Quantum Information in Semiconductors: Noiseless Encoding in a Quantum-Dot Array

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A potential implementation of quantum-computation schemes in semiconductor-based structures is proposed. In particular, an array of quantum dots is shown to be an ideal quantum register for a noiseless information encoding. In addition to the suppression of phase-breaking processes in quantum dots due to the well-known phonon bottleneck, we show that a proper quantum encoding allows one to realize a decoherence-free evolution on a time scale long compared to the femtosecond scale of modern ultrafast laser technology. This result might open the way to the realization of semiconductor-based quantum processors. [S0031-9007(98)07746-1]

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The physical implementation of any computing device taking actual advantage from the additional power provided by quantum theory [1] is extremely demanding. In principle one should be able to perform, on a system with a well-defined state space, long coherent quantum manipulations (*gating*), precise quantum-state synthesis, and detection as well. Ever since the very beginning it has been recognized that the major obstacle arises from the unavoidable open character of any realistic quantum system. The coupling with external (i.e., noncomputational) degrees of freedom spoils the unitary structure of quantum evolution, which is the crucial ingredient in quantum computation (QC). This is the well-known decoherence problem [2]. The possibility to partly overcome such a difficulty by means of the *active stabilization* pursued by quantum error correction is a definite success of theoretical QC [3]. Nevertheless, mostly due to the necessity of low decoherence rates, the up-to-date proposals for experimental realizations of quantum processors are based on quantum optics as well as atomic and molecular systems [1]. Indeed, the extremely advanced technology in these fields allows for the manipulations required in simple QC's. It is, however, generally believed that future applications, if any, of quantum information may hardly be realized in terms of such systems, which do not permit the large-scale integration of existing microelectronics technology. In contrast, in spite of the serious difficulties related to the "fast" decoherence times, a solid-state implementation of QC seems to be the only way to benefit synergetically from the recent progress in ultrafast optoelectronics [4] as well as in nanostructure fabrication and characterization [5]. To this end, the primary goal is to design quantum structures and encoding strategies characterized by "long" decoherence times, compared to the typical time scale of gating. The first well-defined semiconductor-based proposal of QC [6] relies on spin dynamics in quantum dots (QD); it exploits the low decoherence of spin degrees of freedom in comparison to the one of charge excitations. However, the proposed manipulation schemes are based on spin dynamics control that would allow a number of gate operations within the decoherence time smaller than the one desired by theoretical QC. On the other hand gating of charge excitations could be envisioned by resorting to *present* ultrafast laser technology, which is now able to generate electron-hole quantum states on a subpicosecond time scale and to perform on such states a variety of coherent-carrier-control operations [4]. More specifically, this suggests the idea of designing fully optical gating schemes based on interqubit coupling mechanisms as, e.g., optical nonlinearities and dipole-dipole coupling. In this respect decoherence times on nano/microsecond scales can be regarded as long ones.

Following this spirit, in this Letter we investigate a semiconductor-based implementation of the noiseless quantum encoding proposed in [7]. The idea is that, in the presence of a sort of "coherent" environmental noise, one can identify states that are hardly corrupted rather than states that can be easily corrected. More specifically, we show that by choosing as quantum register an array of quantum dots [5] and by preparing the QD system in proper multidot quantum states, it is possible to strongly suppress electron-phonon scattering, which is known to be the primary source of decoherence in semiconductors [8]. The physical system under investigation consists of an array of N identical quantum dots, whose Hamiltonian can be schematically written as $H = H_c + H_p + H_{cp}$. The term $H_c = \sum_i H_c^i = \sum_{i\alpha} \epsilon_\alpha a_{i\alpha}^\dagger a_{i\alpha}$ describes the noninteracting carrier system, i and α being, respectively, the QD index and energy level, while $H_p = \sum_{\lambda q} \hbar\omega_{\lambda q} b_{\lambda \mathbf{q}}^\dagger b_{\lambda \mathbf{q}}$ is the free-phonon Hamiltonian, λ and $\mathbf{q}$ denoting, respectively, the phonon mode and wave vector. The last term accounts for the coupling of the carriers in the QD array with the different phonon modes of the crystal: $H_{cp} = \sum_{i\alpha,i'\alpha';\lambda\mathbf{q}}[g_{i\alpha,i'\alpha';\lambda\mathbf{q}} a_{i\alpha}^\dagger b_{\lambda\mathbf{q}} a_{i'\alpha'} + \mathrm{H.c.}]$. Here, $g_{i\alpha,i'\alpha';\lambda\mathbf{q}} = \tilde{g}_{\lambda\mathbf{q}} \int \phi_{i\alpha}^*(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}} \phi_{i'\alpha'}(\mathbf{r})\, d\mathbf{r}$ are the matrix elements of the phonon potential between the quasi-0D states $i\alpha$ and $i'\alpha'$. The explicit form of the

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coupling constant $\tilde{g}_{\lambda \mathbf{q}}$ depends on the particular phonon mode λ, e.g., acoustic, optical, etc. We will assume that only the two lowest energy levels in each dot ($\alpha = 0, 1$) will play a role in the quantum-computation dynamics [9]. The dynamics of this low-energy sector coupled with the phonon modes of the crystal is therefore mapped onto the one of N two-level systems (*qubits*) linearly coupled with the bosonic degrees of freedom $\lambda \mathbf{q}$, the latter representing the decoherence-inducing environment of the computational (i.e., carrier) subsystem. To describe the so obtained N-qubit register it is then convenient to adopt the spin formalism [10]: $\{\sigma_i^\eta\}_{i=1}^N$ ($\eta = \pm, z$) will denote the Pauli matrices spanning N *local* sl(2) algebras. It is also convenient to introduce the *global* sl(2) algebra generated by the collective spin operator $S^\eta = \sum_i \sigma_i^\eta$, ($\eta = \pm, z$). We will assume no direct phonon coupling between different qubits, i.e., $g_{i\alpha,i'\alpha';\lambda \mathbf{q}} = 0$ for $i \neq i'$; in order to obtain a closed-form equation for the reduced density matrix ρ describing our qubit array, one can trace out the phonon degrees of freedom by means of the standard Born-Markov approximation [11]. The resulting "master equation" is of the form $\rho = \mathcal{L}(\rho)$. Here, the Liouvillian superoperator $\mathcal{L}$ is given by the sum of two contributions: a unitary part $\mathcal{L}_u$, which preserves quantum coherence, plus a dissipative one $\mathcal{L}_d$, describing irreversible decoherence-dissipation processes. To better understand such separation, it is useful to introduce the Hermitian matrix $\Delta_{ii'}^{(\pm)}$ ($\Gamma_{ii'}^{(\pm)}$) as the real (imaginary) part of the matrix

$$L_{ii'}^{(\pm)} = \sum_{\lambda \mathbf{q}} \frac{g_{i,\lambda \mathbf{q}} \bar{g}_{i',\lambda \mathbf{q}}}{E - \hbar \omega_{\lambda \mathbf{q}} - i0^+} [n_{\lambda \mathbf{q}} + \theta(\mp)], \quad (1)$$

where $g_{i,\lambda \mathbf{q}} \equiv g_{i1,i0,\lambda \mathbf{q}}$ and $E = \epsilon_1 - \epsilon_0$. Here, $n_{\lambda \mathbf{q}}$ and θ are, respectively, the Bose thermal distribution and the step function. As we will see, this matrix encodes the spatial correlations of the quantum register defining the *effective topology* that can be probed by the phononic environment. In particular, the spectral data of $\mathbf{L}^{(\pm)}$ contain information about the existence of *subdecoherent* subspaces [11]. More specifically, one finds $\mathcal{L}_u(\rho) = i/\hbar[\rho, H_c + \delta H_c]$, where the phonon-induced renormalization δH_c to our free-qubit Hamiltonian H_c (which in our spin formalism reads $H_c = ES^z$) is given by $\delta H_c = \sum_{\eta=\pm} \sum_{ii'=1}^N \Delta_{ii'}^{(\eta)} \sigma_i^{-\eta} \sigma_{i'}^\eta$. These contributions are usually referred to as the Lamb-shift term. In contrast, the dissipative (nonunitary) component of the Liouvillian is given by $\mathcal{L}_d(\rho) = \sum_{\eta=\pm} \mathcal{L}_d^\eta(\rho)$. where the emission ($\eta = -$) and the absorption ($\eta = +$) terms can be cast into the compact form

$$\mathcal{L}_d^\eta(\rho) = \frac{1}{\hbar} \sum_{ii',\eta=\pm} \Gamma_{ii'}^{(\eta)}([\sigma_i^\eta \rho, \sigma_{i'}^{-\eta}] + [\sigma_i^\eta, \rho \sigma_{i'}^{-\eta}]) . \quad (2)$$

The diagonal ($i = i'$) terms describe the usual carrier-phonon scattering processes in a single quantum dot, as obtained from Fermi's golden rule; in contrast, the off-diagonal elements are not positive-definite and describe collective coupling effects, which play a crucial role in the realization of a decoherence-free evolution [7].

In order to study the corruption of the information encoded in the initial *pure* preparation $\rho = |\psi\rangle\langle\psi|$, it is useful to introduce the following quantity: The *fidelity* $F(t) \equiv \langle\psi|\rho(t)|\psi\rangle$. We define the first order decoherence time (rate) τ_1 (τ_1^{-1}) in terms of the short-time expansion $F(t) = 1 - t/(\tau_1) + o(t^2)$. If $|\psi\rangle$ is a total spin eigenstate, it is easy to check that $\hbar\tau_1^{-1}[|\psi\rangle] = \langle\psi|H_{\rm eff}|\psi\rangle$, where the effective Hamiltonian $H_{\rm eff}$ has the same structure of the Lamb-shift term δH_c with $\mathbf{\Gamma}$ replacing $\mathbf{\Delta}$ [see Eq. (1)]. Notice that (i) $\tau_1 \geq 0$ (i.e., $H_{\rm eff}$ is a positive operator); (ii) in this (first-order) decoherence time the Lamb-shift terms do *not* play any role.

To exemplify the collective nature of the decoherence process let us consider the decoherence rate for the states $|\psi_{\mathcal{D}}\rangle \equiv \otimes_{(i,i')\in\mathcal{D}}(|01\rangle - |10\rangle)_{ii'}$ (here, $\mathcal{D}$ is a dimer partition of the qubit array) that are *singlets* of the global sl(2) algebra [12]. In this case one gets $\tau_1^{-1} = \sum_{\eta=\pm}(2\tau_\eta)^{-1} f_{\mathcal{D}}(\mathbf{\Gamma}^{(\eta)})$, where $\hbar\tau_\eta^{-1} = N\Gamma_{11}^{(\eta)}$ is the (maximal) decoherence rate for N uncorrelated qubits and

$$f_{\mathcal{D}}(\mathbf{\Gamma}) = 1 - \frac{2}{N} \mathrm{Re} \sum_{(i,i')\in\mathcal{D}} \Gamma_{ii'}/\Gamma_{11} . \quad (3)$$

The quantity $f_{\mathcal{D}}$ contains the information about the degree of multiqubit correlation in the decay process. Suppose now that one is able to design our qubit array in such a way that $f_{\mathcal{D}}(\mathbf{\Gamma}) = 0$ then $1/\tau_1 = 0$ that means that our coding state $|\psi\rangle$ is on a short time scale unaffected by decoherence; moreover if $|\psi\rangle$ is annihilated by $\mathcal{L}_u$ as well it turns out to be *noiseless:* it does not suffer any evolution at all. Generally speaking there are two extreme cases in which $H_{\rm eff}$ is easily diagonalized. (i) If $\mathbf{\Gamma}^{(\eta)} \propto \mathbf{I}$, then $H_{\rm eff}$ has a trivial kernel and the qubits decohere independently: no subdecoherent encoding exists. In this limit the environment "sees" a register endowed with a discrete topology. The same is true for an initial *unentangled* preparation (i.e., simple tensor product). (ii) $\Gamma_{ii'}^{(\pm)} =$ const the register gets "pointlike." In this case, the effective Hamiltonian is bilinear in the S^η's and the subdecoherent subspace coincides with singlet sector of the global sl(2) algebra [7].

The above theoretical analysis has been applied to state-of-the-art quasi-0D semiconductor heterostructures. In particular, a linear array of vertically stacked quantum dots—along the growth (z) direction—has been considered; more specifically, the array is formed by GaAs/AlGaAs structures similar to that of [13] alligned on the same z axis. The three-dimensional confinement potential giving rise to the quasi-0D single-particle states ϕ_α is properly described in terms of a quantum-well (QW) profile along the growth direction times a two-dimensional (2D) parabolic potential in the normal plane. Since the width d of the GaAs QW region is typically of the order of few nanometers, the energy splitting due to the quantization along the growth direction is much larger than the confinement energy E induced by the 2D parabolic potential (typically of a few meV). Thus, the two single particle states—states $|0\rangle$ and $|1\rangle$—realizing the *qubit* considered so far are given by products of the QW ground state times the ground or first excited state of the 2D parabolic potential [14], their energy splitting being equal to E.

As a starting point, let us discuss the role of carrier-phonon interaction in a single QD structure with $d = 4$ nm. Figure 1 shows the total (emission plus absorption) carrier-phonon scattering rate ($\Gamma_{ii}^{+} + \Gamma_{ii}^{-}$) at low temperature ($T = 10$ K) as a function of the energy spacing E. Since the energy range considered is smaller than the optical-phonon energy (36 meV in GaAs), due to energy conservation scattering with LO phonons is not allowed. Therefore, the only phonon mode λ which contributes to the rate of Fig. 1 is that of acoustic phonons. The latter has been evaluated starting from the explicit form of the carrier-phonon matrix elements G which, in turn, involve the 3D wave functions as well as the explicit form of the deformation-potential coupling $\tilde{g}$. To this end, a bulk phonon model in the long-wavelength limit has been employed [15]. Again, due to energy conservation, the only phonon wave vectors involved must satisfy $|\mathbf{q}| = E/\hbar c_s \equiv q$, c_s being the GaAs sound velocity. It follows that by increasing the energy spacing E the wave vector q is increased, which reduces the carrier-phonon coupling g entering in the electron-phonon interaction and then the scattering rate. This well-established behavior is typical of a quasi-0D structure. As shown in Fig. 1, for $E = 5$ meV—a standard value for many state-of-the-art QD structures—the carrier-phonon scattering rate is significantly suppressed compared to typical bulk values [8].

We will now show that by means of a proper information encoding, i.e., a proper choice of the initial multisystem quantum state, and a proper design of our QD array, we can strongly suppress phonon-induced decoherence processes, thus further improving the single-dot scenario discussed so far. Let us now consider a four-QD array (i.e., the simplest noiseless qubit register). From the short-time expansion discussed above, we have numerically evaluated the decoherence rate for such QD array choosing as energy splitting $E = 5$ meV (see Fig. 1). As initial state we have chosen the singlet defined by the dimer partition $\mathcal{D}_1 = \{(1,2),(3,4)\}$. The resulting decoherence rate is shown as a solid line in Fig. 2 as a function of the interdot distance a. The uncorrelated-dot decoherence rate is also reported as a dashed line for comparison. Rather surprisingly, in spite of the 3D nature of the sum over $\mathbf{q}$ entering the calculation of the function $\Gamma_{ii'}^{(\pm)}$ [see Eq. (1)], the decoherence rate exhibits a periodic behavior over a range comparable to the typical QD length scale. This effect—which would be the natural for a 1D phonon system—stems from the exponential suppression, in the overlap integral, of the contributions of phononic modes with nonvanishing in-plane component. The 1D behavior is extremely important since it allows one, by suitable choice of the interdot distance a, to realize the symmetric regime (ii) in which all the dots experience the *same* phonon field and therefore decohere collectively. Indeed, by taking $\bar{a} = n2\pi/q$ ($n \in \mathbf{N}$) one finds, for example, that $f_{\mathcal{D}_1}(\bar{a}) \ll 1$. Figure 2 shows that for the particular QD structure considered, case C should correspond to a decoherence-free evolution of a singlet state, which is not the case for A and B (see symbols in the figure). In order to extend the above short-time analysis, we have performed a full time-dependent solution by direct integration of the master equation for the density matrix ρ, taking also into account the Lamb-shift terms. Starting from the same GaAs QD structure

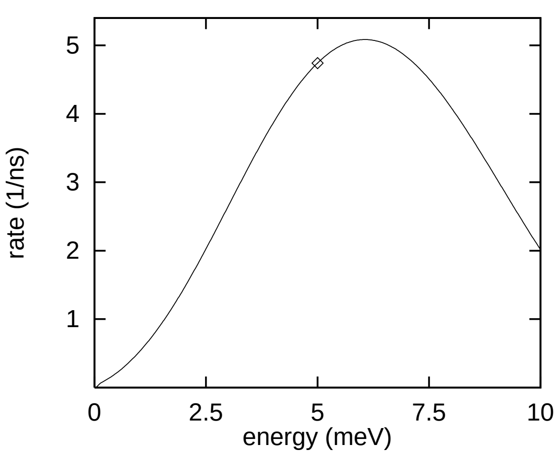

FIG. 1. Carrier-phonon scattering rate for a single QD structure as a function of the energy splitting E at low temperature (see text).

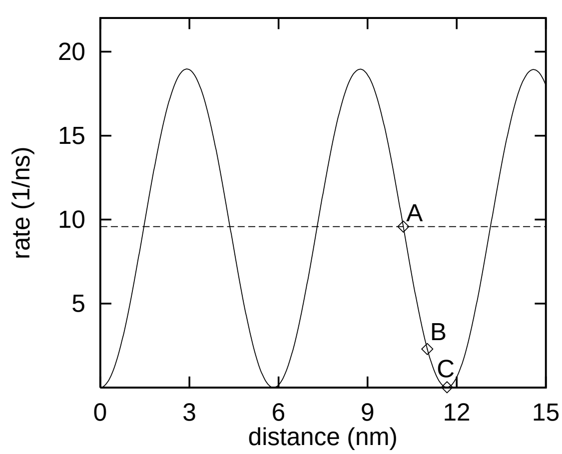

FIG. 2. Phonon-induced decoherence rate for a four-QD array (solid line) as a function of the interdot distance a compared with the corresponding uncorrelated-dot case (see text).

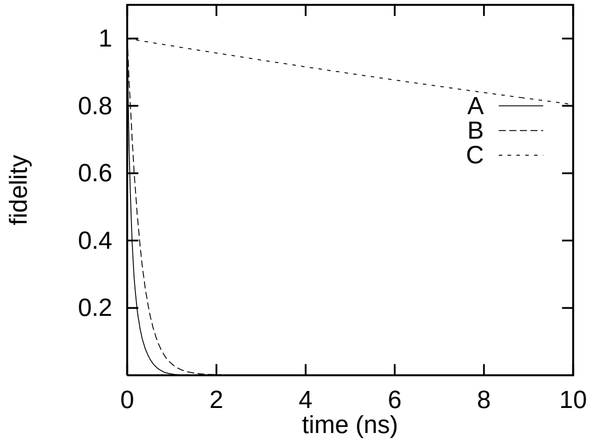

FIG. 3. Fidelity F as a function of time as obtained from a direct numerical solution of the master equation for the relevant case of a four-QD array (see text).

considered so far, we have simulated the above noiseless encoding for a four-QD array. Figure 3 shows the fidelity as a function of time as obtained from our numerical solution of the master equation. In particular, we have performed three different simulations corresponding to the different values of a depicted in Fig. 2. Consistently with our short-time analysis, for case C we find a strong suppression of the decoherence rate which extends the subnanosecond time scale of the B case (corresponding to twice the single-dot rate) to the microsecond time scale. This confirms that by means of the proposed encoding strategy one can realize a decoherence-free evolution over a time scale comparable with typical recombination times in semiconductor materials [8].

At this point a few comments are in order. The actual implementation of the suggested encoding relies, of course, on precise quantum state synthesis and manipulations. This crucial point—that was not the focus of this paper—might be addressed, by resorting, for example, to the ideas of the early QD proposal in [16]. Moreover, it is well known that carrier-phonon scattering is not the only source of decoherence in semiconductors. In conventional bulk materials also carrier-carrier interaction is found to play a crucial role. However, state-of-the-art QD structures—often referred to as semiconductor macroatoms [5]—can be regarded as few-electron systems basically decoupled from the electronic degrees of freedom of the environment. For the semiconductor QD array considered, the main source of Coulomb-induced "noise" may arise from the interdot coupling. However, since such Coulomb coupling vanishes for large values of the QD separation and since the proposed encoding scheme can be realized for values of a much larger than the typical Coulomb-correlation length (see Fig. 2), a proper design of our quantum register may rule out such additional decoherence channels.

In summary, we have investigated a semiconductor-based implementation of a quite general quantum-encoding strategy, which allows one to suppress phonon-induced decoherence on the carrier subsystem. More specifically, we have shown that an array of state-of-the-art QD structures is a suitable qubit register since it allows one to realize a decoherence-free evolution on a time scale long compared with those of modern ultrafast laser-pulse generation and manipulation. Since the latter is the natural candidate for quantum gating of charge excitations in semiconductor nanostructures, this result might constitute an important first step toward a solid-state implementation of quantum computers.

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