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Mathematical Communication: What And How To Develop It In Mathematics Learning?

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Abstract

Mathematics is the language of symbols so that everyone who studied mathematics required having the ability to communicate using the language of these symbols. Mathematical communication skills will make a person could use mathematics for its own sake as well as others, so that will increase positive attitudes towards mathematics. Mathematical communication skills can support mathematical abilities, such as problem solving skills. With good communication skills then the problem will more quickly be represented correctly and this will support in solving problems. Students' mathematical communication skills can be developed in various ways, one with group discussions. Brenner (1998) found that the formation of small groups facilitate the development of mathematical communication skills. This paper describes the mathematical communication and how to develop the mathematical communication skills in learning mathematics. For further clarify the discussion, given also the example of learning that emphasizes the development of mathematical communication skills.

Keywords: Mathematical Communication, Mathematics Learning.

I. INTRODUCTION

Mathematical communication skill in learning mathematics is very necessary to be developed. This is because through the mathematical communication, students can organize mathematical thinking both orally and in writing. In addition, students also can provide an appropriate response between students and the media in the learning process.

Developing mathematical communication skills is also an objective of the national curriculum (KTSP). In KTSP stated that one purpose of mathematics courses is that students have the ability to communicate ideas with symbols, tables, diagrams, or other media to clarify the situation or problem. It is also in accordance with the standards of mathematics education set by the National Council of Teachers of Mathematics (NCTM 2000). In the NCTM 2000, process standards to be achieved are: (1) mathematical communication, (2) mathematical reasoning, (3) mathematical problem solving; (4) mathematical connections, and (5) mathematical representations.

Mathematical communication needed by students to gain understanding. Likewise, students who already have a mathematical understanding is also required to be able to communicate his understanding, so that understanding can be understood by

others. By communicating mathematical ideas to others, a student can enhance mathematical understanding. Huggins (1999) suggested that in order to improve mathematical conceptual understanding, students can communicate mathematical ideas to others.

Develop mathematical communication skills in line with the new paradigm of learning mathematics. In the old paradigm, teachers are more dominant and only be transferring knowledge to students, while the students quietly and passively accept the transfer of knowledge from the teacher. But in the new paradigm of learning mathematics, teachers are leaders of community learning in the classroom, teachers guide students to actively communicate in the classroom. Teachers help students to understand mathematical ideas correctly, and straighten out the students' understanding if one is not true.

However, designing the learning so that students are actively communicating is not easy. In a discussion conducted by researchers with some junior high school teachers revealed that students are still not good in communication, either through oral or written communication. Especially for students in rural areas, students' oral communication skills are still low. Students are difficult to express his opinion, even though the actual idea and the idea already exist in their minds. Teachers expect that students' fear of being wrong in expressing his ideas. In addition students are also less accustomed to communicating ideas orally.

Communication is needed to understand the mathematical ideas correctly. Weak communication skills will result in a lack of other mathematical abilities. Students who have good mathematical communication skills will be able to create a diverse representation, it will be easier in finding alternatives to solving problems that resulted in the increased ability to solve mathematical problems.

Communication capability is an important capability that needs to be owned by students who want to succeed in his studies. According to Kist (Clark, 2005) effective communication skills today is the ability to be possessed by the students for all subjects. So communication skill not only for certain subjects such as language lessons and social

science course. Even in the association community, someone who has good communication skills are more likely to cooperate, which in turn will be a success in life.

II. DISCUSSION (EXPLANATION)

Mathematical Communication

Mathematics is the language of symbols in which every person who learns mathematics required to have the ability to communicate using the language of symbols. Mathematical communication skills will make a person could use mathematics for its own sake as well as others, so that will increase positive attitudes towards mathematics both from within oneself and others. Sumarmo (2000) argued that mathematics as a language of symbols implies that mathematics is universal and can be understood by anyone anytime and anywhere. Each symbol has a clear meaning, and mutually agreed by everyone. For example the symbol '9 ', the operation ' + ' , and ' ×' applies internationally at every level of school anywhere that can be understood by everyone.

According Sumarmo (2000), the development of language and symbols in mathematics aims to communicate the mathematics so that students can:

a. reflect and explain the students' thinking about mathematical ideas and relationships;
b. formulate mathematical definitions and generalizations through the method of the invention;
c. express mathematical ideas orally and in writing;
d. read the discourse of mathematics with understanding;
e. clarify and expand the question to the mathematics he had studied;
f. appreciate the beauty and power of mathematical notation and its role in the development of mathematical ideas.

Mathematical communication support other mathematical skills, such as problem-solving ability. With good communication skills then the problem will sooner be represented correctly and it will be supported to resolve the problem. Hulukati (2005) states that communication skills is a prerequisite for solving mathematical problems,

that is if students can not communicate properly interpret mathematical problems and concepts it can not resolve the problem properly.

Opinions about mathematical communication is also expressed by Schoen, Bean and Ziebarth (Hulukati, 2005). They argued that mathematical communication is the students 'ability to explain an algorithm and a unique way of solving problems, students 'ability to construct and explain real-world phenomena in graphs, sentences, equations, and tables or students' ability to give conjecture about the pictures of geometry .

Broader understanding of mathematical communication proposed by Romberg and Chair (in Sumarmo, 2000) namely: (a) connecting real objects, pictures, and diagrams into mathematical ideas, (b) explain the ideas, situations and mathematical relationships, orally or in writing with real objects, pictures, graphs and algebra, (c) states a daily occurrence in the language of mathematics or mathematical symbols, (d) to listen, discuss, and write about mathematics, (e) read with understanding a written mathematical presentations, make a conjecture , preparing arguments, formulating definitions and generalizations; (f) describe and create questions about the mathematics they have learned.

Baroody (1993) proposed five aspects of communication, the five aspects are:

(1) Representing. Making a representation means making other forms of idea or problem, suppose a table represented in diagram form or otherwise. Representations can help students to explain concepts or ideas and enables them to get problem-solving strategies. Moreover, it can increase flexibility in answering math problems. But starting from the NCTM 2000, the ability of mathematical representations is an ability separate and apart from the mathematical communication skills.

(2) Listening. Aspects of listening is one very important aspect in the discussion. Ability in listening to the topics being discussed will affect students' ability to give an opinion or comment. Students should listen carefully when there are questions and comments from friends. Baroody (1993) suggested that listening carefully to the statements of friends in a group also can help students construct a more complete knowledge of mathematics or mathematical strategies more effective.

(3) Reading. The process of reading is a complex activity, because in it there are aspects of remembering, understanding, comparing, analyzing, and organizing what

is contained in the passage. What is the importance of reading, so in Islam, the first revelation is: "Iqro' " which means "Read!". By reading a person may understand the ideas already set forth in writing another person. By reading, it becomes a mathematical scientific community in which one member with another member of give and take mathematical ideas.

(4) Discussion. In discussions students can express and reflect her thoughts regarding the content being studied. Students can also ask for things that are unknown or are still hesitant. The questions asked of students are directed to find out "How to get solution to the problem?" And not just "What is the solution to the problem?". In the discussion, the questions "How" is more qualified than the question "What" (Huggins, 1999). Baroody (1993) outlines some of the advantages of discussion include: (a) can accelerate the understanding of learning materials and proficiency using the strategy, (b) help students construct mathematical understanding, (c) inform that mathematicians do not usually solve the problem alone but build on the ideas with other experts in a team, and (4) help students analyze and solve problems wisely. Huggins (1999) suggests that one form of mathematical communication is speaking (speaking), this is identical with the discussion of (discussing) raised by these Baroody. Baroody (1993) did not include speaking in mathematical communication element, because it was put in the element of discussion.

(5) Writing. Writing is an activity undertaken by the conscious mind to reveal and reflect, as outlined in the paper media, computer or other media. Writing is a useful tool of thinking as students gain experience mathematics as a creative activity. By writing, students transfer its knowledge into written form. Parker (Huggins, 1999) suggests that writing about something that thought can help students to gain clarity and can reveal the level of understanding of the students. Writing about math concepts can also lead students to discover the level of understanding.

Developing Mathematical Communication in Mathematics Learning

Students' mathematical communication can be developed in learning mathematics through a variety of ways, as described below:

1. Through group discussions. With the group discussion, each member can develop mathematical communication skills more effectively. Brenner (1998) found that the formation of small groups facilitate the development of mathematical communication skills. Given the small groups, then the intensity of one student in expressing their opinions will be higher. This will provide a great opportunity for students to develop mathematical communication skills.

2. Using contextual issues in learning mathematics. Kadir (2010) in his research found that the use of coastal potency-based contextual teaching and learning can improve the ability of junior high school students' mathematical communication.

   In learning mathematics, by designing a form of contextual issues, then to answer it needed an explanation and reasoning and not just the final answer of a standard procedure.

   For example, consider this first problem:

   > In a right triangle, given the length of the hypotenuse is 10 cm and the length of one side at a right angle is 6 cm. How long is the third side ?

   Then consider the second problem follows. :

   > One day Mr. Zaky went to the house of Mrs. Vina on a motorbike. From his home, he must ride a bike in the west as far as 8 km. Then turn left at an angle of 90^o and continue the journey as far as 6 km and came to the house of Mrs. Vina. On his way back, Zaky not through the original path, but through the straight path which directly connects the house of Mrs. Vina and the house of Mr. Zaky. Explain how to measure the total distance traveled by Mr. Zaky for taking all these trips?

   Note that the first problem required little mathematical communication skills, such as reading and writing mathematical ideas. This is different to the second problem. In the second problem, students are trained to develop their communication skills through reading the problem and understand it, then communicate mathematical ideas into writing so that others can understand. To develop students' mathematical communication skills, students can be trained with the problems of the second.

3. By learning that encourages students to express opinions and explain. Pugalee (2001) stated that in order for students to be trained mathematical communication

skills, then students need to get used to provide arguments for each answer and provide feedback on answers given by others, so that what is being learned to be more meaningful to him.

4. By learning that encourages students to understand the text or instructional materials.

   One model of learning that enhance the mathematical communication skills through reading texts or instructional materials is *transactional reading strategy*. In this strategy students are trained ability to read and understand the teaching materials prepared by teachers. Sugiatno (2008) in his research found that learning mathematics with *transactional reading strategy* successfully develop mathematical communication skills of students.

5. By learning that encourages students to write mathematically.

   Aspects of writing, in mathematical communication can be enhanced by the existence of mathematical writing in learning activities, for example in making conclusions, making questions, answers, reasons and so forth. These activities can enhance students' skills in writing mathematical ideas. Mathematical understanding of students who obtained during the learning process can be seen when students express their understanding through communication, especially written communication. Activity to make the questions will make students able to pour things unknown and the known but need more detailed explanations. Those things are communicated in writing so as to improve writing skills for students.

   One of the learning strategies that enhance the ability of mathematical writing is a Think-Talk-Write(TTW) strategy. Ansari (2003) who conducted a study to develop mathematical understanding and communication skills high school students through a strategy of TTW, found that TTW strategy can develop communication skills and mathematical understanding of high school students, especially if done in small groups.

6. By learning that encourages students to make questions. One lesson that encourages students to make questions is reciprocal teaching. Qohar (2010) found that learning mathematics with Reciprocal Teaching can improve mathematical communication

skills in junior high school students. This is because the stages in the Reciprocal Teaching of reading materials, conclude, make inquiries, explain or clarify and make predictions or follow-up question is closely related to the development of mathematical communication skills.

Aspects of reading, in mathematical communication can be enhanced by the stage of reading the text that done before the process of making conclusions. One characteristic of reciprocal teaching is the text materials that teachers should be prepared prior to the learning process begins. Aspects of writing can be enhanced by the existence of the stages of making conclusions, making questions and predictions. While aspects of the discussion can be enhanced by the existence of an explanation or clarification in the process of reciprocal teaching. For students who served as chairman of the group, this phase is very useful to enhance the ability to speak, to give an explanation, and understanding the opinions of other students. For students who are not serving as chairman of the group, can express its opinions, asking things that are not clear, and adds to the explanation already given.

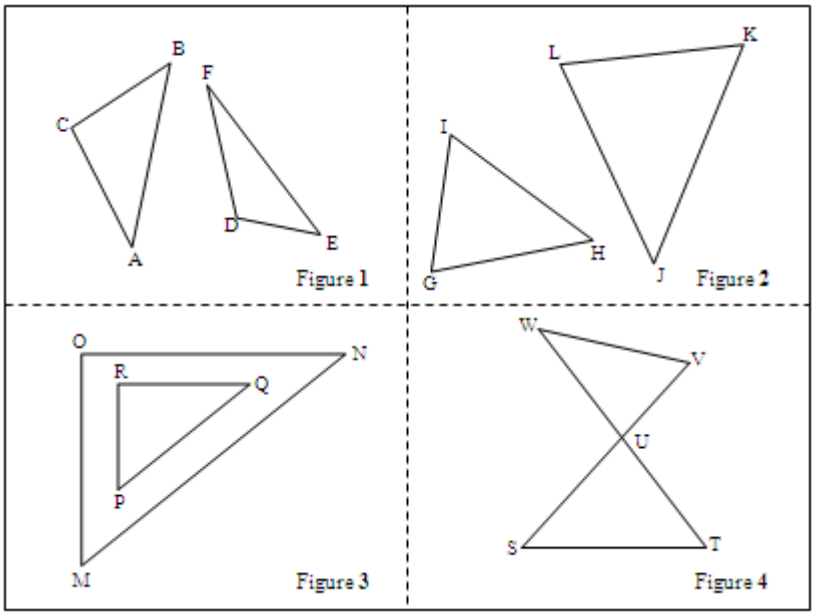
Listening aspects can be enhanced by the process of explanation or clarification. Students who served as chairman of the group, in addition to increasing the ability to speak, this phase is also beneficial to improve the ability to listen to opinions of other students who wish to express their opinion. As for students who are not served as chairman of the group, to hear the clarification from the chairman of the group will improve the ability to listen.

Example of Mathematics Learning to Support Development of Mathematical Communication

Based on the description that has been described above, the following will be given examples of mathematical learning that support the development of students' mathematical communication. Learning material used in this example is congruent triangles.

Learning begins with an introduction by the teacher and doing apperception. Then proceed with cooperative learning in small groups (4-5 students). In groups, students work on the student worksheets as follows:

Look at pictures of the couple triangles as follows:



a. In four of the above picture, which figures contained triangle with angles as large as the angles of the triangle partner?
Answer : .....
.....
....
(Arrange the triangles correspond to the name of the angles corresponding to their partner).

Some couples like the above triangle are examples of two triangles are congruent and written with the symbols Δ ..... $\cong$ Δ .....

Some of the other triangle pairs that the two corners are not equal are .....
.....
....

Some couples like the above triangle are examples of two triangles are not congruent, and written with the symbol: Δ ..... $\not\cong$ Δ .....

b. Note the size of angles and sides of congruent triangles.

Equal angles of two congruent triangles are called corresponding angles. Whereas the corresponding sides are the sides that his position was in front of the corresponding angles.

Write one pair of congruent triangles:

Δ ..... $\cong$ Δ .....

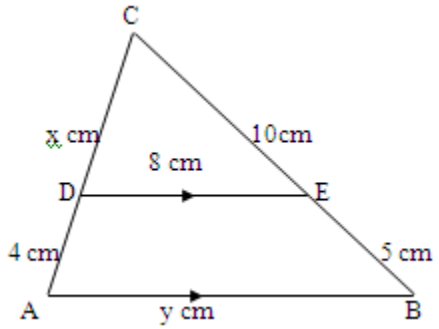
Next, compare the corresponding sides of congruent triangles that you wrote!

c. What is the ratio of corresponding sides of the pairs of congruent triangles that you wrote?
.....

Write back the conclusions obtained on congruent triangles !

Conclusions :

1. Note the picture below, it is known that the DE and AB are parallel. AD = 4 cm, DE = 8 cm, CE = 10 cm, EB = 5 cm.



a. Note the angle on the ΔDEC and ΔABC. Are there corresponding angles found in the two triangles mentioned? Write the name of the angle and explain why!

.....

Based on these data, can be concluded that the ΔDEC and ΔABC congruent? Give reasons for your answer!

.....

b. From the conclusions in point a, then obtained a side comparisons of the two triangles are:

$$\frac{\ldots\ldots}{\ldots\ldots} = \frac{\ldots\ldots}{\ldots\ldots} = \frac{\ldots\ldots}{\ldots\ldots} = \frac{\ldots\ldots}{\ldots\ldots} = \ldots\ldots$$

From the comparison, then specify the length of the unknown.

Answer: .....

c. From the example above, create another corresponding question!

Questions:

d. Chairman led the group in discussing the answers to the questions in item c. If there is still a difficult group, can ask for help and guidance from teachers.

Answers and explanations :

e. Of the problems already described, make a prediction or follow-up question about matters relating to the problems already described.

Prediction:

f. From the predictions you've made, led by chairman of the group, answer that problems and give explanations. If the group still have difficulties, ask for guidance and direction from teachers.

Answers and explanations :

After students finished working on worksheets, the teacher asked one group to present its results. Teachers do the strengthening in the discussion and straighten if something goes wrong concept. At the end of the lesson, the teacher invites students to summarize the course material learned that day, to reflect on and close the lesson.

III. CONCLUSION AND SUGGESTION

Selection of appropriate learning model is an important step for successful learning in mathematics. Application of the learning model that emphasizes the development of students' mathematical communication skills such as reciprocal teaching approach can change the paradigm of learning, from the old paradigm where the teacher as a learning center into a new paradigm in which students become the center of learning, and teacher as a motivator and facilitator. The learning model, also changed the old paradigm where learning is a knowledge transfer (transfer of knowledge) towards a new paradigm where learning is an exploratory activity, interactive, cooperative and constructive to gain new knowledge. Learning process with reciprocal teaching approach will develop communication skills among students, and

students with teachers, so as to foster mutual respect and mutual help in the goodness of the learning process.

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