Lubricated bearings: determination of dynamic coefficients according to Warner's Theory

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Abstract

Nomenclature

D	=	Journal diameter	n	=	Eccentricity ratio	δ	=	Radial clearance
L	=	Journal length	$\dot{n}$	=	Squeeze speed	ε	=	Eccentricity
W	=	Load	z	=	Axial co–ordinate	μ	=	Dynamic viscosity
h	=	Fluid film thickness	$\dot{\beta}$	=	Whirl speed	ω	=	Angular speed
$()_0$	=	Stationary configuration						

1 Introduction

It is well known that rotors supported by lubricated bearings can undergo instability due, in most cases, to the forces exerted by the fluid. Stability is generally analyzed using two different approaches. One approach consists of numerical integration of motion equations so that the rotor behavior can be analyzed as a function of time, while the other approach is characterized by the linearization of the fluid forces acting on the bearings in the neighborhood of a well determined stationary equilibrium position so that asymptotic stability conditions can be determined. In the latter case it is necessary to know the analytical expressions of the stiffness and damping coefficients to linearize the hydrodynamic forces.

It is evident that the first approach can provide more information to study the rotor dynamic behavior, e.g. the possibility to compute minimum fluid film thickness; moreover, this method is not affected by the consequences of linearization. However, it is worth noting that the resulting numerical code can be complex. It is well known the argument arisen between Trumpler and Poritsky [1] about the linearization necessary to determine the dynamic coefficients. However the authors agree with Poritsky when he says that "*while the effect of nonlinearity of the oil–film forces may limit the whirl amplitude to a finite value, (..) nonlinearity by itself can never restore complete stability in a range where the linear theory indicates instability*" [1]. Therefore, if determination of the system's stability is the only goal, it is undoubtedly more convenient to use dynamic coefficients.

The analytical expressions of dynamic coefficients can be determined from Ocvirk's theory, (*short bearing*), or from Sommerfeld's solution (*infinitely long bearing*) or from Warner's solution (*finite bearing*), consisting of Sommerfeld's solution corrected through a suitable "end leakage function".

In the literature it is possible to find the expressions for the dynamic coefficients for Ocvirk's solution [2] and for Sommerfeld's solution [3], but, to the authors' knowledge, expressions of the dynamic coefficients for Warner's solution have never been published. This fact is probably due to the tedious nature of the analysis more than to numerical difficulties. Hence, the aim of the present work is to determine, in a stationary reference frame, the analytical expressions of the dynamic coefficients according to Warner's theory, giving a further tool for stability analysis.

2 Warner's model

Warner's model [4] is based on the hypothesis that the oil pressure inside the bearing can be written as

$$p(\vartheta, z) = p_\infty(\vartheta)\left[1 - \cosh\left(\frac{2z}{L}A\right)/\cosh(A)\right] \tag{1}$$

where $A = \lambda L/D$, z is the axial co–ordinate with the origin located at the middle of the bearing and

$$\lambda^2 = \int_{\vartheta_1}^{\vartheta_2} h^3\left(\frac{dp_\infty}{d\vartheta}\right)^2 d\vartheta \Big/ \int_{\vartheta_1}^{\vartheta_2} h^3 p_\infty^2 \, d\vartheta. \tag{2}$$

The angular displacements ϑ_1 and ϑ_2 identify the bearing arc where pressure is positive and can be determined by solving the system

$$\begin{cases} n\sin\vartheta_1\left(\omega - 2\dot{\beta}\right) - \dot{n}\left(2+n^2\right)\cos\vartheta_1 = 0 \\ n\cos\vartheta_1\left(\omega - 2\dot{\beta}\right) + \dot{n}\left(2+n^2\right)\sin\vartheta_1 \geq 0 \\ \vartheta_2 = \vartheta_1 + \pi \end{cases} \tag{3}$$

In eq.(2), $p_\infty(\vartheta)$ represents the pressure corresponding to an infinitely long bearing, given by

$$p_\infty - p(0) = 6\mu \left(\frac{D}{2\delta}\right)^2 \left[\left(\omega - 2\dot{\beta}\right)\frac{n\sin\vartheta}{2+n^2} - \dot{n}\cos\vartheta\right]\frac{2+n\cos\vartheta}{(1+n\cos\vartheta)^2} \tag{4}$$

The radial and tangential components of the oil force $\vec{W}$ acting on the journal are:

$$W_r = -3\mu D\left(\frac{D}{2\delta}\right)^2\left[\left(\omega - 2\dot{\beta}\right)\frac{n}{2+n^2}I_{r1} - \dot{n}I_{r2}\right]F(\lambda) \tag{5}$$

$$W_\vartheta = -3\mu D\left(\frac{D}{2\delta}\right)^2\left[\left(\omega - 2\dot{\beta}\right)\frac{n}{2+n^2}I_{\vartheta 1} - \dot{n}I_{\vartheta 2}\right]F(\lambda) \tag{6}$$

where

$$I_{r1} = I_{\vartheta 2} = \int_{\vartheta_1}^{\vartheta_2} \sin\vartheta\cos\vartheta\frac{2+n\cos\vartheta}{(1+n\cos\vartheta)^2}d\vartheta\partial \quad I_{r2} = \int_{\vartheta_1}^{\vartheta_2}\cos^2\vartheta\frac{2+n\cos\vartheta}{(1+n\cos\vartheta)^2}d\vartheta \tag{7}$$

$$I_{\vartheta 1} = \int_{\vartheta_1}^{\vartheta_2}\sin^2\vartheta\frac{2+n\cos\vartheta}{(1+n\cos\vartheta)^2}d\vartheta \qquad F(\lambda) = L\left[1 - \frac{\tanh(A)}{A}\right] = L\,G_0 \tag{8}$$

The forces components given in Eqs. (5) and (6), in a stationary reference system (x, y), assume the following expressions:

$$W_x = W_r\cos\gamma - W_\vartheta\sin\gamma \qquad \text{and} \qquad W_y = W_r\sin\gamma + W_\vartheta\cos\gamma \tag{9}$$

It is then possible to compute the stiffness and damping dynamic coefficients, K_{ij} and C_{ij} respectively, in any given equilibrium configuration. It is worth noting that the parameter λ is a function of n, $\dot{n}$ and $\dot{\beta}$; here we give expressions of λ and of its partial derivatives.

$$(\lambda)_0 = \sqrt{\frac{2n^2(2+n^2)}{(1-n^2)^{1/2}(2+3n^2) - 2(1-n^2)}} \qquad \left(\frac{\partial\lambda}{\partial\dot{\beta}}\right)_0 = 0 \tag{10}$$

$$\left(\frac{\partial\lambda}{\partial n}\right)_0 = \frac{4\left(1-n^2\right)^{1/2}\left(n^4 - 2n^2 - 2\right) + 8 + 4n^2 + 6n^4 - 3n^6}{\left[2(2+n^2)\right]^{1/2}(1-n^2)^{5/4}\left[2+3n^2-2(1-n^2)^{1/2}\right]^{3/2}} = G_1 \tag{11}$$

$$\left(\frac{\partial\lambda}{\partial\dot{n}}\right)_0 = 2\,\frac{2+n^2}{1-n^2}\,\frac{4n^2+n^4+\lambda^2n^2\left(1-n^2\right)\left[\frac{2}{3}+\frac{2}{n^2}+\frac{1}{n^3}\ln\frac{1-n}{1+n}\right]}{\omega\lambda\pi\left[2+3n^2-2(1-n^2)^{1/2}\right]} = G_3 \tag{12}$$

In order to simplify the coefficient expressions, let us define:

$$G_2 = \frac{G_1}{\lambda^2}\left[\frac{A}{\cosh^2(A)} - \tanh(A)\right] \quad \text{and} \quad G_4 = \frac{G_3}{\lambda^2}\left[\frac{A}{\cosh^2(A)} - \tanh(A)\right] \tag{13}$$

$$\bar{K}_{ij} = K_{ij}\frac{4}{\mu D\omega}\left(\frac{\delta}{L}\right)^3 \qquad \text{and} \qquad \bar{C}_{ij} = C_{ij}\frac{4}{\mu D}\left(\frac{\delta}{L}\right)^3 \tag{14}$$

Finally, the following expressions hold for the dynamic coefficients:

$$\bar{K}_{xx} = 3\left(\frac{D}{L}\right)^2 G_0 \frac{2\,n\left[\pi^2\left(2-n^2\right)+4\,n^2\right]}{\left(1-n^2\right)\left(2+n^2\right)\left[4\,n^2+\pi^2\left(1-n^2\right)\right]} \tag{15}$$

$$\bar{K}_{xy} = 3\left(\frac{D}{L}\right)^2 G_0\,\pi \frac{\pi^2\left(1-n^2\right)^2-4\,n^4}{\left(1-n^2\right)^{3/2}\left(2+n^2\right)\left[4\,n^2+\pi^2\left(1-n^2\right)\right]} \tag{16}$$

$$\begin{aligned}\bar{K}_{yx} &= 3\left(\frac{D}{L}\right)^2 \left\{\frac{D}{L}\,G_2 \frac{\pi\,n}{\left(1-n^2\right)^{1/2}\left(2+n^2\right)}+\right. \\ &\left. -G_0\,\pi \frac{8\,n^2\left(2+n^4\right)+\pi^2\left(2-3\,n^2+3\,n^4-2\,n^6\right)}{\left(1-n^2\right)^{3/2}\left(2+n^2\right)^2\left[4\,n^2+\pi^2\left(1-n^2\right)\right]}\right\}\end{aligned} \tag{17}$$

$$\begin{aligned}\bar{K}_{yy} &= 3\left(\frac{D}{L}\right)^2 \left\{-\frac{D}{L}\,G_2 \frac{2\,n^2}{\left(1-n^2\right)\left(2+n^2\right)}+\right. \\ &\left. +G_0 \frac{16\,n^3\left(2+n^4\right)+2\,n\,\pi^2\left(2-3\,n^2+3\,n^4-2\,n^6\right)}{\left(1-n^2\right)^2\left(2+n^2\right)^2\left[4\,n^2+\pi^2\left(1-n^2\right)\right]}\right\}\end{aligned} \tag{18}$$

$$\bar{C}_{xx} = 3\left(\frac{D}{L}\right)^2 G_0 \frac{\pi\left(\pi^2-4\,n^2\right)}{\left(1-n^2\right)^{1/2}\left[4\,n^2+\pi^2\left(1-n^2\right)\right]} \tag{19}$$

$$\bar{C}_{xy} = 3\left(\frac{D}{L}\right)^2 G_0 \frac{2\,n\left(4\,n^2-\pi^2\right)}{\left(1-n^2\right)\left[4\,n^2+\pi^2\left(1-n^2\right)\right]} \tag{20}$$

$$\begin{aligned}\bar{C}_{yx} &= 3\left(\frac{D}{L}\right)^2 \left\{\frac{D}{L}\,\omega\,G_4 \frac{\pi\,n}{\left(1-n^2\right)^{1/2}\left(2+n^2\right)}+\right. \\ &\left. -G_0 \frac{2\,n\left(8\,n^2-2\,\pi^2-2\,\pi^2\,n^2+\pi^2\,n^4\right)}{\left(1-n^2\right)\left(2+n^2\right)\left[4\,n^2+\pi^2\left(1-n^2\right)\right]}\right\}\end{aligned} \tag{21}$$

$$\begin{aligned}\bar{C}_{yy} &= 3\left(\frac{D}{L}\right)^2 \left\{-\frac{D}{L}\,\omega\,G_4 \frac{2\,n^2}{\left(1-n^2\right)\left(2+n^2\right)}+\right. \\ &\left. +2\,\pi\,G_0 \frac{2\,n^2\left(6-2\,n^2-n^4\right)+\pi^2\left(1-n^2\right)^2}{\left(1-n^2\right)^{3/2}\left(2+n^2\right)\left[4\,n^2+\pi^2\left(1-n^2\right)\right]}\right\}\end{aligned} \tag{22}$$

References

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