

Thus it follows that

$$\overset{\frown}{K_2'C_2'}=(\rho_1+\rho_2)\tan\theta' - \{\rho_1\tan\theta + \rho_1\tan(\theta'-\theta)\} - \rho_2\theta'. \tag{2}$$

The angular rotation α caused by the distance increase x is

$$\alpha = \frac{\overset{\frown}{K_2'C_2'} - \overset{\frown}{K_2C_2}}{\rho_2}. \tag{3}$$

By introducing (1) and (2) into (3), we get

$$\alpha = \frac{\rho_1+\rho_2}{\rho_2}[\tan(\theta'-\theta') - \tan(\theta-\theta)]. \tag{4}$$

The initial distance between the axes is

$$\overline{O_1O_2} = \frac{\rho_1+\rho_2}{\cos\theta}$$

while the final distance is

$$\overline{O_1O_2'} = \overline{O_1O_2} + x = \frac{\rho_1+\rho_2}{\cos\theta'}.$$

Therefore

$$\frac{1}{\cos\theta'} = \frac{1}{\cos\theta} + \frac{x}{\rho_1+\rho_2}.$$

It is convenient to re-write this expression by introducing the number of teeth z_2, the modulus $m = \dfrac{2\rho}{z\cos\theta}$ and the gear ratio $\tau = \rho_2/\rho_1$. We obtain

$$\frac{1}{\cos\theta'} = \frac{1}{\cos\theta}\left[1 + \frac{x}{m}\frac{2}{z_2\left(1+\dfrac{1}{\tau}\right)}\right]. \tag{5}$$

From equations (4) and (5) it is thus possible to compute α and θ'. The new pressure angle is

$$\cos\theta' = \frac{\cos\theta}{1 + \dfrac{x}{m}\dfrac{2}{z_2\left(1+\dfrac{1}{\tau}\right)}} \tag{6}$$

while the angular rotation α is

$$\alpha = \left(1 + \frac{1}{\tau}\right)\left[(\tan\theta' - \theta') - (\tan\theta - \theta)\right]. \tag{7}$$

These expressions give the value of the angular rotation α as a function of the change of distance x, and are plotted in the diagrams in FIG.2:4, which show the influence of gear parameters on the angular error α. From FIG.2 it can be noted that an increase in the number of teeth causes the angle to decrease; on the contrary a larger value of the variation of the distance between the two gears results in a bigger error α. The same effect occurs when the pressure angle θ varies from 15° to 30°, as visible in FIG.3. Finally, it can be noted that the influence of the gear ratio τ is negligible (see FIG.4).

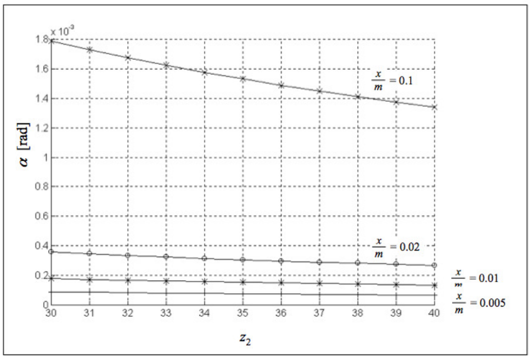

Figure 2: Influence of a change x of distance between the axes on the angular error α, plotted versus the number of teeth z_2. (Gear ratio $\tau = 1$, pressure angle $\theta = 15°$)

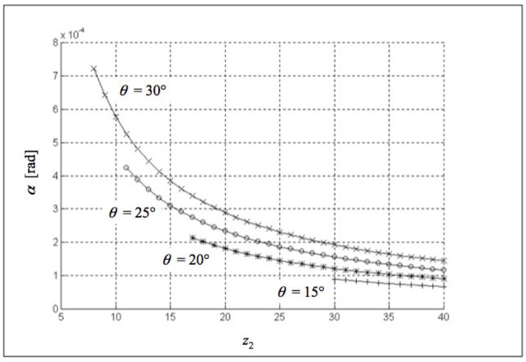

Figure 3: Influence of the pressure angle θ on angular displacement α (dimensionless change of distance $x/m = 0.1$)

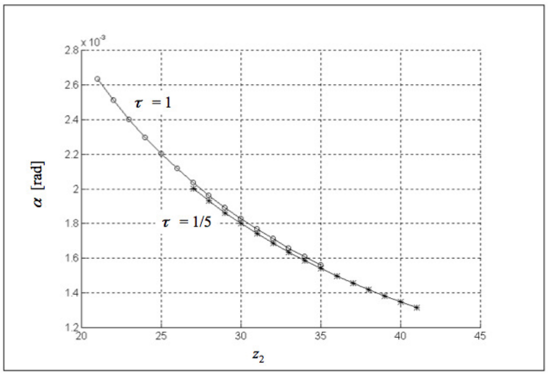

Figure 4: Influence of gear ratio τ on the angular error α for a gear set with $\theta = 15°$ and with a distance change $x/m = 0.1$.

For pinion and rack systems equations (4) and (5) become indeterminate. In this case the angular rotation of the pinion relative to a fixed rack can be easily determined, since there is no change in the pressure angle θ (FIG.5).
If the pinion is moved of a distance x away from the rack, the angular rotation α is given by

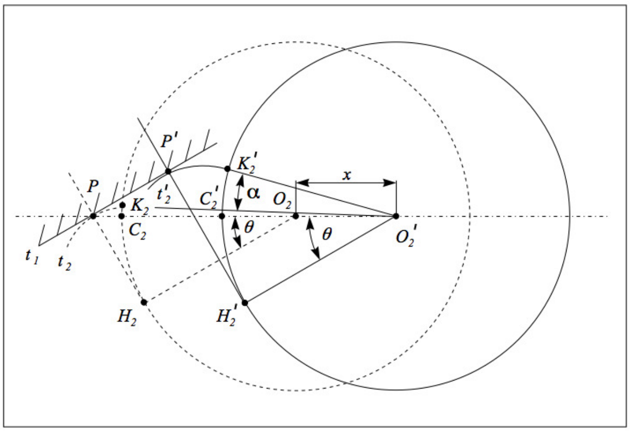

Figure 5: Geometry of rack and pinion

$$\alpha = \frac{\overset{\frown}{K_2'C_2'} - \overset{\frown}{K_2C_2}}{\rho_2}.$$

The two arcs $\overset{\frown}{K_2'C_2'}$ and $\overset{\frown}{K_2C_2}$ are given by

$$\begin{aligned}
\overset{\frown}{K_2C_2} &= \rho_2(\tan\theta - \theta) \\
\overset{\frown}{K_2'C_2'} &= \overset{\frown}{K_2'H_2'} - \overset{\frown}{C_2'H_2'} = \overline{H_2'P'} - \overset{\frown}{C_2'H_2'} = \rho_2 \tan\theta + x\sin\theta - \rho_2\theta.
\end{aligned}$$

Thus we obtain

$$\alpha = \frac{\overset{\frown}{K_2'C_2'} - \overset{\frown}{K_2C_2}}{\rho_2} = \frac{x\sin\theta}{\rho_2},$$

and introducing the number of teeth z_2, we have

$$\alpha = \frac{x}{m}\frac{2}{z_2}\tan\theta. \tag{8}$$

The behavior is similar to the two gear's one: an increase in the number of teeth results in a smaller rotation α, while larger pressure angles cause larger errors.

As an example, we compute the error in the case of two equal gears with $z_1 = z_2 = 15$, $m = 2.5$ mm and $\theta = 20°$. The nominal distance i between axes is $i = 37.5$ mm. An increase of the distance $x = 0.1$ mm, which corresponds approximately to a temperature increase of $200°C$ in a system with steel gears and aluminium housing, leads to an angular error that can be estimated as follows. From eqn. (5) we have

$$\theta' = 20.4146°$$

and from eqn (4):

$$\alpha = 0.002 \text{ rad} = 0.112°.$$

<u>Conclusions</u>

Last result indicates that the position error induced by the temperature increase is about 0.1°, which is a relatively small figure. However, a common requirement of position indication systems using rotary pick-offs is to have an accuracy of 0.5°. Therefore the error caused by the increase of the distance between the axes of the gears corresponds in this example to about 20% of the total accuracy band, which is an appreciable part of the total error allowance.
It is worth noting that the gear ratio doesn't influence the error, while gears with a number of teeth close to the minimum and with large pressure angles are likely to be affected by considerable position errors. Therefore it is suggested to employ gears with little pressure angles and with a number of teeth larger than the minimum required for correct meshing.

References

1. G. Niemann and H. Winter, *Elementi di Macchine*, Scienza e Tecnica, Milano, (1964)
2. G. Henriot, *Traité Théorique et Praticque des Engranages*, Dunod, Paris, (1960)