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RESPONSE OF SANDWICHES UNDERGOING STATIC AND BLAST PULSE LOADING WITH TAILORING OPTIMIZATION AND STITCHING

ICARDI U. °, SOLA F.

Dipartimento di Ingegneria Meccanica e Aerospaziale

Politecnico di Torino - Corso Duca degli Abruzzi 24, 10129 Torino, Italy

° Corresponding author. Tel: +39 (0)110906872; fax: +39 (0)110906899

E-mail addresses: ugo.icardi@polito.it (U. Icardi); federico.sola@polito.it (F. Sola)

Abstract

A numerical study is presented where tailoring optimization and stitching are applied to improve the structural performances of sandwich plates undergoing static and blast pulse pressure loading. The purpose is to recover the critical interlaminar stresses at the interface with the core and contemporaneously keep maximal the flexural stiffness. Optimized distributions of the stiffness properties for the faces are obtained solving an extremal problem whose target is the minimization of the energy due to transverse shear and bending stresses under spatial variation of the stiffness properties, along with the maximization of the energy due to in-plane stresses. The contribution of stitching is computed through 3-D finite element analysis and it is incorporated as modified elastic moduli into the refined, hierarchic zig-zag model employed as structural model to carry out the analysis accurately accounting for the layerwise effects of the out-of-plane transverse shear and transverse normal stresses and deformations. Approximate solutions giving the ply fibre orientation at any point (compatible with the current manufacturing technologies) are considered in the numerical applications. The numerical results show that stitched sandwiches incorporating optimized low-cost glass-fibre plies can achieve the same bending stiffness as sandwiches with uniform stiffness carbon fibre faces, with a consistent reduction of critical out-of-plane stresses. The amplitude of vibrations under blast pulse loading can be consistently reduced with a proper choice of the curvilinear paths of fibres incorporated in the faces.

Keywords: Optimised tailoring; Variable stiffness composites; Stitched composites; Stress relaxation; Blast loading; Hierarchic zig-zag model;

1 Introduction

Sandwiches with laminated faces find use as primary structural components in aerospace and other branches of engineering, thanks to an outstanding specific bending stiffness compared to monolithic composite structures (see e.g. Heimbs et al. [1]). Also their high energy absorption, thermoelastic, thermal insulation, damping and fatigue properties are of great practical interest.

Regrettably, composites suffer from strong out-of-plane stress concentrations at the interfaces, which can have harmful effects on structural performances and service life as exhaustively explained among many others by Liu and Islam [2] and Vachon et al. [3].

Various technical skills have been developed (Heimbs et al. [1], Potluri et al. [4], Judawisastra et al. [5], Wang et al. [6], Vaidya et al. [7] and Nilanjan [8] and Cox and Flanagan [9]) with the aim of improving the damage tolerance of composite aircraft structures, their strength under impact loading, their fatigue behaviour and the detrimental effects of out-of-plane stress concentrations.

However, stitching is the most effective technique and it is easy to use with the current manufacturing technologies as discussed by Dow and Dexter [10] within the framework of the Advanced Composites Technology (ACT) program by NASA.

Another effective way in mitigating the out-of-plane stress concentrations at the interfaces is the tailoring optimization. As examples, the recent papers by Liu et al. [11] and Sliseris and Rocens [12] are cited dealing with optimization design of layup configuration, fibre distribution and discrete varying stiffness. Generally, the optimal lamination stack-up is found for the chosen objective function, under the pertinent constraints, solving the problem through gradient based search techniques or genetic algorithms (see, e.g., Pholdee and Bureerat [13] and Badallò et al. [14], respectively). Both finite element and closed form analytic are used as structural models. To limit the computational burden, often the fibre orientation angle is assumed constant throughout the plies and the core properties are assumed uniform across the thickness. In addition, also simplified structural models like smeared laminate models are adopted, despite they cannot accurately describe the structural behaviour of laminates. With the advent of automated fibre placement techniques (see. e.g. Martin et al. [15] and Evans [16]), variable stiffness composites in which the fibres follow curvilinear paths started to be considered, as they can offer consistent advantages over uniform stiffness plies, such as improvement of structural performances and damage tolerance. The papers by Sliseris and Rocens [12], Khani et al. [17], Honda et al. [18], Sousa et al. [19] and Nik et al. [20] are cited as recent examples. Because automated fibre placement used to manufacture variable stiffness composite laminates can lead to the formation of gaps or overlaps embedded defects that can deteriorate strength and stiffness properties, as considered in [20].

At the authors' best knowledge, no papers can be found where optimization is carried out using structural models that accurately account for the layerwise effects, the transverse shear deformations and the out-of-plane stresses as their complexity can make the computational effort unaffordable for the optimization process. In this paper, the tailoring optimization technique developed and successively refined by the authors in Refs. [21] - [26] is employed to determine the optimal fibre angle variation over the faces of sandwiches, using as structural model the refined zig-zag model with hierarchic representation of displacements across the thickness recently developed by the authors [27]. It can accurately describe the transverse shear deformations and the out-of-plane stresses, therefore the optimization procedure could attain more realistic results than with simplified models. In order to further improve the structural performances, the optimization procedure is coupled with stitching through-the-thickness. The aim here is to assess through a numerical study whether stitching of faces and stitching through-the-thickness of sandwiches coupled with the tailoring optimizations of their face plies can be effective for recovering the critical interlaminar stress concentrations at the interface with the core, but keeping maximal the flexural stiffness. It could be remarked that stitching and variable stiffness optimization have been always treated separately in the literature. The final goal is to verify whether sandwiches with variable stiffness glass fibre faces and stitching can achieve the same bending stiffness of conventional sandwiches with carbon fibre reinforced faces having spatially uniform properties, recovering the critical stress concentrations at the interfaces with core.

This exploratory study will contribute to the discussion on whether low cost materials like glass-fibre laminates, whose performances are improved through currently available technological skills like tailoring optimization and stitching, could offer a possible alternative choice to advanced carbon reinforced composites that may have unaffordable costs for many applications.

The numerical results will confirm the beneficial effects of variable stiffness composites and stitching found by other researcher using different modelling techniques. This paper will also show that sandwiches with variable stiffness glass fibre faces and stitching can reach almost the same performances of the corresponding sandwiches with carbon-fibre faces.

2 Basic remarks

Necessary premise to the description of the structural model and of the optimization technique used in this paper, first their features and motivations are overviewed, along with the technique used for computing the properties of the stitched structure. The readers are referred to [26], [27] and [28] for further details.

The structural model developed by the authors in [27], which is a progressive refinement of those of Refs. [29] - [32], is aimed at reducing the computational effort keeping maximal the accuracy. It features a variable piecewise representation across the thickness of the three displacement components that *a priori* satisfies the continuity of the transverse shear and normal stresses at the material interfaces through continuity functions whose expressions are determined once for all. Its expansion order can be entirely different for any displacement and can vary from point to point across the thickness, in order to adapt to the variation of the material properties, though the number of functional degrees of freedom (d.o.f.) is kept fixed, being just the mid-plane displacements and the shear rotations.

It has the merit of solving the structural problems with the minimal number of unknowns, giving very accurate predictions of the piecewise variation of displacements across the thickness and of out-of-plane stress distributions directly from constitutive equations, as shown in [27], even for problems with different length and elastic scales. Since this model very accurately represents the strain energy, it is suited for the optimization technique here used. It is also suited for carrying out the analysis of optimized configurations being as accurate as the high-order layerwise plate models to date extensively employed, with a much lower computational effort, having less d.o.f. In fact, its memory storage dimension and its processing time are not considerably larger of those required by equivalent single-layer models. It also accurately predicts the through-the-thickness variation of the transverse displacement and of the transverse normal stress, which can have a significant bearing for keeping equilibrium in many practical cases (e.g., thermo-elasticity, cut-outs, free edges, crushing behaviour of sandwiches). The readers are referred to [27] for a more comprehensive discussion of the available structural models for laminates and sandwiches and of their relative merits and drawbacks in terms of accuracy and computational costs.

The tailoring optimization technique (Refs.[21] to [26]) here employed makes simultaneously extreme the strain energy contributions associated to in-plane and out-of-plane stresses and deformations under spatial variation of the stiffness properties. After having considered equilibrium and the boundary conditions of the problem (see Figure 1), a set of equations are determined, the so-called the Euler–Lagrange equations, whose solution in exact or approximate numerical or closed form, depending by the problem, represents the optimal distribution of stiffness properties. The purpose is to obtain plies with spatially variable properties that minimize the transverse shear stresses at the critical interfaces and the bending deformation. However, any other strategy of interest could be applied making minimal or maximal the strain energy contributions of interest, then solving simultaneously the resulting equations giving the optimal distribution of stiffness properties.

It could be noticed that the search of the optimal orientation of fibre paths is carried apart from the computation of the structural response and once for all. As a consequence, layerwise structural models that are impractical if used with the available optimization techniques, their computational effort being unaffordable, can be considered within the present approach in order to give more realistic predictions.

The optimization problem of variable-stiffness composites is reduced to a simple problem of detecting the proper stacking sequence like with straight-fibre composites, which can be efficiently solved by the classical optimization techniques.

Differently to previous works by the authors, functionally graded foam core is not considered, being a potentially effective technology which is still under development. On the contrary, stitching and plies with variable fibre orientation are consolidated technologies (see, e.g. Refs. [4], [15] and [16]) that allows for an immediate application.

According to Prodromou et al. [33], the models accounting for the through-the-thickness reinforcements can be broadly classified as analytical methods (Refs. [34] and [35]), methods based upon inclusion method (Refs. [36], [37]), methods based upon cell method (Refs. [33] and [38]) and finite element methods. The latter approaches are of general validity being capable of treating arbitrary reinforcement configurations and in the form of a material testing by a full scale finite element analysis (Refs. [39] and [40]) they are the most accurate method to date available. Hence, as in Ref. [28], 3D finite element analysis (FEA) is here used for obtaining the homogenized mechanical properties of stitched sandwiches considered as continuum media, whose average stresses and strains are predicted by the hierarchic zig-zag model [27]. Considerations upon the microstructural effects are left to a future study.

3 Structural Modelling, homogenized properties and optimization technique

In this section, the features of the adaptive model [27] and the method adopted to evaluate the homogenized mechanical properties of stitched structures [28] are briefly overviewed.

3.1 The structural model

Sandwiches are here treated as multilayered structures made of an arbitrary number of thin layers constituting the faces and of a thick intermediate layer constituting the core, whose properties are computed from the cellular properties.

The following piecewise representation of the displacement is considered across the thickness:

$$\begin{aligned} u(x,y,z) &= u^0(x,y) + z(\gamma_x^0(x,y) - w^0{}_{,x}) + C_x^i(x,y)z^2 + \\ &+ D_x^i(x,y)z^3 + (Oz^4 \ldots) + \sum_{k=1}^{n_i} \Phi_x^k(x,y)(z-z_k)H_k + \sum_{k=1}^{n_i} C_u^k(x,y)H_k \end{aligned} \tag{1}$$

$$\begin{aligned} v(x,y,z) &= v^0(x,y) + z(\gamma_y^0(x,y) - w^0{}_{,y}) + C_y^i(x,y)z^2 + \\ &+ D_y^i(x,y)z^3 + (Oz^4 \ldots) + \sum_{k=1}^{n_i} \Phi_y^k(x,y)(z-z_k)H_k + \sum_{k=1}^{n_i} C_v^k(x,y)H_k \end{aligned} \tag{2}$$

$$\begin{aligned} w(x,y,z) &= w^0(x,y) + b^i(x,y)z + c^i(x,y)z^2 + d^i(x,y)z^3 + e^i(x,y)z^4 + \\ &+ (Oz^5 \ldots) + \sum_{k=1}^{n_i} \Psi^k(x,y)(z-z_k)H_k + \sum_{k=1}^{n_i} \Omega^k(x,y)(z-z_k)^2 H_k + \sum_{k=1}^{n_i} C_w^k(x,y)H_k \end{aligned} \tag{3}$$

Where the symbols u, v, w represent the elastic displacement components in the direction of x, y and z respectively, while their counterparts for the points on the reference surface Ω are indicated with the superscript 0. Note that the reference system has the in-plane coordinates x, y over the reference middle surface Ω and the coordinate z across the thickness.

The Heaviside unit step function H_k makes active the contribution of Φ_x^k, Φ_y^k, Ψ^k, Ω^k, C_u^k, C_v^k, C_w^k since their pertinent interface, i.e. H_k=1 for $z \geq z_k$ and 0 for $z < z_k$.

The superscripts i mean that these terms are valid only in a single ply of the laminate, thus enabling different representation from point to point.

The terms in the summations, which are the zig-zag contributions, are incorporated in order to fulfil *a priori* the contact conditions prescribed by the elasticity theory. Namely, the two zig-zag contributions Φ_x^k and Φ_y^k are incorporated in the in-plane displacements in order to fulfil the continuity of the out-of-plane shear stresses, while, the terms Ψ^k and Ω^k are incorporated in the transverse displacement in order to meet the stress contact conditions on the transverse normal stress and its gradient. Finally, C_u^k, C_v^k and C_w^k make continuous the displacements at the points across the thickness where the representation is varied.

The coefficients C_x^i, C_y^i, D_x^i, D_y^i, as well as the other high order terms are determined by enforcing equilibrium conditions at discrete points across the thickness, as well as by enforcing the boundary conditions:

$$\sigma_{xz}\,|^{u} = 0 \quad \sigma_{xz}|_{l} = 0 \tag{4}$$

$$\sigma_{yz}\,|^{u} = 0 \quad \sigma_{yz}|_{l} = 0 \tag{5}$$

$$\sigma_{zz}\,|^{u} = p^0\,|^{u} \quad \sigma_{zz}|_{l} = p^0\,|_{l} \tag{6}$$

$$\sigma_{zz,z}\,|^{u} = 0 \quad \sigma_{zz,z}|_{l} = 0 \tag{7}$$

The enforcement of the above mentioned constraints turns in rather intricate algebraic operations, however, thanks to a symbolical calculus tool they are carried out apart once at a time, thus increasing the efficiency of the model.

The model can predict the effects of transverse shear deformations on displacements and stresses directly from constitutive equations with the same accuracy of 3D and high-order layerwise models with just five functional d.o.f. This feature, as shown in Ref. [27], enables to obtain computational times that are 1/10 of those required by a model with the face sheets assimilated to equivalent anisotropic layers and the core assumed as a thick intermediate layer. The readers can find more details in Ref. [27].

3.2 Stitching

The computation of the homogenized mechanical properties is carried out through virtual material testing performed with the mixed solid element of Ref. [41]. It has a computational effort not larger than that required for displacement-based counterpart solid elements, while convergence and accuracy are improved. As a result, it can be accurate even with relatively coarse mesh. The nodal d.o.f. are the three displacements and the three interlaminar stresses, thus obtaining continuous displacements with appropriate discontinuous derivatives, in order to fulfil the continuity of transverse shear and normal stresses. C^0, trilinear, standard serendipity shape functions ($N_i = \frac{1}{8}(1+\xi_0)(1+\eta_0)(1+\zeta_0)$) are used across the element volume. The element matrix [K] and the system of equations are obtained in the standard way from the Hellinger-Reissner functional:

$$
\begin{aligned}
&U_{HR} = \int_V \left(\{\varepsilon^u\}^T \{\sigma\} - \frac{1}{2}\{\varepsilon^s\}^T \{\sigma\} \right) dV = \\
&\{q^e\}^T \int_V \left([D]^T [B] - \frac{1}{2}[B]^T [C]^T [B] \right) dV \{q^e\} = \{q^e\}^T [K] \{q^e\} = \\
&= W_{HR} = \int_{V^e} \left(b_x u^e + b_y v^e + b_z w^e \right) dV
\end{aligned}
\tag{8}
$$

Here b_i is the load component in the i-direction; [C] is the elastic matrix giving stresses from strains, while the other symbols have the following meaning:

$$
[D] = \begin{bmatrix}
\mathbf{N},_x & [0] & [0] & [0] & [0] & [0] \\
[0] & \mathbf{N},_y & [0] & [0] & [0] & [0] \\
[0] & [0] & \mathbf{N},_z & [0] & [0] & [0] \\
\mathbf{N},_y & \mathbf{N},_x & [0] & [0] & [0] & [0] \\
\mathbf{N},_z & [0] & \mathbf{N},_x & [0] & [0] & [0] \\
[0] & \mathbf{N},_z & \mathbf{N},_y & [0] & [0] & [0]
\end{bmatrix}
\tag{9}
$$

(where comma stands for spatial derivation)

$$
\{\sigma\} = \begin{bmatrix}
 & & & [B*][D] & & & \\
0 & 0 & 0 & \mathbf{N} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \mathbf{N} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \mathbf{N} & 0
\end{bmatrix} \{q^e\} = [B]\{q^e\}
\tag{10}
$$

$$
[B*] = \begin{bmatrix}
C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\
C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\
C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36}
\end{bmatrix}
\tag{11}
$$

The homogenized mechanical properties are computed starting from the creation of the geometrical model of the structure (Figure 2a), which is split in 20000 and 15000 elements respectively for the sandwich and the reinforcing thread. Then, the loading schemes described by Okereke and Akpoyomare [42], are applied enabling the evaluation of the stiffness matrix of the structure. Uniaxial load along X- or 1-axis determines the following mechanical properties:

$$E_{11} = \frac{\sigma_{11}}{\varepsilon_{11}}; \quad \upsilon_{12} = -\frac{\varepsilon_{22}}{\varepsilon_{11}}; \quad \upsilon_{13} = -\frac{\varepsilon_{33}}{\varepsilon_{11}} \tag{12}$$

Uniaxial load along Y- or 2-axis determines the following mechanical properties:

$$E_{22} = \frac{\sigma_{22}}{\varepsilon_{22}}; \quad \upsilon_{23} = -\frac{\varepsilon_{33}}{\varepsilon_{22}}; \quad \upsilon_{21} = -\frac{\varepsilon_{11}}{\varepsilon_{22}} \tag{13}$$

Uniaxial load along Z- or 3-axis determines the following mechanical properties:

$$E_{33} = \frac{\sigma_{33}}{\varepsilon_{33}}; \quad \upsilon_{31} = -\frac{\varepsilon_{11}}{\varepsilon_{33}}; \quad \upsilon_{32} = -\frac{\varepsilon_{22}}{\varepsilon_{33}} \tag{14}$$

Shear deformation along XY- or 12-, XZ- or 13- and YZ- or 23-planes determines respectively the following shear properties:

$$G_{12} = \frac{\tau_{12}}{\gamma_{12}}; \quad G_{13} = \frac{\tau_{13}}{\gamma_{13}}; \quad G_{23} = \frac{\tau_{23}}{\gamma_{23}} \tag{15}$$

Since the stitching spacing has a significant bearing in varying the stiffness of the structure, as outlined in [4], this parameter is taken into account using the rule of mixture [36]:

$$\begin{aligned} E_{i\,f} &= E_{i\,0} + V\left(E_{1b} - E_{i\,0}\right), \text{(i} = 1,2,3) \\ \upsilon_{ij\,f} &= \upsilon_{ij\,0} + V\left(\upsilon_{12b} - \upsilon_{ij\,0}\right), \text{(ij} = 12,13,23) \\ G_{ij\,f} &= G_{ij\,0} + V\left(G_{12b} - G_{ij\,0}\right), \text{(ij} = 12,13,23) \end{aligned} \tag{16}$$

The subscripts “f”, “0” and “b” refer respectively to the stitched layer, to the unstitched one and to the binder. The symbol “V” corresponds to the reinforcement percentage volume in the considered layer, thus V allows to consider the effects of the stitching spacing.

Okereke and Akpoyomare [42] showed that the results by the rule of mixture are accurate and similar by the practical viewpoint to those provided by Halpin-Tsai and Hopkins-Chamis rules. Also the authors showed in Ref. [28] that the relations of Eq. (16) allow determining mechanical properties in a very good agreement with experimental and numerical reference results.

4 The optimization technique

A detailed description of the optimization procedure is given in Ref. [25], thus here just the basic steps toward the obtainment of the governing equations are summarized.

The goal of the process is finding a proper distribution of the stiffness properties that minimizes the energy stored by unwanted modes, e.g. the modes involving interlaminar strengths, and maximizes that absorbed by desired modes, e.g., the modes involving membrane strengths, since laminates and sandwiches have larger strength and stiffness in the in-plane direction than in the thickness one.

In effects, the procedure acts as a tuning of the energy absorption properties, energy being transferred from unwanted out-of-plane modes to in-plane modes. As a consequence, while the out-of-plane stresses that are responsible of the damage rise are recovered, the in-plane stresses can increase. Previous applications of this technique [22] - [25] have shown that the delamination failure index can be consistently reduced without risk of in-plane failure. Results in [26] proven that replacing few layers having uniform stiffness with variable stiffness layers having mean stiffness properties equal to those of the replaced plies, an improved overall bending stiffness is achieved while the transverse shear stresses at the critical interfaces are recovered.

As mentioned in Section 2, functionally graded cores are not considered, even if the optimization procedure allows also for variation of the core properties across the thickness. The tailoring optimization process starts writing the strain energy of the plate model, then the stationary conditions for bending and shear energy are imposed accounting for equilibrium and boundary conditions (Lagrange multipliers). Collecting the homologous stiffness contributions, a system of partial differential equations in the stiffness quantities is obtained. The solution to this set of equations is:

$$Q_{ij} = \sum_{p=1}^{P} \left[A_{1p}^{ij} e^{\left(px + {}^{1}\varphi_n^x\right)} + {}^{1}k_x \right] \cdot \left[A_{2i}^{ij} e^{\left(py + {}^{1}\varphi_n^y\right)} + {}^{2}k_y \right] \tag{17}$$

(ij=11, 12, 13, 16, 22, 23, 26, 36, 66). Instead, the elastic coefficient Q_{44}, Q_{45} and Q_{55} are assumed constant. This is physically consistent since the transverse shear properties of the plies are not affected by the orientation of the fibres over the plane (x, y).

The appropriate amplitude, phase, mean value and period in Eq. (17) are determined enforcing conditions such as the stiffness at the bounds, a convex or a concave shape and thermodynamic constraints related to the conservation of energy.

Also the mean values of the mechanical properties (i.e. the mean value of the functions and of their powers of various orders) of the optimized ply should be imposed to be the same of that un-optimized. The mass of the optimized plies can also be enforced to be equal to that of the conventional uniform stiffness counterparts plies, through an appropriate choice of fibre volume fraction. The fibre paths shown in Figure 3 are considered, which represent a solution compatible with the governing equation

(17). In details, the fibre path indicated as MB minimizes the bending component of the strain energy, while that indicated as MS minimizes the shear component of the strain energy in a generic ply. From the practical viewpoint, MB is obtained imposing maximum bending stiffness at the centre of the ply and minimum at the edge, while MS is obtained imposing maximum bending stiffness at the edge of the ply and minimum at the centre.

Please note that the fibre paths of Figure 3 are similar to those presented in Refs. [17] and [20].

5 Numerical applications

In this section numerical applications to simply-supported sandwich beams are considered. The attention is focused on static and dynamic loadings, which are a major hazard for structures (see, e.g. Refs. [43] - [45]). Namely, here a triangular pulse loading is considered (sonic boom):

$$P(t) = \begin{cases} P_m\left(1-\dfrac{t}{t_p}\right) & \textit{for}\ \mathrm{t < r \cdot t_p} \\ 0 & \mathrm{for\ t > r \cdot t_p} \end{cases} \tag{18}$$

Where $t_p = 0{,}02\,\mathrm{s}$, r=2 and $P_m = 1\,\mathrm{N}$.

According to the assumed boundary conditions, the displacements are postulated as:

$$u^0(x) = \sum_{m=1}^{M} A_m \cos\left(\frac{m\pi}{L_x}x\right) \tag{19}$$

$$w^0(x) = \sum_{m=1}^{M} C_m \sin\left(\frac{m\pi}{L_x}x\right) \tag{20}$$

$$\gamma_x{}^0(x) = \sum_{m=1}^{M} D_m \cos\left(\frac{m\pi}{L_x}x\right) \tag{21}$$

The static distributed loading:

$$p^0(x) = \sum_{m=1}^{M} P_m \sin\left(\frac{m\pi}{L_x}x\right) \tag{22}$$

is applied. A set of linear algebraic equations in the unknown amplitudes is obtained using the Rayleigh-Ritz method for both static and dynamic loadings.

The Newmark implicit time integration scheme is used for solving the dynamic equations, considering that alternative explicit time integration schemes need extremely small time steps to be stable.

The following mechanical properties of the core are assumed: $E_1=E_3=0.05$ GPa, $G_{13}=0.0217$ GPa, $\upsilon_{13}=0.15$, $\rho = 2.32 \cdot 10^{-3}\ kg/dm^3$, while the faces are in T300 or in M10. The mechanical properties of T300 are: $E_1=139$ GPa, $E_3=9.4$ GPa, $G_{13}=4.5$ GPa, $\upsilon_{13}=0.0209$, $\rho = 1.86\ kg/dm^3$. The mechanical properties of M10 are: $E_1=36.23$ GPa, $E_3=7.21$ GPa, $G_{13}=5.68$ GPa, $\upsilon_{13}=0.3263$, $\rho = 2.54\ kg/dm^3$.

As discussed in 3.1, sandwich beams are simulated as multilayered beams, the thickness of the layers are $(0.2\ h/0.3\ h)_s$, while the length to thickness ratio (*L/h*) is 10, where *L=100 mm*.

According to the results presented in Ref. [25], the choice of considering this specific value of *L/h* is done considering that for thick structures the effect of the optimization is limited due to their high stiffness. On the other hand, extremely thin structures would not enhance the capability of the adaptive model in describing the effects of the transverse shear deformations and the out-of-plane stresses.

The aim is to verify whether tailoring technique and stitching can improve the performance of a sandwich with M10 faces and initially straight fibres up to that of a sandwich made of T300 faces with straight fibre orientation, here named as reference case. As reported in Figure 3, five lay-ups are considered (OPTI 1-OPTI 5), by varying the position across the thickness of un-optimized, MB and MS plies. Of course, the position of these layers across the thickness is determined directly comparing the response of all the stack-up possible options since few constituent layers are considered. Please notice that the layers MB and MS have a mass variation that does not exceeds 15% with respect to the un-optimized ply.

The numerical results are normalized as follows:

$$
\begin{aligned}
& w^*(x,z) = \frac{w\left(\frac{L_x}{2}, z\right)}{p^0 \cdot h} \quad \sigma^*_{xz}(x,z) = \frac{\sigma_{xz}(0,z)}{p^0} \quad \text{(statics)} \\
& \overline{w}(x,z,t) = \frac{w\left(\frac{L_x}{2}, z, t\right)}{w^*(x,z)} \quad \text{(dynamics)}
\end{aligned}
\tag{23}
$$

Figure 4 shows the results for the lay-ups collected in Figure 3. In Figure 4c only the results from OPTI 1 and OPTI 3 are reported, since the time history of the other optimized lay-ups is not significantly different from that of the un-optimized one.

As outlined in Figure 1, the computation of the structural response through the Rayleigh-Ritz method is performed with the adaptive model of Section 3.1, which is also employed to compute the solution of the optimization problem. In this way, the computational time is kept very low, as only 90 s are required to compute the time history of the beam, and 30 s are required to evaluate the stress and displacement fields. It is remarked that the MB and MS distributions do not consider gaps and overlaps due to the manufacturing process, their consideration being left to a future study. They can be taken into account in a straightforward way using the technique developed by Fayazbakhsh et al. [46].

For the static case, the following observations can be done. The reference case is characterized by a larger value of the shear stress in the faces, while the shear stress at the face-core interface is lower compared to that of the sandwich with M10 faces. In addition, as obvious, the bending stiffness of the reference case is larger than that of the sandwich with M10 faces. However the use of optimized distribution improves the performance of the sandwich with M10 faces. In particular the best lay-ups are OPTI 1 and OPTI 3. The first one obtains a lower value of the transverse displacement and of the shear stress at the face-core interface, while OPTI 3 is better than OPTI 1 in reducing the maximum value of the shear stress. Although OPTI 1 and OPTI 3 improve the performance of the sandwich with M10 faces, further improvement are required for reaching the performance of the reference case. In order to achieve this goal the faces of OPTI 1 and OPTI 3 lay-ups, are stitched using a reinforcing thread in T300.

This strategy improves the bending stiffness of the sandwich with M10 faces as shown in Figure 5b. In fact, for both the lay-ups considered, a 7% reduction of the transverse displacement is obtained. As it can be noticed in Figure 5a, the behaviour of OPTI 1 and OPTI 3 with stitched faces is similar: an increase of the shear stress in the faces is obtained, coupled with a reduction of its value at the face-core interface. The increase of the bending stiffness is also underlined by the time history reported in Figure 5c. In fact, it could be noticed a decrease of the amplitude of the oscillations, as well as a decrease of their period.

In order to further improve the performance of the sandwich with M10 faces, it is possible to consider OPTI 1 and OPTI 3 stitched across all the thickness with a reinforcing thread made of T300 (named as 'OPTI 1 *all stitched (T300)*' and 'OPTI 3 *all stitched (T300)*'). The improvements are evident. In fact, the transverse displacement of the sandwich with M10 faces is now considerably lower than that with T300 faces. The transverse shear stress is considerably lower in the faces, while an increase is obtained in the core. It could be noticed, than differently to the previous cases, OPTI 3 lay-up obtains lower value of transverse displacement than OPTI 1.

The improvements in terms of stiffness are evident also for dynamic loading, as it can be noticed from Figure 5c. In fact the insertion of a reinforcing binder determines a significant decrease of the amplitude of the oscillations, coupled with a decrease of their period.

Bearing in mind the effectiveness of a stitching across the all thickness of the sandwich, it is possible to consider a reinforcing thread, whose mechanical properties are lower than those of one in T300. In particular the results of Figure 5 named as ‘OPTI 1 *all stitched (DISP)*’ and ‘OPTI 3 *all stitched (DISP)*’ are obtained considering a reinforcing binder with E_1=0.72 GPa, E_3=0.21 GPa, G_{13}=0.2 GPa, υ_{13}=0.2613. With this configuration, the transverse displacement of the sandwich with M10 faces is the same of that with T300 faces. As far as the shear stress is concerned, a reduction of its value in the faces is obtained, while, an increase in the core is achieved.

The time history reported in Figure 5c shows that the two lay-ups have similar dynamic behaviour.

From the numerical results previously presented, it is possible to underline a consistent increase of the stiffness given by tailoring optimization and stitching. Applying the delamination criterion of Ref. [47] ‘OPTI 1 *all stitched (DISP)*’ obtains a 4% reduction of the delamination index in the faces and obtains a 7% increase of the delamination index at the face-core interface. For what concerns ‘OPTI *3 all stitched (DISP)*’, a 6% reduction of the delamination index in the faces and a 4% increase of the delamination index at the face-core interface are obtained.

6 Concluding remarks

A new tailoring optimization technique recently developed by the authors was applied for deriving curvilinear fibre paths over the faces of sandwiches, which simultaneously make extreme the strain energy contributions due to either transverse shear or bending deformations under spatial variation of the stiffness properties, by solving the Euler–Lagrange equations. As a result of the tailoring optimization of faces, two curvilinear paths of fibre were obtained: MB and MS that respectively minimize the bending component of the strain energy and the shear component of the strain energy in a generic ply. Applications were presented to sandwich beams undergoing static and impulsive dynamic loading. The critical transverse shear stress concentrations at the interfaces with the core were recovered and the bending stiffness was improved coupling use of variable stiffness faces and through-the-thickness stitching. The equivalent, homogenized elastic properties accounting for the through-the-thickness reinforcement were computed by a three-dimensional finite element analysis carried out with mixed solid elements. The optimization procedure and the computation of the response of sandwich panels were performed in analytical form using a refined zig-zag model with hierarchic representation of displacements across the thickness. This model accurately captures the transverse shear deformations and the out-of-plane stresses directly from the constitutive equations with a much lower computational effort than the currently available high-order layerwise models.

Numerical applications showed that the bending stiffness can be considerably increased incorporating plies of type MB. Of course, the better bending stiffness is obtained with MB plies close to the upper and lower bounding surfaces, as they are far from the neutral axis. Stitching through-the-thickness of the faces gives rise to a growth of transverse shear stresses across the faces, but a reduction of these critical stresses in the core. Stitching through-the-thickness of faces and core determines a relevant increase of the shear stress in the core, as its strength and stiffness are boosted. However, at the same time, it reduces the shear stress in the faces and cuts down the transverse displacement of the sandwich. Thus the present results confirm those previously obtained by other researchers. As a new contribution, configurations were found that through simultaneous incorporation of layers MB and MS and use of stitching give rise to an improved bending stiffness with a recovery of transverse shear stresses at the face-core interfaces and across the core, but keeping unchanged the mass.

As a corroboration of this, numerical results showed that sandwiches with variable stiffness glass fibre faces and through-the-thickness reinforcement can achieve the same bending stiffness of conventional sandwiches with carbon fibre reinforced faces with spatially uniform properties and can recover the critical out-of-plane stress concentrations at the core interfaces.

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Curriculum Ugo Icardi

Prof Ugo Icardi graduated Magna cum Laude at Politecnico di Torino in 1982, where he also received his Ph.D. in 1986. He was first employed in automotive sector, then at the Department of Mechanical and Aerospace Engineering of Politecnico di Torino. His main research area is the simulation of the response of composite materials and smart structures. Another topic he recently explored is that of composites with spatially variable stiffness coefficients, which offer a better impact resistance and can be used as light armour materials using the optimization technique he developed.

Curriculum Sola Federico

Dr Sola Federico is Ph.D. student at Politecnico di Torino. He graduated Magna cum Laude at Politecnico di Torino in October 2012 discussing a thesis titled 'Efficient formulation of a model with adaptive kinematic for optimization of variable stiffness composites'. He has authored some papers in co-operation with Prof Ugo Icardi dealing with variable stiffness composites.