On the Stability of Contention Resolution Diversity Slotted ALOHA (CRDSA)

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Overview

• Introduction

• Framework for Stability Analysis

• Stability Results

• Stability Comparison with SA

• Conclusions
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Introduction

• Random Access techniques
  • Interesting for bursty, unpredictable and small size traffic and delay critical messages
  • Initial Logon-signalling

• Demand Assigned Multiple Access
  Efficient for predictable and large volume traffic sources

• Packet loss due to collisions

• Throughput is load dependent
Retransmission Mechanisms

• **Retransmission** (ReTx) mechanism ensure a **reliable** packet **delivery**

• **Attempt** packet retransmission with **probability** $p_r$ in every transmission opportunity (geometric distribution)

• **Total load** determined by two components:
  - **New** offered traffic (*fresh* traffic) fluctuates statistically
  - **Retransmissions** add on top of fresh transmissions (*backlogged* traffic)
Retransmission Mechanisms (2)

- Problem:
  - Overall load determines the collision probability and throughput
  - Total load depends on user population $M$, traffic generation probability $p_0$ and retransmission probability $p_r$
Instability

• Offered traffic fluctuates statistically
• Operation at the maximum throughput desirable
• Problem:
  • Traffic fluctuations move the operating point
  • Higher load reduces throughput asymptotically to zero
• Without retransmissions:
  • Packets lost, but channel can always return to low load → Stable
Instability (2)

• With retransmissions
  • Retransmission feedback loop keeps load high
• Avalanche effect of more and more retransmissions
• Drives the channel into total saturation
  • Low throughput, high delay
• Low chance to get out of this situation again
• Way out:
  • Reset: Drop all pending retransmissions
  • Often not acceptable (QoS)
How can we know whether there is a risk to get stuck in the low throughput region?

How do we have to set our system parameters to be sure that we never get stuck in the low throughput region?
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Stability Framework for SIC RA channels (1)

- **Stability** well investigated for **classical Slotted ALOHA** (SA)
- Recent **new evolutions** of Slotted ALOHA using Successive Interference Cancellation (SIC) techniques: CRDSA, IRSA (Irregular Repetition Diversity Slotted Aloha [Liva10])
- **Frame based** techniques
  - Throughput can be drastically increased
  - More than one user can get un-backlogged at a time
- **Methods** known from Slotted ALOHA cannot be **applied** anymore
- **Impact on stability** on CRDSA and other SIC based RA techniques unknown as of today

![Diagram showing user interactions and frame vs slot time/frequency]
Stability framework for SIC RA channels (2)

- Definition of stability
  - **Local Stability**
    - An equilibrium point is locally stable if the system, for a small (local) distortion flows back towards it
  - **Global Stability**
    - An equilibrium point is globally stable if the system is always flowing towards it

- Markov Chain model developed to describe the dynamics of the CRDSA RA channel

- Users can be in one of two states: Fresh $x_F$ or Backlogged $x_B$
Stability framework for SIC RA channels (3)

- System can then be represented by a differential equation of first order expressing the drift $d_B$
  \[ \frac{dx_B}{dt} = E\{X_{B}^{l+1} - X_B^l | X_B^l\} = \ldots = (M - x_B) \cdot p_0 - N_S \cdot \overline{S}(x_B) \]

- The average throughput is dependent on the backlog state $x_B$
  \[ \overline{S}(x_B) = \left[ (M - x_B) p_0 + x_B p_r \right] \cdot \overline{P}_s((M - x_B) p_0 + x_B p_r) \]

- Fresh traffic is generated with probability $p_0$
- Packets collide with probability $p_r$
- Frame consists of $N_s$ slots
- Collided packets are retransmitted with probability $p_r$

- User population $M$
Stability Framework for SIC RA channels (4)

- **0 ≤ \( p_0 \) ≤ 0.01**  
  **Stable: 1 Equilibrium Point**  
  - Channel always tends to move back to equilibrium

- **0.01 < \( p_0 \) ≤ 0.11**  
  **Instable: 3 Equilibrium Points**  
  - The channel will remain for some time in the locally stable equilibrium in the high throughput region  
  - Once positive drift, amplifying self-excitation to locally stable saturation point

- **0.11 < \( p_0 \) ≤ 1.0**  
  **Overload:**  
  - Channel moves directly into the equilibrium (saturation point)
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Validation of the model

- Simulation of different channel configurations
- User population $M$
- Packet generation probability $p_0$
- Packet retransmission probability $p_r$
- Comparison with analytical predictions
- Simulations show very good match of expected and observed behavior
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Stability Comparison with SA

• Channel can be stabilized with parameter $p_r$
  • Low $p_r \rightarrow$ Better stability but also higher delay
  • High $p_r \rightarrow$ Worse stability but also lower delay

Which is the optimum $p_r$ that minimizes the delay $D_b$ while guaranteeing stability?

• Expected delay in equilibrium
• With Little’s theorem

$$D_b = \frac{\bar{N}}{S_{out}}$$

Avg. number of packets present in the channel when in equilibrium
Avg. throughput in equilibrium

• 4 system parameters: $M$, $p_0$, $p_r$, $D_b$
• Optimization criteria: Find the optimum $p_r$ to
  • Minimize $D_b$ for fixed $M$ and $p_0$
  • Maximize possible $M$ for fixed $p_0$ and target $D_b$
  • Maximize possible $p_0$ for fixed $M$ and target $D_b$
Minimize Delay

• User population M given, e.g. $M = 400$
• Traffic generation probability $p_0$ given, e.g. $p_0 = 2.63 \cdot 10^{-3}$
• Optimization problem:
  $$p_r^* = \arg\min_{p_r \in [0...1]} D_b(\Psi, p_r)$$
• Whereas
  $$\Psi = \{ M = 400, p_0 = 0.263, d = 2, N_s = 100, I = 10 \}$$

Result

$$D_{b,\text{CRDSA}}^{\min}(p_r = p_{r,\text{CRDSA}}^* : 10^{-2}) = 3.68 \text{ frames} \equiv 368 \text{ slots}$$
$$D_{b,\text{SA}}^{\min}(p_r = p_{r,\text{SA}}^* = 2.5 \cdot 10^{-3}) = 707 \text{ slots} \equiv 7.07 \text{ frames}$$

CRDSA can save 48% of delay while being guaranteed stable
Maximize User Population $M$

- Find max. supported population $M$ to achieve $D_b^0$, if traffic is generated with $p_0$
- Implicit optimization problem

$$g(p_r, M, p_0) = D_b(p_r, M, p_0) - D_b^0 = 0$$

$$L(p_r, M, p_0, \lambda) = D_b(p_r, M, p_0) + \lambda[D_b(p_r, M, p_0) - D_b^0]$$

CRDSA can support 45% more users than SA (363 users vs. 250 users), while achieving an average Delay of 300 slots (3 frames) and being guaranteed stable.
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Conclusions Stability

- Random Access Channels show instability behaviour
- Existing tools from Slotted ALOHA not sufficient to describe new frame-based Random Access mechanisms based on Successive Interference Cancellation
- A Markov chain model was derived to describe the dynamics of the system
- The system stability can be characterized by the number of equilibrium points
  - 1 equilibrium point: Globally stable equilibrium
  - 3 equilibrium points: Instable channel, will earlier or later enter the saturation
- Model allows
  - Analysis of the stability behaviour of a channel
  - Parameterization of the retransmission probability of a channel with user population M and packet generation probability $p_0$ to be guaranteed stable
- Analytical model was verified by simulations
- CRDSA does not only show advantage in peak throughput but also in it stability
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Contestion Resolution ALOHA (CRA)

- 2nd path of research activities ongoing
- SIC was so far only applied to synchronous (=slotted) schemes (CRDSA, IRSA)
- New Idea: Apply SIC to asynchronous scheme (ALOHA like)
  - Contention Resolution ALOHA
- Investigations have shown that this scheme has high potential and offers advantages in several usage scenarios