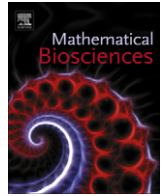


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Mathematical modelling of mosquito dispersal in a heterogeneous environment

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ABSTRACT

Mosquito dispersal is a key behavioural factor that affects the persistence and resurgence of several vector-borne diseases. Spatial heterogeneity of mosquito resources, such as hosts and breeding sites, affects mosquito dispersal behaviour and consequently affects mosquito population structures, human exposure to vectors, and the ability to control disease transmission. In this paper, we develop and simulate a discrete-space continuous-time mathematical model to investigate the impact of dispersal and heterogeneous distribution of resources on the distribution and dynamics of mosquito populations. We build an ordinary differential equation model of the mosquito life cycle and replicate it across a hexagonal grid (multi-patch system) that represents two-dimensional space. We use the model to estimate mosquito dispersal distances and to evaluate the effect of spatial repellents as a vector control strategy. We find evidence of association between heterogeneity, dispersal, spatial distribution of resources, and mosquito population dynamics. Random distribution of repellents reduces the distance moved by mosquitoes, offering a promising strategy for disease control.

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1. Introduction

Mosquitoes transmit malaria, dengue, yellow fever, filariasis, and several other important diseases. Malaria, in particular, shows considerable spatial variation predominantly determined by climatic variation [25], intervention coverage, and human movement [39,55,60,62]. At local scales (i.e. from 100 m to 1 km), mosquito behaviour and ecology play an important role in determining the distribution of transmission [34]. Like other animals, mosquitoes can move in any direction, motivated by resource availability and other drivers of dispersal, but can only travel over limited distances. Control interventions should consider locality and mosquitoes' ability to move, to achieve a high level of effectiveness in reducing the mosquito population.

The impact of vector dispersal in the spread and control of diseases was first highlighted a century ago by Ronald Ross [53], but has received limited attention within the public health community. Ross stipulated that mosquito density within any area is always a function of four variables, which include the reproduction rate, mortality rate, immigration, and emigration rates. A study

by Manga et al. [38] also showed that the spatial variation in the distribution of resources used by mosquitoes affects their reproduction and their rate of dispersal. This in turn contributes to variation in densities [10,24,37,58], human exposure to vectors, and the ability to control disease transmission [55]. The effects of resource availability on transmission can be surprising. For instance, even the presence of non-productive larval habitats may affect biting densities [34]. However, conducting experimental studies of mosquito dispersal [21–23,42] are challenging.

Mathematical models play an important role in understanding and providing solutions to phenomena which are difficult to measure in the field, but few models have incorporated dispersal or heterogeneity when modelling resource availability [17,34,46,49,58,68] or varied the usual assumption of a closed vector population [45,50,67]. Others have sub-divided the adult stage of the mosquitoes into different stages [45,50,54]. To investigate the effects of dispersal and heterogeneity, a model should incorporate features of the mosquito life cycle, the feeding cycle, spatial heterogeneity in mosquito resources, and dispersal.

Spatial models have commonly used the diffusion approach, which considers space as a continuous variable. Despite the existence of diffusion models, which account for heterogeneity [51,63], it is difficult to explicitly incorporate the various factors that affect movement. For example, in areas where resources are located in patches or discrete locations, mosquito dispersal is more conveniently modelled using a metapopulation approach, in which the population is divided into discrete patches. In each patch, the

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population is sub-divided into subgroups, corresponding to different states, leading to a multi-patch, multi-compartment system.

Several models using diffusion approaches [18,19] have incorporated heterogeneity and have shown that the environment has a strong influence on the distribution of disease vectors. However, none of them have included the aquatic stages of the mosquitoes or have provided a general and simple framework for modelling arbitrary spatial patterns of mosquito control interventions. A model framework that includes the aquatic stages and that partitions space into discrete locations allows us to capture the various forms of spatial heterogeneity that exist in our environment.

In this paper, a mathematical model, that includes all of the above features is developed and simulated to investigate the impact of dispersal and heterogeneous distribution of mosquito resources, such as hosts and breeding sites, on the spatial distribution, dynamics, and persistence of mosquito populations. The distance a mosquito can travel from its place of emergence or food source is a critical factor for vector control interventions, thus the model is used to project likely dispersal distances and considers how these might be changed by vector control interventions.

In the following sections, we develop and analyse a model for mosquito population dynamics that does not consider movement of mosquitoes. We then develop a meta-population model for mosquito movements with discrete space in hexagonal patches and compare it to a continuous space model. We then combine the two models and run simulations of a spatially explicit model of the full mosquito life cycle to determine the effect of repellents.

2. Description of the basic model: mosquito dynamics without dispersal

Mosquito life begins with eggs, which hatch into larvae under suitable conditions. The larvae develop into pupae that mature and emerge into adults (see Fig. 1). Female mosquitoes then feed on human or animal blood to provide protein for their eggs. After biting, female mosquitoes rest while their eggs develop. Once eggs are fully developed, the females oviposit and then proceed to find another blood meal thus completing the mosquito feeding cycle [12].

Ignoring the effects of hibernation and breaks in the reproductive cycle, and assuming that eggs deposited at breeding sites proceed through development immediately [56], we consider six compartments of the mosquito life cycle: eggs (E), larval (L), pupal

(P), host seeking adults (A_h), resting adults (A_r), and oviposition site seeking adults (A_o) (Fig. 1). In contrast to other models [36], we distinguish all of these stages because interventions may be applied to any one (or more) of them. Since only female mosquitoes are involved in the transmission of vector-borne diseases, this model ignores males. The six subgroups have different mortality and progression rates. Each subgroup is affected by three processes: increase due to recruitment, decrease due to mortality, and development or progression of survivors into the next state. The parameter b is the average number of female eggs laid during an oviposition and ρ_{A_o} (day^{-1}) is the rate at which new eggs are oviposited (i.e. reproduction rate). Exit from the egg stage is either due to mortality, μ_E (day^{-1}), or hatching into larvae, ρ_E (day^{-1}). In the larval stage, individuals exit by death or progress to pupal stage at a rate, ρ_L (day^{-1}). Assuming a stable environment, inter-competition for food and other resources for larvae may occur, leading to density-dependent mortality, $\mu_L L^2$ (day^{-1} mosquitoes $^{-1}$) or natural death at an intrinsic rate, μ_{L_1} (day^{-1}). Pupae die at a rate, μ_P (day^{-1}) and survivors progress and emerge as adults at rate ρ_P (day^{-1}). In the adult stage, host seeking mosquitoes die at a rate μ_{A_h} (day^{-1}). Those surviving this stage, and if they are successful in feeding, enter the resting stage at a rate ρ_{A_h} (day^{-1}). In the resting stage, mosquitoes die at a rate, μ_{A_r} (day^{-1}). Survivors progress to the oviposition site searching stage at a rate ρ_{A_r} (day^{-1}). Oviposition site searchers die at rate μ_{A_o} (day^{-1}) and after laying eggs return to the host seeking stage. These processes account for the dynamics of each subgroup over time. Although mosquitoes might require more than one blood meal to produce eggs [5], this model assumes the simple case where only one blood meal is enough for eggs to mature. Throughout this work, we use the words oviposition sites and breeding sites interchangeably.

From the description above, we develop the following system of differential equations to describe mosquito dynamics without movement:

$$\begin{aligned}\frac{dE}{dt} &= b\rho_{A_o}A_o - (\mu_E + \rho_E)E, \\ \frac{dL}{dt} &= \rho_E E - (\mu_{L_1} + \mu_L L + \rho_L)L, \\ \frac{dP}{dt} &= \rho_L L - (\mu_P + \rho_P)P, \\ \frac{dA_h}{dt} &= \rho_P P + \rho_{A_o}A_o - (\mu_{A_h} + \rho_{A_h})A_h, \\ \frac{dA_r}{dt} &= \rho_{A_h}A_h - (\mu_{A_r} + \rho_{A_r})A_r, \\ \frac{dA_o}{dt} &= \rho_{A_r}A_r - (\mu_{A_o} + \rho_{A_o})A_o,\end{aligned}\quad (1)$$

with initial conditions $E(0), L(0), P(0), A_h(0), A_r(0)$, and $A_o(0)$. Mosquito survival in each stage and the progression period from one stage to the next are assumed to be exponentially distributed. The definitions of state variables and the associated parameters are given in Tables 1 and 2, respectively.

Since the system in Eq. (1) monitors populations in each stage of mosquito development and because all model parameters (Table 2) are positive, there exists a region \mathbb{D} such that

Table 1
State variable definitions.

Variable	Description
E	density of eggs
L	density of larvae
P	density of pupae
A_h	density of mosquitoes searching for hosts
A_r	density of resting mosquitoes
A_o	density of mosquitoes searching for oviposition sites

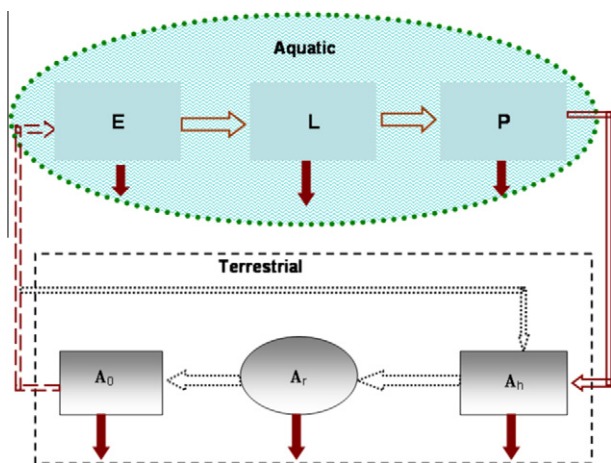


Fig. 1. Schematic representation of *Anopheles* mosquito life cycle and feeding cycle. Model states are Eggs (E), Larvae (L), Pupae (P), host seeking adults (A_h), resting adults (A_r), and oviposition site searching adults (A_o).

Table 2

Description and values of parameters of the model. All parameters are positive and time is measured in days. For the model with dispersal, these parameters are patch dependent.

Parameter	Description	Units	Baseline	Range	Source
b	number of female eggs laid per oviposition	-	100	50 – 300	[56]
ρ_E	egg hatching rate into larvae	day ⁻¹	0.50	0.33 – 1.0	[56,27],69
ρ_L	rate at which larvae develop into pupae	day ⁻¹	0.14	0.08 – 0.17	[56],27,4,32,20
ρ_P	rate at which pupae develop into adult/emergence rate	day ⁻¹	0.50	0.33 – 1.0	[56,27]
μ_E	egg mortality rate	day ⁻¹	0.56	0.32 – 0.80	[47]
μ_{L_1}	density-independent larvae mortality rate	day ⁻¹	0.44	0.30 – 0.58	[47]
μ_{L_2}	density-dependent larvae mortality rate	day ⁻¹ mosq ⁻¹	0.05	0.0 – 1.0	Variable
μ_P	pupae mortality rate	day ⁻¹	0.37	0.22 – 0.52	[47]
ρ_{A_h}	rate at which host seeking mosquitoes enter the resting state	day ⁻¹	0.46	0.322 – 0.598	[13], Estimated
ρ_{A_r}	rate at which resting mosquitoes enter oviposition site searching state	day ⁻¹	0.43	0.30 – 0.56	[13]
ρ_{A_o}	oviposition rate	day ⁻¹	3.0	3.0 – 4.0	[13]
μ_{A_h}	mortality rate of mosquitoes of searching for hosts	day ⁻¹	0.18	0.125 – 0.233	[13], Estimated
μ_{A_r}	mortality rate of resting mosquitoes	day ⁻¹	0.0043	0.0034 – 0.01	[13]
μ_{A_o}	mortality rate of mosquitoes searching for oviposition sites	day ⁻¹	0.41	0.41 – 0.56	[13]

$$\mathbb{D} = \left\{ \begin{pmatrix} E \\ L \\ P \\ A_h \\ A_r \\ A_o \end{pmatrix} \in \mathbb{R}^6 \mid \begin{matrix} E \geq 0, \\ L \geq 0, \\ P \geq 0, \\ A_h \geq 0, \\ A_r \geq 0, \\ A_o \geq 0 \end{matrix} \right\}, \quad (2)$$

where the model is mathematically and biologically meaningful and all solutions of the system (1) with non-negative initial data will remain non-negative in the feasible region \mathbb{D} for all time $t \geq 0$. We use the notation X' to represent $\frac{dX}{dt}$ here and denote the boundary of \mathbb{D} by $\partial\mathbb{D}$.

Theorem 2.1. *If the initial conditions of system (1) lie in region \mathbb{D} , then there exists a unique solution for (1), $E(t), L(t), P(t), A_h(t), A_r(t)$, and $A_o(t)$ that remains in \mathbb{D} for all time $t \geq 0$.*

Proof. The right hand side of the system (1) is continuous with continuous partial derivatives in \mathbb{D} , therefore (1) has a unique solution that exists for all time. It remains to be shown that \mathbb{D} is forward-invariant. We see from system (1) that if $E = 0$, then $E' = b\rho_{A_o}A_o \geq 0$; if $L = 0$, then $L' \geq 0$; if $P = 0$, then $P' \geq 0$; if $A_h = 0$, then $A_h' \geq 0$; if $A_r = 0$, then $A_r' \geq 0$; and if $A_o = 0$, then $A_o' \geq 0$. Therefore all solutions of the system of equations (Eq. (1)) are contained in the region \mathbb{D} . \square

3. Analytical results of the basic model without mosquito dispersal

3.1. Existence of equilibrium points

This section presents existence and stability results of the model (Eq. (1)) of the steady states. An equilibrium point of a given a system of equations ($\dot{\mathbf{X}}(t)$) (where \mathbf{X} is a vector composed by state variables) is a steady-state solution, where $\mathbf{X}(t) = \mathbf{X}^*$ for all t .

Proposition 1. *The model in (1) has exactly one equilibrium point on $\partial\mathbb{D}$ given by $P_0 = (0, 0, 0, 0, 0, 0)$. We label P_0 the mosquito-free equilibrium point.*

Proof. Substituting P_0 into the right hand side of (1) shows that all derivatives are zero so P_0 is an equilibrium point of (1). Setting any of E, L, P, A_h, A_r , or A_o equal to 0, we see that all other remaining state variables must also be equal to zero for the system to be at equilibrium. Therefore, P_0 is the only equilibrium point on $\partial\mathbb{D}$. \square

Similar to White et al. [67], we define the population reproduction number, R_0 , as the expected number of female mosquitoes produced by a single female mosquito in her life time in the absence of density-dependence. In [64], a method for computing the reproduction number for epidemic models was developed. However, it can equivalently be used in ecological models where new births are treated as new infections. We determine the mosquito population reproduction number for model (1) using the next-generation technique [64].

Defining x as a set of all state variables (E, L, P, A_h, A_r, A_o) in the model, then $x = (x_1, x_2, \dots, x_6)^T$ for $i = 1, 2, \dots, 6$. The system in (1) can be written in the form of $\frac{dx_i}{dt} = F_i(x) - V_i(x)$, where F_i is the rate of new recruitment (birth of eggs) in a compartment, $V_i = V_i^- - V_i^+$, with V_i^+ being the rate of transfer of mosquitoes into a compartment and V_i^- is the rate of transfer of mosquitoes out of the compartment. For this model, F , and V are given by:

$$F = \begin{bmatrix} b\rho_{A_o}A_o \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

and

$$V = \begin{bmatrix} (\mu_E + \rho_E)E \\ (\mu_{L_1} + \rho_L)L + \mu_{L_2}L^2 - \rho_EL \\ (\mu_P + \rho_P)P - \rho_LP \\ (\mu_{A_h} + \rho_{A_h})A_h - \rho_PA - \rho_{A_o}A_o \\ (\mu_{A_r} + \rho_{A_r})A_r - \rho_{A_h}A_h \\ (\mu_{A_o} + \rho_{A_o})A_o - \rho_{A_r}A_r \end{bmatrix}.$$

To obtain the next generation operator, \mathbf{FV}^{-1} , we calculate $\mathbf{F}_{ij} = \frac{\partial F_i}{\partial x_j} \Big|_{P_0}$ and $\mathbf{V}_{ij} = \frac{\partial V_i}{\partial x_j} \Big|_{P_0}$ to obtain

$$\mathbf{F} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & b\rho_{A_o} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (3)$$

and

$$V = \begin{bmatrix} (\mu_E + \rho_E) & 0 & 0 & 0 & 0 & 0 \\ -\rho_E & (\mu_{L_1} + \rho_L) & 0 & 0 & 0 & 0 \\ 0 & -\rho_L & (\mu_P + \rho_P) & 0 & 0 & 0 \\ 0 & 0 & -\rho_P & (\mu_{A_h} + \rho_{A_h}) & 0 & -\rho_{A_0} \\ 0 & 0 & 0 & -\rho_{A_h} & (\mu_{A_r} + \rho_{A_r}) & 0 \\ 0 & 0 & 0 & 0 & -\rho_{A_r} & (\mu_{A_0} + \rho_{A_0}) \end{bmatrix}. \quad (4)$$

The population reproduction number, R_0 , is the spectral radius of the next generation operator, $\rho(FV^{-1})$. This value is given by

$$R_0 = \frac{b \prod_j \left(\frac{\rho_j}{\mu_j + \rho_j} \right)}{1 - \prod_{A_i} \left(\frac{\rho_{A_i}}{\mu_{A_i} + \rho_{A_i}} \right)}. \quad (5)$$

where $j = E, L, P, A_h, A_r, A_0$ and $i = h, r$, and $\frac{\rho_j}{\mu_j + \rho_j}$ is the probability that a mosquito in stage j will survive to the next stage. The value $\prod_{A_i} \left(\frac{\rho_{A_i}}{\mu_{A_i} + \rho_{A_i}} \right) \in (0, 1)$ for all i is the probability that an adult mosquito survives the feeding cycle. Although density-dependent mortality of larvae affects mosquito population, R_0 does not depend on density-dependent mortality of larvae.

Theorem 3.1. The system of Eq. (1) has a persistent positive equilibrium solution $P_e = (E^*, L^*, P^*, A_h^*, A_r^*, A_0^*)$, with its components given by

$$\begin{aligned} E^* &= \frac{b \rho_{A_0} A_0^*}{\mu_E + \rho_E}, \\ L^* &= \frac{(\mu_{L_1} + \rho_L)(R_0 - 1)}{\mu_{L_2}}, \\ P^* &= \frac{\rho_L L^*}{\mu_P + \rho_P}, \\ A_h^* &= \frac{\rho_P P^* R_0}{(\mu_{A_h} + \rho_{A_h}) B_1}, \\ A_r^* &= \frac{\rho_{A_h} A_h^*}{\mu_{A_r} + \rho_{A_r}}, \\ A_0^* &= \frac{\rho_{A_r} A_r^*}{\mu_{A_0} + \rho_{A_0}}, \end{aligned} \quad (6)$$

with R_0 given in Eq. (5) and $B_1 = b \prod_j \left(\frac{\rho_j}{\mu_j + \rho_j} \right)$ for $j = E, L, P, A_h, A_r, A_0$, which exist in the interior of \mathbb{D} if $R_0 > 1$.

Proof. Substituting $P_e = (E^*, L^*, P^*, A_h^*, A_r^*, A_0^*)$ into (1) shows that P_e is an equilibrium point of (1). If $R_0 > 1$, we see that all components P_e are positive. Thus, P_e exist in the interior of \mathbb{D} if $R_0 > 1$. \square

3.2. Stability of the equilibrium points

Theorem 3.2. The mosquito-free equilibrium is locally asymptotically stable when $R_0 < 1$ and unstable otherwise.

Proof. Let the new births in the ecological model (1) be equivalent to new infections in the epidemic models studied in van den Driessche and Watmough [64]. The matrices $F(x)$, $V(x)^+$, and $V(x)^-$ satisfy the assumptions A(1)–A(5) [64]. Thus, this theorem is a straightforward application of Theorem 2 given in [64]. \square

Theorem 3.3. The persistent equilibrium is locally asymptotically stable whenever $R_0 > 1$ and unstable when $R_0 < 1$. When $R_0 = 1$, $P_e = P_0$.

Proof. Let J_{P_e} be the Jacobian matrix of system (1) at the mosquito persistent equilibrium given by

$$J_{P_e} = \begin{bmatrix} -(\mu_E + \rho_E) & 0 & 0 & 0 & 0 & b \rho_{A_0} \\ \rho_E & -(\mu_{L_1} + \rho_L) - \Phi & 0 & 0 & 0 & 0 \\ 0 & \rho_L & -(\mu_P + \rho_P) & 0 & 0 & 0 \\ 0 & 0 & \rho_P & -(\mu_{A_h} + \rho_{A_h}) & 0 & \rho_{A_0} \\ 0 & 0 & 0 & \rho_{A_h} & -(\mu_{A_r} + \rho_{A_r}) & 0 \\ 0 & 0 & 0 & 0 & \rho_{A_r} & -(\mu_{A_0} + \rho_{A_0}) \end{bmatrix}, \quad (7)$$

where $\Phi = 2(\mu_{L_1} + \rho_L)(R_0 - 1)$. To obtain the eigenvalues of J_{P_e} , we solve $\det(J_{P_e} - \lambda I) = 0$. We use the concept of block matrices to compute this determinant. Let $J = J_{P_e} - \lambda I$ be a block matrix given by

$$J = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \quad (8)$$

with the following components:

$$A = \begin{pmatrix} -(\mu_E + \rho_E) - \lambda & 0 & 0 \\ \rho_E & -(\mu_{L_1} + \rho_L) - \Phi - \lambda & 0 \\ 0 & \rho_L & -(\mu_P + \rho_P) - \lambda \end{pmatrix},$$

$$B = \begin{pmatrix} 0 & 0 & b \rho_{A_0} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, C = \begin{pmatrix} 0 & 0 & \rho_P \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

and

$$D = \begin{pmatrix} -(\mu_{A_h} + \rho_{A_h}) - \lambda & 0 & \rho_{A_0} \\ \rho_{A_h} & -(\mu_{A_r} + \rho_{A_r}) - \lambda & 0 \\ 0 & \rho_{A_r} & -(\mu_{A_0} + \rho_{A_0}) - \lambda \end{pmatrix}.$$

It follows from the concepts of block matrices that $\det(J) = \det(AD - BC)$. But in this case, BC is a zero matrix leading to $\det(J_{P_e} - \lambda I) = \det(J) = \det(AD) = 0$. By solving the equation, we obtain three of the eigenvalues given by $\lambda_1 = -(\mu_E + \rho_E)$, $\lambda_2 = -(\mu_{L_1} + \rho_L) - \Phi$, and $\lambda_3 = -(\mu_P + \rho_P)$. When $R_0 > 1$, $\lambda_2 < 0$, which forms the necessary condition for a stable equilibrium point. When $R_0 < 1$, $\lambda_2 > 0$, P_e is unstable. The remaining three eigenvalues are given by the roots of the following equation:

$$a_0 \lambda^3 + a_1 \lambda^2 + a_2 \lambda + a_3 = 0, \quad (9)$$

where

$$\begin{aligned} a_0 &= 1, \\ a_1 &= (\mu_{A_h} + \rho_{A_h}) + (\mu_{A_r} + \rho_{A_r}) + (\mu_{A_0} + \rho_{A_0}), \\ a_2 &= (\mu_{A_h} + \rho_{A_h})(\mu_{A_r} + \rho_{A_r}) + (\mu_{A_h} + \rho_{A_h})(\mu_{A_0} + \rho_{A_0}) + (\mu_{A_r} + \rho_{A_r})(\mu_{A_0} + \rho_{A_0}), \\ a_3 &= (\mu_{A_h} + \rho_{A_h})(\mu_{A_r} + \rho_{A_r})(\mu_{A_0} + \rho_{A_0}) - \rho_{A_h} \rho_{A_r} \rho_{A_0} \\ &= (\mu_{A_h} + \rho_{A_h})(\mu_{A_r} + \rho_{A_r})(\mu_{A_0} + \rho_{A_0}) \left(\frac{B_1}{R_0} \right), \end{aligned} \quad (10)$$

where $B_1 = b \prod_j \left(\frac{\rho_j}{\mu_j + \rho_j} \right)$ for $j = E, L, P, A_h, A_r, A_0$. It remains to be shown that when $R_0 > 1$, the eigenvalues have negative real parts. The roots of the polynomial in Eq.(9) are difficult to calculate explicitly, but it is clear from (9) that $a_0 > 0$, $a_1 > 0$, $a_2 > 0$, and $a_3 > 0$ always. By the Routh–Hurwitz criteria [41] we need to show that $a_1 a_2 - a_3 > 0$ for all roots of Eq. (9) to have negative real parts.

$$\begin{aligned}
 a_1 a_2 - a_3 = & [(\mu_{A_h} + \rho_{A_h}) + (\mu_{A_r} + \rho_{A_r}) + (\mu_{A_o} + \rho_{A_o})] \\
 & [(\mu_{A_h} + \rho_{A_h})(\mu_{A_r} + \rho_{A_r}) \\
 & + (\mu_{A_h} + \rho_{A_h})(\mu_{A_o} + \rho_{A_o}) + (\mu_{A_r} + \rho_{A_r})(\mu_{A_o} + \rho_{A_o})] \\
 & - (\mu_{A_h} + \rho_{A_h})(\mu_{A_r} + \rho_{A_r})(\mu_{A_o} + \rho_{A_o}) - \rho_{A_h} \rho_{A_r} \rho_{A_o} \\
 = & (\mu_{A_h} + \rho_{A_h})^2 [(\mu_{A_r} + \rho_{A_r}) + (\mu_{A_o} + \rho_{A_o})] \\
 & + (\mu_{A_r} + \rho_{A_r})^2 [(\mu_{A_h} + \rho_{A_h}) + (\mu_{A_o} + \rho_{A_o})] \\
 & + (\mu_{A_o} + \rho_{A_o})^2 [(\mu_{A_h} + \rho_{A_h}) + (\mu_{A_r} + \rho_{A_r})] \\
 & + 2(\mu_{A_h} + \rho_{A_h})(\mu_{A_r} + \rho_{A_r})(\mu_{A_o} + \rho_{A_o}) - \rho_{A_h} \rho_{A_r} \rho_{A_o} \\
 = & (\mu_{A_h} + \rho_{A_h})^2 [(\mu_{A_r} + \rho_{A_r}) + (\mu_{A_o} + \rho_{A_o})] \\
 & + (\mu_{A_r} + \rho_{A_r})^2 [(\mu_{A_h} + \rho_{A_h}) + (\mu_{A_o} + \rho_{A_o})] \\
 & + (\mu_{A_o} + \rho_{A_o})^2 [(\mu_{A_h} + \rho_{A_h}) + (\mu_{A_r} + \rho_{A_r})] \\
 & + (\mu_{A_h} + \rho_{A_h})(\mu_{A_r} + \rho_{A_r})(\mu_{A_o} + \rho_{A_o}) \left[1 + \frac{B_1}{R_o} \right].
 \end{aligned} \quad (11)$$

From (11) we see that $a_1 a_2 - a_3 > 0$ for all values of R_o . Thus, the roots of (9) have negative real parts. Therefore, when $R_o > 1$, the six eigenvalues have negative real parts and the persistent equilibrium point is locally asymptotically stable. Where, as when $R_o < 1$, $\lambda_2 > 0$. The persistent equilibrium point is unstable. Substituting $R_o = 1$ in (6) shows that at $R_o = 1$, $P_e = P_o$. \square

3.3. Sensitivity Analysis of R_o

Sensitivity analysis determines the effects of parameters on model outcomes [11]. To carry out local sensitivity analysis, we use a simple approach to compute the sensitivity index, which is a partial derivative of the output variable with respect to the input parameters [11,12]. For the base reproduction number, R_o , and p_i , an input parameter, the sensitivity index can be computed as

$\partial R_o / \partial p_i$. The normalized sensitivity index, $\Omega_{p_i}^{R_o}$, of R_o , with respect to parameter p_i at a fixed value, p^0 [11,12] is

$$\Omega_{p_i}^{R_o} = \frac{\partial R_o}{\partial p_i} \times \frac{p_i}{R_o} \bigg|_{p_i=p^0}. \quad (12)$$

Using the parameter values presented in Table 2, we compute the sensitivity indices using Eq. (12). In Fig. 2A we show the impact of each parameter on the reproduction number. The number of female eggs laid per oviposition, b , is the most important parameter in the model ($\Omega_b^{R_o} = 1.00$), indicating a maximum impact on model outcomes. Increasing or decreasing b by 10%, for example, can increase or decrease R_o by 10%. The parameters with the next highest sensitivity indices are ρ_L and μ_{L1} . If the development rate from larval to pupae stage (ρ_L) is increased, we observe a decreased risk of dying of larvae (μ_{L1}) and vice versa. A 10% increase (or decrease) in ρ_L , for example, increases (or decreases) R_o by 7.6%, while a similar increase (or decrease) of μ_{L1} in R_o decreases (or increases) R_o by 7.6%. Other important parameters with higher indices are ρ_{A_h} and μ_{A_h} . Similar to ρ_L and μ_{L1} , these parameters indicate an equal but opposite impact on R_o . Increasing ρ_{A_h} can lead to an increase in R_o . Increasing μ_{A_h} , however would decrease R_o .

Local sensitivity analysis shows the effect of one parameter while all others are kept constant. Global sensitivity analysis estimates the effect of one parameter on the output, while allowing all other parameters to vary, enabling the identification of interactions [11]. Here, we used SaSAT software [26] to carry out the global sensitivity analysis of the mosquito population reproduction number. The Latin Hypercube Sampling Method (LHS), a type of stratified Monte Carlo sampling [6], was used to sample the input parameters using the parameter value ranges provided in Table 2. Due to the absence of data on the distribution function of the parameters used in our model, a uniform distribution for all input parameters was chosen. The sets of input parameter values sampled using the LHS method were used to run 5000 simulations.

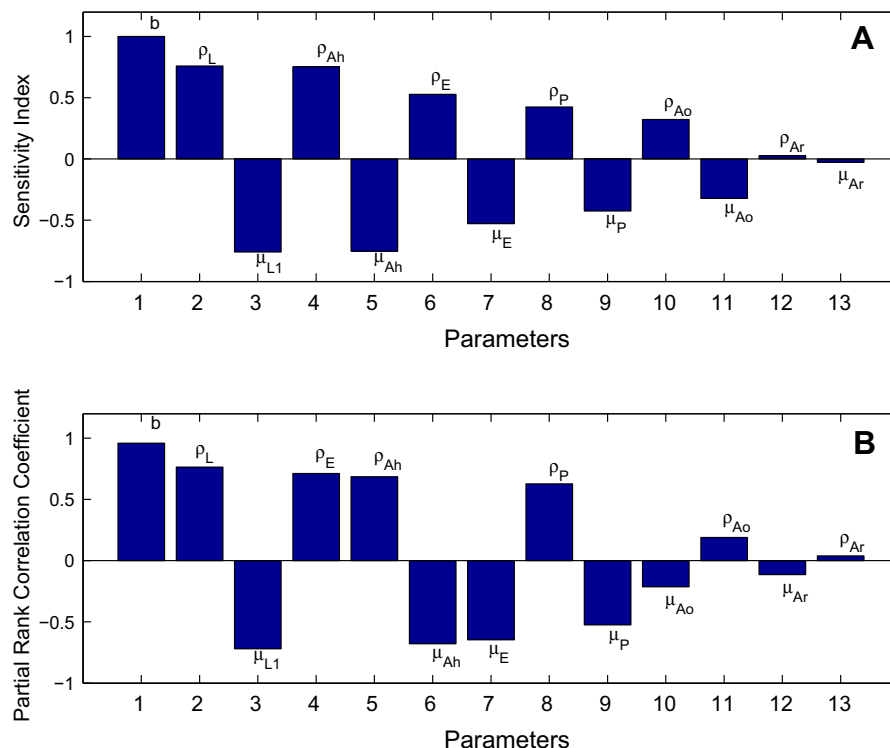


Fig. 2. Sensitivity Analysis of R_o . A: Local sensitivity analysis. Normalized sensitivity indices of R_o to parameters evaluated at the baseline parameter values given in Table 2. B: Global sensitivity analysis. Partial Rank correlation coefficients showing the ranking of parameter influence on R_o .

To identify input parameters with the greatest influence on R_0 , we computed the Partial Rank Correlation Coefficients between the input parameters and our output variable using the SaSAT software.

In Fig. 2B, we present the results of the partial rank correlation coefficients for each of the parameters. Again, results show that birth parameter, b , has the highest influence on the mosquito population reproduction number. Next to b are the parameters associated with the larvae stage, followed by the egg development rate and the parameters related to the host seeking stage. Parameters related to the resting stage of the mosquitoes show the lowest influence on R_0 .

In general, we find that mortality rates are negatively correlated to the population reproduction number, while development rates are positively correlated. Because the population reproduction number gives information on the stability of the equilibrium point and the persistence of the mosquito population, increasing parameters that are positively correlated to the reproduction number would result in the persistence of the mosquito population.

4. Modelling movement

4.1. Continuous space model

Traditional methods of modelling diffusion have involved the use of the heat equation in which the domain is assumed to be continuous. If we assume that the movement of individual mosquitoes is similar to that of Brownian motion, then we can define the rate of change of mosquito density at time t at location (x, y) , $M(x, y, t)$ as

$$\frac{\partial M(x, y, t)}{\partial t} = D^* \nabla^2 M(x, y, t) \quad (13)$$

where $(x, y) \in \mathbb{R}^2$, ∇ represents the partial derivative in 2-dimensional space and $\nabla^2 M = \partial^2 M / \partial x^2 + \partial^2 M / \partial y^2$, and D^* is the diffusion coefficient (metres² time⁻¹). We assume that the initial conditions are given by $M(x, y, 0) = K\delta(x, y)$, where $\delta(x, y)$ is the 2-dimension Dirac delta function, $\delta(x, y) = 0$ for $x^2 + y^2 \neq 0$ and $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(x, y) dx dy = 1$. Therefore, $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} M(x, y, 0) dx dy = K$ represents an initial condition of K mosquitoes released at the origin.

The standard solution to the heat Eq. (13) is given by:

$$M(x, y, t) = \frac{K}{4\pi D^* t} \exp \left[-\frac{(x^2 + y^2)}{4D^* t} \right] \quad (14)$$

for $t > 0$ and $(x, y) \in \mathbb{R}^2$. We convert our solution to polar coordinates with

$$x = r \cos \theta \text{ and } y = r \sin \theta, \text{ implying that } r = \sqrt{x^2 + y^2} \text{ and } dx dy = r dr d\theta. \quad (15)$$

Using (14) and (15) we obtain

$$M(r, \theta, t) = \frac{K}{4\pi D^* t} \exp \left[-\frac{r^2}{4D^* t} \right] \quad (16)$$

for $r \geq 0$ is the radial distance measured from centre. The mosquito density at a given distance, r from the centre is obtained from $\bar{M}(r, t) = \int_0^{2\pi} M(r, \theta, t) r d\theta$, which gives

$$\bar{M}(r, t) = \frac{Kr}{2D^* t} \exp \left[-\frac{r^2}{4D^* t} \right]. \quad (17)$$

Although partial differential equations (PDEs) are a good way of modelling dispersal [18,19], their analysis is usually limited to numerical simulations when modelling environmental heterogeneity. Discrete approaches offer a better and simpler way of modelling heterogeneity [2,3,29], specifically when resources such as hosts and breeding sites are variable across regions. In the next section, we develop a mosquito dispersal model which considers discrete space and describes how we model heterogeneity in resources and its influence on mosquito dispersal.

4.2. Discrete space model spatial structure

We let Ξ be the set of all patches and ξ be any patch in Ξ . We construct the model by dividing 2-dimensional space into a set of discrete hexagonal patches (Fig. 3). We label the hexagonal grid with a coordinate system, (i, j) , where $1 \leq i \leq n$ and $1 \leq j \leq m$ represent the locations of the centre of the patches and $i, j \in \mathbb{N}$.

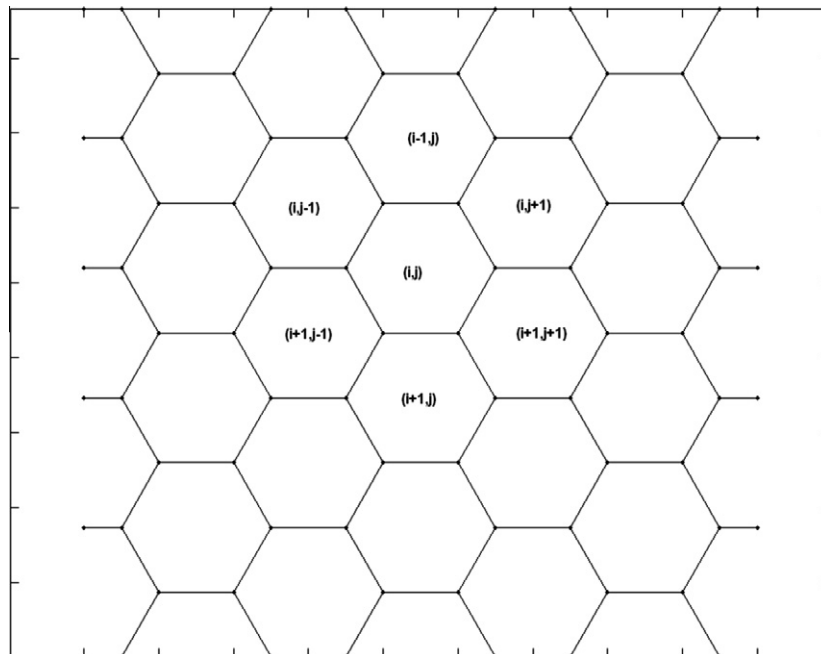


Fig. 3. A Schematic representation of a landscape division into hexagonal patches. Model equations (Eq. (36)) apply in each patch.

We define the neighbourhood, $N(i, j)$, (Fig. 3) of an index patch as an ordered set of six patches given by

$$N(i, j) = \{(i, j+1), (i, j-1), (i+1, j), (i-1, j), (i-1, j+1), (i-1, j-1)\} \quad (18)$$

when j is even or

$$N(i, j) = \{(i, j+1), (i, j-1), (i+1, j-1), (i-1, j), (i+1, j+1), (i+1, j)\} \quad (19)$$

when j is odd. We assume periodic boundary conditions so that patch $(i, 0) = (i, m)$ and $(0, j) = (n, j)$.

4.3. Dispersal in a homogeneous landscape

Mosquitoes disperse while searching for hosts or oviposition sites, causing a link between patches. A given fraction of adults searching for hosts and a fraction of adults searching for oviposition sites leave their original or current patches of residence, while others stay behind. We assume that dispersing adults move from their current patch to enter any of the other six nearest neighbouring patches (Fig. 3) and that long-range dispersal is achieved through a repeated single patch movement. That is, patch jumping is precluded.

Mosquitoes can detect host odour [33,44], but it is unclear whether they have the learning capacity they would need to enable them to return to particular hosts or breeding sites [1]. We make the simplifying assumption that mosquitoes do not preferentially return to their previous locations, so that movement is a Markov process. In the case where all patches have similar characteristics (i.e. a homogeneous landscape), the mosquitoes disperse equally to each of the six neighbouring patches surrounding the current position (Fig. 3) and the dispersal parameter is the same for all patches. If we let $D > 0$ (per time) be the rate at which mosquitoes move from one patch to a neighbouring patch, we can compute its value from:

$$D = \frac{D^*}{A} \quad (20)$$

where D^* is the diffusion coefficient in the absence of all other factors affecting flight. The area A (in metres²) of a hexagon is given by:

$$A = \frac{\sqrt{3}L^2}{2}, \quad (21)$$

with L (in metres) being the patch size defined as the measurement from the centre of one patch to the centre of the neighbouring patch. We let $M_{(ij)}$ be the number of free flying mosquitoes in patch (i, j) . We let mosquitoes move from patch (i, j) (a source or index patch) to a neighbouring patch $\xi \in N(i, j)$. We define the movement rate from patch (i, j) to a neighbouring patch ξ to be $D_{(ij)/\xi}$ and the movement rate from the neighbouring patch to the index patch to be $D_{\xi/(ij)}$. For a homogeneous environment, $D_{(ij)/\xi} = D_{\xi/(ij)} = D$. Assuming that mosquitoes do not reproduce or die during dispersal, the dynamics of free-flying mosquitoes in any patch (i, j) can be represented as

$$\frac{dM_{(ij)}}{dt} = \sum_{\xi \in N(i, j)} DM_{\xi} - \sum_{\xi \in N(i, j)} DM_{(ij)} \quad (22)$$

with initial conditions $M_{(ij)}(0)$. The first term represents mosquitoes moving into the patch and the second term represents mosquitoes moving out of a patch. The movement model in (22) is biologically and mathematically meaningful in the domain $\Omega = M_{(ij)} \in \mathbb{R}^m$, such that $M_{(ij)} \geq 0$.

Theorem 4.1. *If initial conditions lie in region Ω , the movement Eq. (22) has a unique solution that exists and remains in Ω for all time $t \geq 0$.*

Proof. The right hand side of Eq. (22) is continuous with a continuous partial derivative in region Ω and therefore (22) has a unique solution. We then show that Ω is forward-invariant. If $M_{(ij)} = 0$, then $M'_{(ij)} = \sum_{\xi \in N(i, j)} DM_{\xi} \geq 0$ for all (i, j) . Thus, the solution to Eq. (22) is enclosed in Ω and a unique solution exists for all t . \square

4.4. Dispersal in a heterogeneous landscape

Differences in the distribution of resources creates heterogeneity on the grid, since patches may have different degrees of attractiveness to mosquitoes. In this section, we describe how heterogeneity and differences in patch attractiveness to mosquitoes during movement is incorporated.

4.4.1. Dispersal with heterogeneity in host availability

The number of hosts is allowed to differ between patches across the grid, introducing heterogeneity. Because of the neighbour to neighbour dispersal nature of this model, movement of mosquitoes from one patch to other patches is only affected by the patches bordering each neighbourhood. We therefore calculate and use the proportion of hosts in each set of seven patches relative to each other, using the number of hosts on the particular patch and on its six neighbours. However, we assume that host distribution across patches is constant over time.

We recall that Ξ is a set of patches on the grid, ξ is any patch in Ξ , and $N(i, j)$ is a set of neighbours given by (18) and (19) of an index patch (i, j) . We also let c_{ij} be a set of seven patches sharing boundaries, that is, patch (i, j) and its 6 neighbours. $c_{i'j'}$ is a set of seven patches sharing boundaries made up of patch ξ' and its six neighbours, of which one is patch (i, j) . For easy reference, we use the following notations:

- H^{ξ} is the population of hosts in patch ξ
- H_T^{Ξ} is the total population of hosts in c_{ij}
- $H^{\xi'}$ is the population of hosts in patch ξ'
- $H_T^{i'j'}$ is the total population of hosts in $c_{i'j'}$
- \bar{H}_{ξ}^{Ξ} is the proportion of hosts in patch $\xi \in c_{ij}$ out of all hosts in c_{ij}
- $\bar{H}_{\xi'}^{i'j'}$ is the proportion of hosts in patch $\xi' \in c_{i'j'}$ out of all hosts in $c_{i'j'}$
- $\bar{H}_{\xi}^{i'j'}$ is the proportion of hosts in patch $\xi \in c_{i'j'}$ out of all hosts in $c_{i'j'}$
- $\bar{H}_{\xi'}^{\Xi}$ is the proportion of hosts in patch $\xi' \in c_{i'j'}$ out of all hosts in $c_{i'j'}$

We calculate the total number of hosts over these seven patches sharing boundaries from

$$H_T^{\Xi} = \sum_{\xi \in c_{ij}} H^{\xi}, \quad (23)$$

and the proportion of hosts in each $\xi \in c_{ij}$ from

$$\bar{H}_{\xi}^{\Xi} = \frac{H^{\xi}}{H_T^{\Xi}} \quad (24)$$

with

$$\sum_{\xi \in c_{ij}} \bar{H}_{\xi}^{\Xi} = 1 \quad (25)$$

Mosquitoes are attracted to odours released by hosts [15,33,44,61]. This leads to mosquitoes being less likely to leave the patch if their current patch is a home to many hosts and likely

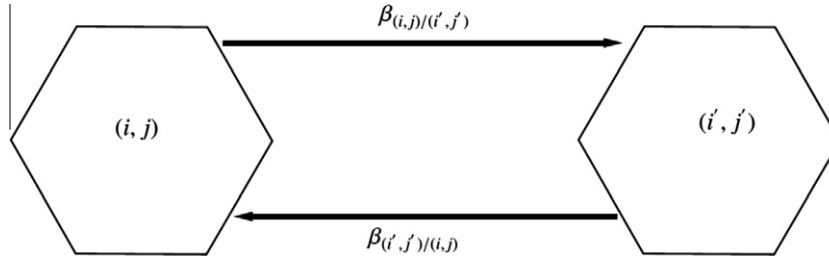


Fig. 4. Diagrammatic representation of mosquito movement between an index patch (source patch (i, j)) and a neighbouring patch $((i', j') = \xi' \in N(i, j))$ where $N(i, j)$ is defined by Eqs. (18) and (19).

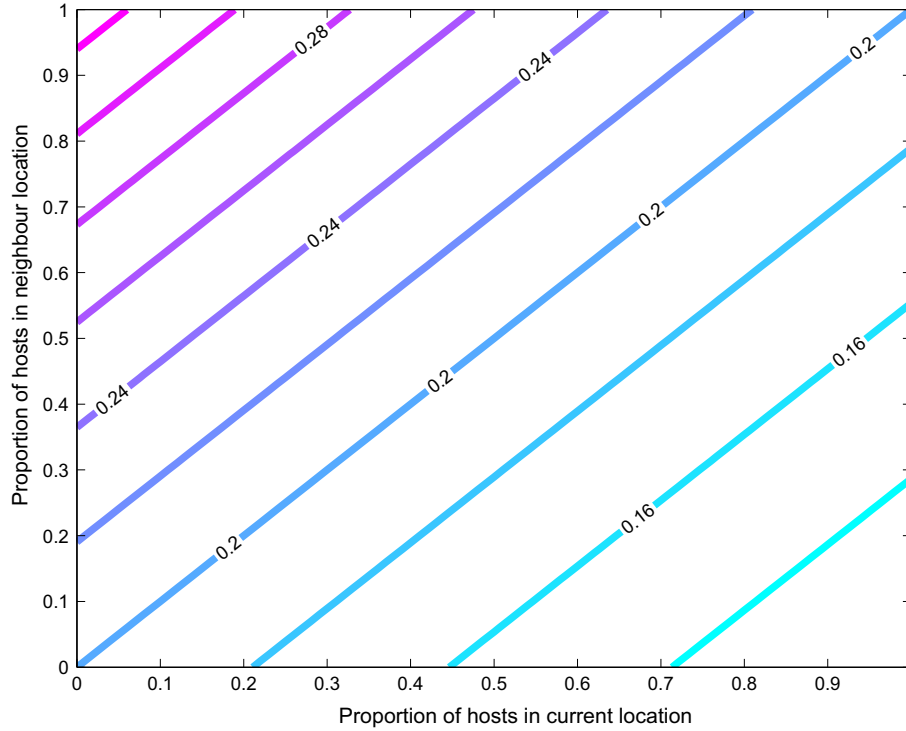


Fig. 5. Behaviour of the dispersal function in Eq. (26) at $D = 0.2$, $\lambda = 0.5$, and $\bar{H} \in [0, 1]$.

to move out of the patch if there are few hosts. To mimic this phenomenon, we use a decreasing exponential function to model the movement rate. If (i, j) is a source patch and its neighbours (Fig. 4), and if we take into account the availability of hosts in each of the patches contained in c_{ij} , we can define the movement out of a patch i, j to a neighbour patch ξ' as

$$\beta_{(i,j)/\xi'}^H = D e^{-\lambda(\bar{H}_{\xi'}^H - \bar{H}_{ij}^H)}. \quad (26)$$

Here, λ is a constant parameter for the decay function and $\bar{H}_{\xi'}^H$ is the proportion of hosts in patch ξ' contained in c_{ij} , which is obtained from

$$\bar{H}_{\xi'}^H = \frac{H_{\xi'}^H}{H_T^H}. \quad (27)$$

The function in Eq. (26) (its behaviour is shown in Fig. 5) represents different possible characteristics of two patches sharing boundaries as follows:

- If $\bar{H}_{\xi'}^H > \bar{H}_{ij}^H$, then $0 < \beta_{(i,j)/\xi'}^H < D$. This condition establishes that the source patch (i, j) contains more hosts compared to patch ξ' . The patch is therefore more attractive to mosquitoes compared to its neighbour and will tend to retain mosquitoes; few mosquitoes will tend to move away from it.

- If $\bar{H}_{\xi'}^H = \bar{H}_{ij}^H$, then $\beta_{(i,j)/\xi'}^H = D$. This implies that the two patches have equal attractiveness to mosquitoes.
- If $\bar{H}_{\xi'}^H < \bar{H}_{ij}^H$, then $\beta_{(i,j)/\xi'}^H > D$. Here, patch (i, j) is less attractive to mosquitoes because it has fewer hosts compared to patch ξ' . The dispersal rate out of the patch is high as more mosquitoes will migrate out to patches that are more attractive.

Similarly, the movement of mosquitoes from patch ξ' to patch (i, j) (Fig. 4), where both ξ' and (i, j) are contained in c_{ij} , is modelled. In this respect, $H_{\xi'}^H$ is calculated using a different set of neighbouring patches, $c_{i'j'}$. In other words, it is the total number of hosts in ξ' and its six neighbours, of which one of them is patch (i, j) . We calculate it using

$$H_{\xi'}^H = \sum_{\xi \in c_{i'j'}} H_{\xi}^H. \quad (28)$$

Therefore, we model the movement rate from any neighbouring patch ξ' into patch (i, j) (as shown in Fig. 4) using

$$\beta_{\xi'/(i,j)}^H = D e^{-\lambda(\bar{H}_{\xi'}^H - \bar{H}_{ij}^H)} \quad (29)$$

where

$$\bar{H}_{\xi'}^H = \frac{H_{\xi'}^H}{H_T^H}, \quad (30)$$

and

$$\bar{H}_{\zeta'}^{ij} = \frac{H_{\zeta'}^{ij}}{H_T^{ij}}. \quad (31)$$

In general, the movement rate from patches with relatively low attraction is higher compared to patches with higher attraction and vice versa. To summarise, we re-write the general movement model presented in Eq. (22) as

$$\frac{dA_{h(i,j)}}{dt} = \left(\sum_{\zeta' \in N(i,j)} \beta_{\zeta'/(i,j)}^H A_{h\zeta'} \right) - \left(\sum_{\zeta' \in N(i,j)} \beta_{(i,j)/\zeta'}^H \right) A_{h(i,j)}, \quad (32)$$

to describe the dynamics of host seeking mosquitoes in the absence of new recruitment and deaths in any of the patches. Here, the dispersal rate takes into account the dependence of dispersal on hosts availability.

4.4.2. Dispersal with heterogeneity in oviposition site availability

Another form of heterogeneity is imposed by the availability of oviposition sites in an area. Mosquitoes searching for breeding sites for egg laying are attracted by the availability of breeding sites [43]. We incorporate this in a manner similar to that for hosts.

If $B_{c_{ij}}$ is the number of oviposition sites in a patch and $\bar{B}_{c_{ij}}$ is the proportion of oviposition sites in a patch relative to its neighbours, the movement rate out of the index patch (i,j) is expressed as

$$\beta_{(i,j)/\zeta'}^B = De^{-\lambda \left(\frac{B_{c_{ij}} - \bar{B}_{c_{ij}}}{B_{c_{ij}}} \right)} \quad (33)$$

and the movement rate into the patch from neighbouring patches:

$$\beta_{\zeta'/(i,j)}^B = De^{-\lambda \left(\frac{B_{c_{\zeta'}} - \bar{B}_{c_{\zeta'}}}{B_{c_{\zeta'}}} \right)} \quad (34)$$

Similarly, the movement rate of mosquitoes from a patch is higher if there are few breeding sites (B) in the patch. We represent the movement of mosquitoes searching for oviposition sites in the following equation

$$\frac{dA_{o(i,j)}}{dt} = \left(\sum_{\zeta' \in N(i,j)} \beta_{\zeta'/(i,j)}^B A_{o\zeta'} \right) - \left(\sum_{\zeta' \in N(i,j)} \beta_{(i,j)/\zeta'}^B \right) A_{o(i,j)}. \quad (35)$$

Since the density of breeding sites is affected by seasonal variations, as temporal sites are created due to rainfall for example, their distribution changes over time. However, in this model, for simplicity, we consider only permanent breeding sites. So the initial distribution of breeding sites does not change over time.

4.5. Full dispersal model equations

In Section 2, we studied the dynamics of mosquito populations in each stage of the mosquito life cycle within a single patch. We extend this model to incorporate dispersal processes. If we allow host seeking and oviposition site searching mosquitoes to move between patches, then we can combine the system of equations in Eq. (1) for patch (i,j) and the movement terms in (32) and (35) to form the following system of equations:

$$\begin{aligned} \frac{dE_{(i,j)}}{dt} &= b_{(i,j)} \psi_{(i,j)}^B \rho_{A_o(i,j)} A_{o(i,j)} - (\mu_{E(i,j)} + \rho_{E(i,j)}) E_{(i,j)} \\ \frac{dL_{(i,j)}}{dt} &= \rho_{E(i,j)} E_{(i,j)} - (\mu_{L_1(i,j)} + \mu_{L_2(i,j)} L_{(i,j)} + \rho_{L(i,j)}) L_{(i,j)} \\ \frac{dP_{(i,j)}}{dt} &= \rho_{L(i,j)} L_{(i,j)} - (\mu_{P(i,j)} + \rho_{P(i,j)}) P_{(i,j)} \\ \frac{dA_{h(i,j)}}{dt} &= \rho_{P(i,j)} P_{(i,j)} + \psi_{(i,j)}^B \rho_{A_o(i,j)} A_{o(i,j)} - (\mu_{A_{h(i,j)}} + \psi_{(i,j)}^H \rho_{A_{h(i,j)}}) A_{h(i,j)} \\ &\quad - \left(\sum_{\zeta' \in N(i,j)} \beta_{(i,j)/\zeta'}^H \right) A_{h(i,j)} + \left(\sum_{\zeta' \in N(i,j)} \beta_{\zeta'/(i,j)}^H A_{h\zeta'} \right) \\ \frac{dA_{r(i,j)}}{dt} &= \psi_{(i,j)}^H \rho_{A_{h(i,j)}} A_{h(i,j)} - (\mu_{A_{r(i,j)}} + \rho_{A_{r(i,j)}}) A_{r(i,j)} \\ \frac{dA_{o(i,j)}}{dt} &= \rho_{A_{r(i,j)}} A_{r(i,j)} - (\mu_{A_{o(i,j)}} + \psi_{(i,j)}^B \rho_{A_{o(i,j)}}) A_{o(i,j)} \\ &\quad - \left(\sum_{\zeta' \in N(i,j)} \beta_{(i,j)/\zeta'}^B \right) A_{o(i,j)} + \left(\sum_{\zeta' \in N(i,j)} \beta_{\zeta'/(i,j)}^B A_{o\zeta'} \right) \end{aligned} \quad (36)$$

with initial conditions $E_{(i,j)}, L_{(i,j)}, P_{(i,j)}, A_{h(i,j)}, A_{r(i,j)}, A_{o(i,j)} \geq 0$ at time $t = 0$. Here, H and B represents hosts and breeding sites respectively. The state variables and some of the parameters carry the same meaning as in system (1) (see Tables 1 and 2). The individual equations in system (36) describe the evolution of eggs, larvae, pupae, host seeking, resting, and oviposition site searching mosquitoes in patch (i,j) .

The progression from the oviposition site searching state, A_o , to the host seeking state, A_h , is possible if and only if oviposition site searching mosquitoes have laid eggs. We introduce a parameter $\psi_{(i,j)}^B$ defined by:

$$\psi_{(i,j)}^B = \begin{cases} 1 & \text{if } B_{(i,j)} > 0 \\ 0 & \text{if } B_{(i,j)} = 0, \end{cases} \quad (37)$$

to control this process, since laying eggs in a patch is possible only if the particular patch contains at least one breeding site. In patches where $B_{(i,j)} = 0$, the initial conditions for $E_{(i,j)}, L_{(i,j)}$, and $P_{(i,j)}$ are 0. Similarly, the progression from host seeking to the resting stage is possible if there are hosts in the patch [31]. As such, we define

$$\psi_{(i,j)}^H = \begin{cases} 1 & \text{if } H_{(i,j)} > 0 \\ 0 & \text{if } H_{(i,j)} = 0. \end{cases} \quad (38)$$

Patches without hosts have initial conditions $A_{r(i,j)} = 0$. All other parameters are patch dependent and their definitions are summarised in Tables 2 and 3.

The total number of mosquitoes in each stage at time t over all patches on the grid is given by the sum over all locations Ξ . That is

$$S(t) = \left(\sum_{\zeta \in \Xi} S_{\zeta}(t) \right) \quad (39)$$

with $S(t)$ representing the stage specific total number of mosquitoes ($E(t), L(t), P(t), A_h(t), A_r(t)$, and $A_o(t)$). The solutions of Eq. (36) remain nonnegative in the region

$$\Gamma = \left\{ \begin{pmatrix} E_{(i,j)} \\ L_{(i,j)} \\ P_{(i,j)} \\ A_{h(i,j)} \\ A_{r(i,j)} \\ A_{o(i,j)} \end{pmatrix} \in \mathbb{R}^{6nm} \mid \begin{matrix} E_{(i,j)} \geq 0, \\ L_{(i,j)} \geq 0, \\ P_{(i,j)} \geq 0, \\ A_{h(i,j)} \geq 0, \\ A_{r(i,j)} \geq 0, \\ A_{o(i,j)} \geq 0 \end{matrix} \right\}, \quad (40)$$

because movement always stops when there are no mosquitoes in a patch. The model is therefore mathematically and biologically well posed.

Table 3
Description of parameters and variables specific to the dispersal model.

Parameter	Description	Units
H	number of hosts	hosts
B	number of breeding sites	breeding sites
β^H	dispersal rate of host seeking mosquitoes	per time
β^B	dispersal rate of mosquitoes searching for breeding sites	per time
β^{H^*}	dispersal rate of mosquitoes in the presence of repellents	per time
L	patch size	metres
D	rate of movement	per time
λ	a constant parameter for the decay function	dimensionless
D^*	diffusion coefficient	metres ² time ⁻¹
P	repellents blocked ability of mosquitoes to enter a patch	dimensionless
ϕ^H	a fraction measuring the strength of a repellent in in patch i, j	unitless

Theorem 4.2. Assuming that initial conditions lie in Γ , the system of equations for the mosquito population dynamics for all patches (36) has a unique solution that exists and remains in Γ for all time $t \geq 0$.

Proof. The right hand side of system (36) is continuous with continuous partial derivatives in region Γ . Thus, there exists a unique solution for (36). We show that region Γ is forward-invariant. From system (36) we see that if $E_{(ij)} = 0$, then $E'_{(ij)} = b_{(ij)}\psi_{(ij)}^B \rho_{A_0(ij)} A_{0(ij)} \geq 0$; if $L_{(ij)} = 0$, then $L'_{(ij)} = \rho_{E(ij)} E_{(ij)} \geq 0$; if $P_{(ij)} = 0$, then $P'_{(ij)} = \rho_{L(ij)} L_{(ij)} \geq 0$; if $A_{h(ij)} = 0$, then $A'_{h(ij)} = \rho_{P(ij)} P_{(ij)} + \psi_{(ij)}^B \rho_{A_0(ij)} A_{0(ij)} + \left(\sum_{e' \in N(ij)} \beta_{e'/(ij)}^H A_{he'}\right) \geq 0$; if $A_{r(ij)} = 0$, then $A'_{r(ij)} = \psi_{(ij)}^H \rho_{A_{hi}(ij)} A_{hi(ij)} \geq 0$; and if $A_{o(ij)} = 0$, then $A'_{o(ij)} = \rho_{A_{ri}(ij)} A_{ri(ij)} + \left(\sum_{e' \in N(ij)} \beta_{e'/(ij)}^B A_{oe'}\right) \geq 0$. Therefore, all solutions of the system of equations in (36) are contained in the region Ξ and a unique solution exists for all t . \square

System (36) is at an equilibrium if the right hand side is zero at all time t . Patch (i, j) is at a mosquito-free equilibrium if $E_{(ij)} = L_{(ij)} = P_{(ij)} = A_{h(ij)} = A_{r(ij)} = A_{o(ij)} = 0$. However, given the complexity of the model, we do not show its stability or show the existence of other invariant subsets and only run numerical simulations of this model.

5. Numerical simulations

The model without dispersal (Eq. (1)) and the model with dispersal (Eq. (36)) are both simulated using Matlab 7.10.0(R2010a) student version [40] and the *ode45* solver for solving differential equations is used. The 25 by 21 grid (see sketch in Fig. 3) is used as a platform to simulate movement of mosquitoes between hexagonal patches. To ensure that boundary conditions do not influence results, periodic boundary conditions are used. This implies a torus topology for the landscape, where edge patches are such that their nearest neighbours on the outside are patches on the opposing edges.

For model simulation and investigation, we use data on stage specific mortality and development rates from the literature (see A), summarised in Table 2. For mosquito dispersal, some studies show that mosquitoes can move up to 800 m a day [21]. Field studies on mark release recapture experiments of *Anopheles gambiae* also show that daily flight range from 200 to 400 m [42]. These results indicate that mosquito dispersal distance is variable. Due to these variations, in Section 6.1 we use our model platform (Section 4) and the movement rate D (Eq. (20)) to produce distributions of dispersed mosquitoes by distance travelled in a day. However, for numerical illustration of the model with dispersal, we set the

distance from the centre of one patch to the centre of the neighbouring patch, L , to 50 m.

We run simulations with total numbers of 2700 eggs, 1900 larvae, 2000 pupae, 2400 host seeking adults, 1800 resting adults, and 1200 oviposition site seeking adults, initially distributed across the grid (Fig. 7). The distribution is based on the whether a patch contains breeding sites or hosts. Five scenarios are set up to simulate the effect of different kinds of heterogeneity (Fig. 6). In the first scenario, all patches contain hosts and breeding sites; the second scenario simulates the case when hosts and breeding sites are randomly distributed on the grid. In the third scenario, all patches contain breeding sites and hosts are only on one side of the grid; while in the fourth scenario, hosts are present in all patches, with breeding sites being on one side of the grid. In scenario five, hosts and breeding sites are placed in clusters that are far apart from each other. Simulations are continued for each scenario until the total mosquito population over the entire grid for each of the stages and their spatial distribution reaches an equilibrium. The final time of analysis for the simulations for all results presented in this work is 250 days, except where stated otherwise.

6. Model application, comparisons, and results

6.1. Dispersal distances

In this section, we use the dispersal model to estimate the distance travelled by an average mosquito. The evolution of Eq. (22) is simulated on a homogeneous grid with uniform attractiveness to mosquitoes. The system is initialized with all mosquitoes placed at a single source patch. We then calculate the total number of mosquitoes per patch and per neighbourhood, the average density of mosquitoes per patch, and the average of the dispersal distance after time 1.

We let $M_n(t)$ be the average density of mosquitoes in a patch at time t , where n measures the distance from the source patch. Here, n is 0, 1, 2, ..., m , with $n = 0$ being the source patch, $n = 1$ being the nearest neighbouring patches (first ring of patches), $n = 2$ being the second ring of patches, $n = 3$ being the third ring of patches, and so on (see Fig. 8). The total number of patches in each of the rings is given by

$$N_n = \begin{cases} 6n & \text{for } n \geq 1 \\ 1 & \text{for } n = 0. \end{cases} \quad (41)$$

The total number of mosquitoes that reached ring n after time t , $P_n(t)$ is

$$P_n(t) = \sum_{k=1}^{N_n} C_k(t), \quad (42)$$

where C_k is the number of mosquitoes in patch k contained in n . From Eq. (42), we obtain the mosquito frequency distance travelled from the source patch for a particular time t . We present the results in Fig. 9A for $t = 1$ day and different values of D .

The average density of mosquitoes per patch, after time t has elapsed, is obtained from:

$$M_n(t) = \frac{P_n(t)}{N_n}, \quad (43)$$

which gives the average density distribution presented in Fig. 9B when $t = 1$ day.

We let S_1 be the initial number of mosquitoes released from the source patch and the weighted average distance travelled by one mosquito at time t , $W_d(t)$, is

$$W_d(t) = \left(\frac{\sum_{n=0}^m n P_n(t)}{S_1} \right) \times L \quad (44)$$

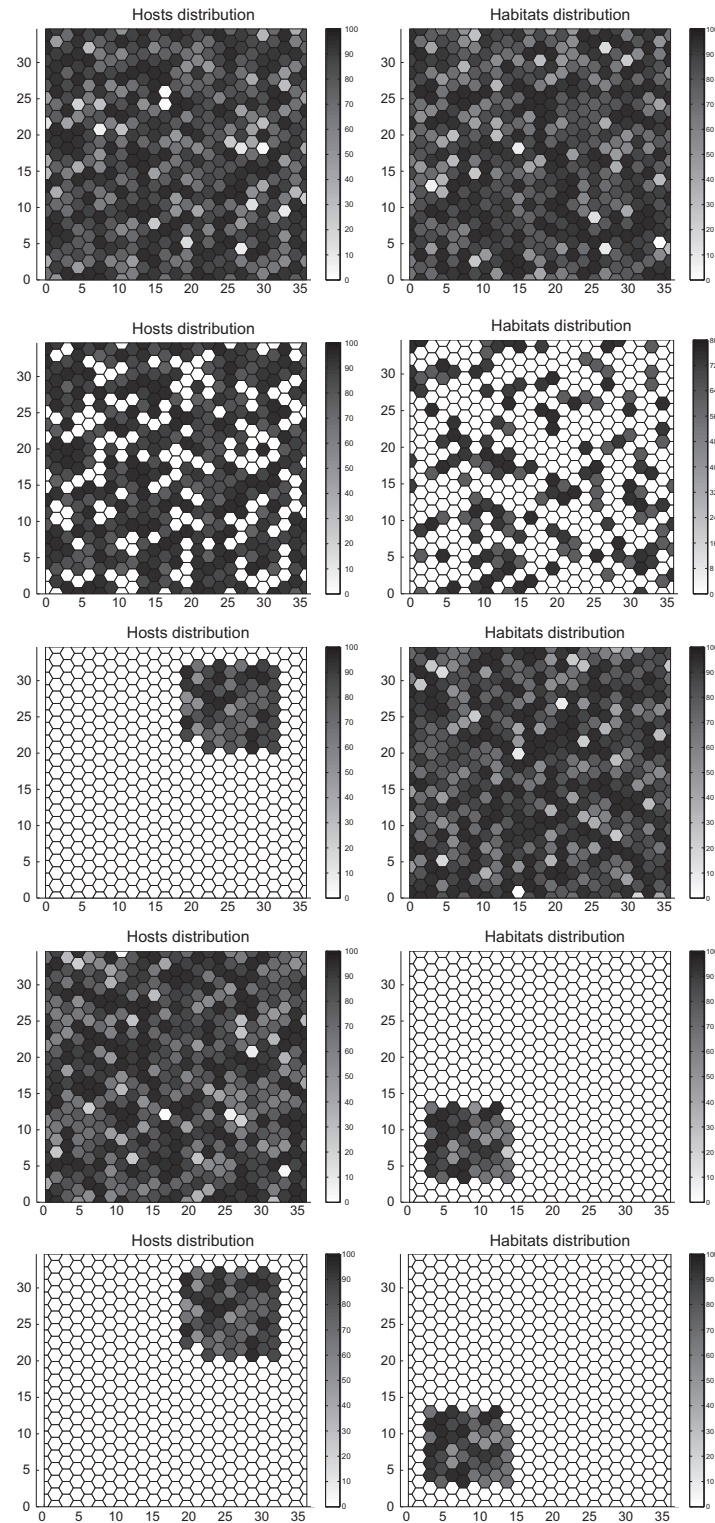


Fig. 6. Spatial arrangements of hosts and breeding sites on the grid showing the set up of scenarios. Scenario 1 (first row): all patches contain hosts and breeding sites. Scenario 2 (second row): random distribution of hosts and breeding sites. Scenario 3 (third row): all patches contain breeding sites but hosts on one side of the grid. Scenario 4 (fourth row): all patches contain hosts but breeding sites are on one side of the grid. Scenario 5 (fifth row): clusters of hosts and breeding sites are far apart from each other.

where L is the patch size. We calculate the weighted average of the dispersal distance travelled by a mosquito in one day, $W_d(1)$.

In Fig. 9A we present the results of the frequency distribution of mosquitoes dispersed in a day by distance from source at different values of the diffusion parameter. As expected, increasing values of D results in mosquitoes moving faster and reaching larger distances. Fig. 9B shows the average density of mosquitoes

per patch by distance moved in a day. After one day, most mosquitoes have moved, but the source still contains the highest density.

From simulations, the weighted mean distance travelled by each mosquito per day (as calculated from Eq. (44)) is estimated to be 43, 79, and 103 m when $L = 50$ m and mosquitoes are allowed to move at a rate, D , of 0.2, 0.5, and 0.8, respectively.

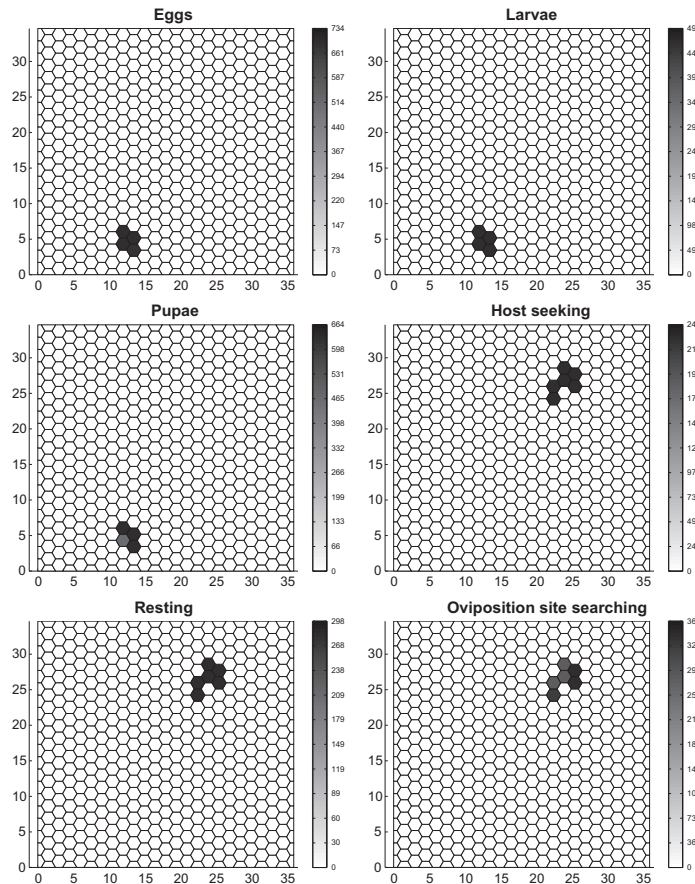


Fig. 7. Spatial population distribution of initial conditions by stage. The distribution of initial conditions is common to all scenarios (Fig. 6) for comparative purposes.

6.2. Comparison between discrete and continuous space form of the models

The nearest neighbours movement approach has been shown to relate closely to diffusion models [3,30]. To evaluate the effects of

using discrete space, we compare the behaviour of the discrete space movement model (Eq. (22)) under homogeneous conditions to that of the model that uses the diffusion approach (Eq. (13)). By comparing the behaviour of the two approaches, we calculate how far a mosquito can travel in a day (and time is set to 1 day in the

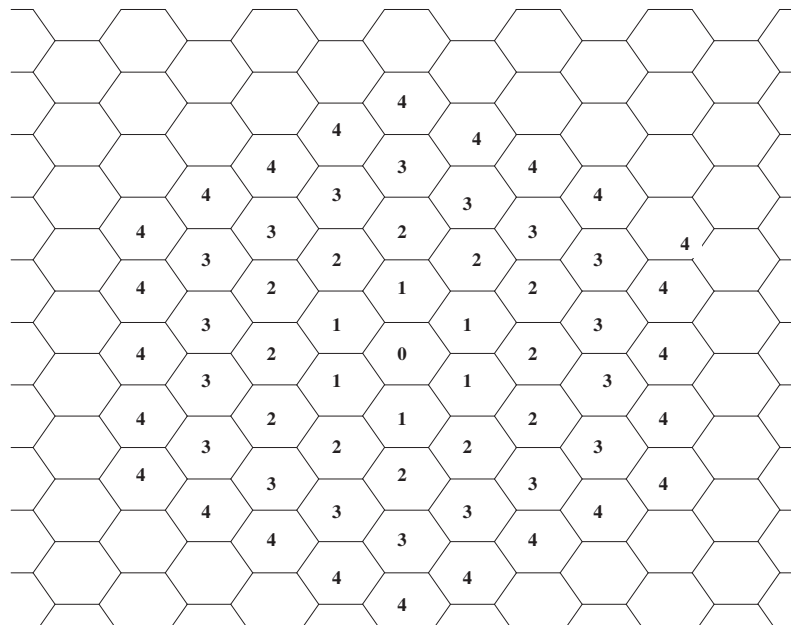


Fig. 8. Sketch diagram showing how the distance from the source patch was obtained. Each ring of neighbours to source patch, $n = 0, 1, 2, \dots, m$ can be multiplied by patch size to obtain the distance.

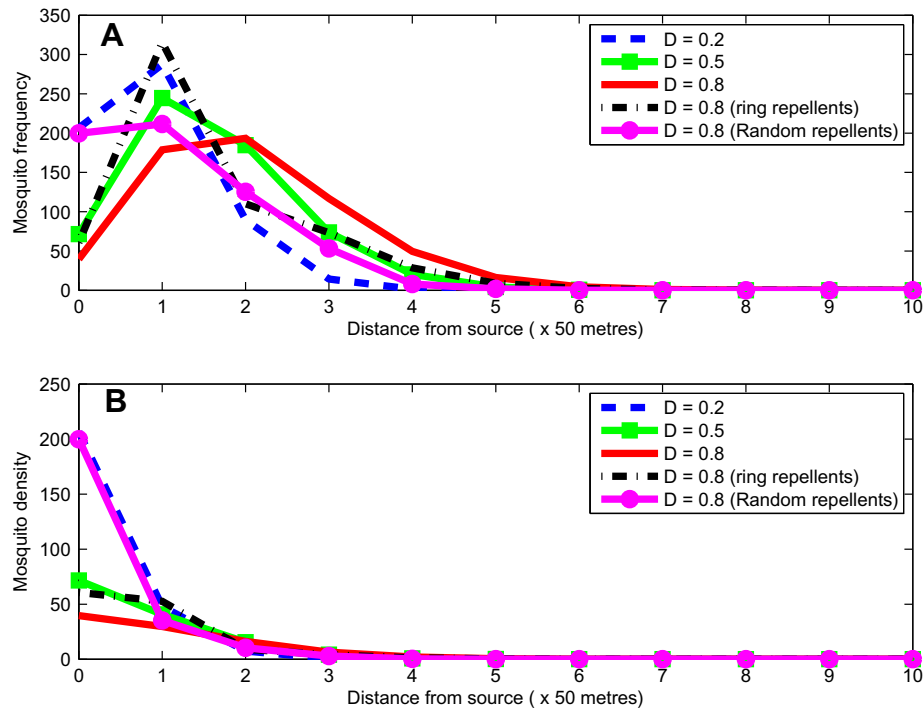


Fig. 9. Distribution of mosquitoes as a function of the distance from centre. A: Frequency distributions of mosquitoes by the distance moved from mosquito source (source patch with $n = 0$) in a day (Eq. (42) in Section 6.1). B: Average density of mosquitoes per patch (Eq. (43)). A total number of 600 (i.e. $S_1 = 600$) mosquitoes were initially placed in one patch and simulated at different values of D and different distributions of repellents for one day with $\lambda = 0.5$.

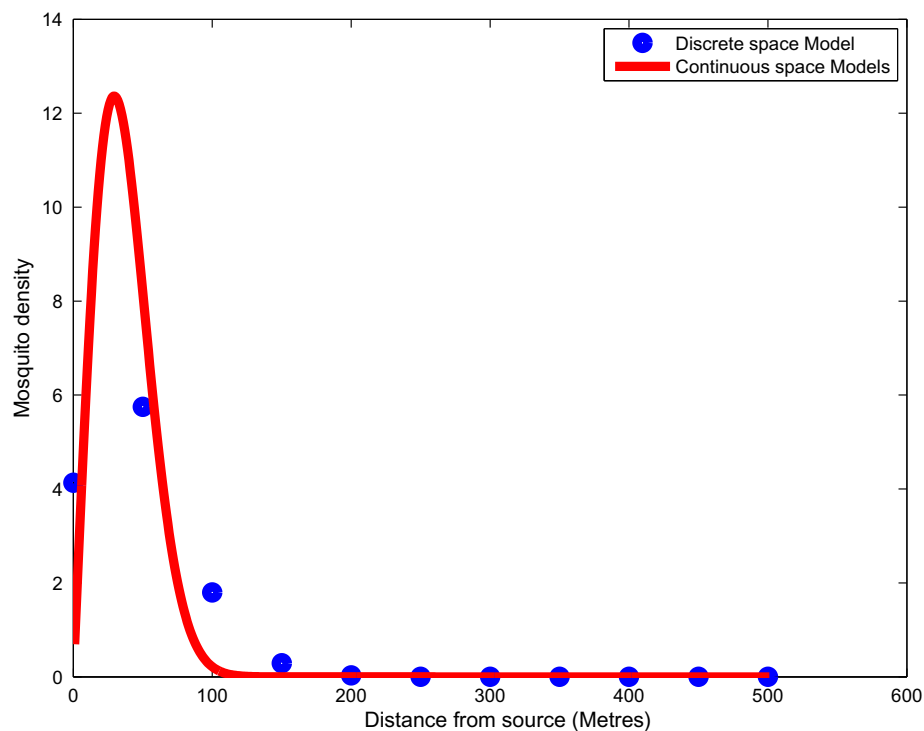


Fig. 10. Comparison between the discrete space Eq. (32) and continuous space Eq. (17) (obtained from Eq. (13)) models. A total number of 600 mosquitoes initially placed at the source patch during the simulation of the discrete space model (with $\lambda = 0.5$) were also used in the continuous space version of the model (i.e. $K = 600$). Mosquitoes were allowed to move at the same rate (i.e. $D = 0.2$) for both forms of the model and time was set to 1 day.

simulations for both models). Fig. 10 presents the results of the discrete (Eq. (22)) and continuous forms (Eq. (17)) of the model. The scenario we compare to the diffusion model is such that all patches contain mosquito resources, creating uniformity in attractiveness to mosquitoes between patches. The probability of a mosquito

moving in any direction is therefore the same. The two models produce slightly different results. However, the distributions show similar properties in terms of the modelled mosquito trajectories between the discrete space and the continuous space models. Both models show peaks in mosquito density near the point of release.

The continuous model shows a higher peak and a higher rate of decrease compared to the discrete model.

6.3. Spatial repellents

Spatial repellents can have different effects on mosquito dispersal, and hence population dynamics, in different areas. These

repellents can be non-physical barriers, such as the treating perimeters with insecticides to protect populations from mosquito bites [8] by reducing the number of biting mosquitoes moving into the area [9]. We use the dispersal model developed in this paper to evaluate the effect of including patches with spatial repellents on the distance travelled by mosquitoes. We include a multiplicative factor $\phi_{(ij)} = 1 - p_{(ij)}$, where $p_{(ij)} \in [0, 1]$ to account for the effect

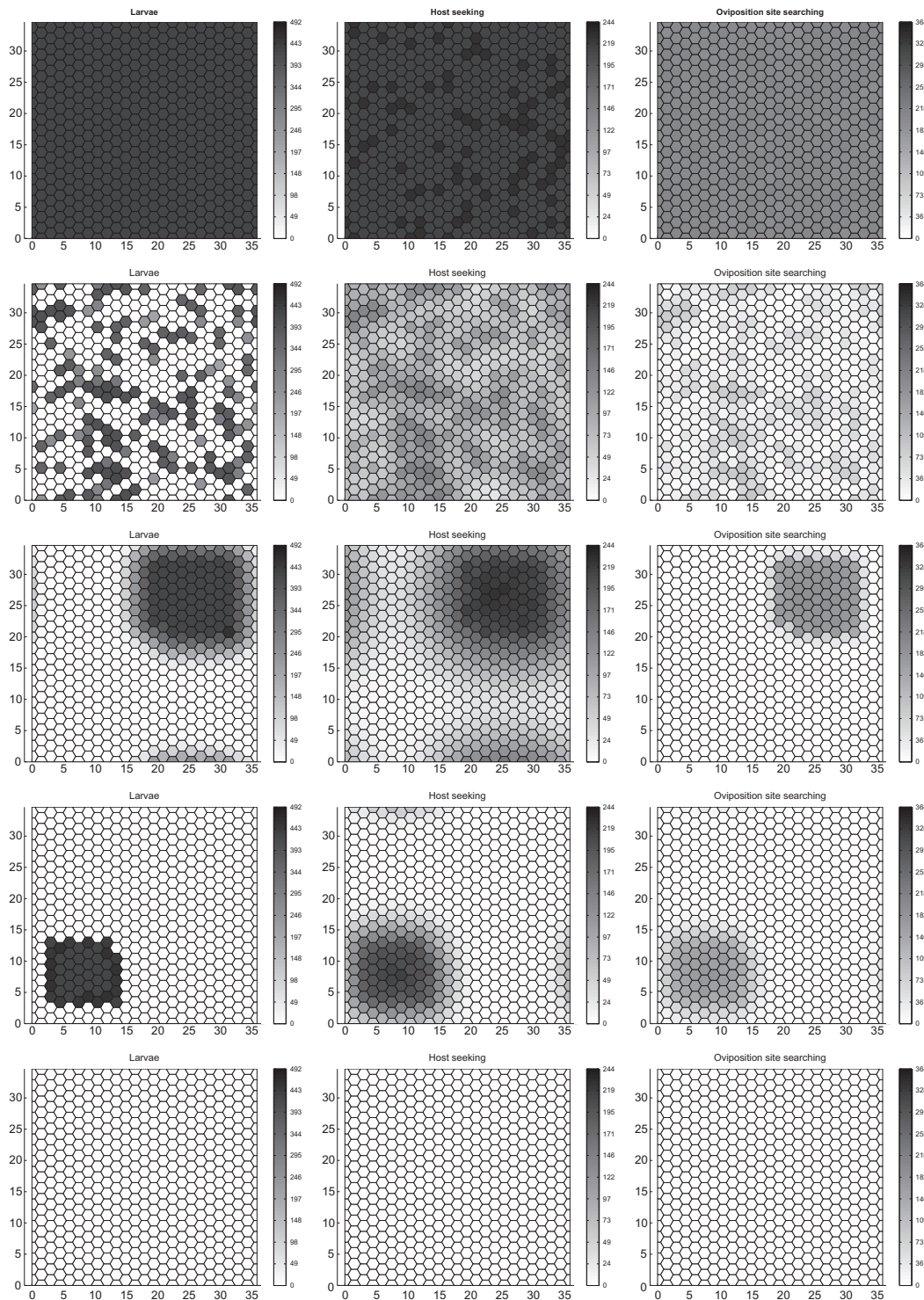


Fig. 11. Spatial population distribution of mosquitoes by scenario (Fig. 6) and stage. Scenario 1 (first row): all patches contain hosts and breeding sites. Scenario 2 (second row): random distribution of hosts and breeding sites. Scenario 3 (third row): all patches contain breeding sites but hosts are on one side of the grid. Scenario 4 (fourth row): all patches contain hosts but breeding sites are on one side of the grid. Scenario 5 (fifth row): clusters of hosts and breeding sites are far apart from each other. These results are a snapshot taken at day 250 when the whole system is at an equilibrium.

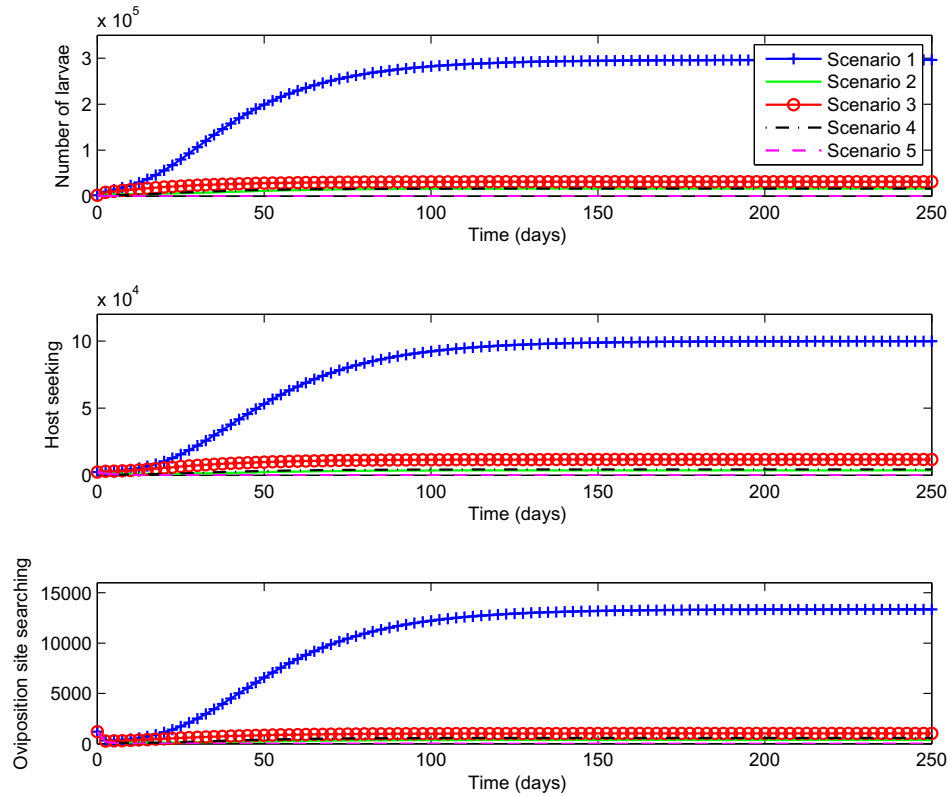


Fig. 12. Population dynamics of mosquitoes across the grid by scenario and stage. Scenario 1: Both hosts and breeding sites were present in all patches. Scenario 2: Both hosts and breeding sites were randomly distributed across the grid. Scenario 3: Breeding sites were placed in all patches, hosts were clustered on one part of the grid. Scenario 4: Hosts were placed in all patches, but breeding sites were placed on one part of the grid. Scenario 5: Both hosts and breeding sites were placed on one part of the grid, far from each other. Fig. 6 shows the set up of the scenarios.

of spatial repellents on flying mosquitoes in some patches. The parameter p can be interpreted as the blocked ability of mosquitoes to enter into a patch. When $p_{(ij)} = 1$, the barrier in the patch acts as an obstacle which completely blocks movement and when $p_{(ij)} = 0$, movement is not impeded. For host seeking mosquitoes, the dispersal rates from the source patch become:

$$\beta_{(ij)/\xi'}^{H*} = \phi_{\xi'} \beta_{(ij)/\xi'}^H \quad (45)$$

and the dispersal rate into the patch changes to

$$\beta_{\xi'/(ij)}^{H*} = \phi_{(ij)} \beta_{\xi'/(ij)}^H. \quad (46)$$

We note that in this way of modelling spatial repellents, emerging adults are not chased away by the repellents unless they have entered the host seeking stage.

We set up two scenarios to simulate the effect of repellents, with $p_{(ij)} = 0.8$. In the first scenario, we place repellents in the second ring to source (i.e. $n = 2$) to form a regular ring distribution. In the second scenario, we randomly distribute repellents over the patches across the landscape. Results from these two scenarios were compared with results produced under homogeneous conditions (without repellents in any of the patches).

The presence of repellents in patches placed at $n = 2$ creates a barrier to mosquitoes (Fig. 9A). Most mosquitoes move away from the source and cluster in the first neighbourhood ($n = 1$). Few mosquitoes are observed in the second neighbourhood. The density of mosquitoes for $n > 2$ are lower, compared to the scenario when there are no repellents. At larger distances from the source patch, the presence of repellents in patches near the source did not show any impact on mosquito dispersal.

The density of mosquitoes in the source patch is found to be higher when D is 0.8, with repellents placed in a ring of patches,

than at $D = 0.2$ with no repellents. From $n = 1$ to $n = 2$, there is no major difference between the two scenarios. For $n > 2$, mosquito density is smaller when D is set to 0.2, compared to when repellents are placed in a ring distribution. In this case, the repellent does not have a strong impact on the movement of mosquitoes and therefore the value of the movement rate has a substantial role in controlling movement to other patches. When there are patches with repellents, the average number of mosquitoes dispersed per patch (Fig. 9B) does not differ much from a scenario where there are no repellents. On the other hand, a small difference is observed for $n < 2$.

A random distribution of repellents results in mosquitoes clustering in the source patch and in the nearest neighbourhoods. Fewer mosquitoes are observed clustering in the patches far from the source patch compared to a situation when there are no repellents.

In the presence of spatial repellents, with $D = 0.8$, the weighted mean distance moved is estimated to be 78 m when repellents are placed at $n = 2$ and 55 m when repellents are randomly distributed across the landscape.

6.4. Impact of heterogeneity on spatial distribution

Fig. 11 shows the effect of heterogeneity on the spatial distribution of larvae, host seeking, and oviposition site searching mosquitoes when the system is at equilibrium. The population distribution is highly dependent on the distribution of both hosts and breeding sites. As expected, when all patches on the grid have both hosts and breeding sites, the entire grid becomes densely populated. Host seeking mosquitoes show a pronounced spread across the grid when hosts and breeding sites are randomly distributed, compared to mosquitoes searching for oviposition sites. When

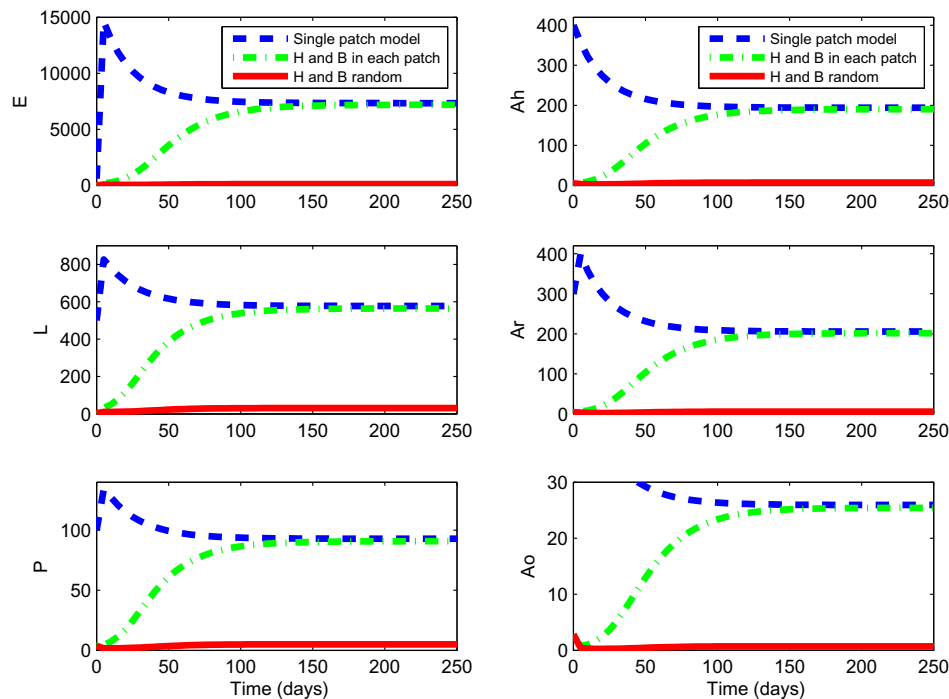


Fig. 13. A comparison of time series plots between the model without (system (1)) and with dispersal ((36)) was simulated under parameter values given in Table 2. For the model with dispersal, the average number of mosquitoes across all patches on the grid are plotted. Two scenarios were simulated for the dispersal model: hosts and breeding sites randomly distributed across the landscape (H and B random), and hosts and breeding sites present in all patches (H and B in each patch).

breeding sites are placed in all patches and hosts are clustered on one part of the grid, host seeking mosquitoes spread over a larger area, compared to a scenario where hosts are present in all patches and breeding sites are clustered on one side of the grid.

6.5. Impact of dispersal on population distribution

Mosquito dispersal becomes more important when the distribution of hosts and breeding sites on the grid is heterogeneous (Fig. 11). Clustering of host seeking mosquitoes towards patches containing both hosts and breeding sites is observed. However, when hosts and breeding sites are located in separate parts of the grid, the population dies out within a few days (given the assumed initial densities of mosquitoes for these simulations).

6.6. Impact of heterogeneity on the dynamics of the total population

Fig. 12 presents the dynamics of the total population (Eq. (39)) over all patches on the grid. Heterogeneous distributions of breeding sites and hosts, to a large extent, reduces the population at equilibrium. When clusters of breeding sites and hosts are placed far from each other, mosquitoes become unable to reproduce as distances required to travel is increased. Hence, population extinction is possible.

6.7. Impact of dispersal and heterogeneity on population dynamics

To evaluate the impact of dispersal and heterogeneity on population dynamics, we carried out numerical simulations using models both without (system (1)) and with dispersal (system (36)). While maintaining the same set up of multiple sources of mosquitoes (Fig. 7) for comparison purposes, we computed the average number of mosquitoes at equilibrium across all patches on the grid for the dispersal model. The two models show slightly different equilibrium values (Fig. 13) (i.e. (7339, 577, 93, 194, 206, 26) for the model with-

out dispersal and (7197, 564, 91, 190, 202, 25) for the dispersal model when all patches have hosts and breeding sites). For randomly distributed mosquito resources, the average equilibrium value across all patches on the grid was (118, 31, 5, 7, 6, 1). This corresponds to an equilibrium population, measured as number of mosquitoes per km^2 , as $(33.2, 2.6, 0.4, 0.9, 0.9, 0.1) \times 10^5$ when hosts and breeding sites were present in all patches and approximately $(54.374, 14.380, 2.314, 3.257, 2.630, 0.333) \times 10^3$ when resources are randomly distributed across the grid.

7. Discussion

Mathematical models for evaluating the impact of the transmission of vector borne diseases do not consider effects on vector mobility, despite evidence that the relative locations of mosquito breeding sites and of human hosts profoundly affect transmission of both malaria [14,59] and the dengue virus [28,65]. One reason for this is that, whereas spatial variation in biting rates is relatively easy to study, rates of movement of mosquitoes can only be studied using challenging mark-recapture techniques, which provide sparse data. Consequently, there is little evidence of the impact of heterogeneity in the distribution of resources used by mosquitoes on the mosquito population size and its spatial distribution. The likely impact of interventions that may affect mosquito movement is thus even less well understood.

Our compartment model of the life cycle and feeding cycle of mosquitoes incorporates spatial heterogeneity both in densities of breeding sites and of human hosts. It also incorporates mosquito movement and can be used to predict the effects of interventions targeting different stages of the mosquito life cycle. We consider effects on population size, on the spatial distribution of mosquitoes, and on how far individual mosquitoes move. We use the example of spatial repellents to illustrate how these parameters can be affected by a relatively simple intervention.

In a homogeneous environment, the model without dispersal indicates that there is a linear relationship between population reproduction numbers and both age-stage specific survival and developmental rates of mosquitoes. This leads to straightforward relationships between the size of the mosquito population, developmental rates from larvae to pupae, and mortality rates of larvae. However, when there are heterogeneities in resource availability, these linear relationships are disturbed, and have far-reaching effects on spatial distribution and population dynamics of mosquitoes [66]. If breeding sites are eliminated from the neighbourhoods of hosts or are not available in most patches, mosquitoes searching for breeding sites are forced to move longer distances in search of oviposition sites, prolonging the feeding cycle [13,35] and increasing mortality during searching [54]. In general, environmental heterogeneity forces mosquitoes to move longer distances and increases their mortality [54]. In our models, we could eliminate mosquito populations by separating breeding sites and hosts.

From the host's perspective, living in proximity to mosquito breeding sites increases exposure to mosquito bites and potentially also to disease. Because the vector-host ratio is higher around breeding sites [34], selectively eliminating breeding sites in areas of human habitation can prevent mosquitoes from using human hosts for blood meals [24]. Similarly, a possible intervention strategy is to deploy interventions such as spatial repellents or bed nets around breeding sites. However, our simulations suggest that such a ring strategy for repellent deployment is advantageous only if mosquito sources are few, clearly defined, and known. In situations where mosquito sources and households are scattered throughout the area, this strategy will not be feasible. However, even random deployment of repellents reduces the distance moved by mosquitoes, making it more difficult for them to complete their life cycle, and hence has beneficial effects.

Spatial heterogeneity in resource availability can thus, on its own, have complex effects on mosquito populations. Even relatively simple interventions, such as spatial repellents, can be deployed in a variety of ways in such environments. We have only just begun to use our model to explore the implications of the resulting multiplicity of combinations of environments with intervention strategies. Analysing of the spatial effects of more complex interventions, such as insecticide treated mosquito nets, which have simultaneous killing and repellent effects, will bring further challenges.

Like any model, ours has limitations. Effects of wind, which can either facilitate or prevent movement [7,16,33,51,57], were not incorporated. We chose to use a discrete hexagonal patches as a representation of space, rather than using a continuous space model [51,63] because this makes it easier to model arbitrary spatial distributions of resources. At the same time, this constrained the modelled mosquito movements to follow a limited set of trajectories. We do not know what trajectories mosquitoes adopt in reality and strategies such as Levy flight [52] may well be used to optimize foraging efficiency. An alternative approach to our discrete space model is to use a PDE model for mosquito dispersal, for example that of Raffy and Tran [51,63]. Here attractiveness is represented via chemotaxis or an advection term, taking into account blood meals, breeding sites, wind, etc. The advantage of the discrete space model proposed in this paper is that one can easily assess vector control strategies, as the discrete space enables easy representation of interventions that cover sets of households or villages.

The differences in the peaks and rates of decrease in mosquito distributions by distances travelled indicate that the choice of the exponential movement rate in the discrete model does not force the results to be the same as those produced by the continuous space approach. However, we could show that although there are differences, mosquito distributions by distances moved have

similar properties (both models show peaks in mosquito density in the regions close to the origin and are zero far away from the release point) to those predicted by a continuous space diffusion model [48], and suggest that our results are broadly applicable no matter what foraging strategies mosquitoes may adopt.

We could also show that the various factors taken into account by the model play an important role in the spatial distribution of mosquitoes. The model could show realistic behaviours in simple theoretical situations on an artificial landscape. Our model, together with field data, could be used to determine areas of high transmission within local settings, evaluate the community effect of interventions, and aid in developing possible and efficient vector control strategies, which can optimize the allocation of scarce resources.

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Appendix A. Data for model parameters

Data for parameterizing the model was obtained from literature. There is variability in the available data as study designs and conditions under which studies were carried out vary from one place to another. A single value was chosen from a range of values as baseline and used for the numerical simulation of the model.

The development of mosquitoes in their early stages is a nonlinear process that depends on water temperature [27,50,17]. However, for simplicity, we assume that the mean development time for each stage is constant over time.

For *Anopheles gambiae*, the duration of egg development (from oviposition to hatching into a larva) ($1/\rho_e$) is about 2 days in field environments [56]. Under laboratory conditions and tropical areas this period extends to 3 days [27,69,56]. In a study by [56], the larval period for mosquitoes of the *Anopheles* genus is found to be 7 days. Other studies have shown that the larval stage may last ($1/\rho_l$) between 6 to 10 days in field environments or 11 to 13 days in laboratory conditions [27] or last between 7 to 15 days in temperate and tropical areas [4,32,20]. It has also been found that the pupal period ($1/\rho_p$) lasts for 1–2 days in field environments but under laboratory conditions the pupal period lasts for about 2 days [27]. In tropical regions the pupal stage for *Anopheles* genus last between 2 to 3 days [56].

We used mean mortality rates of 0.56 ± 0.28 for eggs, 0.51 ± 0.14 for larvae instars I and II, 0.37 ± 0.14 for larvae instars III and IV, and 0.37 ± 0.15 for pupae [47]. The average of the two categories of larvae for the density independent mortality of larvae, $\mu_{l_1} = 0.44 \pm 0.14$. Larval mortality can be resolved into natural mortality rates, μ_{l_1} and density dependent mortality of larvae, μ_{l_2} . For our simulations, we allow μ_{l_2} to take any value between 0 and 1.

Since the model details the adult mosquito life cycle via the mosquito feeding cycle, we derive the estimates of most of the

parameters from studies on the mosquito feeding cycle. The time spent while searching for hosts ($1/\rho_{A_h}$) can be estimated. From [13], we can calculate $\rho_{A_h} = 0.46$. Once mosquitoes survive the host seeking stage and have successfully fed, mosquitoes rest for food digestion and egg maturation. Using $1/\rho_{A_r} = 2.33$ days [13], which is $\rho_{A_r} = 0.43$ per day, we can calculate the value of μ_{A_r} as 0.0043 given that the probability of surviving while resting is $1 - \mu_{A_r}/(\mu_{A_r} + \rho_{A_r}) = \rho_{A_r}/(\mu_{A_r} + \rho_{A_r}) = 0.99$ [13]. If mosquitoes spend $1/\rho_{A_o} = 0.33$ days ovipositing, then $\rho_{A_o} = 3$ per day. The corresponding probability of surviving the oviposition site searching stage $1 - \mu_{A_o}/(\mu_{A_o} + \rho_{A_o}) = \rho_{A_o}/(\mu_{A_o} + \rho_{A_o})$ is 0.88 [13]. From this probability, we obtain $\mu_{A_o} = 0.41$ per day. From [13] we see that the probability of surviving the feeding cycle is $p_f = 0.623$. From our model, this probability can be calculated from $(\rho_{A_h}/(\mu_{A_h} + \rho_{A_h}))(\rho_{A_r}/(\mu_{A_r} + \rho_{A_r}))(\rho_{A_o}/(\mu_{A_o} + \rho_{A_o}))$. Substituting the values for the survival probabilities of the oviposition site search and resting given above in this section, we obtain $\rho_{A_h}/(\mu_{A_h} + \rho_{A_h}) = 0.72$ as the probability of surviving during the host searching. Thus, we obtain $\mu_{A_h} = 0.18$ (Table 2).

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