Three Essays on Oligopoly: Product Bundling, Two-Sided Markets, and Vertical Product Differentiation

Inaugural-Dissertation

zur Erlangung des Grades Doctor oeconomiae publicae (Dr. oec. publ.)

an der Ludwig-Maximilians-Universität München

2004

vorgelegt von

Markus Reisinger

Referent: Prof. Sven Rady, Ph.D.

Koreferent: Prof. Ray Rees, Ph.D.

Promotionsabschlußberatung: 21. Juli 2004

Contents

Acknowledgements

First, and foremost, I would like to thank my supervisor Sven Rady for his encouragement and advise. He read all chapters of this thesis very carefully and gave me numerous suggestions on how to improve them. I am also very grateful to Ray Rees and Klaus Schmidt, the other advisors in my thesis committee, for many helpful comments and suggestions.

I thank all participants of the "internal seminar" where I presented each of my papers in an early stage. I benefitted a lot from the comments and discussions during my talks. Among my colleagues my special thanks goes to Florian Englmaier with whom I wrote a joint paper and who was always available to discuss scientific and non-scientific topics. I also thank my colleagues Gregor Gehauf, Florian Herold, Ingrid Königbauer, Christian Merz, Astrid Selder and Ferdinand von Siemens who supported me in finishing the thesis, each in her or his own special way. My thanks also goes to our secretary Manuela Beckstein for helping me in handling administrative problems.

The second chapter benefitted a lot from comments by Mattias Polborn, Lars Stole, seminar participants at EARIE 2002 in Madrid (especially Steven Berry), the EDGE Jamboree 2002 in Copenhagen and the ESEM 2003 in Stockholm. The third chapter was presented at the conference on "The Economics of Two-Sided Markets" in Toulouse, 2004, and I am grateful to Tommy Gabrielsen, who served as my discussant, and all other participants. The fourth chapter was presented at the annual meeting of the "Verein für Socialpolitik" 2003 in Zürich, and I am indebted to my discussant Armin Schmutzler for his numerous and very helpful suggestions.

Last, but certainly not least, I thank my parents for their permanent support during the time this work was conducted.

Chapter 1

Introduction

The study of oligopoly has a long tradition in economic thought and has stayed up to now a major field which is relevant for both theoretical and practical issues. Many theories of oligopoly developed in the last century have become a framework for the analysis of models in different fields of economics and are used in antitrust and regulatory issues. In this preface I want to give short treatise of the history of oligopoly theory and after that present a summary of the three chapters of the thesis and how they relate to and differ from the existing literature.[1]

The term oligopoly goes back to the year 1516 where the British humanist and political scientist Sir Thomas More coined the term in his *Utopia*. He noted that prices need not fall only because there is more than one supplier.[2] But after More's work it took more than 300 years before Cournot (1838) provided a first formal theory of oligopoly. Probably the reason why no progress could be made in the mean time was, as Schumpeter (1954) noted, that, "as we leave the case of pure monopoly, factors assert themselves that are absent in this case and vanish again as we approach pure competition," thus "the unbroken line from monopoly to competition is a treacherous guide."[3] In a modern terminology this is the problem of strategic interaction.

[1]Nevertheless all chapters can be read independently because a motivation and summary is given in the introduction to each chapter as well.

[2]A reference on More can be found in Schumpeter (1954), p. 305.

[3]See Schumpeter (1954), p.981.

Cournot (1838) presented a model where firms compete in quantities and prices are determined by the interplay of supply and demand. He presents a solution concept which is equivalent to a Nash equilibrium (Nash (1950)). The results of his analysis are that prices are above marginal costs and firms earn positive profits but this profits fall as the number of firms in the market increase. This results are quite intuitive and realistic. But one natural objection against Cournot's model (1838) is that in practice businesses choose prices rather than quantities. But it took 50 years before economists get aware of Cournot's work and Bertrand (1883) brought forward the above criticism. He proposed a model with two similar firms competing in prices where both firms have constant marginal costs. Bertrand (1883) pointed out that each firm has a strong incentive to undercut the other's price to capture the whole demand. In equilibrium prices equal marginal costs and firms earn zero profits. This result became famous as the "Bertrand-Paradoxon" because two firms are enough to get the same outcome as under perfect competition. So economists faced the dilemma that with the strange assumption of quantities as strategic variables we get a realistic result but with the reasonable assumption of prices as strategic variables the result is utterly unrealistic.

But for the Bertrand result to hold many crucial assumptions have to be satisfied, among others, that marginal costs are constant, that firms compete only in a one-shot game, and perhaps the most unnatural one that products are homogeneous and therefore perfect substitutes. Addressing the first point, Edgeworth (1925) presented an analysis where firms face capacity constraints, i.e. an extreme form of decreasing returns to scale, and proved the non-existence of an equilibrium in pure strategies.[4] Later Levitan & Shubik (1972) showed that an equilibrium in mixed strategies exists but pointed out that it is very hard to interpret a mixed strategy equilibrim in prices.[5] For the second critique to the Bertrand model, namely that the result only holds in a one-shot game, it took almost one hundred years until the first formal models of dynamic price competition were provided. The reason is that non-cooperative game theory made rapid progress only in the 1970's and 1980's. This holds especially in the

[4]This result became known in oligopoly theory as Edgeworth cycle.

[5]See also Dasgupta & Maskin (1986).

area of repeated games under asymmetric information which is the appropriate tool to study dynamic price competition.[6] The general result of this literature is that prices above marginal costs can be sustained if firms' discount factors are high enough.[7] The underlying idea is that it does not pay for a firm to undercut its competitor's price today because this triggers a punishment in the following periods. This behaviour is called tacit collusion.[8] Empirical studies, e.g. Porter (1983) or Rees (1993), also seem to confirm the intuition that firms in real world markets engage heavily in tacit collusion.

Let us now turn to the third and perhaps most serious critique on Bertrand's model, namely that products are assumed to be perfect substitutes. In this case a firm can capture the entire market by slightly undercutting the other's price. This is obviously an unrealistic assumption because there is basically always some degree of heterogeneity between goods. The literature on heterogeneity of goods is subsumed under the generic term "product differentiation" and has two branches, the "non-address" branch and the "address" branch,[9] where the latter was the more successful one.[10] The non-address approach started with Chamberlin's book (1933), "the theory of monopolistic competition", in which he tried to explain unexploited scale economies. He analyses a model where firms produce differentiated products and where the degree of competition between each firm is the same. This gave rise to the critique of Kaldor (1935) who argues that each firm has competitors producing similar products and others producing distinct products and that competition between similar products

[6]For the theoretical foundations of the analysis of repeated games used in Industrial Organisation see Abreu (1988), Farrell & Maskin (1986), Fudenberg & Maskin (1986), and the references in Shapiro (1989).

[7]The most influential papers in dynamic price competition are Green & Porter (1984), Rotemberg & Saloner (1986) and the three papers by Maskin & Tirole (1987,1988,1988).

[8]The idea that it is the obvious choice for oligopolists to collude was already emphasized by Bertrand (1883).

[9]For a longer discussion on this classification see Eaton & Lipsey (1989).

[10]The name "address approach" stems from Hotelling's (1929) idea that consumers are located on a position on the main street of a linear city. In the non-address approach consumers are described by some characteristics but not by a location.

should bear the main effect of change in a firm's behaviour. So Kaldor (1935) in some sense proposed the address approach as the more realistic one and his argumentation was widely accepted.[11]

The address approach started with the famous seminal paper of Hotelling (1929). He assumes that each consumer is described by her preference which is expressed through her location on a line. Two competing firms are located on the line as well and if a consumer's distance to firm 1 is shorter than her distance to firm 2 she prefers firm 1's product. Hotelling shows that the higher consumers' "transportation costs", i.e. consumers' disutility from not consuming her ideal product, the higher profits of both firms. So by relaxing the assumption of homogeneous goods one gets a model with prices as strategic variables and firms earning positive profits. Hotelling's model was developed further in many respects.[12] Since the new developments in non-cooperative game theory the model came under scrutiny of many economists. Hotelling assumed that the transportation cost function is linear in the distance between the location of a consumer and a firm. But d'Aspremont, Gabszewicz, & Thisse (1979) showed that if firms' locations are endogenous with such a formulation an equilibrium might not exist because profit functions are discontinuous and non concave. Instead they came up with a quadratic transportation cost function which is now common. They also show that firms position themselves at the opposite ends of the line to minimize price competition. Salop (1979) considers a model where consumers are distributed on a circle instead of a line. With this formulation he allows entry of more than two firms. Irmen & Thisse (1998) consider competition in not only one but in many characteristics and show that firms choose maximal differentiation in the most important characteristic and minimal differentiation in all others.

The model also serves as a building block in many papers which analyse different aspects of competition. Examples are Judd (1985) who used the model to look at the case of product proliferation to deter entry and Keller & Rady (2003) who analyse a

[11] For an extensive discussion of the non-address approach see Anderson, de Palma, & Thisse (1992).

[12] For examle a famous new development was made by Lancaster (1966) who builds a model in which consumers' preferences are defined over characteristics which are embodied in the goods.

dynamic game with firms learning about the degree of product differentiation.

Chapters two, three, and four of this thesis all relate in one or the other way to this model of product differentiation. Each of the three chapters analyses a different problem of pricing behaviour in oligopoly. The second analyses commodity bundling, the third two-sided markets with negative externalities and the fourth quality differentiation and market entry. In what follows I summarise each chapter and point out the important results and the differences to the existing literature.

In the second chapter, **"The Effects of Product Bundling in Duopoly"**, a duopoly model is analysed where both firms sell two goods. These goods can be sold either only independently (independent pricing) or independently and in a package consisting of both goods at some discount (mixed bundling).[13] In the economics literature starting with the seminal paper by Adams & Yellen (1976) bundling for a long time has been seen as a price discrimination device for monopolists. Whether mixed bundling raises a monopolist's profit depends on the correlation of consumers' reservation values for the two goods. If this correlation is negative, i.e. there exists many consumers with high valuation for good A but low valuation for good B and vice versa (extreme preferences), mixed bundling is more profitable than independent pricing. This result was shown by Adams & Yellen (1976) in some examples and was generalised by Schmalensee (1984) to a joint normal distribution and by McAfee, McMillan, & Whinston (1989) to general distribution functions. The intuition behind this result is the following. If correlation is negative there are many consumers with extreme preferences. The monopolist charges high independent prices and these consumers buy only the good for which they have a high valuation. Yet, there are some consumers with middle range valuations for both goods and they buy the bundle. Thus with the instrument of the bundle the monopolist can sort its consumers into three categories instead of only two and can extract more consumer rent. This is especially profitable

[13] There are many examples of this practice. Electronic companies sell stereo systems consisting of CD-player and receiver at a low price. In the USA long distance telephone companies sell internet access together with long distance service. This package is cheaper than if both services are bought independently.

with negative correlation because of the consumers with different valuations for both goods. Those do not exist under positive correlation.

Now let us look at the duopoly situation. Firms compete for consumers with the help of the bundle. So in addition to the sorting effect there is a business-stealing effect which lowers profits. Whether mixed bundling is profitable depends on which effect is dominant.

In my model both firms' products are maximally differentiated on two circles a la Salop (1979). The correlation of consumers' valuations can be expressed through their location on both circles. If many consumers are located near the same firm on both circles they have higher valuations for both goods of that firm than for the goods of the other firm and correlation is positive. If there are many consumers who are located on circle A near firm 1 but on circle B near firm 2 and vice versa, correlation is negative.

First let us look at the case of positive correlation. In this case firms can act in some sense as local monopolists because only few consumers are undecided between both firms. So it does not pay to lower prices much in order to get these consumers at the margin. The sorting effect of bundling dominates the business stealing effect and profits are higher than without the ability to bundle. Thus the consequences are similar to the ones in the monopoly case.

If correlation is negative the situation is completely different and bundling lowers profits. The intuition is the following. If we look only at the bundle many consumers are indifferent between the bundle of firm 1 and firm 2. Thus by undercutting the competitor's bundle price a firm can capture many new consumers. This results in harsh competition and low bundle prices. But this affects the independent prices as well and profits are lower than without the ability to bundle. Firms are in a prisoner's dilemma. The business-stealing effect dominates the sorting effect. The results are exactly opposite to the monopoly case where negative correlation renders mixed bundling profitable.

Concerning welfare I can show that bundling reduces welfare. The reason is that the bundle price is cheaper than the sum of the independent prices. This induces consumers to buy the bundle although they may prefer the goods of different firms. I also analyse the consequences of endogenous firm location. In choosing their locations on the circles firms influence consumers' reservation price correlation. Thus firms try to avoid strong negative correlation. They achieve this by choosing maximal differentiation in the good with the higher transportation costs and minimal differentiation in the good with the lower transportation costs. Thus they forego profits with the second product to avoid the negative consequences arising from additional competition in the bundle.[14]

The literature on commodity bundling in a duopolistic framework is up to now mainly concerned with bundling as strategic foreclosure and entry deterrence. The main question is if a monopolist in one market can monopolise a second market with the help of bundling. This is modelled in different ways by Choi (1996), Nalebuff (2004), and Whinston (1990). Exceptions are the papers by Anderson & Leruth (1992) and Matutes & Regibeau (1993) who analyse duopolistic competition in both goods as in my model. They get the result that a prisoners' dilemma will always arise. But they are not concerned with the consequences of different correlations of reservation prices. I show that this correlation is the driving force for the results in duopoly as well as in monopoly but has completely different effects in duopoly as compared to monopoly.

The third chapter analyses **"Two-Sided Markets with Negative Externalities"**. The term "two-sided market" is a relatively new one in economics and is, to my best knowledge, coined by Rochet & Tirole in an early draft of their 2003 paper "Platform Competition in Two-Sided Markets". It refers to a market where two distinct sides are present interacting with each other on a common platform. The platform's problem now is to "get both sides on board". The real world examples inspiring these literature usually are markets where both sides exert positive externalities on each other.[15] These markets have been studied extensively by Rochet & Tirole (2003) and Wright (2003,2004).

[14] Such firm behaviour can be observed by US telephone companies where the long distance offer in each package is very similar while firms try to differentiate themselves a lot in the offer of internet access.

[15] An example is the market for credit cards. Card holders value their cards only to the extent that the cards are accepted by merchants and merchants benefit from widespread diffusion of cards they accept.

In chapter three I analyse a model where one side causes a negative externality on the other. A prominent example for such an industry are the media. Platforms are e.g. radio stations or internet portals, one side are listeners or users and the other side are advertisers. Users dislike advertising and spend less time to consume platform's services if there are many commercials.[16] By contrast, advertisers wish to gain users' attention to tempt them to buy their products. So users exert a positive externality on advertisers while advertisers cause a negative externality on users. I study a model in which two platforms compete for advertisers and users. In the basic model platforms can only charge advertisers because it is either impossible to set a user fee (like in case of radio stations) or it is not customary to do so (like in the case of internet portals). From the users' point of view platforms are differentiated a la Hotelling while advertisers have no special preference for one of the platforms. Both sides decide for only one platform.[17]

First take a look at the efficient outcome. If the gains from trading advertisers' goods are high compared to users' utility loss from advertising all advertisers should advertise while some should be excluded if users' utility loss is comparatively high. The optimal allocation of advertisers among platforms is even. The reason is that if one platform has more advertisers the externality on its users is high and the overall externality can be reduced if some advertisers switch to the other platform.

Comparing the efficient outcome with the Nash equilibrium of the game I find that there can be too much or too little advertising. The intuition behind this is the following. An additional commercial on a platform causes a negative externality directly on

[16]The assumption that users are disturbed by advertising is strongly confirmed in the literature. See Bagwell (2003) for an overview.

[17]This is a realistic assumption with regard to users who usually use only one platform to do e-mailing or listen to only one radio station at a time. Advertisers instead can advertise on both platforms which is not possible in my model. But this is not crucial to the results because the only thing which matters is that a change in the price of platform 1 changes the number of advertisers on platform 2. So one gets the same result with the assumption that advertisers are "multi-homing"(can advertise on both platforms) but have only a certain budget for advertising expenditures where the last unit can either be spend on one or the other platform.

the users but indirectly on all other advertisers as well because users spend less time on a platform and some switch to the other platform. In their pricing behaviour platforms consider only the indirect externality on advertisers because they cannot charge users. If differentiation between platforms is small many users switch to platform 1 if platform 2 broadcasts many advertisements. This results in high prices for advertising and too little advertising. If differentiation is high advertising prices are low and there is too much advertising.

Platforms' profits depend on the level of differentiation as well. But contrary to standard results in my model a higher level of differentiation can lead to lower profits. This is the case if differentiation is relatively high. The reason is that in this case platforms want to attract many advertisers and set low prices. But since both platforms do so advertising levels stay the same and profits fall. This shows that in a two-sided market a lower level of competition on one side can increase competition on the other side and lead to lower profits. If differentiation is relatively small an increase in differentiation raises profits because more firms advertise.[18]

I also analyse the case if platforms can set a user charge. I show that if this user charge is positive in equilibrium profits always increase. But dependent on parameters it is well possible that the user charge is negative in equilibrium. Platforms subsidise users to make higher profits on advertisers. If this is the case profits are lower and the additional instrument of a user charge hurts platforms.

Nevertheless the equilibrium with two instruments is efficient. The intuition is that platforms have two instruments for two groups and competition induces them to use the instruments efficiently.

Recently several papers dealt with media competition and advertising.[19] The ones closest to my model are Anderson & Coate (2003) and Kind, Nilssen & Sorgard (2003). I replicate the result of their papers that there can be too much or too little advertising. The big difference to my model is that they only consider competition for users but not

[18]In Section 3.5 I present an example of a such pricing behaviour by the internet portals AOL and GMX.

[19]See Section 3.1 for an overview of this papers.

for advertisers while in my model platforms compete for both sides of the market. Thus the result that profits can fall with an increase in differentiation cannot be generated in their model. Kind, Nilssen & Sorgard (2003) do also not analyse the case where a platform can set a user charge. Anderson & Coate (2003) briefly analyse this possibility but obtain different results to my model because in their model platforms decide in addition whether they want to provide the same or different contents.

In contrast to the first two chapters which analyse models of horizontal product differentiation the fourth chapter deals with **"Vertical Product Differentiation, Market Entry, and Welfare"**. In models of horizontal product differentiation consumers have different preferences for the goods. By contrast, in a model of vertical product differentiation everyone agrees on the most preferred good, e.g. because it is of superior quality. The pioneering papers in this field are Gabszewicz & Thisse (1979) and Shaked & Sutton (1982). In both models consumers differ in their income levels and in equilibrium consumers with higher income buy the high quality good at a high price while consumers with a lower income buy the low quality good at a lower price. Shaked & Sutton (1982) show that ex-ante similar firms choose different qualities to relax price competition.

In chapter four I compare two duopoly models with respect to welfare. In one model firms can produce only one quality level while in the other model they can produce a whole quality range and engage in second-degree price discrimination. Both models have a leader-follower game structure. The leading firm (incumbent) chooses a quality or a quality range in the first stage to which it is committed for the rest of the game. After observing this choice the follower (entrant) decides whether it wants to enter the market at some fixed entry costs. If it enters it chooses its quality or quality range.[20] In the third stage firms set their prices depending on the chosen quality levels.

First consider the case where each firm produces only one quality. The incumbent has an incentive to deter entry if its monopoly profit (given entry deterring quality) is higher than the duopoly profit. The way entry can be deterred depends on the

[20]It is assumed that it is profitable for the leader to be the high quality firm. So in equilibrium the entrant always produces a lower quality or quality range than the incumbent.

strategic relation between the two qualities, e.g. if they are strategic complements or strategic substitutes. If qualities are strategic complements welfare in the case of entry deterrence is lower than in the case of pure monopoly. The intuition is that the incumbent produces a lower quality than in monopoly to induce the entrant to produce a lower quality as well which reduces the entrant's profit. If fixed costs are high the entrant stays out of the market. But since quality is lower welfare is reduced. If qualities are strategic substitutes the reverse is true.

Even in the case where entry is accommodated the incumbent might lower its quality as compared to monopoly. This is the case if production costs of quality are high. The reason is that due to competition it is harder for the incumbent to extract consumer rent, inducing it to produce lower quality. If costs are low it produces higher quality to reach a higher level of differentiation in order to reduce price competition. In this case welfare increases.[21]

If firms can produce a quality range and engage in second-degree price discrimination results are different. I show that the lowest quality of the incumbent and the highest quality of the entrant are strategic complements. Thus if the incumbent wants to deter entry he has to expand its quality range which leads to an increase in welfare. So the result is quite different to the one-quality model where welfare in case of entry deterrence can be lower than in monopoly. If entry is accommodated I find that consumer rent unambiguously increases. The reason is that competition leads to lower prices. By contrast, the consequences for welfare are not clear. The intuition is that the incumbent might contract its quality range to avoid fierce price competition. There is a gap between the two quality ranges of the firms. More consumers are served but some consumers buy lower quality than in monopoly. The welfare effects are therefore ambiguous.[22]

Concerning the related literature on product differentiation and entry Donnenfeld

[21] I provide two examples of such firm behaviour from different industries, namely the pharmaceutical industry and the cosmetic industry.

[22] Again, I give two examples for such firm behaviour, one from the airline industry and the other, again, from the pharmaceutical industry.

& Weber (1992,1995) in two papers analyse a situation with two incumbents and one potential entrant, but neither do they provide a welfare analysis nor are they interested in price discrimination. A paper which is very close to my quality range model is the one by Champsaur & Rochet (1989). They analyse a model of simultaneous quality choice. My analysis in some respects draws heavily on theirs. The difference is that in my model firms choose their quality ranges sequentially which gives rise to the question of entry deterrence and that I am mainly interested in the welfare consequences of potential competition while they only look at a pure duopoly situation and do not give a welfare analysis.

I conclude the work with some remarks on the limitations of the models and give an outlook on how they can be interrelated.

Chapter 2

The Effects of Product Bundling in Duopoly

2.1 Introduction

Product bundling refers to the practice of selling two or more goods together in a package at a price which is below the sum of the independent prices. This practice can be observed very often in the real world. For example in the USA internet access is sold by long distance telephone companies. If a consumer buys internet access and long distance service together from the same company this is cheaper than if he buys both services independently. Another well known example is the selling of stereo systems. Big electronic companies always supply a package consisting of CD-player, stereo deck and receiver which is sold at a low price. There are many other examples of bundling in big department stores or cultural organisations, e.g. theaters and concert halls always offer season tickets.

In the industrial organisation literature bundling has been extensively studied for monopolists and it is shown that mixed bundling, that is selling the goods individually and bundled together in a package, will in general increase the monopolist's profit.[23] However, the industry structure in the examples above is clearly not monopolistic.

[23] See Varian (1989) for an overview.

This shows that there is a need to examine bundling in oligopolistic or competitive markets. The objective of this paper is to analyse, how the ability to bundle affects profits and consumer rents in a duopolistic market structure.

It is shown that duopolists generically have an incentive to use mixed bundling, but the consequences on profits are ambiguous. If consumers are homogeneous, i.e. correlation of their reservation prices is positive, firms are better off with bundling. If instead consumers are heterogeneous, i.e. their reservation values are negatively correlated, profits are lower than without bundling. This is in sharp contrast to the monopoly case, where bundling raises the monopolist's profit, especially if consumers are heterogeneous.

The intuition behind this result is the following. First look at the monopoly case. If correlation of reservation values is negative there exist many consumers with extreme preferences, that means with a high valuation for good A but a low valuation for good B and vice versa. The optimal pricing strategy for a monopolist is to charge a high individual price for each good and the consumers with these extreme preferences buy only the good for which they have a high valuation. But still there are some consumers with middle range valuations for both goods and they buy the bundle at some discount. Thus bundling has a sorting effect. It allows the monopolist to sort its consumers into three categories instead of two and it can therefore extract more consumer rent.

Now let us look at a situation with two firms. Each firm must compete for demand and will do this with the help of the bundle. So beside the sorting effect, bundling now causes a second effect, which is called 'business-stealing' effect. This effect goes in the opposite direction than the sorting effect, because it results in a higher degree of competition and thus in lower profits. Whether bundling is profitable for the firms depends on which effect is dominating the other one.

The first result is that there is always an incentive for the duopolists to engage in mixed bundling as long as the correlation of valuations is not perfectly positive. This result is in line with the monopoly case. Since the firms have an additional instrument to sort their consumers they will use it.

Now assume consumers are homogeneous. This means that many of them have a strong preference for both goods of one firm. Therefore firms can act in some sense as local monopolists and can extract more consumer rent with bundling. There are only few consumers who are undecided between both firms. So it does not pay for a firm to undercut its competitor's prices to get these consumers at the margin. Thus prices and profits are relatively high. The sorting effect dominates the business-stealing effect. The consequences of bundling are very similar to the monopoly case.

If instead consumer preferences are heterogeneous the situation is completely different. In this case many consumers prefer good A from firm 1 and good B from firm 2 and vice versa. For simplicity assume first that both firms can sell their goods only in a bundle. These bundles are now almost perfect substitutes to each other. Each firm can gain many new customers by lowering the price of its own bundle. Thus harsh price competition arises. If the firms can sell their products independently as well, this business-stealing effect endures. The price of the bundle is driven down to nearly marginal costs and this influences the unbundled prices which are now very low. Thus profits are low and consumer rent is high. The initial idea of the bundle, namely to price discriminate in a more skilful manner, is dominated by the business-stealing effect. So the result is completely opposite to the monopoly case. In this second case firms are in a prisoner's dilemma situation. It would be better for both of them not to bundle.

There is also an interesting welfare effect. Since the bundle is cheaper than the sum of the two independent prices, consumers are encouraged to buy the bundle. If heterogeneity increases firms react in equilibrium with an increase of their independent prices. Thus more consumers buy the bundle. This results in distributive inefficiency because some consumers prefer the products from different firms. So if markets are covered bundling reduces social welfare as it can only cause consumers to purchase the wrong good.

It is also analysed what will happen if firms can influence the correlation of valuations. This can be done with the introduction of an additional stage in which firms choose their location in the product range. It is shown that firms may choose minimal differentiation in one product and thus forego profits with that product. They do this to avoid competition on the bundle which is very fierce if correlation is negative. Such firm behavior can be observed in the US by telephone companies which sell long distance service and internet access in one package. The long distance service offer is very similar in each package while firms try to differentiate themselves a lot in the offer of internet access with each firm offering different rates and amounts of installation gifts.

In the literature economists' attention on bundling was first drawn by the seminal paper of Adams and Yellen (1976). They show in a series of examples with an atomistic distribution of consumers that selling goods through bundling will raise the profit of a monopolist. This result was generalized by Schmalensee (1984) to a joint normal distribution and by McAfee, McMillan and Whinston (1989) to general distribution functions. They all show that bundling will raise the monopolist's profit, because it is an additional instrument to sort its customers. This is especially the case if reservation values for different goods are negatively correlated.

There are some papers which study bundling in a more competitive environment. The focus of these papers is if and how a multiproduct firm, which has monopoly power in one market, can increase its profit through bundling. Such a strategy is called tying. Whinston (1990) analyses whether a firm which has monopoly power in the first market can monopolize a second market with duopolistic market structure by committing to engage in pure bundling. He shows that this is possible. The reason is that the monopolist prices the bundle aggressively with the consequence that many consumers will now buy the bundle and therefore the profit of the rival is very low, which induces him to exit. Carbajo, deMeza and Seidmann (1990) study a model with the same market structure as in Whinston (1990). They present another idea why a tying commitment can be profitable for a monopolist. This is that with pure bundling products in the second market are differentiated and thus prices are higher. Profits of both firms are increased.[24] Nalebuff (2004) shows that pure bundling is more profitable for an incumbent even if commitment is not possible. This is the case if the entrant

[24]Chen (1997) presents a model with the same intuition only the market structure is different. He assumes duopoly in one market and perfect competition in the second. The duopolists can differentiate themselves by one firm selling the bundle and the other firm selling the goods only independently.

can enter only in one market. The intuition is that the entrant must compete for consumers in the first market as well since the incumbent only offers the bundle. This greatly reduces the profit of the entrant. Choi (1996) analyses the effects of bundling on research & development. In his model there is originally duopoly in both markets but both firms can invest in R&D to lower their production costs before reaching the price competition stage. If the difference in production costs for one good is large after the R&D game the market for this good is monopolised by the low cost firm. Choi (1996) shows that in this case bundling serves as a channel to monopolise the second market. Finally, Mathewson and Winter (1997) study a model with monopoly in one market and perfect competition in the other. They show that requirements tying is profitable for the monopolist provided that demands are stochastically dependent. For a great parameter range the optimal prices are Ramsey prices.[25]

There are two papers which have the same market structure as in my model (duopoly in both markets), namely Matutes and Regibeau (1992) and Anderson and Leruth (1993). The result in both papers is that if firms cannot commit not to bundle in equilibrium they choose mixed bundling. But this results in increased competition and lowers profits. A prisoner's dilemma dilemma arises because profits would be higher without bundling. However, in these papers the driving force of the monopoly case, the correlation of consumers' reservation values, is not modelled. In my model it is shown that this is also the crucial variable for the oligopoly case, but can create opposite effects. Also these papers are not concerned with welfare and location choice.

This chapter is also in the spirit of a relatively new literature which studies the effects of price discriminating methods in a competitive environment. An extensive overview of the different branches of these literature is given in a paper by Stole (2003) which is prepared for the forthcoming volume of the Handbook of Industrial Organization. In the section about bundling Stole (2003) summarizes many of the recent papers which are concerned with the question how bundling affects profits and market structure when commitment is possible or not.

[25] Seidmann (1991) and Denicolo (2000) analyse the consequences of bundling in other market structures.

The structure of the chapter is as follows. Section 2.2 sets out the model. The particular structure of consumer heterogeneity and the correlation of reservation values is presented in Section 2.3. Equilibrium selling and price policy is determined in Section 2.4. Section 2.5 studies the welfare consequences of bundling. Section 2.6 analyses the effects if firms have the possibility to choose their location and influence the reservation price correlation. An application of the model to the US telephone industry is considered in Section 2.7. Section 2.8 concludes the chapter. The proofs of all results are given in the Appendix of this chapter.

2.2 The Model

The model is a variant of Salop's (1979) model of spatial competition on the circle but with two goods.

There are two firms $i = 1, 2$. Both firms produce two differentiated goods $j = A, B$ at the same constant marginal costs c_A and c_B.[26] The product space for each good is taken to be the unit-circumference of a circle. The product variants are then the locations of the firms on each circle. It is assumed that firm 1 is located at point 0 on both circles and firm 2 is located at point $\frac{1}{2}$ on both circles. So there is maximum product differentiation in both goods. The firms have the choice to sell their products not only independently but also together as a bundle. So each firm i can choose between two possible selling strategies. It can sell its goods separately at prices p_A^i and p_B^i (independent pricing) or it can sell the goods independently and as a bundle at prices p_A^i, p_B^i and p_{AB}^i (mixed bundling).[27] Firms have to decide simultaneously about their selling and price strategies. It is assumed that they cannot monitor the purchases of consumers. So the strategy space for each firm i is to quote three prices p_A^i, p_B^i and

[26] The assumption of the same cost function for both firms is made for simplicity and is not crucial to the results.

[27] There can also be a third strategy, namely to sell the goods only as a bundle at price p_{AB}^i. Adams & Yellen (p. 483) and McAfee, McMillan & Whinston (p. 334) have shown that this cannot be the unique optimal strategy because mixed bundling with prices $p_A^i = p_{AB}^i - c_B$ and $p_B^i = p_{AB}^i - c_A$ always does weakly better. This also holds in my model.

p^i_{AB}. If $p^i_{AB} < p^i_A + p^i_B$ firm i engages in mixed bundling while if $p^i_{AB} \geq p^i_A + p^i_B$ firm i practice independent pricing as no consumer would buy the bundle from firm i. Last, resale by consumers is impossible.

There is a continuum of consumers and without loss of generality we normalize its total mass to 1. Each consumer is described by her location on both circles, $\mathbf{x} = (x_A, x_B)^T$. Every consumer has a unit demand for both goods and purchases each good independently of the other. So there is no complementarity between the products. This allows me to focus on the pure strategic effect of bundling. The consumers are uniformly distributed on each circle j. This is mainly for tractability reasons and to compare the results with previous papers.[28] In the next section we give some structure to the joint distribution and present the modelling of the correlation of reservation values.

A consumer who is located at $0 \leq x_A, x_B \leq \frac{1}{2}$ and buys good A from firm 1 and good B from firm 2 enjoys an indirect utility of

$$V(x_A, x_B) = K_A - p^1_A - t_A(x_A)^2 + K_B - p^2_B - t_B\left(\frac{1}{2} - x_B\right)^2. \tag{2.1}$$

A similar expression holds for consumers who are located somewhere else or buy different products. K_A and K_B are the surpluses from consumption (gross of price and transportation cost) of good A and B. p^i_j is the price of variant i of product j. The transportation cost function is the weighted squared distance between the location of the consumer and the variant produced by the firm where she buys. The weight is the salience coefficient for each product, t_j, and without loss of generality we assume that $t_A > t_B > 0$.[29] The reservation price of a consumer for variant i of good j, R^i_j, is thus $K_j - t_j(\mathrm{d}_i)^2$, where d_i is the shortest arc length between the consumer's location and firm i on circle j. It is also assumed that K_j is sufficiently large such that both markets are covered. This means that the reservation values are high enough such that in each price equilibrium all consumers buy both goods. When dealing with welfare considerations this means that there is no welfare loss due to exclusion of consumers who should

[28]For analyses without uniform distributions see Neven (1986), Tabuchi & Thisse (1995) and Anderson, Goeree & Ramer (1997).

[29]The cases $t_B \to t_A$ and $t_B \to 0$ are analysed in Section 2.4.

buy the product from a social point of view. The form of utility in (2.1) looks special but it is the standard form in models with spatial competition if consumers can buy many products.[30]

The consumers thus have the choice between four alternative consumption combinations. They can buy the bundle from firm 1 ($AB1$), the bundle from firm 2 ($AB2$), good A from firm 1, good B from firm 2 ($A1B2$), and good B from firm 1, good A from firm 2 ($B1A2$).

2.3 Dependence between Location and Correlation

In the monopoly case the correlation of reservation values is crucial for the incentive to bundle. It is a known result that especially in case of independence or negative correlation bundling dominates unbundled sales.

In our case it is possible to infer the joint distribution function of reservation values $G(R_A^i, R_B^i)$ for firm i and therefore the correlation between the reservation values from the joint distribution function of consumer location $F(x_A, x_B)$. If for example every consumer has the same location on both circles then the conditional density function of x_A given x_B is

$$f(x_A \mid x_B) = \begin{cases} 0 & \text{if } x_A \neq x_B \\ 1 & \text{if } x_A = x_B. \end{cases}$$

The conditional density function $g(R_A^i \mid R_B^i)$ of reservation values for firm i is then

$$g(R_A^i \mid R_B^i) = \begin{cases} 0 & \text{if } R_A^i - R_B^i \neq K_A - K_B - (t_A - t_B)(\mathrm{d}_i)^2 \\ 1 & \text{if } R_A^i - R_B^i = K_A - K_B - (t_A - t_B)(\mathrm{d}_i)^2. \end{cases}$$

This would imply a reservation price correlation of $\rho[R_A^i, R_B^i] = 1$. This is a simple example and there are possibly infinitely many ways how the consumers can be

[30]In the literature the assumption of a quadratic transportation cost function is usually made to guarantee existence of an equilibrium if firms can choose their locations before setting prices (see e.g. D'Aspremont et al (1979) and Irmen & Thisse (1998)). In my basic model this assumption is not necessary since firms are maximally differentiated and one could also work with a linear transportation cost function. However, in Section 2.6 the model is extended to allow for location choice of firms. To keep the analysis consistent quadratic transportation costs are assumed right from the beginning.

distributed on one circle given the location on the other circle. To keep the model tractable, we have to give some structure to this conditional distribution, which still captures the main point of expressing different correlations. This is done in a very simple way. It is assumed that if a consumer is located at x_A on circle A then she is located at

$$x_B = \begin{cases} x_A + \delta & \text{if } x_A + \delta \leq 1 \\ x_A + \delta - 1 & \text{if } x_A + \delta > 1 \end{cases}$$

on circle B, where $0 \leq \delta \leq \frac{1}{2}$.[31] This means a δ-shift of all consumers on circle B. So a δ of 0 corresponds to the former example. The advantage of doing this is that with this simple structure correlations of values can be obtained easily by altering δ.

Remark 2.1

The function $\rho[R_A, R_B](\delta) = \frac{\text{Cov}[R_A, R_B](\delta)}{\sigma(R_A)\sigma(R_B)}$ is given by $1 - 30\delta^2 + 60\delta^3 - 30\delta^4$.[32]

Thus correlation is strictly decreasing in δ.[33] If $\delta = 0$, $\rho(\delta) = 1$, i.e. perfect positive correlation while if $\delta = 0.5$, $\rho(\delta) = -0.875$.[34] Correlation here relates to the products of one firm. So negative correlation means that a consumer who values product A from firm i highly has a low valuation for product B of firm i.

Obviously this simple structure has important characteristics. First, there is a one-to-one mapping between positions on circles. This implies that there is no stochastic in the model.

Second given the location on circle A the location on circle B is exactly ordered by δ and can not be crisscross.

[31] It suffices to consider δ between 0 and $\frac{1}{2}$. A δ greater than $\frac{1}{2}$ expresses the same correlation as one between 0 and $\frac{1}{2}$. For example a δ of 0.8 expresses the same correlation as a δ of 0.2.

[32] The proof of this and all other results can be found in the appendix of this chapter.

[33] The term correlation does not mean a stochastical correlation in this model, because there is no stochastic element. It describes the relation between known reservation values. So it is a term from descriptive statistics.

[34] We do not get the whole range of correlation coefficients because distance enters quadratically in the utility function. With a linear transportation cost function the whole range of coefficients could be reached but the results of the analysis would stay the same.

However, this structure captures the main point of correlation. With a low δ, there are many consumers having high reservation values for both goods of firm i. For a high δ, many people have extremely different reservation values for both goods of firm i. So this structure represents exactly what is meant with correlation. Its main advantage is that it keeps the model tractable and gives clear cut results.

2.4 Equilibrium Price and Selling Strategies

In this section the equilibrium price and selling strategies of a firm conditional on the correlation of values is analysed.

Before doing this the equilibrium of the game without the bundling option is determined. The result will later be used as a benchmark.

If bundling is not possible there is no connection between the two products. Each market is independent and we are in a standard situation of product differentiation on the circle. The Nash equilibrium can be determined in the usual way. In this equilibrium firms set prices

$$p_A^1 = p_A^2 = p_A^\star = c_A + \frac{1}{4} t_A,$$

$$p_B^1 = p_B^2 = p_B^\star = c_B + \frac{1}{4} t_B$$

and earn profits

$$\Pi_1^\star = \Pi_2^\star = \frac{1}{8}(t_A + t_B).$$

Now assume that bundling is possible. In the following the profit functions of the firms for different correlations are determined. First, the question arises if firms have an incentive to bundle.

Proposition 2.1

If $\delta > 0$, i.e. $\rho < 1$, then in equilibrium both firms choose mixed bundling.

This is in line with the monopoly case. The firms have an additional instrument to sort their customers and so they will use it. The exception is, if $\delta = 0$, i.e perfect positive correlation. In this case all consumers have the same position on each circle. Thus firms do not need a third instrument because consumers cannot be sorted better than with independent prices.

Now the demand structure on the circles in dependence of δ can be derived. The special form of locations allows us to work only with one circle because the location on the other circle is then uniquely determined.

First, assume δ is small and start at a consumer with location $x_A = 0$. She has a high reservation value for both variants 1 and will therefore buy bundle $(AB1)$.[35] If we move clockwise on circle A then the consumer who is indifferent between $(AB1)$ and $(A1B2)$ is defined by

$$x_A = \frac{1}{4} + \frac{p_A^1 + p_B^2 - p_{AB}^1}{t_B} - \delta.$$

The product combination which is bought to the right of $(AB1)$ is $(A1B2)$. It is not bundle 2, because then no one would buy the independent products, which cannot be the case in equilibrium.[36] Moving further to the right the next combination which is bought is $(AB2)$ and the marginal consumer is located at

$$x_A = \frac{1}{4} + \frac{p_{AB}^2 - p_A^1 - p_B^2}{t_A}.$$

If we pass the point $\frac{1}{2}$ and move upward on the left side of the circle, we get the same product structure as on the right side, because of symmetry, only with firm 1 and 2 reversed. Consumers next to $\frac{1}{2}$ buy $(AB2)$, consumers in the middle buy $(A2B1)$ and consumers next to 1 buy $(AB1)$. Figure 2.1 illustrates the product combinations on circle A.

The profit function of firm 1 is therefore

$$\begin{aligned} \Pi_1 = \quad & (p_{AB}^1 - c_A - c_B)(\tfrac{1}{4} + \tfrac{p_A^1 + p_B^2 - p_{AB}^1}{t_B} - \delta + 1 - \tfrac{3}{4} - \tfrac{p_{AB}^1 - p_B^1 - p_A^2}{t_A}) \\ & + (p_A^1 - c_A)(\tfrac{p_{AB}^2 + p_A^1 - p_B^2}{t_A} + \tfrac{p_A^1 + p_B^2 - p_{AB}^1}{t_B} + \delta) \\ & + (p_B^1 - c_B)(\tfrac{p_{AB}^1 + p_A^2 - p_B^1}{t_A} + \tfrac{p_{AB}^2 - p_A^2 - p_B^1}{t_B} + \delta). \end{aligned} \tag{2.2}$$

[35] Product combination (AB1) is only bought if p_{AB}^1 is not too high compared with other prices. In the proof of Proposition 2.2 in the appendix it is shown that this is the case in equilibrium .

[36] Remember that firms always engage in mixed bundling.

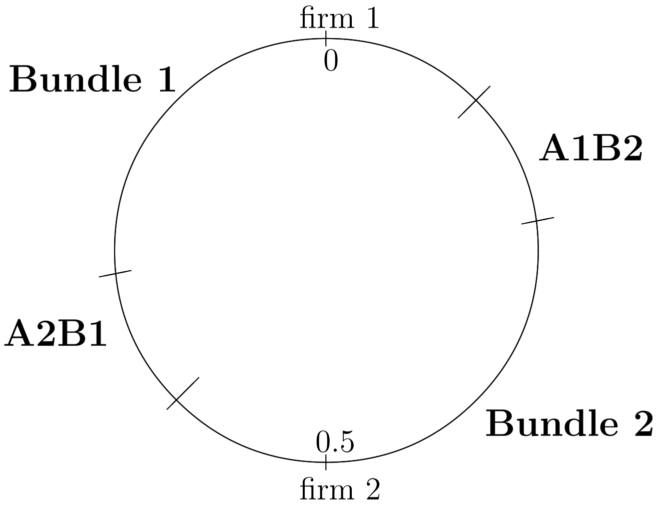

Figure 2.1: Demand structure if $\delta \leq \frac{1}{3} + \frac{t_B}{6t_A}$

Because of symmetry we get a similar function for firm 2. Calculating prices and profits we get

$$\begin{aligned} p_A^\star &= c_A + \tfrac{1}{4}t_A + \tfrac{1}{3}\delta\tfrac{t_A t_B}{t_A+t_B}, \\ p_B^\star &= c_B + \tfrac{1}{4}t_B + \tfrac{1}{3}\delta\tfrac{t_A t_B}{t_A+t_B}, \\ p_{AB}^\star &= c_A + c_B + \tfrac{1}{4}(t_A+t_B), \\ \Pi^\star &= \tfrac{1}{8}(t_A+t_B) + \tfrac{4}{9}\delta^2\tfrac{t_A t_B}{t_A+t_B}. \end{aligned} \tag{2.3}$$

for both firms.

Next assume that δ is large and start again at $x_A = 0$. The consumer located there has the highest reservation value for variant 1 of good A and a high reservation value for variant 2 of good B. If p_A^1 and p_B^2 are not much higher than other prices she will buy $(A1B2)$. Moving clockwise the next combination can only be bundle 1 or bundle 2, because it is shown in Claim 2.1 in the appendix, that $(A2B1)$ can never be in direct rivalry to $(A1B2)$. In equilibrium it will be bundle 1 because the position of the consumer on circle A is nearer to firm 1. Since $t_A > t_B$, the distance on circle A is more important than the one on circle B. The marginal consumer is given by

$$x_A = \frac{3}{4} + \frac{p_{AB}^1 - p_A^1 - p_B^2}{t_B} - \delta.$$

If we move further clockwise the distance to firm 2 becomes shorter than that to firm 1 and so consumers buy bundle 2. The marginal consumer between $(AB1)$ and $(AB2)$

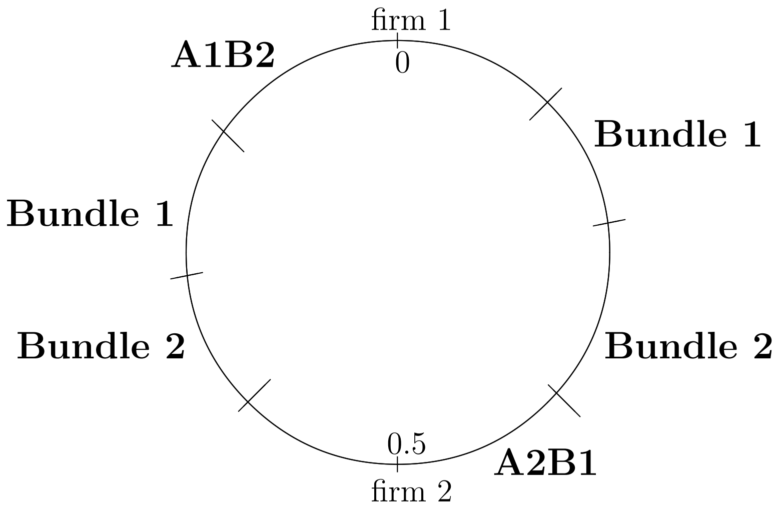

Figure 2.2: Demand structure if $\delta > \frac{1}{3} + \frac{t_B}{6t_A}$

is defined by

$$x_A = \frac{1}{(t_A - t_B)} \left(p_{AB}^2 - p_{AB}^1 + \frac{1}{4}t_A - \frac{3}{4}t_B + t_B\delta \right).$$

Next, consumers located near $\frac{1}{2}$ buy $(A2B1)$. The structure on the left side is the same only with firms reversed. The whole demand structure is illustrated in Figure 2.2.

The profit function of firm 1 is thus

$$\begin{aligned}
\Pi_1 = \; & (p_A^1 - c_A)(\tfrac{3}{4} + \tfrac{p_{AB}^1 - p_A^1 - p_B^2}{t_B} - \delta - \tfrac{1}{4} + \delta - \tfrac{p_A^1 + p_B^2 - p_{AB}^1}{t_B}) \\
& +(p_{AB}^1 - c_A - c_B)\Big(\tfrac{p_{AB}^2 - p_{AB}^1 + \frac{1}{4}t_A - \frac{3}{4}t_B + t_B\delta}{(t_A - t_B)} - \tfrac{3}{4} - \tfrac{p_{AB}^1 - p_A^1 - p_B^2}{t_B} + \delta \\
& +\tfrac{5}{4} + \tfrac{p_A^1 + p_B^2 - p_{AB}^1}{t_B} - \delta - \tfrac{p_{AB}^1 - p_{AB}^2 + \frac{3}{4}t_A - \frac{5}{4}t_B + t_B\delta}{(t_A - t_B)}\Big) \\
& +(p_B^1 - c_B)(\tfrac{5}{4} + \tfrac{p_{AB}^2 - p_B^1 - p_A^2}{t_B} - \delta - \tfrac{3}{4} - \tfrac{p_A^2 + p_B^1 - p_{AB}^2}{t_B} + \delta).
\end{aligned} \tag{2.4}$$

and equilibrium prices and profits are

$$\begin{aligned}
p_A^\star &= c_A + \tfrac{1}{6}t_A - \tfrac{1}{6}t_B, \\
p_B^\star &= c_B + \tfrac{1}{12}t_B, \\
p_{AB}^\star &= c_A + c_B + \tfrac{1}{4}(t_A - t_B), \\
\Pi^\star &= \tfrac{1}{8}t_A - \tfrac{7}{72}t_B.
\end{aligned} \tag{2.5}$$

for both firms. It remains to calculate at which value of δ the profit function is changing. The difference between the two profit functions is that on the right side of the circle the region (A1B2) is followed by (AB2) in profit function (2.2) while in profit function (2.4)

(A1B2) is followed by (AB1). Likewise on the left side (A2B1) is followed by (AB1) in profit function (2.2) but by (AB2) in profit function (2.4). If profit function (2.2) is relevant there is some value of δ at which (A1B2) would no longer be followed by (AB2) but by (AB1) if firms charge equilibrium prices. Calculating this threshold yields $\delta = \frac{3}{2}(\frac{t_A+t_B}{5t_A+t_B})$. At this value both firms begin to lower its prices in such a way that demand structure of Figure 2.1 is still valid. The prices and profits for $\delta > \frac{3}{2}(\frac{t_A+t_B}{5t_A+t_B})$ are given by

$$\begin{aligned} p_A^\star &= c_A + \tfrac{1}{4}t_A + \tfrac{t_A t_B}{2(t_A-t_B)^2}((5t_A+4t_B) - 2\delta(8t_A+t_B)), \\ p_B^\star &= c_B + \tfrac{1}{4}t_B + \tfrac{t_A t_B}{2(t_A-t_B)^2}((5t_A+4t_B) - 2\delta(8t_A+t_B)), \\ p_{AB}^\star &= c_A + c_B + \tfrac{1}{4}(t_A+t_B) + \tfrac{t_A t_B}{2(t_A-t_B)^2}(9(t_A+t_B) - 6\delta(5t_A+t_B)), \\ \Pi^\star &= \tfrac{1}{8}t_A + \tfrac{1}{8}t_B + \tfrac{t_A t_B}{2(t_A-t_B)^2}\left(4(t_A+t_B) - 2\delta(6t_A+t_B) - 4\delta^2 t_A\right). \end{aligned} \tag{2.6}$$

But if δ increases further at some point it is profitable for both firms to deviate from the above strategy and keep their prices constant. At this value the demand structure changes and for all δ above this value profit function (2.4) is valid. Calculating this threshold yields $\delta = \frac{1}{3} + \frac{t_B}{6t_A}$.

The analysis above is summarized in the following proposition.

Proposition 2.2

If $\delta \leq \frac{3}{2}(\frac{t_A+t_B}{5t_A+t_B})$ then in the unique Nash equilibrium firms set prices and earn profits according to (2.3).

If $\delta > \frac{1}{3} + \frac{t_B}{6t_A}$, then in the unique Nash equilibrium firm set prices and earn profits according to (2.5).

If $\frac{3}{2}(\frac{t_A+t_B}{5t_A+t_B}) < \delta \leq \frac{1}{3} + \frac{t_B}{6t_A}$ then in the unique Nash equilibrium firm set prices and earn profits according to (2.6).

So the profit function is continuous but non-monotonic in δ. It is first increasing in δ then decreasing and for high values of δ it is constant. The profit function in dependence of δ is illustrated graphically in Figure 2.3.

What is the intuition behind this result? First look at the case where $\delta \leq \frac{3}{2}(\frac{t_A+t_B}{5t_A+t_B})$. Because δ is small, the locations of consumers on both circles are similar. This means

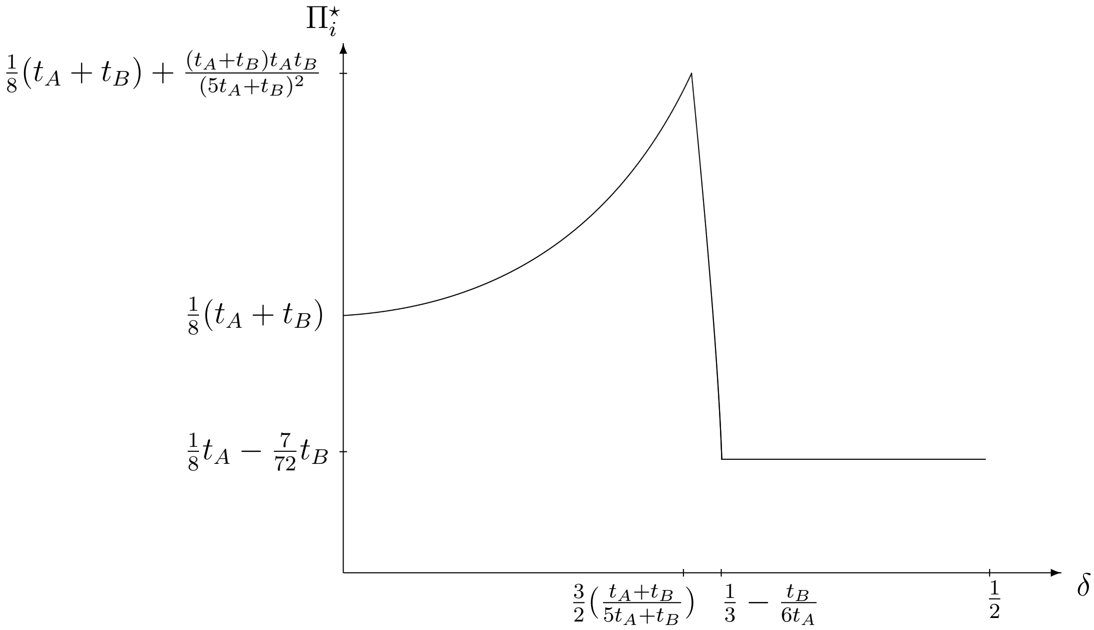

Figure 2.3: Equilibrium profits

that there are a lot of consumers with high reservation values for both goods of one firm. From the perspective of these consumers, firms are very distinct. Thus firms have high market power and price competition is low. One can see this also in Figure 2.1. There are four product combination regions. But there are no bundle regions side by side. This means that if one firm lowers its bundle price, it will get more bundle consumers but also lose demand on its own independent sales. So lowering a price has also a negative effect on a firm's own demand and thus there is only little incentive to lower prices. Note that for $\delta \to 0$ equation (2.3) implies that prices and profits are the same as without bundling. This is in line with Proposition 2.1 where it is shown that if $\delta = 0$, there is no incentive to bundle.

From (2.3), $p_{AB}^\star$ is independent of δ. $p_{AB}^\star$ is the sum of the two prices that arise if bundling is not possible. So consumers buying the bundle have to pay the same amount of money if bundling is possible or not. Consumers located further away from the variants of the firms, thus buying $(A1B2)$ or $(A2B1)$, lose through bundling because $p_A^\star$ and $p_B^\star$ are increasing in δ. Calculating the breadth of the product combination ranges we get that demand for each bundle is $D_{AB1} = D_{AB2} = \frac{1}{2} - \frac{1}{3}\delta$ and demand for each two-variant-combination is $D_{A1B2} = D_{A2B1} = \frac{1}{3}\delta$. Despite the fact that $p_A^\star$ and $p_B^\star$ increase with δ, $D_{A1B2} = D_{A2B1}$ increase with δ as well. The reason is that preferences get more heterogeneous with higher δ and this effect is stronger than the price increase. Because of this increasing heterogeneity firms gain through product bundling. They charge higher independent prices and can better sort their consumers. Profits rise with δ and consumer rent decreases.

If on the opposite $\delta > \frac{1}{3} + \frac{t_B}{6t_A}$, then profits are low. It is apparent from (2.3) that profits are lower than without bundling. This can be explained in the following way. Assume that firms can only offer the bundle. In this case the reservation value of a consumer for both bundles is nearly the same. An extreme case would be $\delta = \frac{1}{2}$ and $t_A = t_B$. Then each consumer has the same valuation for both bundles. Firms can gain many new consumers by lowering the bundle price. So competition in the bundle is very harsh and this affects also the unbundled prices. This business-stealing effect of bundling drives profits down. In terms of strategic substitutes and complements defined by Bulow, Geanakoplos and Klemperer (1985), the two bundles are direct strategic complements, $\frac{\partial^2 \Pi^i}{\partial p_{AB}^1 \partial p_{AB}^2} > 0$. So if one firm lowers its bundle price, the other will do the same. This can also be seen in Figure 2.2. On the right as well as on the left side of the circle there is a region, where bundle 1 is side by side with bundle 2. If a firm lowers its bundle price then it gets new consumers, who formerly did not buy either good of that firm. Such a region does not exist in Figure 2.1. In case of profit function (2.2) there is no direct strategic complementarity.

This result is in sharp contrast to the monopoly case. In monopoly the bundle helps the firm to reduce the dispersion of reservation values to get more consumer rent. This is especially profitable if correlation is negative. In duopoly there is the same effect, but with completely different consequences. The bundle also reduces dispersion, but competition gets harsher and profits lower.

In this region prices and profits are low and do not change with δ. The reason is that there is no incentive to decrease prices because they are already low and thus the gains from decreasing prices are low compared with the losses. There is also no incentive to increase prices because a firm would lose some consumers who have formerly bought the bundle and would buy both goods from the rival after the price increase.

In the remaining region $\frac{3}{2}(\frac{t_A+t_B}{5t_A+t_B}) < \delta \leq \frac{1}{3} + \frac{t_B}{6t_A}$ prices are decreasing with δ. As δ is already high consumers are more homogeneous. Each firm has an incentive to exploit this and reduce its price to induce more consumers to buy the bundle. So both firms lower their prices. But since δ is not very high and consumers' bundle valuations are still heterogeneous the demand structure does not change. This effect of lowering prices becomes stronger the higher δ is. Thus prices and profits decrease with δ.

It is interesting to compare profits in case of bundling with profits if bundling is not possible. If bundling is not possible profits are $\Pi^\star = \frac{1}{8}t_A + \frac{1}{8}t_B$. Thus if $\delta \leq \frac{3}{2}(\frac{t_A+t_B}{5t_A+t_B})$ bundling raises profits while if $\delta > \frac{1}{3} + \frac{t_B}{6t_A}$ profits are lower with bundling. Since the profit function is strictly and continuously decreasing in δ in the region $\frac{3}{2}(\frac{t_A+t_B}{5t_A+t_B}) < \delta \leq \frac{1}{3} + \frac{t_B}{6t_A}$ there is one value of δ for which profits are the same. Calculating this value by comparing profits yields the following lemma.

Lemma 2.1

If $\delta > \frac{\sqrt{52t_A^2+28t_At_B+t_B^2}}{4t_A} - \frac{3}{2} - \frac{t_B}{4t_A}$ profits are lower than without bundling and firms are in a prisoner's dilemma.

Firms are in a prisoner's dilemma situation because as is shown in Proposition 2.1 they both choose to bundle. But this results in lower profits than if they did not bundle. Thus firms would be better off without the possibility to bundle.

It is also possible to analyse the thresholds where the profit function has kinks. The first threshold is given by $\delta_1^{TS} = \frac{3}{2}(\frac{t_A+t_B}{5t_A+t_B})$. Since $t_A > t_B > 0$ the threshold lies in the range $\delta_1^{TS} \in]\frac{3}{10}, \frac{1}{2}[$. The maximal profit of the firms is reached at this threshold and is given by $\Pi^\star = \frac{1}{8}t_A + \frac{1}{8}t_B + \frac{(t_At_B)(t_A+t_B)}{(5t_A+t_B)^2}$. The second threshold is given by $\delta_2^{TS} = \frac{1}{3} + \frac{t_B}{6t_A}$. At this threshold the demand structure changes. Since $t_A > t_B > 0$ this threshold lies in the range $\delta_2^{TS} \in]\frac{1}{3}, \frac{1}{2}[$. Thus the intermediate region where the profit decreases is very small. Its maximal breadth is approximately 0.03. This is the case when $t_B \to 0$ which implies $\delta_1^{TS} = \frac{3}{10}$ and $\delta_2^{TS} = \frac{1}{3}$. Thus the profit decreases sharply from a high level to a level that is even lower than without bundling.

It is also interesting to look at two extreme cases of the transportation costs. First let us see what will happen if $t_B \to 0$. In this case $\lim_{t_B \to 0} \delta_2^{TS} = \frac{1}{3}$. In this case no consumer has a special preference for product B of one firm. The standard Bertrand argument leads to $p_B^* = c_B$. But also if firms bundle they can only make profits on good A. A look at the profit functions shows that $\Pi_i^* = \frac{1}{8} t_A$ independent of which profit function arises. The bundle has neither a sorting nor an additional competition effect since good B is offered in perfect competition. Another extreme is if $t_B \to t_A$ which results in $\lim_{t_B \to t_A} \delta_2^{TS} = \frac{1}{2}$. This shows that in this case only profit function (2.2) is relevant. Thus only the price discrimination effect of bundling is valid and profits are always increasing the more negative the correlation is. But for all values of t_B between 0 and t_A whenever $\delta > \frac{\sqrt{52t_A^2 + 28t_A t_B + t_B^2}}{4t_A} - \frac{3}{2} - \frac{t_B}{4t_A}$ the ability to bundle reduces profits.

2.5 Welfare Consequences

The model has also interesting welfare implications. It is assumed that the reservation price of every consumer is high enough, so that in each price equilibrium all consumers are served. Thus there is no inefficiency that results from consumers whose valuations are higher than marginal costs and who do not buy the goods. But there is a distributive inefficiency. It arises because some consumers do not buy their preferred product.[37]

As a benchmark we can first calculate maximal welfare. Welfare is maximized if transportation costs are minimized. This is the case if on both circles consumers at $0 \le x_j \le \frac{1}{4}$ and $\frac{3}{4} \le x_j \le 1$ buy from firm 1 and consumers at $\frac{1}{4} \le x_j \le \frac{3}{4}$ buy from firm 2. The resulting welfare is

$$WF^{max} = K_A + K_B - c_A - c_B - \frac{1}{48}[t_A + t_B].$$

Maximal welfare is reached if the firms do not bundle.

If bundling is possible welfare depends on δ.

[37]Distributive inefficiency is also present in the monopoly case. Here some consumers who value a good higher than others do not buy it while the latter individuals do. See Adams & Yellen (1976).

Proposition 2.3

If $\delta \leq \frac{3}{2}(\frac{t_A+t_B}{5t_A+t_B})$ then

$$WF = K_A + K_B - c_A - c_B - \frac{1}{48}(t_A + t_B) - \frac{4}{9}\delta^2 \frac{t_A t_B}{t_A + t_B}. \tag{2.7}$$

If $\frac{3}{2}(\frac{t_A+t_B}{5t_A+t_B}) < \delta \leq \frac{1}{3} + \frac{t_B}{6t_A}$ then

$$WF = K_A + K_B - c_A - c_B - \frac{1}{48}(t_A + t_B) - (\frac{1}{4} - \delta^2 + \delta)\frac{(t_A + t_B)t_A t_B}{(t_A - t_B)^2}. \tag{2.8}$$

If $\delta > \frac{1}{3} + \frac{t_B}{6t_A}$ then

$$WF = K_A + K_B - c_A - c_B - \frac{1}{48}(t_A + t_B) - \frac{1}{36}(t_A + t_B)\frac{t_B}{t_A}. \tag{2.9}$$

Thus welfare in case of bundling is always lower than without bundling. The reason is that the price of the bundle is lower than the sum of the independent prices. This induces some consumers to buy the bundle and therefore both goods from one firm although they prefer the goods from different firms. Bundling always causes a welfare loss if markets are covered.

In case of $\delta \leq \frac{3}{2}(\frac{t_A+t_B}{5t_A+t_B})$ welfare decreases with δ. With an increase in δ consumers get more heterogeneous. This means that they wish to buy the goods from different firms. But in equilibrium independent prices are increasing in δ while the bundle price is constant. The difference between the independent prices and the bundle price is therefore increasing in δ. This tempts consumers to buy the bundle. Thus distributive inefficiency increases with δ.

In the region $\frac{3}{2}(\frac{t_A+t_B}{5t_A+t_B}) < \delta \leq \frac{1}{3} + \frac{t_B}{6t_A}$ welfare slightly increases with δ because $\delta < \frac{1}{2}$. All three prices are decreasing in δ because competition rises. This reduces the distributive inefficiency slightly.

If $\delta > \frac{1}{3} + \frac{t_B}{6t_A}$ welfare is independent of δ because all prices are independent of δ as well.

As the profit functions the welfare function is also continuous but non-monotonic in δ. Figure 2.4 illustrates the shape of the welfare function, where $WF_{\text{gr}} = K_A + K_B - c_A - c_B$.

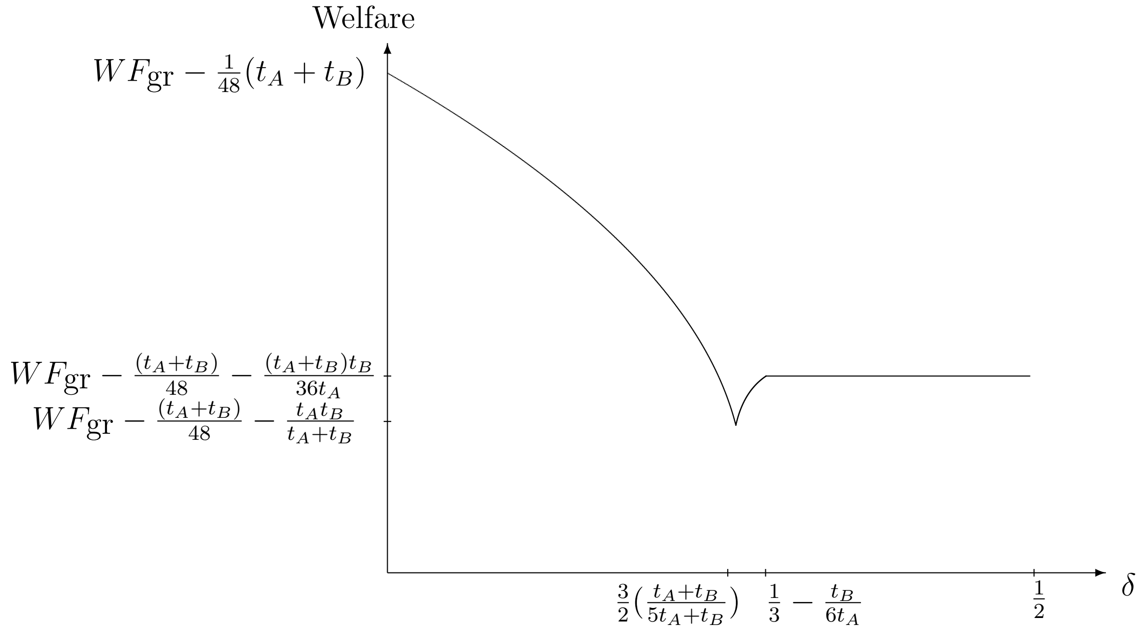

Figure 2.4: Welfare function

This shows that the shape of the welfare function in the first two regions is exactly opposite to the shape of the profit function. The intuition is the following. If δ is small an increase in consumer heterogeneity helps firms to extract more consumer rent through bundling. But this is done by increasing the independent prices thereby inducing consumers to buy the bundle which reduces welfare. If δ is high consumers are heterogeneous and their valuation for both bundles is almost the same. Price competition is fierce and profits are low. But the difference between the sum of the independent prices and the bundle price is almost the same as with a δ in the middle range. Thus welfare stays unchanged.

2.6 Location Choice

In this section the model is extended by endogenizing the extent of product differentiation. In choosing the locations the firms not only change the differentiation and with that the degree of competition but also the correlation of values. This effect of correlation change has interesting implications on firms' location choice.

Before analysing this let us look at a generalisation of the basic model where products are no longer maximally differentiated. This is also a first step towards the later analysis of location choice.

The location of firm 1 is point 0 on both circles as before. Firm 2 is now located at α on both circles with $0 \leq \alpha \leq \frac{1}{2}$.[38] In calculating marginal consumers the same analysis as in Section 2.4 can be conducted. This yields the following proposition.

Proposition 2.4

If $\delta \leq \frac{3}{2}(\frac{t_A+t_B}{3t_A-t_B+4\alpha(t_A+t_B)})$ prices of the firms are

$$\begin{aligned} p_A^\star &= c_A+\alpha(1-\alpha)\left(t_A+\tfrac{4}{3}\delta\tfrac{t_At_B}{t_A+t_B}\right), \\ p_B^\star &= c_B+\alpha(1-\alpha)\left(t_B+\tfrac{4}{3}\delta\tfrac{t_At_B}{t_A+t_B}\right), \\ p_{AB}^\star &= c_A+c_B+\alpha(1-\alpha)(t_A+t_B), \end{aligned}$$

and profits of the firms are given by

$$\Pi^\star = \frac{1}{2}\alpha(1-\alpha)((t_A+t_B)+\frac{8}{9}\delta^2\frac{t_At_B}{t_A+t_B}).$$

If $\delta > \frac{1}{2}-\frac{\alpha}{3}+\frac{t_B(1-\alpha)}{3t_A}$ prices of the firms are

$$\begin{aligned} p_A^\star &= c_A+\alpha(1-\alpha)\left(t_A-\tfrac{2}{3}t_B\right), \\ p_B^\star &= c_B+\alpha(1-\alpha)\tfrac{1}{3}t_B, \\ p_{AB}^\star &= c_A+c_B+\alpha(1-\alpha)(t_A-t_B) \end{aligned}$$

and profits of the firms are given by

$$\Pi^\star = \frac{1}{2}\alpha(1-\alpha)(t_A-\frac{7}{9}t_B).$$

If $\frac{3}{2}(\frac{t_A+t_B}{3t_A-t_B+4\alpha(t_A+t_B)}) < \delta \leq \frac{1}{2}-\frac{\alpha}{3}+\frac{t_B(1-\alpha)}{3t_A}$ prices of the firms are

$$\begin{aligned} p_A^\star &= c_A+\alpha(1-\alpha)\left(t_A+\tfrac{t_At_B}{(t_A-t_B)^2}(2(5t_A+4t_B)-4\delta(8t_A+t_B))\right), \\ p_B^\star &= c_B+\alpha(1-\alpha)\left(t_B+\tfrac{t_At_B}{(t_A-t_B)^2}(2(5t_A+4t_B)-4\delta(8t_A+t_B))\right), \\ p_{AB}^\star &= c_A+c_B+\alpha(1-\alpha)\left((t_A+t_B)+\tfrac{t_At_B}{(t_A-t_B)^2}(18(t_A+t_B)-12\delta(5t_A+t_B))\right) \end{aligned}$$

[38] Assuming α between $\frac{1}{2}$ and 1 would give the same results since e.g. $\alpha = 0.8$ represents the same game as $\alpha = 0.2$.

and profits of the firms are given by

$$\Pi^\star = \frac{1}{2}\alpha(1-\alpha)((t_A+t_B)+\frac{t_At_B}{(t_A-t_B)^2}\Big(16(t_A+t_B)-8\delta(6t_A+t_B)-16\delta^2t_A\Big)).$$

The method of proof is the same as in Section 2.4 and the proof is therefore omitted.

This shows that the results are qualitatively similar to the results with maximal product differentiation. The profit function is non-monotonic in δ and negative correlation hurts firms. The only difference is that profits are lower if $\alpha < \frac{1}{2}$. This is a result one would expect. Since product differentiation is no longer maximal the degree of competition is higher and thus prices are lower.

Now let us turn to the location choice of firms. As is standard in the literature this is modelled in a two-stage-game. In the first stage location is chosen, in the second stage firms set prices after observing the location choices. To keep the model tractable we have to make two additional assumptions which are not very restrictive. The first is that in the first stage only firm 2 chooses its location α_A, α_B on both circles while firm 1's location is fixed. This assumption is not crucial although it sounds asymmetric. The reason is that in a model on the circle there is no possibility for one firm to have a better position than the other one.[39] Even with the connection between the circles through the bundle there is no advantage for firm 2 and in equilibrium both firms earn the same profits. The second assumption is that firm 1 is still located at $(0,0)^T$. This assumption is a bit more restrictive because the equilibrium values would be different if the exogenous positions of firm 1 were different from each other.[40] Yet, the qualitative results would be the same; only the values of the equilibrium prices and profits would be different but the location choice of firm 2 in the first stage would be the same. To compare the results with the former analysis a location of firm 1 at $(0,0)^T$ is assumed. The game is solved by backward induction. In the second stage optimal prices can be

[39] This stands in contrast to competition on the line where such a modelling would give firm 2 a huge advantage.

[40] The exception is if the distance is $\frac{1}{2}$. This would yield the same results as an equal location on the circles.

calculated given α_A, α_B and in the first stage firm 2 chooses α_A and α_B. This is done in the appendix.

Proposition 2.5

If $\delta \leq \frac{\sqrt{53t_A^2+26t_At_B+2t_B^2}}{4t_A} - \frac{3}{2} - \frac{t_B}{4t_A} = \delta'$ firm 2 chooses maximal product differentiation for both goods ($\alpha_A = \alpha_B = \frac{1}{2}$).

If $\delta > \frac{\sqrt{53t_A^2+26t_At_B+2t_B^2}}{4t_A} - \frac{3}{2} - \frac{t_B}{4t_A} = \delta'$ firm 2 chooses maximal product differentiation on circle A ($\alpha_A = \frac{1}{2}$) and minimal product differentiation on circle B ($\alpha_B = 0$).

Thus if $\delta \leq \delta'$ there is maximal product differentiation on both circles. But if $\delta > \delta'$ we have a sudden shift to minimal differentiation on the circle with lower transportation costs. What is the intuition behind this result?

If δ is small the result is not surprising. With maximal product differentiation firms have high market power and competition is best reduced with a location which is most distant. If δ is high we know from Proposition 2.2 that competition is fierce. This is the case because from the point of view of the bundle consumers are nearly homogeneous if firms are maximally differentiated. With the same location on circle B firms avoid the additional competition resulting from this homogeneity. They make no longer profits with good B because $p_B^* = c_B$. But consumer homogeneity is reduced because on circle A each consumer has a strict preference for one firm. Thus the business stealing effect of bundling is reduced and each firm earns profits of $\Pi_i^* = \frac{1}{8}t_A$.

The threshold value $\delta' = \frac{\sqrt{53t_A^2+26t_At_B+2t_B^2}}{4t_A} - \frac{3}{2} - \frac{t_B}{4t_A}$ can be compared with the value of δ at which the profit with mixed bundling is lower than the profit without bundling. From Lemma 2.1 this value of δ is given by $\frac{\sqrt{52t_A^2+28t_At_B+t_B^2}}{4t_A} - \frac{3}{2} - \frac{t_B}{4t_A}$. Thus δ' is slightly above this value. The reason is that in choosing minimal differentiation on circle B firms forego all profits with good B. Firm 2 therefore chooses $\alpha_B = 0$ only when profits with maximal differentiation are lower than $\frac{1}{8}t_A$. But the profits without bundling are given by $\frac{1}{8}t_A + \frac{1}{8}t_B$. Thus δ' is higher.

With the location choice the firms change the correlation of values. They have to balance the effect of increasing competition because of smaller differentiation with the effect of increasing competition because of homogeneity of the bundle. If the latter effect is dominating firms choose minimal differentiation in one product.

The result can also compared with the result of Irmen & Thisse (1998). They analyse a model with one product where firms have to compete in multidimensional characteristics. Each characteristic is independent from each other. Irmen & Thisse (1998) find that firms choose maximal differentiation in the characteristic with the highest salience coefficient and minimal differentiation in all others. The intuition is that price competition is relaxed with differentiation in one characteristic but firms enjoy the advantage of a central location in all others. The argument for minimal differentiation is quite different in my model where firms want to avoid additional competition on the bundle that would arise with differentiation.

2.7 Application

In this section an application of the model to US telephone companies is presented. In the US many of these companies sell internet access and long distance service together in one package. The price of this package is by far lower than if both services are bought independently.

Here I look at three companies, AT&T, birch telecom, and Verizon. Each of them offers such a package. The long distant service in each package is almost the same, so there are no essential differences in offers. But internet access is supplied quite differently in each bundle. AT&T offers only 20 hours per month but gives a free installation kit and free live support. By contrast, birch telecom offers unlimited access but gives only standard support and no gifts. Verizon offers also unlimited access and free live support but no installation kit. In addition, consumers can choose at Verizon if they want to buy DSL or wireless where wireless is a bit more expensive.

This fits the results of the model in the last section, maximal differentiation in one good and minimal in the other, quite well. It is empirically hard to estimate in which good firms are more differentiated, which is represented by higher transportation costs, but the example points to the fact that it is more important for consumers from which firm they get internet access than which one offers them long distance service.

2.8 Conclusion

This paper has shown that commodity bundling in duopoly has inherently different consequences than in the monopoly case. In duopoly there is a high incentive to bundle. But if the correlation of reservation values is negative, profits of the firms decrease through bundling. This is contrary to the monopoly case where bundling is particularly profitable if correlation is negative. The decrease in consumer heterogeneity which renders bundling profitable in monopoly creates a higher degree of competition in duopoly and lowers profits. Thus firms are in a prisoner's dilemma situation. It has also been shown that welfare decreases with bundling because of distributive inefficiency. If firms can choose their location and thus influence the correlation they want to avoid high negative correlation of reservation values and choose minimal product differentiation in one good.

An interesting way in which the model could be extended is to introduce uncertainty. I assumed a one-to-one mapping of consumer locations on both circles to get clear cut results. A possible way to introduce uncertainty might be to assume that a consumer's location on circle B conditional on her location on circle A is uniformly distributed between $x_A + \delta - \epsilon$ and $x_A + \delta + \epsilon$, with $\epsilon \in [0, 1/2]$. So an ϵ of zero is the model analysed in this paper while $\epsilon = 1/2$ means that x_B is independent of x_A. My intuition is that if ϵ is small the qualitative results would not change because uncertainty is small. If instead ϵ is high one may get different results. So the model also offers a framework to deal with questions of uncertainty.

2.9 Appendix

Proof of Remark 2.1

The goal is to calculate the function $\rho[R_A, R_B](\delta) = \frac{\text{Cov}[R_A,R_B](\delta)}{\sigma(R_A)\sigma(R_B)}$. The proof is done from the perspective of firm 1 but we get the same result for firm 2 because of symmetry.

The gross utility from buying the good, $K_j, j = 1, 2$, is constant and the same for all consumers. It can thus be ignored in the calculation of $\sigma(R_A), \sigma(R_B)$ and $\text{Cov}(R_A, R_B)$.

First we calculate of $\sigma(R_A) = \int_0^1 t_A^2(\text{d}(x_A))^2 dx_A - \bar{\text{d}}_A^2$, where $\bar{\text{d}}_A$ is the expected value of the transportation costs. We start with calculating $\bar{\text{d}}_A$,

$$\bar{\text{d}}_A = t_A \int_0^{\frac{1}{2}} (x_A)^2 dx_A + t_A \int_{\frac{1}{2}}^1 (1 - x_A)^2 dx_A = \frac{1}{12} t_A.$$

Next, calculating $\int_0^1 t_A^2(\text{d}(x_A))^2 dx_A$ yields

$$\int_0^1 t_A^2(\text{d}(x_A))^2 dx_A = t_A^2 \int_0^{\frac{1}{2}} x_A^4 dx_A + t_A^2 \int_{\frac{1}{2}}^1 (1 - x_A)^4 dx_A = \frac{1}{80} t_A^2.$$

Thus

$$\sigma(R_A) = \frac{1}{80} t_A^2 - \frac{1}{144} t_A^2 = \frac{1}{180} t_A^2.$$

Turning to circle B, $\bar{\text{d}}_B$ is given by

$$\bar{\text{d}}_B = t_B \int_0^{\frac{1}{2}-\delta} (x_A+\delta)^2 dx_A + t_B \int_{\frac{1}{2}-\delta}^{1-\delta} (1-x_A-\delta)^2 dx_A + t_B \int_{1-\delta}^1 (x_A+\delta-1)^2 dx_A = \frac{1}{12} t_B.$$

Calculating $\sigma(R_B)$ gives

$$\begin{aligned} \sigma(R_B) = t_B^2 \textstyle\int_0^{\frac{1}{2}-\delta} (x_A + \delta)^4 dx_A + t_B^2 \int_{\frac{1}{2}-\delta}^{1-\delta} (1 - x_A - \delta)^4 dx_A \\ + t_B^2 \textstyle\int_{1-\delta}^1 (x_A + \delta - 1)^4 dx_A - (\frac{1}{12})^2 t_B^2 = \frac{1}{180} t_B^2. \end{aligned}$$

The covariance $\text{Cov}(R_A, R_B)$ is thus given by

$$\begin{aligned} \text{Cov}(R_A, R_B)(\delta) = \textstyle\int_0^{\frac{1}{2}-\delta} (t_A x_A^2 - \frac{1}{12} t_A)(t_B(x_A + \delta)^2 - \frac{1}{12} t_B) dx_A \\ + \textstyle\int_{\frac{1}{2}-\delta}^{\frac{1}{2}} (t_A x_A^2 - \frac{1}{12} t_A)(t_B(1 - x_A - \delta)^2 - \frac{1}{12} t_B) dx_A \\ + \textstyle\int_{\frac{1}{2}}^{1-\delta} (t_A(1 - x_A)^2 - \frac{1}{12} t_A)(t_B(1 - x_A - \delta)^2 - \frac{1}{12} t_B) dx_A \\ + \textstyle\int_{1-\delta}^1 (t_A(1 - x_A)^2 - \frac{1}{12} t_A)(t_B(x_A + \delta - 1)^2 - \frac{1}{12} t_B) dx_A \end{aligned}$$

which after some manipulations yields

$$\mathrm{Cov}(R_A, R_B)(\delta) = t_A t_B[\frac{1}{180} - \frac{1}{6}\delta^2 + \frac{1}{3}\delta^3 - \frac{1}{6}\delta^4].$$

Thus

$$\rho(R_A, R_B)(\delta) = 1 - 30\delta^2 + 60\delta^3 - 30\delta^4.$$

q.e.d.

Proof of Proposition 2.1

Consider the case where both firms do not bundle. Since the equilibrium is symmetric both firms charge the same independent prices, p_A^{ind} and p_B^{ind}, and earn profits of $\Pi_i^* = \frac{1}{2}(p_A^{ind} - c_A + p_B^{ind} - c_B)$.

Now let us look if there is an incentive for firm 1 to introduce a bundle, that means selling both goods together at a price $p_{AB}^1 < p_A^1 + p_B^1$. We analyse the case where $p_{AB}^1 = p_A^{ind} + p_B^{ind}$ and $p_j^1 = p_j^{ind} + \epsilon_1$, with $\epsilon_1 > 0$, but small. So firm 1 increases its independent prices by ϵ_1 and sets the bundle price equal to the sum of the prices if firms do not bundle.

We have to distinguish between two cases, either if δ is "near" $\frac{1}{2}$ or not, because this changes the demand structure on the circles. First look at the case where δ is not near $\frac{1}{2}$. If firms do not bundle there are four demand regions on the circles, namely $(AB1), (A1B2), (AB2)$ and $(A2B1)$. The frontiers between this regions (or the marginal consumers) are the following,

1. frontier between $(AB1)$ and $(A1B2)$: $\frac{1}{4} - \delta$,

2. frontier between $(A1B2)$ and $(AB2)$: $\frac{1}{4}$,

3. frontier between $(AB2)$ and $(A2B1)$: $\frac{3}{4} - \delta$,

4. frontier between $(A2B1)$ and $(AB1)$: $\frac{3}{4}$.

If firm 1 introduces the bundle the frontiers are changed to

1. frontier between $(AB1)$ and $(A1B2)$: $\frac{1}{4} - \delta + \frac{\epsilon_1}{t_B}$,

2. frontier between $(A1B2)$ and $(AB2)$: $\frac{1}{4} - \frac{\epsilon_1}{t_A}$,

3. frontier between $(AB2)$ and $(A2B1)$: $\frac{3}{4} - \delta + \frac{\epsilon_1}{t_B}$,

4. frontier between $(A2B1)$ and $(AB1)$: $\frac{3}{4} - \frac{\epsilon_1}{t_A}$.

The new profit function of firm 1 is

$$\Pi_1^{**} = (p_A^1 + p_B^1 - c_A - c_B)(\tfrac{1}{2} - \delta + \epsilon_1(\tfrac{1}{t_A} + \tfrac{1}{t_B}))$$
$$+(p_A^1 - c_A + \epsilon_1)(\delta - \epsilon_1(\tfrac{1}{t_A} + \tfrac{1}{t_B})) + (p_B^1 - c_B + \epsilon_1)(\delta - \epsilon_1(\tfrac{1}{t_A} + \tfrac{1}{t_B}))$$

or

$$\Pi_1^{**} = (p_A^1 - c_A + p_B^1 - c_B)\tfrac{1}{2} + 2\delta\epsilon_1 - 2(\epsilon_1)^2(\tfrac{1}{t_A} + \tfrac{1}{t_B})$$
$$= \Pi_1^* + 2\delta\epsilon_1 - 2(\epsilon_1)^2(\tfrac{1}{t_A} + \tfrac{1}{t_B}).$$

This is always higher than the old profit Π_1^* as long as $\delta > 0$, because ϵ_1 can made arbitrary small and so $(\epsilon_1)^2$ tends faster to 0 then ϵ_1.

Up to now we have shown that firm 1 has an incentive to introduce a bundle. The question is now if firm 2 has an incentive to bundle if firm 1 is already bundling. The profit of firm 2 if firm 1 bundles while firm 2 not is given by

$$\Pi_2^* = (p_A^2 + p_B^2 - c_A - c_B)(\tfrac{1}{2} - \delta + \epsilon_1(\tfrac{1}{t_A} + \tfrac{1}{t_B}))$$
$$+(p_A^2 - c_A)(\delta - \epsilon_1(\tfrac{1}{t_A} + \tfrac{1}{t_B})) + (p_B^2 - c_B)(\delta - \epsilon_1(\tfrac{1}{t_A} + \tfrac{1}{t_B}))$$
$$= (p_A^2 + p_B^2 - c_A - c_B)\tfrac{1}{2}.$$

If firm 2 chooses to bundle and set $p_{AB}^2 = p_A^{ind} + p_B^{ind}$ and $p_j^2 = p_j^{ind} + \epsilon_2$, with $\epsilon_2 > 0$, but small, the frontiers are given by

1. frontier between $(AB1)$ and $(A1B2)$: $\frac{1}{4} - \delta + \frac{\epsilon_1+\epsilon_2}{t_B}$,

2. frontier between $(A1B2)$ and $(AB2)$: $\frac{1}{4} - \frac{\epsilon_1+\epsilon_2}{t_A}$,

3. frontier between $(AB2)$ and $(A2B1)$: $\frac{3}{4} - \delta + \frac{\epsilon_1+\epsilon_2}{t_B}$,

4. frontier between $(A2B1)$ and $(AB1)$: $\frac{3}{4} - \frac{\epsilon_1+\epsilon_2}{t_A}$.

The new profit of firm 2 is then

$$\Pi_2^{**} = (p_A^2 + p_B^2 - c_A - c_B)(\tfrac{1}{2}\delta + (\epsilon_1 + \epsilon_2)(\tfrac{1}{t_A} + \tfrac{1}{t_B}))$$
$$+(p_A^1 - c_A + \epsilon_2)(\delta - (\epsilon_1 + \epsilon_2)(\tfrac{1}{t_A} + \tfrac{1}{t_B})) + (p_B^1 - c_B + \epsilon_2)(\delta - (\epsilon_1 + \epsilon_2)(\tfrac{1}{t_A} + \tfrac{1}{t_B}))$$
$$= \Pi_2^* + 2\epsilon_2\delta - 2[(\epsilon_2)^2 + \epsilon_1\epsilon_2](\tfrac{1}{t_A} + \tfrac{1}{t_B}).$$

Thus for ϵ_1 and ϵ_2 small, bundling is profitable if $\delta > 0$ since $(\epsilon_2)^2$ and $\epsilon_1\epsilon_2$ tends faster to 0 then ϵ_2.

Now let us turn the case where δ is near $\frac{1}{2}$ and look if firm 1 has an incentive to introduce a bundle. The difference to the former analysis is that in the surrounding of $x_A = \frac{1}{4}$ there are now some consumers who buy $(AB1)$ because they have almost the same preferences for all combinations but the bundle has a lower price than all other combinations. Thus moving clockwise on circle A starting at point zero the product combination $(A1B2)$ is followed by $(AB1)$ and no one buys $(AB2)$. The frontiers are given by

1. frontier between $(A1B2)$ and $(AB1)$: $\frac{3}{4} - \delta - \frac{\epsilon_1}{t_B}$,
2. frontier between $(AB1)$ and $(A2B1)$: $\frac{1}{4} + \frac{\epsilon_1}{t_A}$,
3. frontier between $(A2B1)$ and $(AB1)$: $\frac{3}{4} - \frac{\epsilon_1}{t_A}$,
4. frontier between $(AB1)$ and $(A1B2)$: $\frac{5}{4} - \delta + \frac{\epsilon_1}{t_B}$.

The profit of firm 1 if it bundles is

$$\Pi_1^{**} = (p_A^1 + p_B^1 - c_A - c_B)(2\epsilon_1(\tfrac{1}{t_A} + \tfrac{1}{t_B}))$$
$$+(p_A^1 - c_A + \epsilon_1)(\tfrac{1}{2} - 2\epsilon_1(\tfrac{1}{t_B}) + (p_B^1 - c_B + \epsilon_1)(\tfrac{1}{2} - 2\epsilon_1(\tfrac{1}{t_A})$$

or

$$\Pi_1^{**} = \Pi_1^* + 2(p_A^1 - c_A)\frac{\epsilon_1}{t_A} + 2(p_B^1 - c_B)\frac{\epsilon_1}{t_B} + \epsilon_1 - 2\frac{(\epsilon_1)^2}{t_A} - 2\frac{(\epsilon_1)^2}{t_B}.$$

Thus Π_1^{**} is independent of δ and always greater than Π_1^* if ϵ_1 is small.

Let us now look at firm 2 if firm 1 is already bundling. If firm 2 chooses not to bundle its profit is

$$\Pi_2^* = (p_A^1 - c_A)(\frac{1}{2} - 2\epsilon_1(\frac{1}{t_A})) + (p_B^1 - c_B)(\frac{1}{2} - 2\epsilon_1(\frac{1}{t_B})).$$

If firm 2 introduces a bundle itself the region where consumers buy that bundle returns and frontiers are given by

1. frontier between $(A1B2)$ and $(AB1)$: $\frac{3}{4} - \delta - \frac{\epsilon_1+\epsilon_2}{t_B}$,

2. frontier between $(AB1)$ and $(AB2)$: $\frac{1}{t_A - t_B}(\frac{1}{4}t_A - \frac{3}{4}t_B + \delta t_B)$,

3. frontier between $(AB2)$ and $(A2B1)$: $\frac{3}{4} - \delta + \frac{\epsilon_1+\epsilon_2}{t_B}$,

4. frontier between $(A2B1)$ and $(AB2)$: $\frac{5}{4} - \delta - \frac{\epsilon_1+\epsilon_2}{t_B}$,

5. frontier between $(AB2)$ and $(AB1)$: $\frac{1}{t_A - t_B}(\frac{3}{4}t_A - \frac{5}{4}t_B + \delta t_B)$,

6. frontier between $(AB1)$ and $(A1B2)$: $\frac{5}{4}\delta + \frac{\epsilon_1+\epsilon_2}{t_B}$.

Profit of firm 2 if both firms bundle is then

$$\begin{aligned}\Pi_2^{**} = (p_A^2 + p_B^2 - c_A - c_B)(2(\tfrac{\epsilon_1+\epsilon_2}{t_B}) - \tfrac{1}{2} + \tfrac{1}{t_A - t_B}(\tfrac{3}{4}t_A - \tfrac{5}{4}t_B - \tfrac{1}{4}t_A + \tfrac{3}{4}t_B)\\ +(p_A^2 - c_A + \epsilon_2)(\tfrac{1}{2} - 2(\epsilon_1 + \epsilon_2)(\tfrac{1}{t_B}) + (p_B^2 - c_B + \epsilon_2)(\tfrac{1}{2} - 2(\epsilon_1 + \epsilon_2)(\tfrac{1}{t_B})\\ = \Pi_2^* + \epsilon_2 - 4[\tfrac{(\epsilon_2)^2 + \epsilon_1\epsilon_2}{t_B}] + 2(p_A^2 + p_B^2 - c_A - c_B)\tfrac{\epsilon_1}{t_B}.\end{aligned}$$

If ϵ_1 and ϵ_2 are small $\Pi_2^{**} > \Pi_2^*$, so firm 2 also has an incentive to bundle.

q.e.d.

Proof of Proposition 2.2

Before proving Proposition 2.2 we have to establish several claims:

Claim 2.1

There cannot exist direct rivalry between product combination $(A1B2)$ and $(A2B1)$.

Proof:

Assume that the consumer on x_A with x_A between 0 and $\frac{1}{2} - \delta$ is the marginal consumer between product combination $(A1B2)$ and $(A2B1)$ and she buys either of these alternatives. Thus $(A2B1)$ must be better for her then $(AB2)$. This is only the case if

$$p_A^2 + p_B^1 + t_B(x_A + \delta)^2 \leq p_{AB}^2 + t_B\left(\frac{1}{2} - x_A - \delta\right)^2 \tag{2.10}$$

or

$$p_A^2 + p_B^1 \leq p_{AB}^2 + \frac{1}{4}t_B x_A t_B - \delta t_B. \tag{2.11}$$

Since in equilibrium both firms bundle we know that $p^2_{AB} < p^2_A + p^2_B$. Thus we can write $p^2_A + p^2_B - \kappa$ with $\kappa > 0$ instead of p^2_{AB}. Then from (3.2) we get

$$p^1_B \leq p^2_B - \kappa + \frac{1}{4}t_B - x_A t_B - \delta t_B. \tag{2.12}$$

For the consumer indifferent between (A1B2) and (A2B1) it must also be optimal to buy ($A1B2$) instead of ($AB2$). This is only the case if (knowing that $p^1_{AB} = p^1_A + p^1_B - \lambda$ with $\lambda > 0$)

$$p^1_A + p^1_B - \lambda + t_B\,(x_A + \delta)^2 \geq p^1_A + p^2_B + t_B\left(\frac{1}{2} - x_A - \delta\right)^2.$$

or

$$p^1_B - \lambda \geq p^2_B + \frac{1}{4}t_B - x_A t_B - \delta t_B.$$

But this is a contradiction to (2.12) because $\kappa, \lambda > 0$. Therefore it cannot be optimal for a consumer at x_A to buy ($A1B2$).
One can show that the same holds for x_A between $\frac{1}{2} - \delta$ and $\frac{1}{2}$. Because of symmetry a similar condition holds on the second half of the circle.

q.e.d.

Claim 2.2

(i) Take x_A and x'_A with $0 \leq x_A, x'_A \leq \frac{1}{2}$ and $x'_A < x_A$.
If ($AB1$) is optimal at x_A then at x'_A ($AB2$) can never be optimal.

(ii) Take x_A and x'_A with $\frac{1}{2} \leq x_A, x'_A \leq 1$ and $x'_A < x_A$.
If ($AB2$) is optimal at x_A then at x'_A ($AB1$) can never be optimal.

Proof:

Assume that x_A lies between 0 and $\frac{1}{2} - \delta$. At x_A we have

$$p^1_{AB} + t_A(x_A)^2 + t_B(x_A + \delta)^2 \leq p^2_{AB} + t_A\left(\frac{1}{2} - x_A\right)^2 + t_B\left(\frac{1}{2} - x_A - \delta\right)^2$$

and therefore

$$(t_A + t_B)x_A \leq p^2_{AB} - p^1_{AB} - t_B\delta + \frac{1}{4}\left(t_A + t_B\right).$$

If ($AB2$) were optimal at x'_A then

$$(t_A + t_B)x'_A \geq p^2_{AB} - p^1_{AB} - t_B\delta + \frac{1}{4}\left(t_A + t_B\right).$$

But since $x'_A < x_A$ this cannot be the case.

One gets a similar condition for $\frac{1}{2} - \delta \leq x_A \leq \frac{1}{2}$. If $\frac{1}{2} \leq x_A, x'_A \leq 1$ the method of proof is exactly similar only with $(AB1)$ and $(AB2)$ reversed.

q.e.d.

Claim 2.3

(i) Take x_A and x'_A with $0 \leq x_A, x'_A \leq \frac{1}{2}$ and $x'_A < x_A$.

If $(A1B2)$ is optimal at x_A then at x'_A $(A2B1)$ can never be optimal.

(ii) Take x_A and x'_A with $\frac{1}{2} \leq x_A, x'_A \leq 1$ and $x'_A < x_A$.

If $(A2B1)$ is optimal at x_A then at x'_A $(A1B2)$ can never be optimal.

Proof:

Assume that x_A lies between 0 and $\frac{1}{2} - \delta$. At x_A we have

$$p_A^1 + p_B^2 + t_A(x_A)^2 + t_B\left(\frac{1}{2} - x_A - \delta\right)^2 \leq p_B^1 + p_A^2 + t_A\left(\frac{1}{2} - x_A\right)^2 + t_B\left(x_A + \delta\right)^2$$

and therefore

$$(t_A - t_B)x_A \leq p_B^1 + p_A^2 - p_A^1 - p_B^2 + t_B\delta + \frac{1}{4}\left(t_A - t_B\right).$$

If $(A2B1)$ were optimal at x'_A then

$$(t_A - t_B)x'_A \geq p_B^1 + p_A^2 - p_A^1 - p_B^2 + t_B\delta + \frac{1}{4}\left(t_A - t_B\right).$$

But since $x'_A < x_A$ this cannot be the case.

One gets a similar condition for $\frac{1}{2} - \delta \leq x_A \leq \frac{1}{2}$.

If $\frac{1}{2} \leq x_A, x'_A \leq 1$ the method of proof is exactly similar only with $(A1B2)$ and $(A2B1)$ reversed.

q.e.d.

Claim 2.4

(i) Take x_A and x'_A with $0 \leq x_A, x'_A \leq \frac{1}{2}$ and $x'_A < x_A$.

If $(AB1)$ is optimal at x_A then at x'_A $(A2B1)$ can never be optimal.

(ii) Take x_A and x'_A with $\frac{1}{2} \leq x_A, x'_A \leq 1$ and $x'_A < x_A$.

If $(A2B1)$ is optimal at x_A then at x'_A $(AB1)$ can never be optimal.

Proof:

Assume that x_A lies between 0 and $\frac{1}{2} - \delta$. At x_A we have

$$p_A^1 + p_B^1 - \lambda + t_A(x_A)^2 + t_B(x_A + \delta)^2 \leq p_B^1 + p_A^2 + t_A\left(\frac{1}{2} - x_A\right)^2 + t_B(x_A + \delta)^2$$

and therefore

$$t_A x_A \leq p_A^2 - p_A^1 + \lambda + \frac{1}{4}t_A. \tag{2.13}$$

If $(A2B1)$ were better than $(AB1)$ at x'_A then we would have

$$t_A x'_A \geq p_A^2 - p_A^1 + \lambda + \frac{1}{4}t_A.$$

But since $x'_A < x_A$ this is a contradiction to (2.9).

One gets a similar condition for $\frac{1}{2} - \delta \leq x_A \leq \frac{1}{2}$.

If $\frac{1}{2} \leq x_A, x'_A \leq 1$ the method of proof is exactly similar only with $(AB1)$ and $(A2B1)$ reversed.

q.e.d.

Claim 2.5

(i) Take x_A and x'_A with $0 \leq x_A, x'_A \leq \frac{1}{2}$ and $x'_A < x_A$.
If $(A1B2)$ is optimal at x_A then at x'_A $(AB2)$ can never be optimal.

(ii) Take x_A and x'_A with $\frac{1}{2} \leq x_A, x'_A \leq 1$ and $x'_A < x_A$.
If $(AB2)$ is optimal at x_A then at x'_A $(A1B2)$ can never be optimal.

Proof:

Assume that x_A lies between 0 and $\frac{1}{2} - \delta$. At x_A we have

$$p_A^1 + p_B^2 + t_A(x_A)^2 + t_B\left(\frac{1}{2} - x_A - \delta\right)^2 \leq p_A^2 + p_B^2 - \kappa + t_A\left(\frac{1}{2} - x_A\right)^2 + t_B\left(\frac{1}{2} - x_A\delta\right)^2$$

and therefore

$$t_A x_A \leq p_B^2 - p_A^1 - \kappa + \frac{1}{4}t_A. \tag{2.14}$$

If $(AB2)$ were optimal at x'_A then we would have

$$t_A x'_A \geq p_B^2 - p_A^1 + \kappa + \frac{1}{4}t_A.$$

But since $x'_A < x_A$ this is not possible.
One gets a similar condition for $\frac{1}{2} - \delta \leq x_A \leq \frac{1}{2}$.
If $\frac{1}{2} \leq x_A, x'_A \leq 1$ the method of proof is exactly similar only with $(A1B2)$ and $(AB2)$ reversed.

q.e.d.

As a result in equilibrium there can only be three possible demand structures on the circle A.[41]

(i) $(AB1), (A1B2), (AB2), (A2B1), (AB1)$
(ii) $(A1B2), (AB2), (A2B1), (AB1), (A1B2)$
(iii) $(A1B2), (AB1), (AB2), (A2B1), (AB2), (AB1), (A1B2)$

Calculating the profit function for each demand structure we get profit function (2.2) for demand structures (i) and (ii) and profit function (2.4) for demand structure (iii). Maximising each profit function with respect to p^1_{AB}, p^1_A and p^1_B yields equation (2.3) for profit function (2.2) and equation (2.5) for profit function (2.4).

It remains to calculate for which values of δ the profit functions are valid.
For profit function (2.2) to arise (A1B2) must be followed by (AB2) and not by (AB1). The frontier between (A1B2) and (AB2) at the equilibrium prices is given by

$$x_A = \frac{1}{4} - \frac{2}{3}\delta\frac{t_B}{t_A + t_B}. \tag{2.15}$$

The frontier between (A1B2) and (AB1) at the equilibrium prices is given by

$$x_A = \frac{3}{4} - \delta\frac{5t_A + 3t_B}{3(t_A + t_B)}. \tag{2.16}$$

For demand structure (i) or (ii) to arise (2.15) must be smaller than (2.16). This gives the first threshold

$$\delta_1^{TS} = \frac{3}{2}(\frac{t_A + t_B}{5t_A + t_B}).$$

[41] This means that e.g. at demand structure (i) at point zero we have product combination $(AB1)$ followed clockwise by product combination $(A1B2)$ which in turn is follwed by $(AB2)$. $(AB2)$ is followed by $(A2B1)$ and arriving at point 1 we again have $(AB1)$.

For profit function (2.4) to arise (A1B2) must be followed by (AB1) and not by (AB2). Calculating in the same way as before by inserting the equilibrium prices of profit function (2.4) gives that demand structure (iii) arises only if

$$\delta_2^{TS} > \frac{1}{3} + \frac{t_B}{6t_A}.$$

This gives the second threshold.

In the region in between $\frac{3}{2}\frac{t_A+t_B}{5t_A+t_B} < \delta \leq \frac{1}{3} + \frac{t_B}{6t_A}$ firms set their prices in such a way that demand structure (ii) arises. Routine manipulations show that equilibrium prices and profits are given by (2.6). They exactly satisfy the constraint that

$$\frac{1}{4} + \frac{p_{AB}^2 - p_A^1 - p_B^2}{t_A} \geq \frac{3}{4} - \delta + \frac{p_{AB}^1 - p_A^1 - p_B^2}{t_B},$$

which says that (A1B2) is followed by (AB1) and not (AB2).

This completes the proof.

q.e.d.

Proof of Proposition 2.3

Welfare is calculated by inserting the equilibrium prices in the formulas for the frontiers of each product combination and calculating the resulting transportation costs on each circle. If $\delta < \frac{3}{2}(\frac{t_A+t_B}{5t_A+t_B})$ welfare is given by

$$\begin{aligned} WF = \;& K_A + K_B - c_A - c_B \\ & -t_A\{\int_0^{\frac{1}{4}-\frac{2}{3}\delta\frac{t_B}{t_A+t_B}} (x)^2dx + \int_{\frac{1}{4}-\frac{2}{3}\delta\frac{t_B}{t_A+t_B}}^{\frac{1}{2}} (\tfrac{1}{2}-x)^2dx \\ & \int_{\frac{1}{2}}^{\frac{3}{4}-\frac{2}{3}\delta\frac{t_B}{t_A+t_B}} (x-\tfrac{1}{2})^2dx + \int_{\frac{3}{4}-\frac{2}{3}\delta\frac{t_B}{t_A+t_B}}^{1} (1-x)^2dx\} \\ & -t_B\{\int_0^{\frac{1}{4}+\frac{2}{3}\delta\frac{t_A}{t_A+t_B}} (x)^2dx + \int_{\frac{1}{4}+\frac{2}{3}\delta\frac{t_A}{t_A+t_B}}^{\frac{1}{2}} (\tfrac{1}{2}-x)^2dx \\ & \int_{\frac{1}{2}}^{\frac{3}{4}+\frac{2}{3}\delta\frac{t_A}{t_A+t_B}} (x-\tfrac{1}{2})^2dx + \int_{\frac{3}{4}+\frac{2}{3}\delta\frac{t_A}{t_A+t_B}}^{1} (1-x)dx\}, \end{aligned}$$

which after some manipulations yields

$$WF = K_A + K_B - c_A - c_B - \frac{1}{48}(t_A + t_B) - \frac{4}{9}\delta^2\frac{t_At_B}{t_A + t_B}$$

which is equation (2.7).

Welfare is calculated in the same way if $\frac{3}{2}(\frac{t_A+t_B}{5t_A+t_B}) < \delta \leq \frac{1}{3}+\frac{t_B}{6t_A}$ and if $\delta > \frac{1}{3}+\frac{t_B}{6t_A}$ which gives equations (2.8) in the first case and (2.9) in the second.

q.e.d.

Proof of Proposition 2.5

Calculating prices and profits for arbitrary values of α_A and α_B is done in the standard way. This yields profits of

$$\Pi^\star = \frac{1}{2}\alpha_A(1-\alpha_A)t_A + \frac{1}{2}\alpha_B(1-\alpha_B)t_B + \frac{16}{9}\delta^2\frac{\alpha_A(1-\alpha_A)t_A\alpha_B(1-\alpha_B)t_B}{\alpha_A(1-\alpha_A)t_A+\alpha_B(1-\alpha_B)t_B},$$

if $\delta \leq \frac{3(t_A\alpha_A(1-\alpha_A)+t_B\alpha_B(1-\alpha_B))}{2\alpha_B(1-\alpha_A)[6t_A\alpha_A-2t_B(1-\alpha_B+8(t_A+t_B)\alpha_A(1-\alpha_B))]}$

Differentiating $\Pi^\star$ with respect to α_A and α_B yields that profit is maximal if $\alpha_A = \alpha_B = \frac{1}{2}$.

If $\delta > \frac{1}{2}(1-\alpha_A)+\alpha_B\left(\frac{1}{6}+\frac{t_B}{3t_A}\right)$ profits are given by

$$\Pi^\star = \frac{1}{2}t_A\alpha_A(1-\alpha_A) - \frac{7}{18}t_B\alpha_B(1-\alpha_B).$$

Differentiating this profit with respect to α_A and α_B yields that profit is maximal if $\alpha_A = \frac{1}{2}$ and $\alpha_B = 0$ since α_B can only be between 0 and $\frac{1}{2}$.

If $\delta \leq \frac{3(t_A\alpha_A(1-\alpha_A)+t_B\alpha_B(1-\alpha_B))}{2\alpha_B(1-\alpha_A)[6t_A\alpha_A-2t_B(1-\alpha_B+8(t_A+t_B)\alpha_A(1-\alpha_B))]} < \delta \leq \frac{1}{2}(1-\alpha_A)+\alpha_B\left(\frac{1}{6}+\frac{t_B}{3t_A}\right)$ profits are given by

$$\Pi^\star = \tfrac{1}{2}((\alpha_A(1-\alpha_A)t_A + (\alpha_B(1-\alpha_B)t_B))+$$
$$\tfrac{t_At_B}{(t_A-t_B)^2}\left(64\alpha_A(1-\alpha_B)(t_A+t_B) - 4\alpha_B(1-\alpha_A)\delta(6t_A+t_B) - 8\alpha_A(1-\alpha_A)\delta^2t_A\right).$$

Differentiating this profit with respect to α_A yields that profit is always maximal if $\alpha_A = \frac{1}{2}$. Differentiating with respect to α_B yields that $\alpha_B = \frac{1}{2}$ if $\delta \leq \frac{\sqrt{53t_A^2+26t_At_B+2t_B^2}}{2t_A} - \frac{3}{2} - \frac{t_B}{4t_A}$ and $\alpha_B = 0$ if $\delta > \frac{\sqrt{53t_A^2+26t_At_B+2t_B^2}}{2t_A} - \frac{3}{2} - \frac{t_B}{4t_A}$.

q.e.d.

Chapter 3

Two-Sided Markets with Negative Externalities

3.1 Introduction

There are many markets where companies produce services for a group of agents who do not pay for it or pay only a low price. Instead these companies get revenues from advertisers who wish to gain access to potential consumers via the services of these companies. Examples are private radio or television stations[42] which often interrupt their programme to broadcast advertisement.[43] Search engines like Google or Yahoo! or internet portals like GMX often have a multitude of advertisements on their web sites.[44] In the case of radio it is technically impossible to charge listeners for the broadcasting of programmes. In the case of search engines it is not customary to charge users for the services.

---

[42]To get an idea of the expenditures on advertising, in 2002 approximately $ 50 billion were spent on TV advertising in the US only (Kind, Nilssen & Sorgard (2003)) and a 30 second commercial on FOX had an average price of $ 150,000 (Prime Time Pricing Survey, The Advertising Age (2002)).

[43]In the US advertising time ranged from approximately 10 to 15 minutes per hour in 1999 (Television Commercial Monitoring Report (1999)).

[44]For an internet portal advertising is the most important source of revenue since it does not charge users. For example, the internet portal GMX sells a banner on its web site for Euro 20,000 per week (http://www.gmx.de).

This chapter studies a model of platform competition in which users dislike advertisement and therefore spend less time to consume services of platforms. Advertisers wish to gain users' attention to tempt them to buy their products. In equilibrium the level of advertising might be too high or too low compared with the socially optimal one because platform pricing does not internalise the externality which is exerted on users by more advertising. Concerning platform profits a higher degree of competition for users can increase profits because price competition becomes less fierce. Thus a different level of competition on one side influences the level of competition on the other side and may have consequences on profits which are different to a one-sided market. If platforms can charge users as well there might be an incentive to subsidise users, i.e. set a negative fee, to attract more users. But since both platforms do so this strategic effect lowers their profits. A prisoner's dilemma situation arises. If the user fee is positive the additional instrument increases profits. The equilibrium with fees for both sides of the market is always efficient. The reason is that with the user fee platforms do now take into account users' utility in their pricing behaviour.

More specifically, we assume that two platforms compete for user time and advertisers. For the advertisers platforms are completely similar while platforms compete for users in a standard Hotelling style. Both sides of the market choose only one platform.[45] Profits of advertisers are increasing in the time users spend on a platform. Users' utility and the time they spend on a platform are decreasing with advertising.[46] Therefore an advertiser causes a negative externality on users of that platform directly and also on all other advertisers on that platform indirectly. If the gains from trading advertisers' goods are high compared to users' utility loss all producers should advertise from a social point of view. If the utility loss is high some of the advertisers should

[45]The assumption that advertisers single-home (use only one platform) is not crucial to the results but simplifies the modeling. See the next section for a longer discussion.

[46]Ferrando, Gabszewicz, Laussel, and Sonnac (2003) analyse a model in which some people are advertisement-avoiders while others are advertisement-lovers. But normally commercials are considered a nuisance for users. See Bagwell (2003) and Dukes & Gal-Or (2002).

be excluded. The optimal distribution of advertisers among platforms is even. The intuition is that if one platform has more than half of the advertisers the externality on all of them is high and can be reduced if some advertisers shift to the other platform.

In a Nash equilibrium the number of advertisers on both platforms is the same but might be too high or too low compared with efficiency. Platforms only internalise the indirect externality that one advertiser exerts on other advertisers but not the direct utility loss of users. This is the case because this externality is incorporated in their pricing behaviour while the externality on users does not influence prices. So if the degree of differentiation between platforms is low competition for users is fierce. But platforms compete for users by reducing their advertisement levels. Thus with low differentiation there is little advertising on platforms. But if the gains from trading advertisers' goods are high this level of advertising is too low compared with the social optimum. If instead platforms are highly differentiated they will lose only few users if they advertise more. In this case the level of advertising is too high.

Platforms' profits depend on the level of differentiation as well. If differentiation is relatively high profits fall with an increase in differentiation. The intuition is that platforms have a higher incentive to attract advertisers because users do not switch easily to the other platform. This results in lower prices for advertising. But since both platforms lower their prices the level of advertising stays the same while profits are decreasing. So the strategic effect hurts platforms. This shows that in a two-sided market a lower level of competition on one side can increase the competition on the other side and lead to lower profits. If differentiation is low and competition for users is fierce an increase in the differentiation leads to rising profits. The reason is that advertising levels are low and with a price decrease this level rises, which increases profits.

I also analyse what happens if platforms can charge a user fee. If this user fee is unrestricted, e.g. can either be positive or negative, the efficient outcome is reached. With the possibility of a user fee platforms have two different instruments at hand to make profits. They therefore take users' utility into account as well. Since platforms compete for both sides this leads to the efficient outcome.

In equilibrium it might be the case that this user fee is negative because platforms want to attract users in order to make more profits on advertisers. In this case the additional instrument of a user fee hurts platforms and their profits are lower. If the user fee is positive profits are higher than without a user fee. If the fee is restricted to be positive the efficient outcome cannot be reached in general but only in the case when the user charge would be positive in equilibrium.

Most of the papers in the two-sided markets literature are concerned with participants exerting positive externalities on each other like in the market for credit cards. Examples of these papers are Rochet & Tirole (2003) or Wright (2003). In Section 6 of their paper Rochet & Tirole (2003) briefly analyse a model in which platforms earn revenues from users and advertisers. Platforms are able to use a two-part tariff for both groups of participants. Rochet & Tirole (2003) show that in general both prices depend on the relations between own- and cross-price elasticities.[47]

Recently there has been a growing literature on platform competition for advertisers. A seminal contribution to this literature is the paper of Anderson & Coate (2003). They analyse a model of TV broadcasting and are interested in the question whether two channels will offer the same or different programmes and how much advertisement they will broadcast. They find that dependent on parameter values there can be too little but also too much advertising and also too low or too high a variety of programmes. In their model viewers suffer from advertising with the consequence that they switch to their less preferred programme if this has fewer advertisements. As a result an even distribution of advertisers on platforms is efficient because otherwise some viewers would not watch their preferred programme.

My paper revisits their first result in a different type of model. The difference to their paper is that in my model platforms compete directly for advertisers while in their model a change in the commercial price of channel 1 does not influence the commercial price of channel 2. This allows me to analyse the consequences of different

[47] For a detailed overview how to model different forms of competition and externalities in two-sided markets see Armstrong (2004). For a model with a monopoly platform see Baye & Morgan (2001).

degrees of competition on one side for the degree of competition on the other side and on platforms' profits. Anderson & Coate (2003) also analyse the case in which viewers can be charged for watching the programmes. They find that advertising levels are usually lower in this case.

Kind, Nilssen & Sorgard (2003) analyse the broadcasting market as well and are also concerned with the question if competition between channels leads to over- or underprovision of commercials. Like Anderson & Coate (2003) they do not assume direct competition for advertisers. Kind, Nilssen & Sorgard (2003) also find that there can be underprovision of advertising for low degrees of differentiation between platforms. They show as well that a merger between the two channels can improve welfare as it leads to more advertisements.[48]

In a paper of Gal-Or & Dukes (2003) differentiated TV or radio stations also compete for viewers/listeners. They analyse the conditions under which a merger of two stations can be profitable. In their model consumers are averse to advertising but may profit from advertisements by the fact that they are better informed about prices.[49] If two firms merge this results in a higher level of advertising which can drive producers' prices and profits down. Therefore producers can pay less for advertising. This might render a merger unprofitable.

In contrast to the above cited papers my paper analyses a model with competition for both sides, users and advertisers, and not only users. I look at the consequences on pricing behaviour and profits of platforms. As is shown this behaviour can be very different in a two-sided market compared with a one-sided one. It has also different consequences than competing for only one side.

[48] A paper with a similar basic model is Barros, Kind & Sorgard (2002). They are interested in the consequences of a vertical merger between a platform and a producer. They show that such a merger can be harmful for both firms. This is the case if platforms are close substitutes because the independent platform acts as a free rider on the merger and increases its advertising price.

[49] A problem in their model is that this gain for viewers/listeners is not included in the utility function. The reason is that this would complicate the model dramatically and would change some results.

The remainder of the chapter is organised as follows. The next section sets out the basic model. In Section 3.3 the efficient outcome is presented. Section 3.4 analyses the equilibrium and compares it with efficiency. In Section 3.5 an example of pricing behaviour of internet portals is given. Section 3.6 presents the equilibrium with the possibility of a user charge. A short conclusion is given in Section 3.7.

3.2 The Model

The goal is to develop a model in which platforms compete for users (consumers) and advertisers (producers). It is assumed that if platforms are internet portals, radio stations, or television channels consumers have the hardware to get access to these platforms. Advertising causes a negative externality on users but advertisers' profits are increasing in the number of users. In the following the basic model is presented.

Platforms

There are two platforms $i = 1, 2$. Users cannot be excluded from using the platforms. Therefore platforms cannot make profits from users directly. Instead platforms make profits on advertisers. The profit function of platform i is

$$\Pi_i = p_i n_i.$$

p_i is the price that platform i is demanding from an advertiser for an advert and n_i is the number of advertisers on platform i. Each advertiser can place only one advertisement and has to decide exclusively on which platform she wants to advertise. Thus there is rivalry for advertisers. It is assumed that platform pricing is linear. We also assume that the costs of platforms are zero.[50]

Users

There is a mass of users M. Users are uniformly distributed on a line with length

[50]This assumption is made for simplicity. Relaxing it would change the calculations but not the qualitative results of the model.

one where platform 1 is located at point 0 and platform 2 located at point 1. Each user decides in favour of only one platform.[51] The utility a user derives from spending time t on platform i is $v(t)$ where $v(t)$ is an increasing and strictly concave function. Users' utility is decreasing in the number of advertisements n_i on platform i. The whole amount of disposable leisure time a user has is $\bar{T}$. So $\bar{T} - t$ is the time a user spends on doing other things during his leisure time. We normalize the utility a user gets from doing this other things to 1 per unit of time.

The maximisation problem of a user who is located at x can be written as[52]

$$\max_{i,t} \qquad U_i = \bar{T} - t + v(t) - \gamma t n_i^{\lambda} - \tau_U |x - x_i| \tag{3.1}$$

γ is a measure of the nuisance costs of advertising and is the same for all users. The parameter λ measures the curvature of the utility function in n_i. It is assumed that $\lambda \geq 1$ so utility is weakly concave in n_i. This is a realistic assumption, e.g. one or two commercials on a homepage are not very disturbing but if a web site is full of adverts this disturbs users a lot and the time which is spent on these web site decreases drastically with additional commercials. Lastly, τ_U is the transportation cost parameter and represents the degree of differentiation between both platforms.

If a user has decided in favour of one platform differentiating with respect to t yields

$$v'(t) = 1 + \gamma n_i^{\lambda}. \tag{3.2}$$

$t^*(n_i)$ is implicitly given by (3.2) and represents the demand function for time on platform i dependent on n_i. From the Implicit Function Theorem we get the slope of this demand function

$$\frac{\partial t}{\partial n_i} = \frac{\gamma \lambda n_i^{\lambda - 1}}{v''(t)} < 0. \tag{3.3}$$

[51]This formalisation fits the market for internet portals or TV broadcasting well. Users or viewers decide in favour of only one portal to do e-mailing or can only watch one programme at the same time.

[52]The advantage of this formulation is that the decision of users how much time to spend on a platform is separated from the decision which platform to use. See Anderson, de Palma & Thisse (1992).

So the amount of time on platform i is decreasing in n_i.

The indirect utility function of a user x is given by

$$U(x, n_i) = \bar{T} - t(n_i) + v(t_i(n_i)) - \gamma t(n_i) n_i^{\lambda} - \tau_U |x - x_i|.$$

In the following we set $\bar{T} - t(n_i) + v(t_i(n_i)) - \gamma t(n_i) n_i^{\lambda} = U_B(n_i)$ so $U(x, n_i) = U_B(n_i) - \tau_U |x - x_i|$. The marginal consumer who is indifferent between both platforms is given by

$$x_m = \frac{1}{2} + \frac{1}{2\tau_U}(U_B(n_1) - U_B(n_2)).$$

We assume that τ_U is small enough so that in equilibrium all users use one platform, e.g. $\tau_U \leq 2U_B(N/2)$. Thus a mass of $X_i = Mx_m$ chooses platform 1 and the remaining mass $M(1 - x_m)$ chooses platform 2.

With advertising a producer informs users about the nature and the price of its products. After having seen an advert a consumer knows his willingness to pay for the good. It is assumed that this valuation is the same for all consumers and is K with probability β and 0 with probability $1 - \beta$. For simplicity it is assumed that it is the same for each advertiser's good.[53] This modeling follows Anderson & Coate (2003). Although this formulation is specific it has the advantage that advertising cannot have a positive value for users because each producer will sell its product at a price K. A lower price does not improve the possibility of a sale. Thus the advertiser's price is equivalent to consumers' valuation and therefore a user's utility of getting aware of a new good is zero. The implication of this formulation is that users do not get informational benefits from using a platform with much advertising.

Advertisers

There is a mass of advertisers N. Ex ante advertisers are indifferent between both platforms. Advertisers choose only one platform to advertise on. This assumption represents an easy way to model that platforms have to compete for advertisers.[54] If

[53]This stochastic structure is chosen to make the model more realistic and to express that not every user has a positive valuation for each new good he gets aware of through advertising.

[54]The results of the model do not depend on the assumption that advertisers single-home (choose

platform i is chosen by an advertiser her profit is

$$P_i = X_i \beta K t(n_i) - p_i.$$

If she decides not to advertise she gets a profit of zero. The value of an advertisement on platform i does positively depend on the time users spend on that platform. The idea is that the more time a user spend on platform i the higher is the possibility that he gets aware of that advertisement and buys the product in the end. The gross value of an advertisement on platform i is thus $X_i \beta K t(n_i)$. The advertiser has to pay p_i for an advertisement on i. For simplicity it is assumed that production costs for advertisements and products are zero. Again this assumption does not change the qualitative results.

Game Structure

In the first stage the two platforms decide simultaneously about their prices p_1 and p_2. In the second stage advertisers decide on which platform they want to advertise if on any and users decide how much time they spend on each platform. Then profits and utilities are realised. This completes the description of the model.

In the analysis to follow we maintain the following assumption:

$$A1: \quad \beta K > \frac{U_B''(n_i)}{-\left(2\frac{\partial t}{\partial n_i} + \frac{\partial^2 t}{\partial n_i^2} n_i\right)} \quad \forall n_i.$$

The role of this assumption is to guarantee that the gain from trading advertisers' goods is high enough so that $n_i = 0$ is never efficient, e.g. no advertising is never efficient.

only one platform). What is necessary is that with a price change of platform i the number of advertisers on platform j changes. So if platform i lowers p_i, n_i increases and n_j decreases. One can get the same results with the assumption that advertising firms multi-home (advertise on both platforms) but have only a certain budget for advertising expenditures. So the last unit of this budget can either be spent on one or the other platform. Thus advertisers multi-home but put more commercials on the platform with the lower price.

3.3 Efficiency

In this section the optimal number of advertisements on each platform is derived. This result is later compared with the equilibrium outcome of the pricing game.

In the analysis of efficiency there are two effects to consider. Firstly, a higher number of advertisements increases the possibility of trade of advertisers' products. Secondly, a higher number of advertisements decreases users' utility and exerts a negative externality on other advertisers. Total welfare is given by

$$\begin{aligned} WF = \;& M\beta K n_1[\tfrac{1}{2} + \tfrac{1}{2\tau_U}(U_B(n_1) - U_B(n_2))]t(n_1) \\ & + M\beta K n_2[\tfrac{1}{2} + \tfrac{1}{2\tau_U}(U_B(n_2) - U_B(n_1))]t(n_2) \\ & + MU_B(n_1)[\tfrac{1}{2} + \tfrac{1}{2\tau_U}(U_B(n_1) - U_B(n_2))] \\ & + MU_B(n_2)[\tfrac{1}{2} + \tfrac{1}{2\tau_U}(U_B(n_2) - U_B(n_1))] \\ & - \tau_U \int_0^{\frac{1}{2}+\frac{1}{2\tau_U}(U_B(n_1)-U_B(n_2))} x dx \\ & - \tau_U \int_{\frac{1}{2}+\frac{1}{2\tau_U}(U_B(n_1)-U_B(n_2))}^{1} (1-x) dx. \end{aligned} \tag{3.4}$$

The first two terms are the welfare from trading advertisers' products. The third and the fourth term represent the utility of users gross of transportation costs and the fifth and the sixth term are the transportation costs. Differentiating (3.4) with respect to n_i, $i = 1, 2$ yields the first order conditions

$$\begin{gathered} \tfrac{1}{2\tau_U} U_B'(n_i) M[\beta K n_i t(n_i) + U_B(n_i)] \\ + \tfrac{1}{2} M[\beta K t(n_i) + \beta K n_i t'(n_i) + U_B'(n_i)] \\ - \tfrac{1}{2\tau_U} U_B'(n_i) M[\beta K n_j t(n_j) + U_B(n_j)] = 0, \\ i, j = 1, 2 \qquad \text{and} \qquad i \neq j. \end{gathered} \tag{3.5}$$

So the first order condition is the same for both n_1 and n_2. Thus it is efficient if $n_1 = n_2$. The second order condition is globally satisfied because of $A1$. Simplifying (3.5) yields the following proposition.

Proposition 3.1

If

$$\beta K t(N/2) + \beta K (N/2) t'(N/2) + U_B'(N/2) > 0, \tag{3.6}$$

$n_i^{eff} = \frac{N}{2}$ is efficient.

Otherwise the efficient number of advertisers $n_i^{eff}, i = 1, 2$ is implicitly given by

$$\beta K t(n_i) + \beta K n_i t'(n_i) + U_B'(n_i) = 0. \tag{3.7}$$

It is therefore efficient if advertisers allocate equally among platforms. The intuition behind this is simple. If we look only at the gains from trade the externality that an advertiser causes on another one is increasing convexly. So if one platform has many advertisers users spend little time on this platform and thus many advertisers gain little attention. To reduce this externality as well as possible it is optimal that each platform has the same number of advertisers. Transportation costs can be reduced with an even partition as well. If βK is high which means that the probability and the welfare gains from trade are high all producers should advertise and $n_i = N/2$. If these gains are lower compared to the utility loss of users, $n_1 + n_2 < N$.

3.4 Nash Equilibrium

In this section we solve for the Nash equilibrium of the pricing game.

Since platforms can only quote prices to advertisers and are not differentiated from their point of view we are in a standard Bertrand game. The difference is that with a negative externality one platform cannot win all advertisers by undercutting its competitor's price. The platform with the lower price gets more advertisers but this results in a higher externality on all of them and reduces their profits. It is thus optimal for some advertisers to stay on the other platform. Thus platforms earn positive profits in equilibrium.[55]

It turns out that the model is solvable in a similar way as the product differentiation model of Hotelling.

[55] It should be mentioned that this result is completely different in a model with positive externalities. If in such models buyers (in our model advertisers) can coordinate on the platform that gives them the highest surplus prices would be driven down to zero because of the standard Bertrand argument. For an overview of this literature see Farrell & Klemperer (2001) or Katz & Shapiro (1994).

To see this let us assume first that all N producers advertise. Since all advertisers are the same in equilibrium each advertiser must be indifferent between platform 1 and 2. If this would not be the case one platform can increase its price without losing any advertisers which cannot be an equilibrium. Thus we can determine the marginal advertiser who is indifferent between both platforms. She is described by

$$\begin{aligned} M\beta Kt(n_1)[\tfrac{1}{2} + \tfrac{1}{2\tau_U}(U_B(n_1) - U_B(N-n_1))] - p_1 &= \\ M\beta Kt(N-n_1)[\tfrac{1}{2} + \tfrac{1}{2\tau_U}(U_B(N-n_1) - U_B(n_1))] - p_2 & \end{aligned} \tag{3.8}$$

The left hand side is the profit of an advertiser on platform 1 and the right hand side the profit of an advertiser on platform 2 if the number of advertisers are n_1 and $n_2 = N - n_1$.

Contrary to standard analysis it is not possible to solve (3.8) for n_1 because users' utility is concave in n_1. To get a solution (3.8) is solved for p_1 which yields a maximisation problem of platform 1 of

$$\begin{aligned} \max_{n_1} \qquad \Pi_i = \quad & \{p_2 + M\beta Kt(n_1)[\tfrac{1}{2} + \tfrac{1}{2\tau_U}(U_B(n_1) - U_B(N-n_1))] - \\ & M\beta Kt(N-n_1)[\tfrac{1}{2} + \tfrac{1}{2\tau_U}(U_B(N-n_1) - U_B(n_1))]\}n_1. \end{aligned} \tag{3.9}$$

Maximising profits for both firms yields two first order conditions. These first order conditions in combination with equation (3.8) and equation (3.8) with 1 and 2 reversed yields the equilibrium values of n_i and p_i. After applying the Envelope Theorem, $U_B'(n_i) = -\gamma\lambda n_i^{\lambda-1}t(n_i)$, we get

$$n_i^* = \frac{N}{2}$$

(which is obvious because of symmetry) and

$$p_i^* = \beta KMN\gamma\lambda(N/2)^{\lambda-1}[\frac{t(N/2)^2}{t_U} - \frac{1}{2v''(t(N/2))}].$$

It remains to calculate the equilibrium if $n_1 + n_2 < N$.[56]

The equilibrium of the game is described in the following proposition.

[56]The method of solution is similar to a standard product differentiation game where consumers' gross surplus from buying is so low that firms are local monopolists. See e.g. Gabszewicz & Thisse (1986).

Proposition 3.2

If $\tau_U \leq \tau_U^1 = \frac{(N/2)^\lambda \gamma\lambda t(N/2)^2}{t(N/2)+\frac{\gamma\lambda(N/2)^\lambda}{v''(t(N/2))}}$ in the unique Nash equilibrium n_i^* is implicitly given by

$$t(n_i) + \frac{\partial t(n_i)}{\partial n_i} n_i - \frac{t(n_i)^2 \gamma\lambda n_i^\lambda}{\tau_U} = 0, \tag{3.10}$$

where a unique solution $n_i^* \in (0, N/2)$ exists, and

$$p_i^* = \frac{M\beta K t(n_i^*)}{2}. \tag{3.11}$$

Profits of the platforms are

$$\Pi_i^* = \frac{M\beta K t(n_i^*)}{2} n_i^*. \tag{3.12}$$

If $\tau_U^1 < \tau_U \leq \tau_U^2 = \frac{4(N/2)^\lambda \gamma\lambda t(N/2)^2}{t(N/2)+\frac{2\gamma\lambda(N/2)^\lambda}{v''(t(N/2))}}$ in the unique Nash equilibrium $n_i^* = \frac{N}{2}$ and

$$p_i^* = \frac{M\beta K t(N/2)}{2}. \tag{3.13}$$

Profits of the platforms are

$$\Pi_i^* = \frac{M\beta K t(N/2)}{2} N/2. \tag{3.14}$$

If $\tau_U > \tau_U^2$ in the unique Nash equilibrium

$$n_i^* = \frac{N}{2} \tag{3.15}$$

and

$$p_i^* = \beta K M N \gamma\lambda (N/2)^{\lambda-1} [\frac{t(N/2)^2}{\tau_U} - \frac{1}{2v''(t(N/2))}] \tag{3.16}$$

Profits of the platforms are

$$\Pi_i^* = \beta K M N \gamma\lambda (N/2)^{\lambda} [\frac{t(N/2)^2}{\tau_U} - \frac{1}{2v''(t(N/2))}]. \tag{3.17}$$

Proof

When calculating the marginal advertiser in equation (3.8) it was assumed that all producers advertise. But this is only the case if it pays the 'Nth' producer to advertise instead of not advertising and getting profits of zero.

Thus with a price $p_i^* = \beta KMN\gamma\lambda(N/2)^{\lambda-1}[\frac{t(N/2)^2}{\tau_U} - \frac{1}{2v''(t(N/2))}]$ this is only the case if

$$\frac{M}{2}\beta Kt(N/2) > \beta KMN\gamma\lambda(N/2)^{\lambda-1}[\frac{t(N/2)^2}{\tau_U} - \frac{1}{2v''(t(N/2))}]$$

or

$$\tau_U > \tau_U^2 = 4\frac{(N/2)^\lambda\gamma\lambda t(N/2)^2}{t(N/2) + \frac{2\gamma\lambda(N/2)^\lambda}{v''(t(N/2))}}.$$

The next question is what the optimal price of a platform is if it does not have to compete for advertisers because $n_1 + n_2 < N$. In this case the number of advertisers on platform i depends on p_i and is given by $M\beta Kt(n_i)[\frac{1}{2} + \frac{1}{2\tau_U}(U_B(n_i) - U_B(n_j))] - p_i = 0$. So advertiser n_i is the last one whose profit is not negative given a price of p_i. Thus the profit of platform i is

$$\Pi = M\beta Kt(n_i)n_i[\frac{1}{2} + \frac{1}{2\tau_U}(U_B(n_i) - U_B(n_j))]. \tag{3.18}$$

Maximising this with respect to n_i for both platforms yields that n_i^* is given by

$$t(n_i) + \frac{\partial t(n_i)}{\partial n_i}n_i - \frac{t(n_i)^2\gamma\lambda n_i^\lambda}{\tau_U} = 0.$$

which is equation (3.10).

If $n_i = 0$ the left hand side of (3.10) is positive because $t(0) > 0$. If $n_i = N/2$ the left hand side is negative because profit function (3.18) is only relevant if $\tau_U < \tau_U^1$. Thus a solution with $n_i^* \in (0, N/2)$ exists. Since all terms of (3.10) are continuous functions of n_i this solution is unique.

This n_i^* equals $\frac{N}{2}$ if

$$t(N/2) + \frac{\partial t(N/2)}{\partial n_i}N/2 - \frac{t(N/2)^2\gamma\lambda(N/2)^\lambda}{\tau_U} = 0$$

or

$$\tau_U = \frac{(N/2)^\lambda\gamma\lambda t(N/2)^2}{t(N/2) + \frac{\gamma\lambda(N/2)^\lambda}{v''(t(N/2))}} = \tau_U^1.$$

So for $\tau_U \leq \tau_U^1 = \frac{(N/2)^\lambda\gamma\lambda t(N/2)^2}{t(N/2)+\frac{\gamma\lambda(N/2)^\lambda}{v''(t(N/2))}}$ n_i^* is given by $t(n_i) + \frac{\partial t(n_i)}{\partial n_i}n_i - \frac{t(n_i)^2\gamma\lambda n_i^\lambda}{\tau_U} = 0$ and $p_i^* = \frac{M\beta Kt(n_i^*)}{2}$.

It remains to calculate what happens if $\tau_U^1 < \tau_U \leq \tau_U^2 = \frac{4(N/2)^\lambda \gamma\lambda t(N/2)^2}{t(N/2)+\frac{2\gamma\lambda(N/2)^\lambda}{v''(t(N/2))}}$. In this case $n_1^* = n_2^* = N/2$ and both platforms set their prices such that the advertisers have zero utility, e.g.

$$p_i^* = \frac{M\beta Kt(N/2)}{2}$$

which is equation (3.13).

q.e.d.

The profit function is continuous but it has two kinks. In the following we provide some comparative static analyses. First let us look at a change in the transportation cost parameter τ_U.

Proposition 3.3

Platform profits are increasing in τ_U as long as $\tau_U \leq \tau_U^1$.
Profits are independent of τ_U if $\tau_U^1 < \tau_U \leq \tau_U^2$ and profits are decreasing in τ_U if $\tau_U > \tau_U^2$.

Proof

If $\tau_U \leq \tau_U^1$ profit is given by (3.12) and the optimal number of advertisers is given by (3.10). As was shown in the proof of Proposition 2, (3.10) is the first order condition for the maximisation problem of platform i with respect to n_i. Applying the Implicit Function Theorem to (3.10) yields that sign$(\frac{\partial n_i}{\partial \tau_U})$ = sign$(\frac{\partial (3.10)}{\partial \tau_U})$ = $-\frac{1}{2(\tau_U)^2}\beta KMn_i t(ni)U_B'(n_i)$, which is greater than zero. Differentiating equation (3.12) with respect to τ_U gives $\frac{\partial \Pi_i^*}{\partial \tau_U} = \frac{M\beta K}{2}(t(n_i) + \frac{\partial t(n_i)}{\partial n_i}n_i)\frac{\partial n_i}{\partial \tau_U}$. So sign$(\frac{\partial \Pi_i}{\partial \tau_U})$ = sign$(t(n_i) + \frac{\partial t(n_i)}{\partial n_i}n_i)$. By equation (3.10), $t(n_i) + \frac{\partial t(n_i)}{\partial n_i}n_i - \frac{t(n_i)^2\gamma\lambda n_i^\lambda}{\tau_U} = 0$. Since the last term of the left hand side is negative $t(n_i) + \frac{\partial t(n_i)}{\partial n_i}n_i > 0$ which yields $\frac{\partial \Pi_i}{\partial \tau_U} > 0$.

If $\tau_U^1 < \tau_U \leq \tau_U^2$ profit is given by (3.14). Here $n_i^* = \frac{N}{2}$ and therefore (3.14) is independent of τ_U.

If $\tau_U > \tau_U^2$ profit is given by (3.17). In this case $\frac{\partial \Pi_i}{\partial t_U} = -\beta KMN\gamma\lambda(N/2)^\lambda[\frac{t(N/2)^2}{(\tau_U)^2}] < 0$.

q.e.d.

The intuition behind this result is the following. τ_U represents the level of differentiation between the two platforms from the perspective of the users. If τ_U is small platforms have to compete fiercely for users. They do this by reducing their amount of advertising. Thus prices are high and only few producers advertise. If τ_U increases prices decrease. But profit is rising because more advertiser choose to advertise on the platforms.[57] In this region platforms do not compete for advertisers since $n_1 + n_2 < N$. But when τ_U reaches τ_U^1 all producers are advertising and competition for advertisers starts. In the region between τ_U^1 and τ_U^2 profits stay the same since it does not pay for one platform to lower prices. But if τ_U rises further competition for advertisers lowers prices. The reason is that it pays platforms to attract more advertisers because fewer consumers will switch to the other platform. This strategic effect drives prices down. But also profits are lower because both firms lower their prices and n_i^* stays the same.

This shows that in a two-sided market with negative externalities a lower degree of competition on one side can increase the competition on the other side and lead to lower profits. This is never possible in a standard market with only one side.

It is also possible to derive a comparative static result with respect to γ, the nuisance cost of advertising.

Proposition 3.4

If $\tau_U \leq \tau_U^2$ platform profits are decreasing in γ but if $\tau_U > \tau_U^2$ the effect of a change in γ on profits is ambiguous.

Proof

First look at the case $\tau_U \leq \tau_U^1$. Equation (3.10) is the first order condition of the maximisation problem of platform i. By applying the Implicit Function Theorem we

[57]This result is also obtained by Barros, Kind & Sorgard (2003) in a different model.

have

$$\begin{aligned}
&\text{sign}(\tfrac{\partial n_i}{\partial \gamma}) \\
&= \text{sign}(\tfrac{\partial t(n_i)}{\partial \gamma} + n_i \tfrac{\partial^2 t(n_i)}{\partial n_i \partial \gamma} + n_i U_B'(n_i) \tfrac{1}{\tau_U} \tfrac{\partial t(n_i)}{\partial \gamma}) \\
&= \text{sign}(\tfrac{n_i^\lambda (1+\lambda)\tau_U - n_i^{2\lambda} t(n_i)\gamma\lambda}{v''(t(n_i))}) \\
&= \text{sign}(-(1+\lambda)\tau_U + n_i^\lambda \gamma \lambda t(n_i)).
\end{aligned}$$

Multiplying (3.10) with $\frac{\tau_U}{t(n_i)}$ yields

$$\tau_U + \frac{\partial t(n_i)}{\partial n_i} n_i \frac{\tau_U}{t(n_i)} - t(n_i)\gamma\lambda n_i^\lambda = 0.$$

Thus $\tau_U > t(n_i)\gamma\lambda n_i^\lambda$ and therefore $\tau_U(1+\lambda) > t(n_i)\gamma\lambda n_i^\lambda$. This shows that $\frac{\partial n_i}{\partial \gamma} < 0$. If $\tau_U \leq \tau_U^1$ profit is given by (3.12). Differentiating (3.12) with respect to γ yields $\frac{\partial \Pi_i}{\partial \gamma} = \frac{M\beta K}{2}[t(n_i)\frac{\partial n_i}{\partial \gamma} + \frac{\partial t(n_i)}{\partial \gamma} n_i]$.

Differentiating $\frac{\partial t(n_i)}{\partial \gamma}$ yields $\frac{n_i^\lambda}{v''(t(n_i))} < 0$ and thus $\frac{\partial \Pi_i}{\partial \gamma} < 0$.

If $\tau_U^1 < \tau_U \leq \tau_U^2$ profit is given by (3.14). Differentiating yields $\frac{\partial \Pi_i}{\partial \gamma} = \frac{M\beta KN}{4}\frac{\partial t(n_i)}{\partial \gamma} < 0$.

If $\tau_U > \tau_U^2$ profit is given by (3.17). Differentiating profit with respect to γ yields $\text{sign}(\frac{\partial \Pi_i}{\partial \gamma}) = \text{sign}(2t(n_i)^2(v''(t(n_i)))^2 - t(n_i)v''(t(n_i)) + 4t(n_i)\gamma v''(t(n_i))^2 \frac{\partial t(n_i)}{\partial \gamma} + \gamma\tau_U \frac{\partial t(n_i)}{\partial \gamma} v'''(t(n_i)))$.

The first two terms are positive the third term is negative and the fourth term is unclear. So profit may increase or decrease in γ.

q.e.d.

γ represents the nuisance costs of advertising. So one would guess that profit should decrease in γ because consumers spend less time on the platforms. Proposition 3.4 states that this is only true if platforms do not compete for advertisers, i.e. if τ_U is low. In this case each user spends less time on platforms which results in a lower possibility of trade of advertisers' goods and thus in lower prices. But if τ_U is high and platforms compete for advertisers, profit might increase in γ. The intuition is that with a high τ_U platforms have an incentive to lower their prices to attract new advertisers. This reduces profits. With a higher nuisance cost this effect is dampened because each platform makes lower profits on new advertisers and thus prices might be higher compared with a lower γ. Thus a higher γ causes two effects on prices. The first is that users are more disturbed by commercials which reduces prices. The second is that competition is reduces which increases prices. The consequences on profits are therefore not clear cut.

Now let us turn to the comparison of the Nash equilibrium with the efficient outcome.

Proposition 3.5

If $\beta Kt(N/2) + \beta KN/2t'(N/2) + U'_B(N/2) > 0$ advertising is efficient if $\tau_U \geq \tau_U^1$ and there is too little advertising if $\tau_U < \tau_U^1$.

If there exists n_i s.t.$\beta Kt(n_i) + \beta Kn_i t'(n_i) + U'_B(n_i) = 0$, there can be too much or too little advertising in equilibrium.
There is too little advertising if $\tau_U < \min[\tau_U^1, \beta Kn_i^{eff}t(n_i^{eff})]$ and too much if $\tau_U > \min[\tau_U^1, \beta Kn_i^{eff}t(n_i^{eff})]$.
Only if $\tau_U = \beta Kn_i^{eff}t(n_i^{eff}) \leq \tau_U^1$ the equilibrium is efficient.

Proof

From Proposition 3.1 the optimal number of advertisers on each platform is given by (3.6) or (3.7). First look at the case where there exists an $n_i < N/2$ such that (3.7) holds.

In the Nash equilibrium of the game $n_i^{eq} = N/2$ if $\tau_U > \tau_U^1$. Thus it follows that $n_i^{eq} > n_i^{eff}$ if $\tau_U > \tau_U^1$.

If $\tau_U < \tau_U^1$, n_i^{eq} is given by the first order condition (3.10). If we insert n_i^{eff} in this first order condition we get from (3.7)

$$\frac{\gamma\lambda t(n_i^{eff})(n_i^{eff})^{\lambda-1}}{\beta K} - \frac{\gamma\lambda t(n_i^{eff})^2 n_i^{eff}}{\tau_U} \quad \begin{pmatrix} > \\ = \\ < \end{pmatrix} \quad 0$$

or

$$\tau_U \quad \begin{pmatrix} > \\ = \\ < \end{pmatrix} \quad \beta Kn_i^{eff}t(n_i^{eff}).$$

So if $\tau_U > \beta K n_i^{eff} t(n_i^{eff})$ the left hand side of equation (3.10) is zero at n_i^{eq} but it is greater zero at n_i^{eff}. Thus $n_i^{eq} > n_i^{eff}$.

If $\tau_U < \beta K n_i^{eff} t(n_i^{eff})$ the left hand side of equation (3.10) is zero at n_i^{eq} but it is lower than zero at n_i^{eff}. Thus $n_i^{eq} < n_i^{eff}$.

It therefore follows that $n_i^{eq} > n_i^{eff}$ if $\tau_U > \min[\tau_U^1, \beta K n_i^{eff} t(n_i^{eff})]$ and $n_i^{eq} < n_i^{eff}$ if $\tau_U < \min[\tau_U^1, \beta K n_i^{eff} t(n_i^{eff})]$. Only in the case when $\tau_U = \beta K n_i^{eff} t(n_i^{eff}) \leq \tau_U^1$ the equilibrium is efficient.

Now look at the case where $n_i^{eff} = N/2$. We know that in equilibrium $n_i^{eq} = N/2$ if $\tau_U \geq \tau_U^1$ and $n_i^{eq} < N/2$ if $\tau_U < \tau_U^1$.

The proposition follows.

q.e.d.

This shows that it depends on the level of τ_U whether the equilibrium is efficient or not. If τ_U is low competition for users is fierce and therefore advertising levels are low. Platforms do not take into account users' utility loss from an additional commercial but only the indirect externality on all advertisers. The reason is that only this indirect externality can be reflected in their pricing behaviour. If competition for users is harsh many users switch to the competitor if one platform has an additional advertiser. Thus the advertising level is lower than the efficient level. From (3.6) and (3.7) we know that τ_U does not play a role in determining the efficient advertising level. But it is the important variable for platform competition. In the case when not all producers should advertise there can be too much advertising if τ_U is high because competition for users is low.

If all producers should advertise there can only be too little advertising. This is the case if competition is fierce with the same line of reasoning as before.

3.5 Pricing Behaviour of Internet Portals

In this section we discuss the pricing behaviour of two internet portals, namely AOL and GMX. We argue that the structure of their commercial prices fits the results of the preceding section quite well.

Both AOL and GMX are portals where members have access to free e-mail, get informed about cheap offers of products and can inform themselves about specific topics in so called affinity groups. It is costless to become a member of theses portals. The portals get revenues from members only if the members buy some services from the portals, like sending SMS or printing pictures. Usually these services are sold at low prices.

The most important source for profits of the portals is advertising. There are different forms of advertising on both portals but the most common ones are banners on their web site. AOL sells a full-size banner on its homepage for 15 Euros per thousand eye-balls, a half-size banner is sold for 10 Euros. A full-size banner on the logout-page of AOL costs only 7 Euros, a half-size banner 5 Euros.[58] A similar pricing structure can be observed at GMX.[59] At GMX a logout banner costs 15 Euros while a comparable banner on the homepage costs 24 Euros.[60]

Where does this difference come from? Since these prices are per thousand eye-balls one cannot argue that homepage prices are higher because more people are watching the homepage. Instead a reason can be found from the arguments of the preceding section. To attract advertisers portals have to attract users at first. But before a user decides which portal to use he will compare the homepages of the portals. If one site is plain while the other one is full of commercials while both portals can be used for free he will most likely decide in favour of the plain one. Thus competition for users occurs mainly on the homepages. This can explain the high prices for the homepage banners. Thus homepages of portals do usually have few advertisements on it.

By contrast, only if a user has already decided to use a portal he will see the logout page. So there is no more competition for users and prices for logout-banners are cheap. For example, on the portal GMX usually four advert banners are on the logout page but at most one the start page.

[58]See http://www.aol.de/mediaspace/preise/preistabelle/contentview.jsp

[59]See http://media.gmx.net/de/cgi/preise?LANG=de&AREA=homepage.

[60]Prices are higher at GMX than at AOL because banners are bigger and the form of advertising is fancier.

This provides some evidence that the degree of competition for users has a high influence on commercial prices.

3.6 User Charge

In some markets it is not only possible for platforms to make money on advertisers but to charge users for the consumption of platforms' services as well. Examples are pay-TV channels and newspapers. For example in Europe direct broadcast satellite channels like Canal Plus or Premiere are partially financed by user charges. This is also of policy interest since in the TV case it is becoming technically easier to exclude viewers.

In our model the possibility of a user charge can be incorporated in an easy way. In the following we assume that each platform i can charge users a fee c_i for its services. Then platform profit is given by

$$\Pi_i = p_i n_i + X_i c_i.$$

The indirect utility of a user who is located at x and uses platform i is given by

$$U(x, n_i) = T - t(n_i) + v(t_i) - \gamma t(n_i) n_i^\lambda - c_i - \tau_U |x - x_i|.$$

Again, as in Section 3.2 we set $T - t(n_i) + v(t_i) - \gamma t(n_i) n_i^\lambda = U_B(n_i)$ so $U(x, n_i) = U_B(n_i) - c_i - \tau_U |x - x_i|$. The assumption that all users choose one platform is maintained so $\tau_U \leq 2(U_B(N/2) - c_i^*)$.

The marginal user is then given by

$$x = \frac{1}{2} + \frac{1}{2\tau_U}(U_B(n_1) - U_B(n_2) + c_2 - c_1).$$

Conducting the same analysis as before gives a maximisation problem of platform 1 of

$$\begin{aligned} \max_{n_1, c_1} \qquad \Pi_1 = \ & \{p_2 + \beta K M t(n_1)[\tfrac{1}{2} + \tfrac{1}{2\tau_U}(U_B(n_1) - U_B(N - n_1)) + c_2 - c_1] - \\ & M\beta K t(N - n_1)[\tfrac{1}{2} + \tfrac{1}{2\tau_U}(U_B(N - n_1) - U_B(n_1))]\} n_1 \\ & + c_1 M(\tfrac{1}{2} + \tfrac{1}{2\tau_U}(U_B(n_1) - U_B(n_2) + c_2 - c_1)) \end{aligned}$$

if all producers advertise. Formulating the first order condition and solving for p_i and c_i yields

$$p_i^* = M\gamma\lambda(N/2)^{\lambda-1}[t(N/2) - \frac{N\beta K}{2v''(t(N/2))}]$$

and

$$c_i^* = \tau_U - \beta K N t(N/2).^{61}$$

The profit of the platform is given by

$$\Pi_i^* = M\gamma\lambda(N/2)^{\lambda}[t(N/2) - \frac{N\beta K}{2v''(t(N/2))}] + \frac{M}{2}[\tau_U - t(N/2)N\beta K].$$

Comparing this profit with the profit without a user charge we get

$$\Pi_{\text{with charge}} = \Pi_{\text{without charge}} + [1 - t(N/2)\frac{N\beta K}{\tau_U}][M\lambda\gamma t(N/2)(N/2)^{\lambda} + \tau_U\frac{M}{2}].$$

Thus the profit with user charge is higher if $1 - t(N/2)\frac{N\beta K}{\tau_U} > 0$. But this is exactly the formula for the user charge to be positive.

The profits in the case that not all producers advertise are computed in the same way as in Section 3.3. This leads to the following equilibrium.

Proposition 3.6

If $\beta K \leq \frac{(N/2)^{\lambda-1}\gamma\lambda t(N/2)}{t(N/2)+\frac{\gamma\lambda(N/2)^{\lambda}}{v''(t(N/2))}}$ then n_i^* is implicitly given by

$$t(n_i) + ni\frac{\partial t}{\partial n_i} - t(n_i)\frac{n_i^{\lambda-1}\gamma\lambda}{\beta K} = 0, \tag{3.19}$$

where a unique solution $n_i^* \in (0, N/2)$ exists, and

$$p_i^* = \frac{M}{2}\beta K t(n_i^*) \tag{3.20}$$

and

$$c_i^* = \tau_U - \beta K n_i^* t(n_i^*). \tag{3.21}$$

Profits of the platforms are

$$\Pi_i^* = \frac{1}{2}M\tau_U \tag{3.22}$$

[61] For the moment we assume that c_i can be positive or negative.

If $\frac{(N/2)^{\lambda-1}\gamma\lambda t(N/2)}{t(N/2)+\frac{\gamma\lambda(N/2)^{\lambda}}{v''(t(N/2))}} < \beta K \leq \frac{2(N/2)^{\lambda-1}\gamma\lambda t(N/2)}{t(N/2)+\frac{\gamma\lambda(N/2)^{\lambda}}{2v''(t(N/2))}}$

then $n_i^* = \frac{N}{2}$,

$$p_i^* = \frac{M}{2}\beta K t(N/2) \tag{3.23}$$

and

$$c_i^* = \tau_U - \beta K N t(N/2). \tag{3.24}$$

Profits of the platforms are

$$\Pi_i^* = \frac{1}{2}M\tau_U. \tag{3.25}$$

If $\beta K > \frac{2(N/2)^{\lambda-1}\gamma\lambda t(N/2)}{t(N/2)+\frac{\gamma\lambda(N/2)^{\lambda}}{2v''(t(N/2))}}$

then $n_i^* = \frac{N}{2}$,

$$p_i^* = M\gamma\lambda(N/2)^{\lambda-1}[t(N/2) - \frac{N\beta K}{2v''(t(N/2))}] \tag{3.26}$$

and

$$c_i^* = \tau_U - \beta K N t(N/2). \tag{3.27}$$

Profits of the platforms are

$$\Pi_i^* = M\gamma\lambda(N/2)^{\lambda}[t(N/2) - \frac{N\beta K}{2v''(t(N/2))}] + \frac{M}{2}[\tau_U - t(N/2)N\beta K]. \tag{3.28}$$

Proof

If platforms set prices $p_i^* = M\gamma\lambda(N/2)^{\lambda-1}[t(N/2) - t(N/2)\frac{N\beta K}{2v''(t(N/2))}]$ the condition under which N producers advertise is given by

$$\frac{M}{2}\beta K t(N/2) - p_i^* = \frac{M}{2}\beta K t(N/2) - M\gamma\lambda(N/2)^{\lambda-1}[t(N/2) - \frac{N\beta K}{2v''(t(N/2))}] > 0$$

or

$$\beta K > \frac{2(N/2)^{\lambda-1}\gamma\lambda t(N/2)}{t(N/2) + \frac{\gamma\lambda(N/2)^{\lambda}}{2v''(t(N/2))}}.$$

In this case $p_i^* =$ and c_i^* are given by (3.26) and (3.27).

If not all N producers advertise there is no competition for advertisers. Thus each platform set the price $p_i = \frac{M}{2}\beta K t(n_i)$. The maximisation problem of platform i is thus

$$\max_{n_i, c_i} \qquad \Pi_i = n_i \frac{M}{2} \beta K t(n_i)$$

which yields that n_i^* is implicitly given by (3.19) and $c_i^* = \tau_U - \beta K N t(n_i^*)$.
For the same reason is in the proof of Proposition 3.2 a unique solution $n_i^* \in (0, N/2)$ exists.

Inserting $n_i = N/2$ in (3.19) gives $t(N/2) + N/2\frac{\partial t}{\partial n_i} - t(N/2)\frac{(N/2)^{\lambda-1}\gamma\lambda}{\beta K} = 0$ or $\beta K = \frac{(N/2)^{\lambda-1}\gamma\lambda t(N/2)}{t(N/2)+\frac{\gamma\lambda(N/2)^{\lambda}}{v''(t(N/2))}}$.

Thus if $\beta K \leq \frac{(N/2)^{\lambda-1}\gamma\lambda t(N/2)}{t(N/2)+\frac{\gamma\lambda(N/2)^{\lambda}}{v''(t(N/2))}}$ then n_i^* is given by (3.19). If $\beta K > \frac{(N/2)^{\lambda-1}\gamma\lambda t(N/2)}{t(N/2)+\frac{\gamma\lambda(N/2)^{\lambda}}{v''(t(N/2))}}$ then $n_i^* = N/2$ and $p_i^* =$ and c_i^* are given by (3.23) and (3.24).

q.e.d.

The profit can now be compared with the profit if a user charge is not possible.

Proposition 3.7

Suppose that platforms can set an unrestricted user charge. If this user charge is positive in equilibrium profits are higher than without the user charge.

Proof

To prove the proposition we compare the highest profit without a user charge with the lowest profit with user charge.

Because of Proposition 3.2 the highest profit without a user charge is given by $\Pi_{\text{without charge}} = \frac{M\beta K t(N/2)}{2} N/2$.

The lowest profit with user charge is $\Pi_{\text{with charge}} = \frac{1}{2}M\tau_U$. This is the case because $M\gamma\lambda(N/2)^{\lambda}[t(N/2) - \frac{N\beta K}{2v''(t(N/2))}] + \frac{M}{2}[\tau_U - t(N/2)N\beta K]$, which is the profit with user charge if $\beta K > \frac{2(N/2)^{\lambda-1}\gamma\lambda t(N/2)}{t(N/2)+\frac{\gamma\lambda(N/2)^{\lambda}}{2v''(t(N/2))}}$, is higher than $\frac{1}{2}M\tau_U$ if $\beta K > \frac{2(N/2)^{\lambda-1}\gamma\lambda t(N/2)}{t(N/2)+\frac{\gamma\lambda(N/2)^{\lambda}}{2v''(t(N/2))}}$.

Now comparing $\Pi_{\text{with charge}} = \frac{1}{2}M\tau_U$ with $\Pi_{\text{without charge}} = \frac{M\beta Kt(N/2)}{2}N/2$ yields that $\Pi_{\text{with charge}} > \Pi_{\text{without charge}}$ if $\tau_U > N/2\beta Kt(N/2)$. But this exactly the condition for the user fee to be positive.

q.e.d.

We have shown that if the user charge is positive profits always increase. But if it is negative profits might be lower than without this possibility. The intuition behind this result is the following. If platforms have the possibility to set a user charge there are two different ways to do that. The first is to set a higher commercial price to get rid of some advertisers in order to make profits on users with a positive user charge. This is the case if $\tau_U > \beta KNt(N/2)$. Both platforms set a higher p_i^* so none of them loses many advertisers. But they set $c_i^* > 0$ as well which results in higher profits. The second possibility is to subsidise users with a negative fee in order to attract more advertisers.[62] But since both platforms do so in equilibrium they reduce their advertiser price as well and profits are lower than without a user charge. Thus a prisoner's dilemma situation arises.[63] Profits would be higher if the additional instrument of the user charge were not available.

Differentiating with respect to τ_U yields that $\frac{\partial \Pi_i^*}{\partial \tau_U} = \frac{M}{2} > 0$. So in contrast to the case without user charge profits are always increasing in τ_U. The reason is that p_i^* is independent of τ_U while c_i^* is increasing in τ_U. Thus if platforms can charge both sides of the market the degree of competition on one side is only reflected in the price of that side.

Let us turn now to the welfare analysis.

[62] A similar way of reasoning is given by Rochet & Tirole (2003). In two-sided markets the platforms charge prices such that the side with the higher demand elasticity is subsidised by the side with the lower demand elasticity.

[63] Similar effects are at work in Anderson & Leruth (1993) and Thisse & Vives (1988) where an additional pricing instrument hurt firms.

Proposition 3.8

If platforms can set an unrestricted user charge the equilibrium is efficient.

Proof

If $\beta K \leq \frac{(N/2)^{\lambda-1}\gamma\lambda t(N/2)}{t(N/2)+\frac{\gamma\lambda(N/2)^{\lambda}}{v''(t(N/2))}}$ then in equilibrium n_i^* is given by (3.19). But since $U_B'(n_i) = -\gamma\lambda n_i^{\lambda-1}t(n_i)$ equation (3.19) is the same as $\beta K t(n_i) + \beta K n_i t'(n_i) + U_B'(n_i) = 0$ which is equation (3.7), the condition for efficiency if $n_i^{eff} < N/2$.

If $\beta K > \frac{(N/2)^{\lambda-1}\gamma\lambda t(N/2)}{t(N/2)+\frac{\gamma\lambda(N/2)^{\lambda}}{v''(t(N/2))}}$ then $n_i^* = N/2$. Routine manipulations of the inequality yields that this inequality is the same as $\beta K t(N/2) + \beta K N/2 t'(N/2) + U_B'(N/2) > 0$. But this is inequality (3.6), the condition for efficiency if $n_i^{eff} = N/2$.

q.e.d.

Why does the additional instrument of a user charge lead to the efficient outcome? The intuition is that platforms now take users' utility directly into account and not only indirectly in the commercial prices. Since we have competition for users both platforms set the fees in such a way that users allocate efficiently. On the side of the advertisers there is Bertrand competition (although profits are positive). Advertisers are allocated efficiently as well. The reason is that the efficient allocation helps both firms to get higher revenue. Thus we show that with a second instrument at hand competition for users and advertisers leads to the efficient outcome.

Up to now we assumed that the user charge is unrestricted so it can be negative. But in many situations this is not practicable. TV watchers are not paid by stations or internet users are not subsidised by portals. If the user fee is restricted to be positive this means in our analysis that $c_i^* = \max\{0, \tau_U - \beta K N t(n_i^*)\}$. If $\tau_U < \beta K N t(n_i^*)$ this constraint is binding. In this case it would be optimal for platforms in a symmetric equilibrium to set $c_i^* = 0$. But this exactly what we observe in many markets. Take again the case of an internet portal. For them it would be technically no problem to charge a user if he wants to get access to their site. Instead they do not require users to pay a fee in order to attract many users and make profits on advertisers.

In terms of welfare if a user charge has to be positive the equilibrium is no longer generally efficient but only if $\tau_U > \beta KNt(n_i^*)$. So if the constraint on c_i^* is binding the result is the same as in the case without a user charge. To reach efficiency both pricing instruments have to be unrestricted.[64]

3.7 Conclusion

This paper analysed a model of platform competition in which each advertiser exerts a direct negative externality on users and an indirect one on all other advertisers on the same platform. It was shown that the number of advertisements in equilibrium can be too high or too low compared with the efficient one. Profits of platforms can increase if they become less differentiated because this leads to lower competition on the advertisers' side. If platforms can set a user charge as well profits increase only if this charge is positive in equilibrium. A prisoner's dilemma result is possible. But welfare is always higher with a user charge. We have also given anecdotal evidence that supports our results in an example of pricing behaviour of internet portals .

An interesting suggestion for further research might be to analyse the dynamics of such a two-sided market. Usually if people are used to one internet portal or read a newspaper for several years they would not switch easily if another one has fewer advertisements. People form habits. It would be interesting to analyse how such habit formation might change the results. A new platform which enters the market after the others (such as Google in the search engine case) needs a very low level of advertising to induce consumers to switch. This is what was actually observed for Google. So the question arises if this low level of advertising will persist or vanish over time.

[64] For a discussion of policy implications for two-sided markets see Evans (2004).

Chapter 4

Vertical Product Differentiation, Market Entry, and Welfare

4.1 Introduction

There are a lot of different ways how an incumbent firm reacts when facing the threat of entry. For example, in the pharmaceutical market after patent expiration some formerly protected monopolists introduced their own generics to keep competitors out of the market[65] while others abstained from such practice and increased its price after entry generic competitors.[66]

Another example is the airline industry. In Canada in fall 2000 the low cost carrier CanJet Airlines entered the Toronto-Halifax market. The reaction of Air Canada, the incumbent, was not to increase its price like in the pharmaceutical industry but to lower its fares.[67] A quite different strategy was pursued by British Airways. Its reaction on the entry of low cost carriers on long haul routes was to reduce economy class capacity and enlarge premium class capacity thereby increasing its average prices.[68] Many flag carriers instead tried to deter entry of low cost airlines by establishing their own 'no-

[65] See Hollis (2003).

[66] See Grabowski & Vernon (1992) or Frank & Salkever (1997).

[67] See Gillen (2002).

[68] See Johnson & Myatt (2003).

frills'-airline. This was done by British Airways on short-haul routes with the subsidiary GO. In 2000 the Dutch carrier KLM followed and established Basiq Air and in 2002 the low cost carrier Germanwings was founded.[69] Germanwings is an affiliate company of Eurowings. In turn, Eurowings is controlled by the German flag carrier Lufthansa.

So a couple of questions arise. Why do incumbents pursue so many different strategies to seemingly the same problem, namely threat of entry? Does an incumbent's strategy differ if it can produce only one quality level or a whole quality range? What are the welfare consequences of this potential competition, i.e. does welfare always increase in such a scenario or is it possible that a protected monopoly is better?

This chapter tries to answer these questions in a vertical product differentiation framework. We compare a model where each firm can produce a single quality with one where price discrimination over a quality range is possible. We show that in the single quality case welfare with potential competition can be lower than in monopoly. The intuition is that if qualities are strategic complements the incumbent lowers its quality in comparison to monopoly and produces some middle range quality to deter entry because it is impossible then for an entrant to find a profitable entry segment. Even in case of entry such a quality reduction might be profitable, causing the entrant to produce a low quality and reducing price competition. If qualities are strategic substitutes the incumbent produces higher quality and welfare increases.

If firms can produce a quality range we find that consumer rent with potential competition is higher than under monopoly. The intuition is that in order to deter entry the incumbent enlarges its product line to occupy the lower segment as well. In this case welfare increases as well. If entry cannot be deterred there is a gap between the two firms's quality ranges which reduces competition. In this case consumer rent always increases because of lower prices while the consequences on welfare are unclear. The reason is that some consumers buy a higher quality but others buy a lower one.

Specifically, we analyse a model of vertical product differentiation with entry. In the first stage the incumbent produces a quality which cannot be changed in the sequel. After observing this quality level the entrant decides if it wants to enter and if so which

[69] See Gilroy, Lukas, & Volpert (2003).

quality level it wants to produce. In the third stage firms compete in prices dependent on the produced quality levels.[70]

We compare this situation of potential competition with a situation of monopoly. In monopoly the firm produces too low a level of quality. The reason is that the monopolist can only charge one price which is the valuation of the marginal consumer. The valuation of the inframarginal consumer is higher but cannot be represented in the price. In the scenario of potential entry the incumbent can deter entry by varying its quality level. If qualities are strategic complements in the sense of Bulow, Geanakoplos, & Klemperer (1985) a reduction of the incumbent's quality leads to a reduction of the entrant's quality which lowers the entrant's profit.[71] If fixed costs of entry are high enough entry is deterred by a quality reduction and welfare is lower than under monopoly. Even in the case where entry is accommodated it might be profitable for the incumbent to reduce its quality. The entrant lowers its quality as well which results in lessened price competition. So even in case of competition it is possible that welfare is lower than under monopoly. If products are strategic substitutes welfare rises in both cases (entry deterrence and accommodation) because the incumbent increases its quality.[72] We also show that if marginal costs of production are low quality of the incumbent in case of entry is higher than in monopoly. The intuition is that the incumbent wishes to differentiate itself from its competitor by producing a higher quality. If marginal costs are low it is not very costly to do so and quality in case of entry is higher.

We also analyse a model where each firm can produce a whole range of different qualities and engage in second-degree price discrimination. This model is compared with the single quality case and we find that the results differ in some respects. In the model with price discrimination the lowest quality of the incumbent and the highest

[70]Throughout the paper we assume that it is more profitable for the incumbent to be the high quality firm than the low quality firm.

[71]In a different terminology which is used by Fudenberg & Tirole (1984) the strategy where the incumbent reduces quality to deter entry is called the 'lean and hungry look'.

[72]In the terminology of Fudenberg & Tirole (1984) this strategy is called 'Top Dog'.

quality of the entrant are strategic complements. So if the incumbent enlarges its quality range the profit of the entrant decreases. Thus the incumbent's entry deterrence strategy is to expand its product line which results in a welfare increase because more consumers are served. This is different from the single quality case where welfare in case of entry deterrence can be lower if qualities are strategic complements. If fixed costs of entry are low and the incumbent accommodates entry then we always get a gap between the two product lines of incumbent and entrant in order to reduce price competition. Thus some qualities in the middle range which are produced in monopoly are no longer produced in duopoly. But more qualities in the lower segment are produced in duopoly. The result is that some consumers buy higher quality in duopoly while others buy lower quality. Therefore the consequence on welfare is not clear. By contrast, it can be shown that consumer rent always increases in case of entry due to increased price competition.

For both models, single quality case and price discrimination, we provide two empirical examples from different industries where firms' behaviour is similar to that predicted by our model.

The remainder of the chapter is organised as follows. In the next section our model is related to the existing literature. Section 4.3 presents the model and the equilibrium without price discrimination. Some anecdotal evidence that supports the results is given in Section 4.4. Section 4.5 presents the model, the equilibrium, and the welfare consequences if price discrimination is possible. In Section 4.6 two practical examples for such firm behaviour are given. Section 4.7 gives a short conclusion and some policy implications. Most proofs of the results are presented in the Appendix.

4.2 Related Literature

Our model relates to the literatures on vertical product differentiation, second-degree price discrimination, and market entry. We will give the relation to each of the three branches and how our model differs from these literatures in turn.

The literature on quality competition started with the pioneering work of Gabszewicz & Thisse (1979) and Shaked & Sutton (1982). In their models firms are restricted to produce one quality level and compete in prices. In Gabszewicz & Thisse (1979) firms' qualities are exogenously given while in Shaked & Sutton (1982) firms decide simultaneously about their quality levels in the stage before price competition. Shaked & Sutton (1982) show that firms will produce different quality levels to avoid fierce competition in the last stage of the game. Under some parameter constellations only two firms are active in the market if there exist costs of entry. Shaked & Sutton (1982) were the first to analyse the now common game structure where firms are committed to their quality levels when competing in prices because prices can be changed at will while a quality change involves modifications of the production facilities.

Ronnen (1991) analyses a model with a similar framework as Shaked & Sutton (1982) but where a regulation authority can set a minimum quality standard before firms compete in qualities. In his model qualities are strategic complements. Thus if the minimum quality standard is set (slightly) above the quality which is produced by the low quality firm in a game without restriction, both qualities will rise in equilibrium. Price competition is intensified and all consumers are better off while the high quality firm loses. Ronnen (1991) shows that with an appropriately chosen standard social welfare improves.

Cabrales (2003) looks at the consequence of a price ceiling. He shows that with a lower price ceiling the market share of the high quality variant increases. The reason is that market share depends on the ratio of price to quality. But the quality responds less than proportionally to the price ceiling if the cost function is convex. He applies his model to regulation issues in the pharmaceutical market.

In contrast to these models my paper analyses a sequential move game in the quality decision. It might therefore be possible for the first mover to deter entry by an appropriate quality choice. Also welfare in this sequential structure is compared with a pure monopoly situation.

There are several papers which analyse competition between multiproduct firms.[73]

[73]Spulber (1989) analyses a model where firms are horizontally differentiated on a Hotelling line. He shows that each firm produces the first best quality for the consumer who is located exactly at

The closest to the model considered here are Champsaur & Rochet (1989) and Johnson & Myatt (2003). Champsaur & Rochet (1989) analyse a duopoly where firms commit in the first stage to a quality range and in the second stage compete in prices for each produced quality. They show that firms produce non overlapping quality ranges (there is always a gap between the two product lines) to reduce price competition. This result appears in my paper as well. The difference is that in my paper quality decisions are taken sequentially and one firm has a first mover advantage. This influences prices and quality ranges and may results in entry deterrence. I also provide a welfare analysis.

Johnson & Myatt (2003) analyse an asymmetric duopoly. One firm (which is called 'incumbent' by Johnson & Myatt (2003)) can produce the entire range of qualities while the other (the 'entrant') is limited to some range with an upper quality level. So the incumbent can produce upgrade versions. Firms compete simultaneously in quantities for each quality level. As is shown by Johnson & Myatt (2003) the incumbent may produce fewer qualities ('product line pruning') or more qualities ('fighting brands') in duopoly than in monopoly dependent on the cost function. If marginal revenue is decreasing the quality range is reduced while the quality range might be broader if marginal revenue is increasing in some regions.

A model of market entry in a vertical product differentiation framework is analysed by Donnenfeld & Weber (1995).[74] In their model there are two incumbents who face the entry threat of a third firm. They show that the equilibrium depends on the level of the fixed costs of entry. If these fixed costs are low entry is accommodated and the incumbents select extreme qualities to reduce price competition. The entrant chooses a quality in the middle.[75] If fixed costs are in some middle range incumbents deter entry. They do this by producing similar qualities which leads to harsh competition

the firm's position while qualities for all other consumers are distorted downwards. Stole (1995) in addition to Spulber (1989) considers the case where firms are uncertain about vertical preferences. He finds that a similar result holds in this case.

[74]For a model of entry deterrence and horizontal preferences see Bonanno (1987).

[75]A similar result is obtained in Donnenfeld & Weber (1992) in the case without fixed costs. They show that in this case the entrant's profit is higher than the profit of the incumbent which produces the lower quality.

and low profits. If fixed costs are so high that entry is blockaded incumbents choose sharply differentiated products to reduce competition. In contrast to Donnenfeld & Weber (1995), my model analyses the behaviour of only one incumbent but firms can produce quality ranges and engage in second degree price discrimination.

In short, models of vertical product differentiation usually do not consider the possibility of price discrimination if entry is possible. So this paper makes a first attempt to analyse the equilibrium and the welfare consequences of such a strategy.

4.3 The Model without Price Discrimination

This section presents the model where each firm can produce only one quality level.

Description of the Model

There is a continuum of consumers of mass 1. Each consumer purchases a single unit of a good. If a consumer decides to purchase from firm i she gets a good of quality q_i at price p_i. Consumers' tastes are described by the parameter θ which is distributed between 0 and 1 with distribution function $F(\theta)$ and density function $f(\theta)$. The utility from purchasing from firm i can therefore be denoted as

$$U(q_i, \theta, p_i) = u(q_i, \theta) - p_i,$$

where u is assumed to be strictly concave in q and in θ and thrice continuously differentiable. Consumers' reservation value from not buying is normalised to zero.

We proceed by making a few assumptions on the utility and the distribution function.

$$\begin{aligned} A1: &\quad \text{Single Crossing Property}: u_{q\theta}(q,\theta) > 0 \\ A2: &\quad u_{q\theta\theta}(q,\theta) \leq 0, \qquad u_{qq\theta}(q,\theta) \geq 0 \\ A3: &\quad \text{Monotone Hazard Rate Condition}: \tfrac{\partial}{\partial\theta}\left(\tfrac{1-F(\theta)}{f(\theta)}\right) \leq 0. \end{aligned}$$

A1 is the single crossing property. It states that utility and marginal utility go in the same direction if θ increases. It implies that indifference curves cross only once. This assumption is standard in the literature. A2 imposes two technical assumptions that guarantee that the second order conditions are satisfied. A3 is a standard assumption in the adverse selection literature and is called monotone hazard rate condition. It is satisfied by many distribution functions like the uniform distribution, the normal distribution etc.

There are two firms $i = 1, 2$. Firm 1 is the incumbent and firm 2 the potential entrant. If a firm decides to produce quality q it has to incur development costs $c(q)$ with $c'(q) > 0$ and $c''(q) > 0$.[76] $c(q)$ is the same for both firms. Marginal costs are denoted v and are the same for both firms as well.

The game structure is as follows. The game has three stages. In stage 1 firm 1 chooses q_1. Firm 2 decides about market entry in stage 2 after observing the choice of firm 1. If firm 2 decides not to enter firm 1 is a monopolist in stage three and decides about p_1. If firm 2 enters it has to incur fixed costs of market entry of F[77] and chooses q_2 in stage 2. Firm 1 observes q_2 and in stage 3 both firms set their prices p_1 and p_2 conditional on q_1 and q_2.

The important feature of the model is that both firms are committed to the quality they produce. In particular it is not possible for firm 1 to make a later change in the quality to which it has committed in stage 1.[78] This time structure represents the idea that it is easy and almost costlessly possible to change prices but it takes a considerable amount of time and costs to change the quality of a good.[79]

Monopoly Situation

First let us look at the monopoly case as a benchmark which is later compared with the results of the entry game. So suppose firm 1 is a monopolist and there is no

[76] $c(q)$ satisfies the standard Inada-conditions $\lim_{q\to 0} c'(q) = 0$ and $\lim_{q\to\infty} c'(q) = \infty$.

[77] These entry costs might contain advertising expenditures to inform consumers about the entrant's product, investment in transportation channels and so on.

[78] For a model where such commitment is only partially possible see Henkel (2003).

[79] This line of reasoning is followed in most models of vertical product differentiation, see for example Shaked & Sutton (1982) or Ronnen (1991).

potential entrant. In other words stage 2 of the game does not exist and firm 1 chooses first q_1 and then p_1. Let the marginal consumer who is served by the monopolist be called θ_m^{mon}. If quality is q_1 this marginal consumer is given by $u(q_1, \theta_m^{mon}) - p_1 = 0$. So all types $\theta_m^{mon} \leq \theta \leq 1$ are buying from the monopolist while all types $\theta < \theta_m^{mon}$ are not buying. In the last stage the monopolist chooses its price given quality q_1. The maximisation problem is thus

$$\max_{p_1} \qquad \Pi_1 = \int_{\theta_m^{mon}}^{1} [p_1 - vq_1] f(\theta) d\theta - c(q_1).$$

Since θ_m^{mon} is determined by p_1 it is convenient to make a change in the decision variables and let θ_m^{mon} be the decision variable. Thus we have

$$\max_{\theta_m^{mon}} \qquad \Pi_1 = \int_{\theta_m^{mon}}^{1} [u(q_1, \theta_m^{mon}) - vq_1] f(\theta) d\theta - c(q_1).$$

This results in a first order condition of

$$\frac{\partial \Pi_1}{\partial \theta_m^{mon}} = -f(\theta_m^{mon})[u(q_1, \theta_m^{mon}) - vq_1] + (1 - F(\theta_m^{mon})) u_\theta(q_1, \theta_m^{mon}) = 0. \tag{4.1}$$

Because of Assumption A3 the second order condition is globally satisfied.

The first order condition as usual states that the marginal gain from serving an additional consumer type (first term) is equal to the loss on all other consumers because of the price reduction (second term).

Turning to the first stage where the firm decides about quality q_1 we get a first order condition of[80]

$$\frac{\partial \Pi_1}{\partial q_1} = (1 - F(\theta_m^{mon}))[u_{q_1}(q_1, \theta_m^{mon}) - v] - c'(q_1) = 0. \tag{4.2}$$

The second order condition is globally satisfied because of $u_{q_1 q_1}(q_1, \theta_m^{mon}) < 0$ and $c''(q_1) > 0$. Thus we get that θ_m^{mon*} is given by (4.1), $p^{mon*} = u(q_1^{mon*}, \theta_m^{mon*})$ and q_1^{mon*} is given by (4.2).

A comparison of the monopolistic outcome with the welfare maximising outcome yields

[80]Because of the Envelope Theorem terms with $\frac{\partial \Pi_1}{\partial \theta_m^{mon}} \frac{\partial \theta_m^{mon}}{\partial q_1} = 0$ and can therefore be ignored in the first order condition.

Proposition 4.1

Compared with the welfare-maximizing θ_m^{WF} and q^{WF} a monopolist serves too few consumers, $\theta_m^{mon*} > \theta_m^{WF}$, and provides too low a quality $q^{WF} > q_1^{mon*}$.

Proof

See the Appendix.

The result that too few consumers are served by a monopolist is standard. The intuition for the quality distortion is that the monopolist can charge only one price namely $p_1^{mon*} = u(q_1^{mon*}, \theta_m^{mon*})$ for its produced quality. So by increasing quality it can only increase its price by the amount that the utility of the marginal consumer rises. But the utility of all types $\theta > \theta_m^{mon*}$ rises more from a quality increase than the utility of the marginal consumer because of the single crossing property. Thus from a welfare point of view quality in monopoly is too low. Since the monopolist also serves too few consumers the downward distortion of quality is intensified.

Potential Competition

Now let us turn to the three stage game in which firm 2 can enter the market in stage 2. In the following let us define $q_2(q_1)$ as the best answer of firm 2 if it enters in response to firm 1 producing q_1. Before starting with the analysis we need two additional assumptions:

$$\begin{aligned} A4: &\quad \Pi_2(q_1^{mon*}, q_2(q_1^{mon*})) > 0 \\ A5: &\quad \Pi_1(q_1^H, q_2(q_1^H)) > \Pi_1(q_1^L, q_2(q_1^L)) \end{aligned}$$

$$\text{whenever} \qquad q_1^H > q_2(q_1^H) \qquad \text{and} \qquad q_1^L < q_2(q_1^L).$$

The first assumption states that the profit of firm 2 is positive if firm 1 produces its optimal monopoly quality. The assumption is made to avoid the uninteresting case that it is an equilibrium if firm 1 produces its monopoly quality and firm 2 stays out of the market. In the terminology of Bain (1956) this would mean that entry is blockaded.

Assumption $A5$ states that firm 1's profit is higher if it is the high quality firm, i.e. produces such a quality in stage 1 that the optimal response of firm 2 is to produce a lower quality in stage 2.

As usual the game is solved by backwards induction.

In the third stage there are two possibilities. Either firm 2 has entered in stage 2 and there is competition or firm 2 stayed out of the market and firm 1 is a monopolist. If firm 1 is a monopolist the marginal consumer is determined in the same way as in the last subsection and θ_m^{mon} is given by (4.1) given the quality q_1 firm 1 has produced in stage 1 (which is different from q_1^{mon*} because of Assumption A4.)

If firm 2 has entered the market in stage 2 firms compete for consumers in stage 3. Because of Assumption $A5$ firm 1 will always produce a quality q_1 such that it is optimal for firm 2 to produce $q_2 < q_1$. It is therefore apparent that firm 1 will serve higher consumer types. The marginal consumer θ_{m1}^{duo} who is indifferent between buying from firm 1 and buying from firm 2 is given by $u(q_1, \theta_{m1}^{duo}) - p_1 = u(q_2, \theta_{m1}^{duo}) - p_2$ or $p_1 = p_2 + u(q_1, \theta_{m1}^{duo}) - u(q_2, \theta_{m1}^{duo})$. Thus firm 1's profit function is given by

$$\Pi_1 = \int_{\theta_{m1}^{duo}}^{1} [p_2 + u(q_1, \theta_{m1}^{duo}) - u(q_2, \theta_{m1}^{duo}) - vq_1] f(\theta) d\theta - c(q_1).$$

Maximising this with respect to θ_{m1}^{duo} yields

$$\begin{gathered} \tfrac{\partial \Pi_1}{\partial \theta_{m1}^{duo}} = -f(\theta_{m1}^{duo})[p_2 + u(q_1, \theta_{m1}^{duo}) - u(q_2, \theta_{m1}^{duo}) - vq_1] + \\ (1 - F(\theta_{m1}^{duo}))(u_\theta(q_1, \theta_{m1}^{duo}) - u_\theta(q_2, \theta_{m1}^{duo})) = 0. \end{gathered} \tag{4.3}$$

The second order condition is globally satisfied because of Assumptions A2 and A3.

Concerning firm 2 the marginal consumer θ_{m2}^{duo} who is indifferent between buying at firm 2 and buying nothing is given by $u(q_2, \theta_{m2}^{duo}) - p_2 = 0$ or $p_2 = u(q_2, \theta_{m2}^{duo})$. Thus the profit function of firm 2 is

$$\Pi_2 = \int_{\theta_{m1}^{duo}}^{\theta_{m2}^{duo}} [u(q_2, \theta_{m2}^{duo}) - vq_2] f(\theta) d\theta - c(q_2) - F.$$

The first order condition is

$$\frac{\partial \Pi_2}{\partial \theta_{m2}^{duo}} = -f(\theta_{m2}^{duo})[u(q_2, \theta_{m2}^{duo}) - vq_2] + (1 - F(\theta_{m2}^{duo}))u_\theta(q_2, \theta_{m2}^{duo}) = 0. \tag{4.4}$$

Again because of Assumption A3 the second order condition is satisfied.

In equilibrium marginal consumers θ^*_{m1} and θ^*_{m2} are given by (4.3) and (4.4) and equilibrium prices are given by $p^*_1 = u(q_2, \theta^*_{m2}) + u(q_1, \theta^*_{m1}) - u(q_2, \theta^*_{m1})$ and $p^*_2 = u(q_2, \theta^*_{m2})$.[81]

Now let us look at stage 2 and suppose for the moment that firm 2 has entered. In this case firm 2 maximises its profit with respect to q_2 which yields

$$\frac{\partial \Pi_2}{\partial q_2} = (F(\theta^*_{m1}) - F(\theta^*_{m2}))(u_q(q_2, \theta^*_{m2}) - v) + [u(q_2, \theta^*_{m2}) - vq_2]f(\theta)\frac{\partial \theta^*_{m1}}{\partial q_2} - c'(q_2) = 0. \tag{4.5}$$

The second order condition is satisfied because of $u_{qq}(q, \theta) < 0$ and $c''(q) > 0$. q^*_2 is given by (4.5) and since θ^*_{m1} is dependent on q_1, q^*_2 is dependent on q_1 as well.

Firm 2 only enters if

$$\int_{\theta^*_{m1}}^{\theta^*_{m2}} [u(q^*_2, \theta^*_{m2}) - vq^*_2]f(\theta)d\theta - c(q^*_2) > F.$$

Firm 1 in stage 1 does now take into account that q^*_2 depends on q_1. Its first order condition if firm 2 enters is given by

$$\begin{aligned} \tfrac{\partial \Pi_1}{\partial q_1} &= (1 - F(\theta^*_{m1}))(u_{q_1}(q_1, \theta^*_{m1}) - v \\ -[u_{q_2}(q^*_2, \theta^*_{m1}) - u_{q_2}(q^*_2, \theta^*_{m2}) &- u_\theta(q^*_2, \theta^*_{m2})]\tfrac{\partial \theta^*_{m2}}{\partial q^*_2}\tfrac{\partial q^*_2}{\partial q_1}) - c'(q_1) = 0. \end{aligned} \tag{4.6}$$

But if F is high enough then firm 1 also has the possibility to choose q_1 in such a way that firm 2 does not enter. Let us denote the quality that deters entry of firm 2 by q^{ED}_1. It is given by

$$\int_{\theta^*_{m1}}^{\theta^*_{m2}} [u(q^*_2(q^{ED}_1), \theta^*_{m2}) - vq^*_2(q^{ED}_1)]f(\theta)d\theta - c(q^*_2(q^{ED}_1)) = F.$$

If firm 1 produces this q^{ED}_1 it is a monopolist in stage 3 and earns profits of

$$\Pi^{ED}_1 = \int_{\theta^*_m(q^{ED}_1)}^{1} [u(q^{ED}_1, \theta^*_m(q^{ED}_1)) - vq^{ED}_1]f(\theta)d\theta - c(q^{ED}_1).$$

Thus firm 1 engages in entry deterrence if and only if

$$\begin{gathered} \Pi^{ED}_1 = \textstyle\int_{\theta^*_m(q^{ED}_1)}^{1} [u(q^{ED}_1, \theta^*_m(q^{ED}_1)) - vq^{ED}_1]f(\theta)d\theta - c(q^{ED}_1) > \\ \textstyle\int_{\theta^*_{m1}}^{1} [u(q^*_2, \theta^*_{m2}) + u(q^*_1, \theta^*_{m1}) - u(q^*_2, \theta^*_{m1}) - vq^*_1]f(\theta)d\theta - c(q^*_1) = \Pi^{duo}_1. \end{gathered}$$

[81] Variables marked with a star indicate equilibrium values of the game after firm 2 has entered.

We are now in a position to state the equilibrium of the game:

- If $\Pi_1^{ED} > \Pi_1^{duo}$ then firm 1 chooses q_1^{ED}, firm 2 does not enter in stage 2 and $p_1^* = u(q_1^{ED}, \theta_m^*)$ where θ_m^* is given by (4.1) with $q_1 = q_1^{ED}$.

- If $\Pi_1^{ED} \leq \Pi_1^{duo}$ then q_1^* is given by (4.6), firm 2 enters in stage 2 and q_2^* is given by (4.5). θ_{m1}^* and θ_{m2}^* are given by (4.3) and (4.4) and $p_1^* = u(q_2^*, \theta_{m2}^*) + u(q_1^*, \theta_{m1}^*) - u(q_2^*, \theta_{m1}^*)$ and $p_2^* = u(q_2^*, \theta_{m2}^*)$.

Now this equilibrium with potential competition can be compared with the monopoly equilibrium. First look at the case where firm 2 enters. In this case fixed costs of market entry are so low that it does not pay for firm 1 to choose q_1^{ED} such that firm 2 does not enter. Instead firm 1 sets q_1^* according to (4.6).

Proposition 4.2

$q_1^* > q_1^{mon*}$ if and only if

$$\begin{aligned} v < u_{q_1}(q_1^{mon*}, \theta_{m1}^*) - \tfrac{\partial q_2^*}{\partial q_1}[(u_{q_2}(q_2^*, \theta_{m1}^*) + u_{q_2}(q_2^*, \theta_{m2}^*) \\ -u_\theta(q_2^*, \theta_{m2}^*))\tfrac{\partial \theta_{m2}^*}{\partial q_2^*})](\tfrac{1}{F(\theta_m^{mon*}) - F(\theta_{m1}^*)}) \end{aligned} \tag{4.7}$$

Proof

q_1^{mon*} is given by (4.2),

$$\frac{\partial \Pi_1}{\partial q_1} = (1 - F(\theta_m^{mon*}))(u_{q_1}(q_1, \theta_m^{mon*}) - v) - c'(q_1) = 0,$$

while q_1^* is given by (4.6),

$$\begin{aligned} \tfrac{\partial \Pi_1}{\partial q_1} = (1 - F(\theta_{m1}^*))(u_{q_1}(q_1, \theta_{m1}^*) - v - (u_{q_2}(q_2^*, \theta_{m1}^*) + u_{q_2}(q_2^*, \theta_{m2}^*) \\ -u_\theta(q_2^*, \theta_{m2}^*))\tfrac{\partial \theta_{m2}}{\partial q_2^*}\tfrac{\partial q_2^*}{\partial q_1} - c'(q_1) = 0. \end{aligned}$$

Evaluated at q_1^{mon*}, (4.6) becomes

$$\begin{aligned} [F(\theta_m^{mon*}) - F(\theta_{m1}^*)](u_{q_1}(q_1^{mon*}, \theta_{m1}^*) - v) - \\ ((u_{q_2}(q_2^*, \theta_{m1}^*) + u_{q_2}(q_2^*, \theta_{m2}^*) - u_\theta(q_2^*, \theta_{m2}^*))\tfrac{\partial \theta}{\partial q_2^*})\tfrac{\partial q_2^*}{\partial q_1}, \end{aligned}$$

which can be greater or smaller than zero. Solving for v yields

$$u_{q_1}(q_1^{mon*}, \theta_{m1}^*) - [(u_q(q_2^*, \theta_{m1}^*) + u_{q_2}(q_2^*, \theta_{m2}^*) \\ -u_\theta(q_2^*, \theta_{m2}^*))\frac{\partial\theta}{\partial q_2^*}\frac{\partial q_2^*}{\partial q_1}]\left(\frac{1}{F(\theta_m^{mon*})-F(\theta_{m1}^*)}\right)\begin{pmatrix} > \\ = \\ < \end{pmatrix} v.$$

If $' >'$ is true the first derivative of the profit function of firm 1 after entry is increasing at q_1^{mon*} while it is zero at q_1^*. Since the function is globally concave $q_1^{mon*} < q_1^{duo*}$.

q.e.d.

This shows that the quality level of the incumbent increases after entry if and only if marginal costs are lower than a given threshold. At first glance one may would have guessed that the quality level of firm 1 in duopoly is always higher achieving a higher degree of differentiation from the entrant's quality. But with high marginal costs this is not true. The reason is that in case of competition it is harder for the incumbent to extract consumer rent. Thus it does not pay to produce high quality if this comes at high costs.

More specifically, let us have a closer look at inequality (4.7). It is obvious from equation (4.2) that $u_{q_1}(q_1^{mon*}, \theta_{m1}^*) > v$. Thus if the term $[-\frac{\partial q_2^*}{\partial q_1}[(u_{q_2}(q_2^*, \theta_{m1}^*) + u_{q_2}(q_2^*, \theta_{m2}^*) - u_\theta(q_2^*, \theta_{m2}^*))\frac{\partial \theta_{m2}^*}{\partial q_2^*})](\frac{1}{F(\theta_m^{mon*})-F(\theta_{m1}^*)})]$ is greater than zero the right hand side of (4.7) is higher than the left hand side and we have $q_1^* > q_1^{mon*}$. To get an intuition for the result suppose that $\theta_{m1}^* < \theta_m^{mon*}$ (and thus $F(\theta_{m1}^*) < F(\theta_m^{mon*})$).[82] Then this term is positive if $\frac{\partial q_2^*}{\partial q_1} < 0$, i.e. qualities are strategic substitutes. In this case an increase in q_1^* has a favourable impact for firm 1 on q_2^*, namely a reduction of q_2^*. Thus q_1^* unambiguously increases with competition. If instead $\frac{\partial q_2^*}{\partial q_1^*} > 0$ the qualities are strategic complements. In this case it might be optimal for firm 1 to set $q_1^* < q_1^{mon*}$ to induce firm 2 to lower its quality as well. Firm 1 will do so if variable costs are high because then costs can be reduced and competition is lowered by the reaction of firm 2.

[82]In the next proposition it is shown that this is always the case if $u(q, \theta) = \theta q$.

To gain some insights into welfare comparisons between monopoly and potential competition we have to give a bit more structure to the model.

Proposition 4.3

Let $u(q,\theta) = \theta q$. If qualities are strategic substitutes welfare unambiguously rises with entry.

Proof

See the Appendix.

If $u(q,\theta) = \theta q$ firm 1 serves more consumers in duopoly than in monopoly. The reason is that the quality deflated price $\frac{p_1^*}{q_1^*}$ is lower.[83] This follows from the fact that $\theta_{m1}^* < \theta_m^{mon*}$ which for the specific utility function means that $\frac{p_1^*-p_2^*}{q_1^*-q_2^*} < \frac{p_1^{mon*}}{q_1^{mon*}}$, and the fact that $\frac{p_2^*}{q_2^*} < \frac{p_1^{mon*}}{q_1^{mon*}}$. Taken together this implies that $\frac{p_1^*}{q_1^*} < \frac{p_1^{mon*}}{q_1^{mon*}}$. Thus more consumers are buying from the incumbent. If its quality in duopoly is higher as well then welfare in duopoly is for sure higher. This is the case if qualities are strategic substitutes because then firm 2 reduces its quality as reaction to a quality increase of firm 1, which is profitable for firm 1. It should be mentioned that if qualities are strategic substitutes welfare necessarily increases. But the "only if" statement is not true. Even in case if qualities are strategic complements welfare can rise because more consumers are buying in duopoly. But it is also possible that welfare decreases because the incumbent reduces its quality and this quality reduction effect dominates the effect that more consumers are served.

Now let us turn to the case where firm 1 deters entry of firm 2. In this case firm 1 produces q_1^{ED} and is a monopolist thereafter. From Proposition 4.1 we know that a monopolist distorts quality downwards. So whether welfare in case of entry deterrence is higher than welfare in a pure monopoly situation depends on q_1^{ED} in comparison with q_1^{mon*}. If $q_1^{ED} > (<) q_1^{mon*}$ welfare in case of entry deterrence is higher (lower).

[83] This result is obtained in many models of quality competition, see e.g. Bae & Choi (2003) or Banerjee (2003). In these models quality is exogenous. In the paper here it is shown that this result holds for endogenous quality choice as well.

But this depends on the reaction of q_2^* on q_1^*. If e.g. $\frac{\partial q_2^*}{\partial q_1^*} < 0$ the incumbent has to increase its quality to keep the entrant out of the market. p_1^{mon*} is always given by $p_1^{mon} = u(q_1^{mon*}, \theta_m^{mon*})$. Thus a change in q_1^{mon*} leads to a change in p_1^{mon*} of $u_{q_1^{mon*}}(q_1^{mon*}, \theta_m^{mon*})$ but θ_m^{mon*} stays unchanged and we get the following proposition.

Proposition 4.4

If qualities are strategic substitutes welfare in case of entry deterrence is higher than in a protected monopoly. If qualities are strategic complements the reverse is true.

This shows that the threat of entry can either increase or decrease welfare depending on the strategic reaction of firm 2 to the quality of firm 1. In the most general model it is impossible to assess whether qualities are strategic substitutes or complements. But we can make a general conclusion in the specific framework of Mussa & Rosen (1978). In their model θ is uniformly distributed, $u(q, \theta) = \theta q$, $v = 0$, and $c(q) = \frac{1}{2}q^2$.

Proposition 4.5

In the linear-uniform-quadratic case of Mussa & Rosen (1978) qualities are strategic complements.

Proof

See the Appendix.

So in the uniform-linear-quadratic case welfare decreases with potential entry if fixed costs from entry are high enough such that entry is deterred. The reason is that the incumbent distorts its quality further downwards so that it is not profitable for the entrant to occupy the low quality segment and therefore the entrant stays out of the market. But this downward distortion of quality lowers welfare. In Section 4.5 this result will be contrasted with a model where both firms can produce many different quality levels.

4.4 Discussion

The preceding analysis points to cost-and demand-function-based reasons for an incumbent to increase or decrease its quality and price after entry. In this section we turn to a discussion of some empirical examples from different markets that give anecdotal evidence for our results.

4.4.1 Pricing of Pharmaceuticals after Generic Entry

In the market for pharmaceuticals, patents protect drug developers after the development of a new pharmaceutical. The aim of these patents is to give developing firms an incentive to develop new pharmaceuticals because they can earn monopoly rents during the patent period. After the expiry of the patent, entry of generic drugs is possible. In the US the Watchman-Hax Act in 1984 makes it easier for generic firms to enter the market.[84] This makes the pharmaceutical market a suitable example for applying the results of the previous section.

By Proposition 4.2 our theory predicts that if variable marginal costs of production are low, quality and price of the brand-name drug should increase after entry. In the production of pharmaceuticals marginal costs are very low compared with research and development costs. For example, in the US the pharmaceutical industry has spent the largest fraction of its sales receipts to research and development among all US industries with comparable data (US Federal Trade Commission (1985)). So one would predict that prices increase after generic entry. This is confirmed by empirical studies. Scherer (2000) gives an example of the expiry of the product patent covering the cephalosporin antibiotic cephalexin in April 1987. This was sold under the brand name Keflex. After entry the price of Keflex rose from around \$60 (per 100 capsules) to \$85 in 1990. During this time the prices of generics went down from \$30 to \$15. Frank & Salkever

[84]The reason is that testing requirements for generics have been relaxed. It is only necessary to demonstrate that the drug has the same ingredients as the original, that the formulation was absorbed in the blood stream at more or less the same time, and to document good manufacturing practices of the generic firm. See Scherer (2000), p. 1321.

(1997) looked at 45 drugs which faced generic competition for the first time after the Watchman-Hax Act. They found that brand-name prices increased by 50% five years after generic entry. Similar pricing patterns were obtained in the studies by Grabowski & Vernon (1992) and Scherer (1996). This supports the prediction that if variable marginal costs are low prices will rise after entry.

Rising quality is a bit harder to explain because normally quality of drugs stayed unchanged. But the brand-name producers tried to increase consumers' perceived quality via advertising during the period of patent protection. Scherer (2000) states that in 1998 in the USA producers spend about $1 billion on direct-to-consumer advertising.[85] This amount can not only be seen as informative advertising but is also done to convince consumers of the product's quality and to separate from generics. After entry, advertisement was reduced because of the fear that this would also spur the sales of the new competitors. Thus in the market for pharmaceuticals brand-name producers did not increase the real quality of the drugs. Instead they increased perceived quality when faced with the threat of entry.

4.4.2 The Market for Fragrance and Cosmetics

In Singapore for a long time cosmetics were sold exclusively by authorised distributors and listed retailers. These firms demand high prices and had high price-cost margins. For example, consumers had to pay $35 to $38 for a lipstick at cosmetic counters of department stores but it costs only US $0.50 to manufacture a lipstick.[86] These lipsticks are imported from the US or Europe so one had to add transportation costs. Still price cost margins were high.

In the late 1980s the parallel importer B&N entered the market. B&N imported the same products as the authorised distributors but had a simple business strategy, namely price cuts. It sold a Christian Dior lipstick at $19 or $20[87] and in general offered the cosmetics up to 50% below the prices of listed retailers. The products are

[85] See also Caves, Whinston & Hurwitz (1991).

[86] See Lee, Lim & Tan (2001).

[87] "Parallel Importers Make Cosmetic Firms See Red", The Straits Time, October 7, 1994, p.44.

qualitatively similar but disadvantages for B&N were that the company was unknown at the beginning of their business and that authorised distributors placed their products on premium space and had set up cosmetic counters at department stores. What was the reaction of distributors to the entry of B&N? Beside negative advertising about parallel imports and lawsuits their main response consisted in price cuts. For example they lowered the lipstick price from \$34 to \$28.[88]

In contrast to the pharmaceutical market in the market for fragrance and cosmetics marginal costs play the important role compared to development costs. The only source of development costs is the building up of connections to importers. But the main bulk of costs a retailer has to bear are the delivering costs of lipsticks, the advertising costs, and the rents to be paid to department stores for display on premium space. In this respect the retailers also reduced the quality of their offers. They set up fewer cosmetic counters in stores and spend less money on costly advertising.[89] But especially with cosmetics and fragrance the conveyed life-style of the products is very important and this can be mainly given by advertising. Since the retailers do not manufacture the cosmetics themselves the physical quality if the products stays the same. But the quality was reduced from the perspective of the consumers since the products are no longer displayed on premium space and are less advertised. Thus the observations in this market go in line with the predictions of our theory that an incumbent's price and quality decrease if marginal variable costs are high.

4.5 The Model with Price Discrimination

This section analyses a model where firms can produce many different qualities which can be sold at different prices. The results of this model are later compared with the results of Section 4.3.

[88]"Parallel Imports: Copyright Owners Fight Back", The Straits Time, August 12, 1996, p.31.

[89]See Lee, Lim & Tan (2001).

Model Framework

Consumers' utility functions, the distribution of preferences, firms' cost functions, and the game structure is the same as in Section 4.3. The only difference is that each firm can now produce not only one quality but many different qualities which are sold at different prices. We are therefore in a problem of adverse selection. We assume that for each quality a firm produces it has to bear development costs $c(q)$[90] and variable costs v. Assumptions A1, A2, and A3 are kept as well.

Monopoly Situation

As in Section 4.3 before solving the game consider the benchmark case where firm 1 is a monopolist. In this case we are in a standard mechanism design problem of second-degree price discrimination. The firm's problem is to choose the optimal quality-payment schedule and the marginal consumer θ_m^{mon} subject to the standard participation and incentive compatibility constraints,

$$\begin{array}{lll} \max_{q(\theta),p(\theta),\theta_m^{mon}} & \Pi_1 = \int_{\theta_m^{mon}}^1 [p(\theta) - vq(\theta)]f(\theta)d\theta - \int_{\theta_m^{mon}}^1 c(q)d\theta & \\ \text{s.t.} & u(q(\theta),\theta) - p(\theta) \geq 0 & \forall\theta \geq \theta_m^{mon} \\ & u(q(\theta),\theta) - p(\theta) \geq u(q(\hat{\theta}),\theta) - p(\hat{\theta}) & \forall\theta,\hat{\theta} \geq \theta_m^{mon}. \end{array}$$

The equilibrium is characterised in the following lemma:

Lemma 1

The optimal $q(\theta)^{mon\star}, p(\theta)^{mon\star}, \theta_m^{mon\star}$ are given by the following equations:

$$p^{mon\star}(\theta) = u(q^{mon\star}(\theta),\theta) - \int_{\theta_m^{mon\star}}^{\theta} \frac{\partial u(q^{mon\star}(\tau),\tau)}{\partial\theta} d\tau, \tag{4.8}$$

$$\frac{\partial u(q^{mon\star}(\theta),\theta)}{\partial q} - \Big(\frac{1-F(\theta)}{f(\theta)}\Big)\frac{\partial^2 u(q^{mon\star}(\theta),\theta)}{\partial q\partial\theta} - v - \frac{c'(q^{mon\star}(\theta))}{f(\theta)} = 0, \tag{4.9}$$

[90] Theoretically the assumption of development costs for each quality is necessary to avoid that firm 1 can costlessly commit to the whole range of qualities. If this is possible we get trivial equilibria in which firm 2 is always kept out of the market.

$$\begin{aligned}[u(q^{mon\star}(\theta_m^{mon\star}), \theta_m^{mon\star}) - vq^{mon\star}(\theta_m^{mon\star})]f(\theta_m^{mon\star}) - c(q^{mon\star}(\theta)) \\ = (1 - F(\theta_m^{mon\star}))\tfrac{\partial u(q^{mon\star}(\theta_m^{mon\star}), \theta_m^\star)}{\partial \theta_m^{mon\star}}.\end{aligned} \tag{4.10}$$

Proof

See the Appendix.

The first two equations are standard in second degree price discrimination. The first states that the price for each type θ is the utility type θ gets from buying a good of quality $q^{mon\star}(\theta)$ minus a term which is increasing in θ. So higher types get a higher utility to prevent them from choosing the contract designed for the lower types. The second equation states that the quality a type θ gets is increasing in θ but is always lower than the optimal quality except for $\theta = 1$. This is the famous 'no-distortion-at-the-top-result'. The third equation states that the marginal consumer is characterised in such a way that the net gain of serving him (the left hand side of (4.10)) is exactly equal to the loss that occurs to the firm because it has to give a higher rent to the inframarginal consumers (the right hand side of (4.10)).

Concerning welfare the firm offers a whole range of qualities where higher types get higher quality. But except for the highest type quality is distorted downwards.

Analysis of the Duopoly Situation

In the following we denote the quality range of firm 1 $Q_1 = [q_1^-, q_1^+]$ and the quality range of firm 2 $Q_2 = [q_2^-, q_2^+]$. $Q_2(Q_1)$, as in Section 4.3, is the best response of firm 2 after entry if firm 1 produces a quality range Q_1. If the quality ranges do not overlap, i.e. the lowest quality of firm i, q_i^-, is higher than the highest quality of firm j, q_j^+, we say that $Q_i > Q_j$.[91]

Again before starting with the analysis of the entry game we make two assumptions which are modifications of assumptions $A4$ and $A5$ of Section 4.3.

$$A4' : \;\; \Pi_2(Q_1^{mon}, Q_2(Q_1^{mon})) > 0.$$

[91] In Lemma 4.2 we show that in equilibrium this is always the case.

Q_1^{mon} is the quality range firm 1 produces in the monopoly case and $A4'$ states that firm 2 enters if firm 1 produces Q_1^{mon}.

$$A5': \quad \Pi_1(Q_1^H, Q_2(Q_1^H)) > \Pi_1(Q_1^L, Q_2(Q_1^L))$$

$$\text{whenever} \qquad Q_1^H > Q_2(Q_1^H) \qquad \text{and} \qquad Q_1^L < Q_2(Q_1^L).$$

Assumption $A5'$ states that it is profitable for firm 1 to be the high quality firm, i.e. producing a quality range which is above the one of firm 2.

Again the game is solved by backwards induction. First look at the case where firm 2 did not enter in stage 2. By the same calculations as in the monopoly case we get that prices are $p^\star(\theta) = u(q(\theta),\theta) - \int_{\theta_m^\star}^{\theta} \frac{\partial u(q(\tau),\tau)}{\partial \theta} d\tau$. This prices are independent of the ends of the quality range firm 1 has produced in stage 1.

Now let us turn to the case where firm 2 entered in stage 2.[92] We have to determine the prices given that firm 1 produces quality range Q_1 and firm 2 produces quality range Q_2.[93]

For simplicity let us assume first that $q_1^- > q_2^+$. We will later show that this is always the case. The marginal consumer θ_m who is indifferent between buying q_1^- and q_2^+ is given by $u(q_1^-,\theta_m) - p_1(\theta(q_1^-)) = u(q_2^+,\theta_m) - p_2(\theta(q_2^+))$ or $p_1(\theta(q_1^-)) = p_2(\theta(q_2^+)) + u(q_1^-,\theta_m) - u(q_2^+,\theta_m)$. Firm 1's maximisation problem in stage 3 can be written as

$$\begin{array}{lll}
\max_{p_1(\theta),p_1(q_1^-)} & \Pi_1 = \int_{\theta(q_1^-)}^{1}[p_1(\theta) - vq(\theta)]f(\theta)d(\theta) + & \\
 & \int_{\theta_m}^{\theta(q_1^-)}[p_1(q_1^-) - vq_1^-]f(\theta)d\theta - \int_{\theta(q_1^-)}^{1} c(q(\theta))d\theta & \\
\text{s.t.} & u(q(\theta),\theta) - p_1(\theta) \geq u(q_2^+,\theta) - p_2(q_2^+) & \forall \theta \geq \theta_m \\
 & u(q(\theta),\theta) - p_1(\theta) \geq u(q(\hat\theta),\theta) - p_1(\hat\theta) & \forall \theta,\hat\theta \geq \theta_m,
\end{array}$$

[92]The analysis in this section draws heavily on Champsaur & Rochet (1989). The difference is that firms choose qualities simultaneously in Champsaur & Rochet (1989) while in my model qualities are chosen sequentially. But the analysis of the second and the third stage is quite similar.

[93]In principle we should analyse the third stage for arbitrary (Q_1, Q_2). However, this is clearly impossible to do. But one can put the restriction on (Q_1, Q_2) that there is never a whole in one of two quality ranges for the same reason as for the monopolist. For a discussion on that issue and why it is reasonable to conduct the analysis in the way as it is done in this chapter see Champsaur & Rochet (1989).

where $\theta(q_1^-)$ is the highest type who buys quality q_1^-.

Firm 2's maximisation problem in stage 3 is

$$\begin{array}{lll} \max_{p_2(\theta),p_2(q_2^+)} & \Pi_2 = \int_{\theta_{m2}}^{\theta(q_2^+)}[p_2(\theta) - vq(\theta)]f(\theta)d(\theta) & \\ & + \int_{\theta(q_2^+)}^{\theta_m}[p_2(q_2^+) - vq_2^+]f(\theta)d\theta - \int_{\theta_{m2}}^{\theta(q_2^+)} c(q(\theta))d\theta & \\ \text{s.t.} & u(q(\theta),\theta) - p_2(\theta) \geq u(q_1^-,\theta) - p_1(q_1^-) & \forall\theta < \theta_m \\ & u(q(\theta),\theta) - p_2(\theta) \geq u(q(\hat{\theta}),\theta) - p_2(\hat{\theta}) & \forall\theta,\hat{\theta} < \theta_m, \end{array}$$

where $\theta(q_2^+)$ is the lowest type who buys quality q_2^+.

Solving for $p_1(\theta)$ and $p_2(\theta)$ yields for the same reasons as in Lemma 4.1

$$p_1(\theta) = u(q(\theta),\theta) - \int_{\theta(q_1^-)}^{\theta} \frac{\partial u(q(\tau),\tau)}{\partial\theta}d\tau + p_1(q_1^-) - u(q_1^-,\theta(q_1^-)), \tag{4.11}$$

and

$$p_2(\theta) = u(q(\theta),\theta) - \int_{\theta_{m2}}^{\theta} \frac{\partial u(q(\tau),\tau)}{\partial\theta}d\tau + p_2(q_2^+) - u(q_2^+,\theta(q_2^+)). \tag{4.12}$$

Plugging this back in the profit function and solving for $p_1(q_1^-)$ and $p_2(q_2^+)$ gives

$$p_1^*(q_1^-) = vq_1^- + \frac{1-F(\theta_m)}{f(\theta_m)}[u_\theta(q_1^-,\theta_m) - u_\theta(q_2^+,\theta_m)], \tag{4.13}$$

$$p_2^*(q_2^+) = vq_2^+ + \frac{F(\theta_m)-F(\theta_{m2})}{f(\theta_m)}[u_\theta(q_1^-,\theta_m) - u_\theta(q_2^+,\theta_m)]. \tag{4.14}$$

Having solved stage 3 of the game we can go back one stage to stage 2 where firm 2 chooses its optimal quality range. The problem of firm 2 is thus

$$\begin{array}{ll} \max_{q(\theta),q_2^+,\theta_{m2}} & \Pi_2 = \int_{\theta_{m2}}^{\theta(q_2^+)}[u(q(\theta),\theta) - \int_{\theta_{m2}}^{\theta} \frac{\partial u(q(\tau),\tau)}{\partial\theta}d\tau \\ & +p_2^*(q_2^+) - u(q_2^+,\theta(q_2^+)) - vq(\theta) - \frac{c(q(\theta))}{f(\theta)}]f(\theta)d(\theta) + \\ & \int_{\theta(q_2^+)}^{\theta_m}[p_2^*(q_2^+) - vq_2^+]f(\theta)d\theta. \end{array}$$

Differentiating with respect to θ_{m2} and $q(\theta)$ yields

$$\begin{array}{l} (F(\theta_m) - F(\theta_{m2}))(\frac{\partial u(q^\star(\theta_m^\star),\theta_m^\star)}{\partial\theta_m^\star}) - \frac{f(\theta_{m2})}{f(\theta_m)}[u_\theta(q_1^-,\theta_m) - u_\theta(q_2^+,\theta_m)] = \\ f(\theta_{m2})[u(q(\theta_{m2}),\theta_{m2}) - \frac{c(q(\theta_{m2}))}{f(\theta_{m2})} + p_2^*(q_2^+) - u(q_2^+,\theta(q_2^+)) - vq(\theta_{m2})] \end{array} \tag{4.15}$$

and

$$\begin{array}{c} \frac{\partial u(q^\star(\theta),\theta)}{\partial q} - \left(\frac{1-F(\theta)}{f(\theta)}\right)\frac{\partial^2 u(q^\star(\theta),\theta)}{\partial q\partial\theta} - v - \frac{c'(q^*(\theta))}{f(\theta)} = 0, \\ \forall\theta \quad \text{with} \quad \theta(q_2^+) > \theta \geq \theta_{m2}. \end{array} \tag{4.16}$$

Before differentiating with respect to q_2^+ it is helpful to decompose the profit function as Champsaur & Rochet (1989) do. Inserting $p_2^*(q_2^+)$ in Π_2 yields

$$\begin{gathered}\Pi_2 = \tfrac{(F(\theta_m)-F(\theta_{m2}))^2}{f(\theta_m)}[u_\theta(q_1^-,\theta_m) - u_\theta(q_2^+,\theta_m)]+ \\ \int_{\theta_{m2}}^{\theta(q_2^+)}[u(q(\theta),\theta)) - \int_{\theta_{m2}}^{\theta} \tfrac{\partial u(q(\tau),\tau)}{\partial\theta} d\tau - u(q_2^+,\theta(q_2^+)) - \\ vq(\theta) + vq_2^+ - \tfrac{c(q(\theta))}{f(\theta)}]]f(\theta)d(\theta).\end{gathered} \tag{4.17}$$

The first term is dependent on q_1^- and q_2^+ while the second term (the integral term) is independent of q_1^-.[94] In the following we denote the integral term by $I(q_2^+)$. This decomposition also shows that q_2^+ is only dependent on q_1^- but not on the other qualities firm 1 produces. The first order condition for q_2^+ is thus given by

$$-\frac{(F(\theta_m)-F(\theta_{m2}))^2}{f(\theta_m)}u_{\theta q}(q_2^+,\theta_m) + \frac{\partial I(q_2^+)}{\partial q} = 0 \tag{4.18}$$

It is now possible to show that $q_2^+ < q_1^-$.

Lemma 4.2

There is always a gap between the quality ranges of firm 1 and firm 2.

Proof

See the Appendix.

This result is different to Champsaur & Rochet (1989). If firms decide simultaneously about their qualities there can be equilibria where the quality ranges overlap and firms make zero profits with these overlapping qualities.[95]

We can get an additional result. Differentiating equation (4.17) with respect to q_1^- we get by using the Envelope Theorem

$$\frac{\partial \Pi_2}{\partial q_1^-} = \frac{(F(\theta_m)-F(\theta_{m2}))^2}{f(\theta_m)}u_{\theta q}(q_1^-,\theta_m) > 0, \tag{4.19}$$

[94] Champsaur & Rochet (1989) call the first term pure differentiation profit and the second term pure segmentation profit.

[95] Champsaur & Rochet (1989) assume that there are no development costs, i.e. $c(q) = 0$. If such development costs exists firms would make losses with overlapping qualities and they may decide not to produce them even in the simultaneous move game. Despite this, in the sequential move game even if $c(q) = 0$ product ranges would never overlap.

where the inequality comes from the Single Crossing Property.

So firm 2's profit is increasing if firm 2 produces a smaller quality range. But this also implies that $\frac{\partial q_2^+}{\partial q_1^-} > 0$ so the lowest quality of firm 1 and the highest quality of firm 2 are strategic complements. Firm 1 will take this into account in its decision of Q_1 in stage 1.

Let us now turn to stage 1. As in Section 4.3 firm 1 has two possibilities either to accommodate entry or to deter entry. Let us look at each case in turn. If fixed costs are low firm 1 finds it optimal to accommodate entry. Decomposing firm 1's profit function in the same way as firm 2's profit function before we get a maximisation problem of

$$\begin{aligned}\max_{q(\theta),q_1^-} \quad \Pi_1 = \tfrac{(1-F(\theta_m))^2}{f(\theta_m)}[u_\theta(q_1^-,\theta_m) - u_\theta(q_2^+,\theta_m)] + \\ \textstyle\int_{\theta(q_1^-)}^1 [u(q(\theta),\theta)) - \int_{\theta(q_1^-)}^{\theta} \tfrac{\partial u(q(\tau),\tau)}{\partial \theta} d\tau - u(q_1^-,\theta(q_1^-)) \\ -vq(\theta) + vq_1^- - \tfrac{c(q(\theta))}{f(\theta)}]]f(\theta)d(\theta).\end{aligned}$$

In the following we call the integral term $I(q_1^-)$.

We get two first order conditions

$$\begin{gathered}\tfrac{\partial u(q^\star(\theta),\theta)}{\partial q} - \left(\tfrac{1-F(\theta)}{f(\theta)}\right)\tfrac{\partial^2 u(q^\star(\theta),\theta)}{\partial q \partial \theta} - v - \tfrac{c'(q^*(\theta))}{f(\theta)} = 0, \\ \forall\theta \qquad \text{with} \qquad \theta(q_1^-) \le \theta \le 1.\end{gathered} \tag{4.20}$$

and

$$-\frac{1-F(\theta_m))^2}{f(\theta_m)}(u_{\theta q}(q_1^-,\theta_m) - u_{\theta q}(q_2^+,\theta_m)\frac{\partial q_2^+}{\partial q_1^-}) + \frac{\partial I(q_1^-)}{\partial q} = 0. \tag{4.21}$$

From the first of these two equations it is apparent that all types $\theta(q_1^-) \le \theta \le 1$ get the same quality as in monopoly because this equation coincides with equation (4.9). All types $\theta_m \le \theta < \theta(q_1^-)$ get a higher quality because they buy q_1^- which is above $q(\theta)$ in the monopoly case given by equation (4.9).

The term $u_{\theta q}(q_2^+,\theta_m)\frac{\partial q_2^+}{\partial q_1^-}$ in equation (4.21) is greater than zero because we know that $\frac{\partial q_2^+}{\partial q_1^-} > 0$. This expresses that with a change in q_1^- firm 1 can change firm 2's reaction in stage 2. In the model of Champsaur & Rochet (1989) this term does not exist because qualities are chosen simultaneously. Thus the incumbent produces a larger quality range than with a simultaneous quality choice to shift firm 2's upper quality downwards.

Let us now look at the case where firm 1 deters entry of firm 2. From equation (4.17) we know that q_2^+ does only depend on q_1^- and from equation (4.19) $\frac{\partial \Pi_2}{\partial q_1^-} > 0$. So if firm 1 wants to deter entry it has to enlarge its quality range compared with the monopoly situation. The intuition is straightforward. There is less space in the product range left for firm 2 because firm 1 has occupied more quality levels and if fixed entry costs F are high enough firm 2 founds it not profitable to enter. Let us denote the quality range Q_1 which deters entry by $Q_1^{ED} = [q_1^{ED}, q_1^+]$ where Q_1^{ED} is given by $\Pi_2(Q_1^{ED}, Q_2(Q_1^{ED})) = 0$.

We are now in a position to describe the equilibrium of the game:

- If $\Pi_1^{ED} > \Pi_1^{duo}$ then $Q_1^* = [q_1^{ED}, q_1^+]$, where $q^*(\theta)$ is given by (4.9), firm 2 does not enter in stage 2 and prices are given by $p^\star(\theta) = u(q^\star(\theta), \theta) - \int_{\theta_m^{ED}}^{\theta} \frac{\partial u(q^\star(\tau), \tau)}{\partial \theta} d\tau$.
- If $\Pi_1^{ED} \leq \Pi_1^{duo}$ then $Q_1^* = [q_1^-, q_1^+]$ where $q^*(\theta)$ is given by (4.20), q_1^- is given by (4.21). Firm 2 enters in stage 2 and produces a quality range of $Q_2^* = [q_2^-, q_2^+]$ where $q^*(\theta)$ is given by (4.16), q_2^+ is given by (4.18) and θ_{m2}^* is given by (4.15). Prices of the firm are given by (4.11), (4.12), (4.13), and (4.14).

This equilibrium can be compared with the monopoly outcome with regard to consumer rent and welfare. First we analyse the case where firm 2 enters. Comparing welfare of market entry with welfare under pure monopoly we get the following proposition.

Proposition 4.6

Welfare in case of market entry is higher than under monopoly if and only if

$$\begin{aligned}\int_{\theta_m^{duo*}}^{\theta(q_1^-)} [u(q_1^-, \theta(q_1^-)) - u(q(\theta), \theta) - vq_1^- + vq(\theta) + \tfrac{c(q(\theta))}{f(\theta)}] f(\theta) d\theta - c(q_1^-) - F \\ + \int_{\theta_{m2}^*}^{\theta_m^{mon*}} [u(q(\theta), \theta) - vq(\theta) - \tfrac{c(q(\theta))}{f(\theta)}] f(\theta) d\theta \\ > \int_{\theta(q_2^+)}^{\theta_m^{duo*}} [u(q(\theta), \theta) - u(q_2^+, \theta(q_2^+)) + vq_1^- - vq(\theta) - \tfrac{c(q(\theta))}{f(\theta)}] f(\theta) d\theta + c(q_2^+).\end{aligned} \tag{4.22}$$

Proof

See the Appendix.

If firm 2 enters some consumers stay with firm 1, others switch to firm 2, while a third group which has not bought in monopoly does now buy from firm 2. Types θ with $\theta(q_1^-) \leq \theta < 1$ stay at firm 1 and get the same quality as in monopoly. This can be seen from the first order conditions of the quality maximisation, (4.9) and (4.20). Consumers between θ_m^{duo*} and $\theta(q_1^-)$ are consuming higher quality in duopoly, namely q_1^-, than in monopoly. This leads to a rise in welfare. But consumers between θ_m^{duo*} and $\theta(q_2^+)$ are now getting a lower quality, q_2^+, than in monopoly because they buy from firm 2. Consumers with a θ below $\theta(q_2^+)$ but above θ_m^{mon*} buy the same quality as before since equations (4.9) and (4.16) coincide. Customer types $\theta_m^{mon*} > \theta \geq \theta_{m2}^*$ have not bought in monopoly but are buying now from firm 2.
Thus we have two sources for a welfare increase, namely that more consumers are served and that types between θ_m^{duo*} and $\theta(q_1^-)$ buy higher quality. But there are two sources for a welfare loss as well, namely that types between $\theta(q_2^+)$ and θ_m^{duo*} buy lower quality and the fixed costs of entry F. The overall effect on welfare is therefore ambiguous.

But we can say more about consumer rent.

Proposition 4.7

Consumer rent in case of market entry is always higher than in monopoly.

Proof

See the Appendix.

The intuition behind this result is simple. In monopoly the marginal consumer θ_m^{mon*} gets zero rent. But in duopoly there is competition for this consumer. Thus he gets a positive utility. But because the incentive compatibility constraints have to be satisfied this leads to an increase of the rents for all types above. Since more consumers are served in duopoly utility for the types below θ_m^{mon*} weakly increases as well.

Now let us turn to the case where firm 1 deters entry. As was already mentioned firm 1 deters entry by enlarging its product line and producing more qualities than in the monopoly case. So more people are served. But since the incentive compatibility constraints must be satisfied this results in lower prices for all consumers who bought already in the monopoly case. Thus we get the following proposition.

Proposition 4.8

If the incumbent can produce a range of qualities welfare and consumer rent in case of entry deterrence are higher than in case of pure monopoly.

The proof is omitted.

This result can be contrasted with the result of Section 4.3 where firms can produce only one quality level. If in that case qualities are strategic complements welfare in case of entry deterrence is lower because the incumbent distorts its quality downwards. In case of price discrimination the lowest quality of the incumbent and the highest one of the entrant are strategic complements. This results in an enlargement of the quality range in the segment of low qualities and increases welfare. The rent for every consumer who buys is higher than in monopoly as well because only the marginal one gets zero utility and prices for the 'old' consumers are lower to prevent them from buying lower qualities.

It is also interesting to investigate under which conditions it is more profitable for an incumbent to deter entry than to accommodate entry.

Proposition 4.9

There exists a threshold value v'. If $v < v'$ the incumbent deters entry, if $v \geq v'$ entry is accommodated.

Proof

The incumbent's profit if entry is deterred is given by

$$\Pi_1^{ED} = \int_{\theta_m(q_1^{ED})}^{1} [u(q(\theta), \theta) - vq(\theta) - \int_{\theta_m^{ED}}^{\theta} \frac{\partial u(q^\star(\tau), \tau)}{\partial \theta} d\tau - \frac{c(q(\theta))}{f(\theta)}] f(\theta) d\theta.$$

If entry is accommodated profit is given by

$$\begin{aligned}\Pi_1^{duo} = \quad & \textstyle\int_{\theta_m^{duo}}^{\theta(q_1^-)} [\frac{1-F(\theta_m)}{f(\theta_m)}][u_\theta(q_1^-, \theta_m) - u_\theta(q_2^+, \theta_m)] f(\theta) d\theta + \\ & \textstyle\int_{\theta(q_1^-)}^{1} [u(q(\theta), \theta)) - \int_{\theta(q_1^-)}^{\theta} \frac{\partial u(q(\tau), \tau)}{\partial \theta} d\tau - u(q_1^-, \theta(q_1^-)) - \frac{c(q(\theta))}{f(\theta)} - \\ & \textstyle vq(\theta) + vq_1^- + \frac{1-F(\theta_m)}{f(\theta_m)} [u_\theta(q_1^-, \theta_m) - u_\theta(q_2^+, \theta_m) - \frac{c(q(\theta))}{f(\theta)}] f(\theta) d(\theta).\end{aligned}$$

Thus entry is deterred if $\Pi_1^{ED} > \Pi_1^{duo}$. Rearranging terms yields

$$\begin{aligned}
&\textstyle\int_{\theta_1^{ED}}^{\theta(q_1^-)}[u(q(\theta),\theta)) - \int_{\theta(q_1^-)}^{\theta}\frac{\partial u(q(\tau),\tau)}{\partial\theta}d\tau - vq(\theta) - \frac{c(q(\theta))}{f(\theta)}]f(\theta)d(\theta)\\
&+\textstyle\int_{\theta(q_1^-)}^{1}[u(q_1^-,\theta(q_1^-)) - vq_1^- - \frac{c(q(\theta))}{f(\theta)} - \frac{1-F(\theta_m)}{f(\theta_m)}[u_\theta(q_1^-,\theta_m) - u_\theta(q_2^+,\theta_m)]f(\theta)d(\theta)\\
&\geq \textstyle\int_{\theta(q_1^-)}^{1}[\frac{1-F(\theta_m)}{f(\theta_m)}][u_\theta(q_1^-,\theta_m) - u_\theta(q_2^+,\theta_m)]f(\theta)d\theta.
\end{aligned}$$

The right hand side is independent of v while the left hand side is strictly decreasing in v. Thus there exists a value v' below which the left hand side is higher and above which the right hand side is higher.

q.e.d.

Thus if v is small the incumbent deters entry. The intuition is that in order to deter entry the incumbent has to enlarge its product line. This is costly for him. But if costs are small it pays the incumbent to bear these costs to enjoy monopoly power afterwards. If instead costs are high this enlargement is not profitable. The incumbent reduces its product line to save on costs but faces competition from the entrant. In the next section we provide two examples that seem to fit very well with the results of our theory.

4.6 Empirical Examples

As in Section 4.4 in this section we present two empirical examples from different industries that seem to resonate well with our theory.

4.6.1 Airline Industry

In Europe deregulation of the air transportation market started in the late 1980's and lasted till 1993. The European Council of Ministers decided to launch three 'liberalisation packages' but only the last one which was launched in 1993 really caused market liberalisation. After this package each airline was allowed to offer services with no restrictions either on prices or on routes.[96]

[96]See Doganis (2001).

One of the most striking developments of this deregulation was the entry of the so called 'no-frills'-airlines or low cost carriers starting in summer 1995 with Ryanair.[97] These low cost carriers offer little or no services but demand prices which are very cheap.[98] Also the low cost carriers mainly fly to secondary airports like Stansted instead of Heathrow in London or Frankfurt-Hahn instead of Frankfurt. So the quality of these low cost carriers is obviously below that of the established airlines.

Usually all established airlines engage in second degree price discrimination. So there can be two possible reactions of the established airlines to this entry threat. They can either expand their quality range to deter entry in the low quality segment or accommodate entry and reduce their quality range to lessen competition. From Proposition 4.9 we would predict that if variable costs are high the reaction would be a contraction of the quality range while if costs are low entry would be deterred by introducing an own low cost carrier. In the airline industry there are examples of both practices.

On long-haul routes the U.K. carrier British Airways focused on the business traveller segment and reduced its quality range.[99] The aim of British Airways was to offer premium services and facilities to charge higher prices and attract a higher number of business travellers. The segment of the leisure travellers was given away to the low cost carriers.

On short-haul routes costs are to some extent cheaper than on long-haul routes. For example, on intercontinental flights by regulation three or four pilots are needed instead of only two as on continental flights and also more board personnel. This results in higher personnel costs. After a long-haul flight an airline is obliged to maintain the aircraft because the engine has worked for a long time and the risk of a crash is increased.[100] This causes fewer capacity utilization of a long-haul plane and therefore

[97] For an extensive overview of low cost carriers in Europe see Gilroy, Lukas, & Volpart (2003).

[98] Recently there was an offer of Ryanair to fly from Salzburg (Austria) to London with return flight for 1 Cent. Although the time of the flight was not attractive it is hard to imagine such an offer five years ago.

[99] See Johnson & Myatt (2003), p. 708.

[100] See Doganis (2001).

higher costs. As predicted by our theory the strategy of many established airlines on short-haul routes was very different than the one on long-haul routes. As an example we take the case of Lufthansa in Germany. In October 2002 the low cost carrier Germanwings was founded which is an affiliate company of Eurowings. In turn, Lufthansa holds 24.9% of Eurowings and has the option to enlarge its share up to 49%.[101] Germanwings operates mainly on routes in Germany which are offered by Lufthansa as well. So the foundation of Germanwings can be seen as an entry deterrence strategy of Lufthansa to occupy the lower market segment and to deter entry of competing low cost carriers.[102]

A different interpretation for the introduction of a low cost carrier by an established airline is given by Johnson & Myatt (2003). They argue that these low cost carriers are introduced as fighting brands to other competitive low cost airlines. Without entry of these competitors the subsidiary would not have been founded because of negative effects on core operations but after entry the low quality segment is opened and the established airline finds it profitable to enter. This might be true in case of GO which was purchased by Easyjet in 2002. But in case of Lufthansa, Germanwings was clearly introduced to deter entry of other low cost airlines and up to now no independent low cost airline has entered the German market.

4.6.2 Brand-Controlled Generics in the Pharmaceutical Market

In Subsection 4.4.2 we gave some evidence that prices of brand-name drugs increased after the entry of generics. However, some patent-holding firms pursued a different strategy namely to introduce a 'branded generic', i.e. the same drug under a different label. These branded generics were introduced shortly before patent expiration and were priced below the prices of the branded drugs. Hollis (2003) reports that the success of these branded generics in Canada was very impressive. While in the 1980's

[101] See Gilroy, Lukas, & Volpert (2003).

[102] As mentioned in the introduction a similar strategy was pursued by British Airways and KLM.

they had only a tiny share of total generic sales this share has grown to 34.6% in 1999 which is an amount in money terms of approximately $500 million.[103] The reason was obviously to deter entry of generic competitors as Scherer (2000) states:

> In this way they (brand-name firms) gained a "first mover advantage" in the generic market, secured the leading share of generic sales, and perhaps thereby discouraged some would-be generic suppliers from entering and driving prices even lower.

However, not all brand name producers introduced these pseudo-generics. In the US a study of the U.S. Congressional Budget Office (1998) reports that among 112 drugs with generic competition only 13 sold its own generic products. But this is in line with the predictions of our theory that not all firms expand their product line to deter entry but only those with low costs. In Canada in the 1990's, Altimed, a joint venture of three brand-name firms, was created. The purpose of this joint venture was to sell branded generics. For this three firms after the joint venture it was easily and cheaply possible to sell generics. In contrast, in the US such a joint venture was not created so brand name pharmaceutical firms have to bear higher costs of introducing their own generics.[104] This might be a reason why many of them found it profitable to accommodate entry of generic competitors.

4.7 Conclusion

The reactions of incumbents on entry threats are very different. Some firms accommodate entry and prune their product line while others deter entry and expand their product line. In the single quality case post-entry prices of incumbents in some markets are higher than pre-entry prices while in other markets they are lower.

This paper analysed a model of vertical product differentiation where an incumbent and an entrant can either produce a single quality or a quality range. We show that in

[103] See Hollis (2003).

[104] An important source for these costs is the fear of destroying the brand name. This fear was not by present in case of Altimed because it emerged as an own brand rapidly.

the single quality case the behaviour of the incumbent depends on the cost function and on the nature of strategic competition (whether qualities are strategic complements or strategic substitutes). We have shown that if qualities are strategic complements the incumbent deters entry by reducing its quality which leads to a welfare loss compared with monopoly. In case of entry accommodation quality might be lowered as well to cause a quality reduction of the entrant and reduce price competition. With low marginal costs quality of the incumbent increases after entry which results in a welfare gain. Also if qualities are strategic substitutes the incumbent increases its quality to differentiate itself from the entrant. If firms can produce a quality range the results are different. To deter entry the incumbent has to enlarge its quality range and this leads to a welfare increase. If entry is accommodated the consequences on welfare are not clear because some consumers buy a higher quality while others buy a lower one.

We have not provided a substantial empirical analysis but have given examples from different industries that seem to fit well with the predictions of our theory. Since we relate the results to firm's cost functions which are observable in many industries we give predictions which are potentially testable.

To conclude the paper we want to discuss some policy implications resulting from our theory. Let us first look at the case where production of a quality range is possible. In this case we find that the effects on welfare are positive in case of entry deterrence and unclear in case of entry accommodation but consumer rent increases in both cases.[105] This leads to the conclusion that deregulation and potential entry have positive consequences in industries in which it is possible to produce a quality range. Thus governments should pursue the policy of free market entry and reduce legal barriers like it was done in the deregulation of the airline industry in the US and Europe.

The effects in the single quality case are not so clear. Whether welfare increases

[105] We have not done a welfare comparison between the case of entry deterrence and entry accommodation. This is an interesting topic for further research because it can provide some policy implications, e.g. if it should be allowed for incumbents to establish a subsidiary brand which produces a downgrade version of the product.

with potential entry depends heavily on the nature of competition. But normally it is hard to assess if products are strategic complements or substitutes. Thus governments should be careful in deregulating such markets because potential competition does not necessarily lead to a welfare gain.

4.8 Appendix

Proof of Proposition 4.1

We first show that the monopolist provides too low quality.

Welfare is given by

$$WF = \int_{\theta_m^{WF}}^{1} [u(q,\theta) - vq] f(\theta) d\theta - c(q).$$

For a given q welfare is maximised if

$$\frac{\partial WF}{\partial \theta_m^{WF}} = u(q, \theta_m^{WF}) - vq = 0. \tag{4.23}$$

In monopoly θ_m^{mon} is given by

$$u(q, \theta_m^{mon}) - vq = \frac{1 - F(\theta_m^{mon})}{f(\theta_m^{mon})} u_\theta(q, \theta_m^{mon}). \tag{4.24}$$

The left hand side of equation (4.24) is greater 0 while it is 0 in equation (4.23). Since $u_\theta(q, \theta_m^{mon}) > 0$ it follows that $\theta_m^{WF} < \theta_m^{mon}$. Thus for a given q the monopolist serves too few consumers.

Maximising welfare with respect to quality yields

$$\frac{\partial \int_{\theta_m^{WF}}^{1} [u(q,\theta) f(\theta) d\theta]}{\partial q} = (1 - F(\theta_m^{WF}))v + c'(q). \tag{4.25}$$

The equivalent formula for the monopolist is

$$(1 - F(\theta_m^{mon}))(u_q(q, \theta_m^{mon}) - v) = c'(q). \tag{4.26}$$

If both qualities were the same we can solve both equations (4.25) and (4.26) for $c(q)$ and get

$$\frac{\partial \int_{\theta_m^{WF}}^{1} [u(q,\theta) f(\theta) d\theta]}{\partial q} - (1 - F(\theta_m^{WF}))v = (1 - F(\theta_m^{mon}))(u_q(q, \theta_m^{mon}) - v).$$

This can be written as

$$\frac{\partial \int_{\theta_m^{mon}}^{1}[(u(q,\theta)-u(q,\theta_m^{mon}))f(\theta)d\theta]}{\partial q}+\frac{\partial \int_{\theta_m^{WF}}^{\theta_m^{mon}}[u(q,\theta)f(\theta)d\theta]}{\partial q}=(F(\theta_m^{mon})-F(\theta_m^{WF}))v.$$

The second term on the left hand side is the increase in utility for all consumers between θ_m^{mon} and θ_m^{WF} from a marginal increase in q. The term on the right hand side is the increase in variable costs if consumers between θ_m^{mon} and θ_m^{WF} are served. Thus the second term on the left hand side must be higher than the right hand side because otherwise it would not have been welfare maximising to serve consumers between θ_m^{mon} and θ_m^{WF}. Since the first term on the left hand side is positive as well we get that the first order condition for q^{WF} is positive at q^{mon}. Thus $q^{WF} > q^{mon}$.

Turning back to the comparison of marginal consumers we have shown in equations (4.23) and (4.24) that if $q^{WF} = q^{mon}$ then $\theta_m^{WF} < \theta_m^{mon}$. But now we know that $q^{WF} > q^{mon}$. A comparison of the left hand sides of (4.23) and (4.24) shows that for $\theta_m^{WF} = \theta_m^{mon}$ the left hand side of (4.23) is higher. But since the right hand side of (4.24) is higher it follows that $\theta_m^{WF} < \theta_m^{mon}$.

q.e.d.

Proof of Proposition 4.3

If $u(q,\theta)=\theta q$ the marginal consumer in the monopoly case is given by $\theta_m q_1^{mon} - p_1^{mon} = 0$ or $\theta_m = p_1^{mon}/q_1^{mon}$. This yields a first order condition for θ_m of

$$1-F(\theta_m^{mon})-f(\theta_m^{mon})(\theta_m^{mon}-v)=0.$$

In duopoly the marginal consumer θ_{m1}^{duo} who is indifferent between buying from firm 1 and buying from firm 2 is given by $\theta_{m1}^{duo}q_1 - p_1 = \theta_{m1}^{duo}q_2 - p_2$ or $\theta_{m1}^{duo} = \frac{p_1-p_2}{q_1-q_2}$. The first order condition for the incumbent is then

$$F\left(\frac{p_1-p_2}{q_1-q_2}\right)-f\left(\frac{p_1-p_2}{q_1-q_2}\right)\frac{p_2-vq_2}{q_1-q_2}-F\left(\frac{p_2}{q_2}\right)-f\left(\frac{p_2}{q_2}\right)\left(\frac{p_2}{q_2}-v\right)=0$$

or

$$1-F(\theta_{m1}^{*})-f(\theta_{m1}^{*})(\theta_{m1}^{*}-\frac{p_2-v}{q_1-q_2})=0. \tag{4.27}$$

Evaluating (4.27) at θ_m^{mon*} yields

$$vq_2 - p_2 < 0.$$

Since the profit function is globally concave in θ this shows that $\theta_m^{mon*} > \theta_{m1}^*$ so more consumers are buying from firm 1 in duopoly.

Now we know that $F(\theta_m^{mon*}) > F(\theta_{m1}^*)$. Thus the term $\frac{1}{F(\theta_m^{mon*})-F(\theta_{m1}^*)}$ on the right hand side in inequality (4.7) is positive. If qualities are strategic substitutes, $\frac{dq_2^*}{dq_1} < 0$, the right hand side of inequality (4.7) is always higher than the left hand side since $v < u_{q_1}(q_1^{mon*}, \theta_{m1}^*)$. It follows that $q_1^* > q_1^{mon*}$.

Up to now we have shown that in duopoly quality of the incumbent is higher than in monopoly and that in duopoly more consumers are served by the incumbent. Because firm 2 is present as well there are some people who are not consuming in monopoly but consume in duopoly from firm 2. So the only source for a welfare loss can be the fixed costs F. But firm 2 only enters if $\Pi_2 > 0$. Since $p_2^* = u(q_2^*, \theta_{m2}^*)$, Π_2 must be lower than the welfare gain because consumers between θ_{m2}^* and θ_{m1}^* still get a rent. Thus the welfare gain which is induced by firm 2 is higher than F. Altogether welfare must have been increased.

q.e.d.

Proof of Proposition 4.5

Let us look at the case θ uniformly distributed, $u(q,\theta) = \theta q$, $v = 0$, and $c(q) = \frac{1}{2}q^2$. Solving the first order conditions in the third stage of the game, equations (4.3) and (4.4), we get

$$p_1 = \frac{2q_1(q_1 - q_2)}{4q_1 - q_2} \qquad p_2 = \frac{q_2(q_1 - q_2}{4q_1 - q_2}.$$

Inserting these values in the first order condition of firm 2 in stage 2, we get from equation (4.5)

$$\frac{2q_1-q_2}{4q_1-q_2} - \frac{q_1-q_2}{4q_1-q_2}\left(\frac{2q_1-q_2}{4q_1-q_2} - \frac{6q_1^2}{(4q_1-q_2)^2}\right)$$
$$+\frac{q_1-q_2}{4q_1-q_2}\left(\frac{-q_1}{4q_1-q_2} + \frac{6q_1^2}{(4q_1-q_2)^2}\right) - q_2 = 0.$$

Simplifying and totally differentiating yields

$$dq_1[64q_1^2(1-q_2)+2q_2^2(2-q_2)+q_1q_2(64q_2-50)]$$
$$= dq_2[64q_1^2(q_1-q_2)+25q_1^2+36q_1q_2^2-4q_2(q_1+q_2^2)].$$

Both terms in brackets are always positive since $q_1 > q_2$. Thus we get $\frac{dq_2}{dq_1} > 0$.

q.e.d.

Proof of Lemma 4.1

The first step in this proof is to replace the incentive compatibility constraint

$$u(q(\theta),\theta)-p(\theta) \geq u(q(\hat{\theta}),\theta)-p(\hat{\theta}) \qquad \forall \theta,\hat{\theta} \geq \theta_m^{mon}$$

by

$$\frac{dq(\theta)}{d\theta} \geq 0 \qquad \forall \theta \in [\theta_m^{mon},1] \tag{4.28}$$

and

$$\frac{\partial u(q(\theta),\theta)}{\partial q}\frac{dq(\theta)}{d\theta}+\frac{dp(\theta)}{d\theta}=0 \qquad \forall \theta \in [\theta_m^{mon},1]. \tag{4.29}$$

This step is a standard one in the theory of adverse selection and the proof of it can be found in many textbooks. See e.g. Fudenberg & Tirole (1991, chapter 7) or Schmidt (1995, chapter 4).

We know that $U(\theta)=u(q(\theta),\theta)-p(\theta)$.

Using (4.29) we get

$$\frac{dU(\theta)}{d\theta}=\frac{\partial u(q(\theta),\theta)}{\partial q}\frac{dq(\theta)}{d\theta}+\frac{\partial u(q(\theta),\theta)}{\partial \theta}+\frac{dp(\theta)}{d\theta}=\frac{\partial u(q(\theta),\theta)}{\partial \theta}.$$

Integrating both sides of this equation yields

$$U(\theta)=U(\theta_m^{mon})+\int_{\theta_m^{mon}}^{\theta}\frac{\partial u(q(\tau),\tau)}{\partial \theta}d\tau.$$

Because firm 1 wants to maximise the payoff from consumers, the participation constraint must bind for $\theta=\theta_m^{mon}$, which implies $U(\theta_m^{mon})=0$ and therefore

$$U(\theta)=\int_{\theta_m^{mon}}^{\theta}\frac{\partial u(q(\tau),\tau)}{\partial \theta}d\tau.$$

Equation (4.8) follows.

Now we have determined the prices for a given quality range. In the first stage the marginal consumer θ_m^{mon} and the quality $q(\theta)$ assigned to each type has to be determined.

The maximisation problem of firm 1 can be written as

$$\begin{array}{ll} \max_{q(\theta),\theta_m^{mon}} & \int_{\theta_m^{mon}}^1 [u(q(\theta),\theta) - vq(\theta) - \int_{\theta_m^{mon}}^{\theta} \frac{\partial u(q(\tau),\tau)}{\partial \tau} d\tau] f(\theta) d\theta - \int_{\theta_m^{mon}}^1 c(q(\theta)) d\theta \\ \text{s.t.} & \frac{dq(\theta)}{d\theta} \geq 0. \end{array}$$

After integration by parts we get

$$\max_{q(\theta),\theta_m^{mon}} \quad \int_{\theta_m^{mon}}^1 [u(q(\theta),\theta) - vq(\theta) - \frac{1-F(\theta)}{f(\theta)} \frac{\partial u(q(\theta),\theta)}{\partial \theta} - \frac{c(q(\theta))}{f(\theta)}] f(\theta) d\theta. \quad (4.30)$$

Pointwise differentiation with respect to $q(\theta)$ yields (4.9).

Differentiation with respect to θ_m^{mon} yields (4.10).

Because of Assumptions in $A1, A2$, and $A3$ all second order conditions and condition (4.28) are satisfied.

q.e.d.

Proof of Lemma 4.2

From equation (4.18) we know that the first order condition for q_2^+ is given by

$$-\frac{(F(\theta_m) - F(\theta_{m2}))^2}{f(\theta_m)} u_{\theta q}(q_2^+, \theta_m) + \frac{\partial I(q_2^+)}{\partial q} = 0.$$

We have to show that the derivative of the profit function with respect to q_2^+ is negative at $q_2^+ = q_1^-$. Integrating by parts and differentiating the term in the integral, $I(q_2^+)$, with respect to q_2^+ we get

$$\begin{aligned} \frac{\partial I(q_2^+)}{\partial q_2^+} &= -[u_q(q_2^+, \theta(q_2^+)) - v] f(\theta(q_2^+)) - u_{\theta q}(q_2^+, \theta(q_2^+))[F(\theta(q_2^+)) - F(\theta_{m2})] - \frac{c'(q_2^+)}{f(\theta)} \\ &= \frac{\partial}{\partial \theta}([-u_q(q_2^+, \theta(q_2^+)) + v][F(\theta(q_2^+)) - F(\theta_{m2})]) - \frac{c'(q_2^+)}{f(\theta)}. \end{aligned}$$

Thus

$$\begin{aligned} &\frac{\partial \Pi_2(q_1^-, q_2^+ = q_1^-)}{\partial q_2^+} = \\ &-[u_q(q_2^+, \theta(q_2^+)) - v][F(\theta(q_2^+)) - F(\theta_{m2})] - \frac{(F(\theta_m) - F(\theta_{m2}))^2}{f(\theta_m)} u_{\theta q}(q_2^+, \theta_m) - \frac{c'(q_2^+)}{f(\theta)} \\ &= \frac{(F(\theta(q_2^+)) - F(\theta_{m2}))^2}{f(\theta(q_2^+))} u_{\theta q}(q_2^+, \theta(q_2^+)) - \frac{(F(\theta_m) - F(\theta_{m2}))^2}{f(\theta_m)} u_{\theta q}(q_2^+, \theta_m), \end{aligned}$$

where the first equality follows from the fact that

$$\int_{\theta_m}^{\theta(q_2^+)} \frac{\partial}{\partial\theta}([-u_q(q_2^+,\theta(q_2^+))+v][F(\theta(q_2^+))-F(\theta_{m2})])f(\theta)d\theta = [-u_q(q_2^+,\theta(q_2^+))+v][F(\theta(q_2^+))-F(\theta_{m2})]$$

and the second equality follows from equation (4.16).
We know that $\theta(q_2^+) < \theta_m$ so it remains to check that $u_{\theta q}\frac{(F(\theta)-F(\theta_{m2}))^2}{f(\theta)}$ is increasing in θ.
We have

$$\frac{\partial}{\partial\theta}[u_{\theta q}\frac{(F(\theta)-F(\theta_{m2}))^2}{f(\theta)}] = \\ u_{\theta q}[2(F(\theta)-F(\theta_{m2})) - \frac{(F(\theta)-F(\theta_{m2}))^2 f'(\theta)}{(f(\theta))^2}] + u_{\theta\theta q}(\frac{(F(\theta)-F(\theta_{m2}))^2}{f(\theta)}) > 0$$

because of Assumptions $A2$ and $A3$.

q.e.d.

Proof of Proposition 4.6

We first show that $\theta_{m2} < \theta_m^{mon}$.
θ_{m2} is given by the first order condition

$$(F(\theta_m^*) - F(\theta_{m2}))(\frac{\partial u(q^\star(\theta_m^\star),\theta_m^\star)}{\partial\theta_m^\star} - \frac{f(\theta_{m2})}{f(\theta_m^*)}[u_\theta(q_1^-,\theta_m^*) - u_\theta(q_2^+,\theta_m)] = \\ f(\theta_{m2})[u(q(\theta_{m2}),\theta_{m2}) - \frac{c(q(\theta_{m2}))}{f(\theta_{m2})} + p_2(\theta_m^*) - u(q_2^+,\theta(q_2^+)) - vq(\theta_{m2})].$$

θ_m^{mon} is given by the first order condition

$$[u(q^{mon\star}(\theta_m^{mon\star}),\theta_m^{mon\star}) - vq^{mon\star}(\theta_m^{mon\star})]f(\theta_m^{mon\star}) - c(q^{mon*}(\theta^*)) \\ = (1-F(\theta_m^{mon\star}))\frac{\partial u(q^{mon\star}(\theta_m^{mon\star}),\theta_m^{mon\star})}{\partial\theta_m^{mon\star}}.$$

Inserting $\theta_m^{mon\star}$ in the first order condition for θ_{m2} yields

$$-\frac{f(\theta_m^{mon\star})}{f(\theta_m)}[u_\theta(q_1^-,\theta_m) - u_\theta(q_2^+,\theta_m)] \\ < f(\theta_m^{mon\star})p_2(\theta_m) - u(q_2^+,\theta(q_2^+)) + (1-F(\theta_m))\frac{\partial u(q^{mon\star}(\theta_m^{mon\star}),\theta_m^{mon\star})}{\partial\theta_m^{mon\star}}.$$

Thus $\theta_{m2} < \theta_m^{mon}$, more consumers are served after entry than in pure monopoly.

Now let us turn to the welfare comparison. Consumers with $\theta(q_1^-) \leq \theta \leq 1$ and with $\theta_m^{mon\star} \leq \theta < \theta(q_2^+)$ get the same quality under monopoly and under duopoly. This is obvious because equations (4.9) and (4.16) and also equations (4.9) and (4.20) coincide. Consumers between θ_m^{duo} and $\theta(q_1^-)$ consume a higher quality in duopoly, namely q_1^-, than in monopoly, while consumers between $\theta(q_2^+)$ and θ_m^{duo} consume a lower one, namely (q_2^+). Therefore we have that welfare under market entry is only higher if

$$\begin{gathered}\int_{\theta_m^{duo}}^{\theta(q_1^-)}[u(q_1^-,\theta(q_1^-))-vq_1^-]f(\theta)d\theta-c(q_1^-)-F\\+\int_{\theta(q_2^+)}^{\theta_m^{duo}}[u(q_2^+,\theta(q_2^+))-vq_2^+]f(\theta)d\theta-c(q_2^+)+\int_{\theta_{m2}^*}^{\theta_m^{mon}}[u(q(\theta),\theta)-vq(\theta)-\tfrac{c(q(\theta))}{f(\theta)}]f(\theta)d\theta\\>\int_{\theta(q_2^+)}^{\theta(q_1^-)}[u(q(\theta),\theta)-vq(\theta)-\tfrac{c(q(\theta))}{f(\theta)}]f(\theta)d\theta.\end{gathered}$$

Rearranging terms yields equation (4.22).

q.e.d.

Proof of Proposition 4.7

All types $\theta(q_1^-)<\theta\leq 1$ get the same quality in duopoly than monopoly but have to pay a price of

$$p_1^{duo}(\theta)=u(q(\theta),\theta)-\int_{\theta(q_1^-)}^{\theta}\frac{\partial u(q(\tau),\tau)}{\partial\theta}d\tau+p_1(\theta_m)-u(q_1^-,\theta(q_1^-)).$$

This can also be written as $p_1^{duo}(\theta)=p_1^{mon}(\theta)+p_1(\theta_m^{duo})-u(q_1^-,\theta(q_1^-))<p_1^{mon}(\theta)$. Thus the price in duopoly is lower than in monopoly.

Types $\theta\leq\theta(q_2^+)$ get the same quality in duopoly as in monopoly if they are served in both cases. The price under duopoly is $p_2^{duo}(\theta)=p_1^{mon}(\theta)+p_2(\theta_m^{duo})-u(q_2^+,\theta(q_2^+))<p_1^{mon}(\theta)$ and thus below the price in monopoly. Since in duopoly more consumer types are served as well, the consumer rent for types $\theta\leq\theta(q_2^+)$ is weakly higher in duopoly than in monopoly.

The utility for types $\theta(q_1^-)\geq\theta>\theta(q_2^+)$ in monopoly is given by $\int_{\theta_m^{mon}}^{\theta}\frac{\partial u(q(\tau),\tau)}{\partial\theta}d\tau$. with θ increasing utility is increasing by $\frac{\partial u(q(\theta),\theta)}{\partial\theta}$. In duopoly for types $\theta>\theta(q_2^+)$ utility is $u(q_2^+,\theta(q_2^+))-p_2(q_2^+)$, and for types $\theta(q_1^-)\geq\theta$ utility is given by $u(q_1^-,\theta(q_1^-))-p_1(q_1^-)$. Starting at type $\theta(q_2^+)$, if θ increases utility increases by $u_\theta(q_2^+,\theta)$ up to θ_m^{duo} and by $u_\theta(q_1^-,\theta)$ from θ_m^{duo} up to $\theta(q_1^-)$. But since we know that $U(\theta(q_2^+))^{duo}>U(\theta(q_2^+))^{mon}$ and $U(\theta(q_1^-))^{duo}>U(\theta(q_1^-))^{mon}$ for all types in between $\theta(q_2^+)$ and $\theta(q_1^-)$ utility in duopoly must be higher than in monopoly as well. Thus consumer rent increases.

q.e.d.

Chapter 5

Concluding Remarks

This chapter provides a few concluding remarks on the three models presented in this thesis. In the first part of the chapter I point out some limitations of the models and a few interesting issues that are not addressed and give a direction for future research. In the second part I present some ideas how the models can be interrelated.

In the model on commodity bundling I pointed out that the correlation of reservation prices is the crucial variable in determining profits not only in monopoly but in duopoly as well, yet, with opposite consequences. Such an analysis was not done before and the model can be seen as a first step in this direction. The correlation was modelled in a special way to keep the model tractable. It would be interesting to see if the model can be generalised. This could be done in a similar framework by dropping the assumption of a one-to-one mapping of locations and allowing for uncertainty as described in Section 2.8. But one might also find a different framework than the product differentiation one which is appropriate to model the consequences of different correlations on competition with bundling. However, my intuition is that the results of the model are quite robust. The reason is that the bundle always makes the sum of consumers' valuations more similar. While this helps a monopolist to extract more consumer rent it leads to intensified competition in duopoly.

The field of two-sided markets is a relatively new one in economics and there are several new and interesting ideas for further research. In my model advertisers are local monopolists on the product market and consumers do not get a positive utility from getting aware of a new product because firms can extract all consumer rent. This assumption greatly simplifies the analysis and all models in this literature use it. But it does not allow for product market competition between advertisers. This might be a fruitful direction for further research. In this case one side of the market, the advertisers, interact via two channels (on platforms and in the product market) and this may lead to new results.[107] A second aspect which is not considered in my model is that platforms can invest their revenues from advertising in the content they provide. So platforms that broadcast advertising which yields higher revenues can therefore be "qualitatively" better.[108] To the best of my knowledge, no model exists that addresses this issue. Such a model may yield the realistic result of an asymmetric equlibrium in which one platform has little advertising earnings and provides low quality while the other platform's advertising yields higher revenues and enables it to provide higher quality.

In the model on vertical product differentiation the welfare effects of allowing for price discrimination or not are studied. I consider a special comparison namely unit pricing versus second-degree price discrimination. But it would also be interesting to study the effects of other pricing regimes. As an example consider the case of third-degree price discrimination, e.g. firms can distinguish between students and non-students. Then it might be possible that the entrant chooses to enter only one market, namely the non-student one where consumers have higher valuations. But if third-degree price discrimination were not possible the incumbent could prevent entry because it produces a middle quality range as in my model. In this case the ability to price discriminate hurts the incumbent and benefits the entrant.

Let us now look at possible interrelations between the models.

Consider for example the bundling practice of US telephone companies which sell long

[107] An attempt in this direction is the paper by Gal-Or & Dukes (2003) who model this interaction in a radio-station example. However, the problem in their model is that although listeners get informed by a commercial about cheap prices and derive positive utility from it, this is not represented in their utility function. Thus their model is inconsistent in this respect.

[108] Quality here means that this platform's content attracts more users.

distance service and internet access in a package. Verizon offers a bundle which is obviously designed for lower value consumers (”Verizon Freedom Options” at a price of $39.95) but also another one for higher value consumers (”Verizon Freedom with DSL” at a price of $69.90). Thus it engages in second-degree price discrimination with bundles. So there is a connection between the models of chapter two and four. We have vertical differentiation between bundles but horizontal differentiation between the goods of which the bundles consist of. Since correlation matters in case of only horizontal differentiation one might guess that it should also matter in case of vertical product differentiation, e.g. if a consumer who values internet access highly also has a high valuation for long distance services. So a detailed analysis might reveal under which circumstances it is profitable to offer vertical differentiated bundles both in monopoly and in duopoly.[109]

There can also be an interrelation between the models in chapter two and three. Consider the market for credit cards. In this market Visa or MasterCard offer in addition to their usual credit card (e.g. Visa classic) some upgrade cards (e.g. Visa gold) that are only attractive for consumers with higher income. In contrast to the standard analysis of second-degree price discrimination this practice now has consequences for the other side of the market, here the merchants. Several questions arise. What are the effects on prices on the other side of the market? Will they be lowered (increased) in case of positive (negative) externalities in order to make higher profits with the side where price discrimination is possible? What are the welfare consequences of this practice? Is it easier to deter entry in this case or not? All these questions are of both theoretical and practical relevance. I am convinced that a lot of fruitful research can be done in this area.

[109]For a starting point of research in the direction of monopoly see Armstrong (1999).

References

D. Abreu (1988): "On the Theory of Infinitely Repeated Games with Discounting," *Econometrica*, 56, 383-396.

W.A. Adams and J.L. Yellen (1976): "Commodity Bundling and the Burden of Monopoly," *Quarterly Journal of Economics*, 91, 475-498.

S.P. Anderson and S. Coate (2003): "Market Provision of Broadcasting: A Welfare Analysis," Working Paper, University of Virginia.

S.P. Anderson, A. de Palma, and J.-F. Thisse (1992): *Discrete Choice Theory of Product Differentiation*, Cambridge, Massachusetts: MIT Press.

S.P. Anderson, J.K. Goeree, and R. Ramer (1997): "Location, Location, Location," *Journal of Economic Theory*, 77, 102-127.

S.P. Anderson and L. Leruth (1993): "Why Firms Prefer not to Price Discriminate via Mixed Bundling," *International Journal of Industrial Organization*, 11, 49-61.

M. Armstrong (1999): "Price Discrimination by a Many-Product Firm," *Review of Economic Studies*, 66, 151-168.

M. Armstrong (2004): "Competition in Two-Sided-Markets," Working Paper, University College London.

S.-H. Bae and J.P. Choi (2003): "A Model of Piracy," Working Paper, Michigan State University.

K. Bagwell (2003): "The Economic Analysis of Advertising," Working Paper, Columbia University.

J. Bain (1956): *Barriers to New Competition*, Cambridge: Cambridge University Press.

D.S. Banerjee (2003): "Software Piracy: A Strategic Analysis and Policy Instruments," *International Journal of Industrial Organisation*, 21, 97-127.

P. Barros, H.J. Kind, and L. Sorgard (2002): "Media Competition when the Audience Dislikes Advertising: A Theory of Vertical Alliances on the Internet," Working Paper, Norwegian School of Economics and Business Administration, Bergen.

M.R. Baye and J. Morgan (2001): "Information Gatekeepers on the Internet and the Competitiveness of Homogeneous Product Markets," *American Economic Review*, 91, 454-474.

J. Bertrand (1883): "Theorie Mathematique de la Richesse Sociale," *Journal des Savants*, 67, 499-508.

G. Bonanno (1987): "Location Choice, Product Proliferation, and Entry Deterrence," *Review of Economic Studies*, 54, 37-45.

J. Bulow, J. Geanakoplos, and P. Klemperer (1985): "Multimarket Oligopoly: Strategic Substitutes and Complements," *Journal of Political Economy*, 93, 488-511.

A. Cabrales (2003): "Pharmaceutical Generics, Vertical Product Differentiation, and Public Policy," Working Paper, Universitat Pompeu Fabra, Barcelona.

J. Carbajo, D. deMeza, and D.J. Seidmann (1990): "A Strategic Motivation for Commodity Bundling," *Journal of Industrial Economics*, 38, 283-298.

R.E. Caves, M. Whinston, and M. Hurwitz (1991): "Patent Expiration, Entry, and Competition in the U.S. Pharmaceutical Industry," *Brookings Papers on Economic Activity: Microeconomics*, 1-48.

E.H. Chamberlin (1933): *The Theory of Monopolistic Competition*, Cambridge: Harvard University Press.

P. Champsaur and J.-C. Rochet (1989): "Multiproduct Duopolists," *Econometrica*, 57, 533-557.

Y. Chen (1997): "Equilibrium Product Bundling," *Journal of Business*, 70, 85-103.

J.P. Choi (1996): "Preemptive R&D, Rent Dissipation, and the Leverage Theory," *Quarterly Journal of Economics*, 111, 1153-1183.

A. Cournot (1838): "Recherches sur les Principes Mathematiques de la Theorie des Richesses," English Edition, *Researches into the Mathematical Principles of the Theory of Wealth*, N.Bacon (ed.), New York: Macmillan.

P. Dasgupta and E. Maskin (1986): "The Existence of Equilibrium in Discontinuous Economic Games," *Review of Economic Studies*, 53, 1-26.

C. D'Aspremont, J.J. Gabszewicz, and J.-F. Thisse (1979): "On Hotelling's Stability in Competition," *Econometrica*, 47, 1145-1150.

V. Denicolo (2000): "Compatibility and Bundling with Generalist and Specialist Firms," *Journal of Industrial Economics*, 48, 177-188.

R. Doganis (2001): *Airline Business in the 21st Century*, New York: Routledge.

S. Donnenfeld and S. Weber (1992): "Vertical Product Differentiation with Entry," *International Journal of Industrial Organisation*, 10, 449-472.

S. Donnenfeld and S. Weber (1995): "Limit Qualities and Entry Deterrence," *RAND Journal of Economics*, 26, 113-130.

A. Dukes and E. Gal-Or (2003): "Negotiations and Exclusivity Contracts for Advertising," *Marketing Science*, 22, 222-245.

B.C. Eaton and R.G. Lipsey (1989): "Product Differentiation," in: *The Handbook of Industrial Organization*, I, R. Schmalensee & R.D. Willig (ed.), Amsterdam, North-Holland: Elsevier Science Publishers, B.V.

F.Y. Edgeworth (1925): "The Pure Theory of Monopoly," *Papers Relating to Political Economy*, 1, 111-142.

D.S. Evans (2004): "The Antitrust Economics of Two-Sided Markets," *Yale Journal of Regulation*, forthcoming.

J. Farrell and P. Klemperer (2001): "Coordination and Lock-In: Competition with Switching Costs and Network Effects," Working Paper, Nuffield College, Oxford University, forthcoming in: *The Handbook of Industrial Organization*, III.

J. Farrell and E. Maskin (1986): "Renegotiation in Repeated Games," *Games and Economic Behavior*, 1, 327-360.

J. Ferrando, J.J. Gabszewicz, D. Laussel, and N. Sonnac (2003): "Two-Sided Network Effects and Competition: An Application to Media Industries," Working Paper, Universite Catholique de Louvain.

R.G. Frank and D.S. Salkever (1997): "Generic Entry and the Pricing of Pharmaceuticals," *Journal of Ecocomics & Management Strategy*, 6, 75-90

D. Fudenberg and E. Maskin (1986): "The Folk Theorem in Repeated Games with Discounting or with Incomplete Information," *Econometrica*, 54, 533-556.

D. Fudenberg and J. Tirole (1984): "The Fat Cat Effect, the Puppy Dog Ploy and the Lean and Hungry Look," *American Economic Review, Papers and Proceedings*, 74, 361-368.

D. Fudenberg and J. Tirole (1991): *Game Theory*, Cambridge, Massachusetts: M.I.T. Press.

J. J. Gabszewicz and J.-F. Thisse (1979): "Price Competition, Quality, and Income Disparities," *Journal of Economic Theory*, 20, 340-359.

J. J. Gabszewicz and J.-F. Thisse (1986): "Spatial Competition and the Location of Firms," in: *Location Theory*, R. Arnott (ed.), London: Harwood Academic Press.

E. Gal-Or and A. Dukes (2003): "On the Profitability of Media Mergers," Working Paper, Katz Graduate School of Business, Pittsburgh University.

D. Gillen (2003): "Predation is Alive and Well in Canadian Skies: Myth or Reality?," forthcoming in: *Competition versus Predation in Aviation Markets: A Survey of Experience on North America, Europe and Australia*, P. Forsyth, D. Gillen, O.G. Mayer & H.-M. Niemeier (ed.), Ashgate Publishers.

B.M. Gilroy, E. Lukas, and T. Volpert (2003): "The European "No-Frills"-Aviation Market: Current and Future Developments," forthcoming in: *Competition versus Predation in Aviation Markets: A Survey of Experience on North America, Europe and Australia*, P. Forsyth, D. Gillen, O.G. Mayer & H.-M. Niemeier (ed.), Ashgate Publishers.

H.G. Grabowski and J.M. Vernon (1992): "Brand Loyality, Entry, and Price Competition in Pharmaceuticals after the 1984 Drug Act," *Journal of Law and Economics*, 35, 331-350.

E.J. Green and R.H. Porter (1984): "Noncooperative Collusion under Imperfect Price Information," *Econometrica*, 52, 87-100.

J. Henkel (2003): "The 1.5th Mover Advantage," *RAND Journal of Economics*, 33, 156-170.

A. Hollis (2003): "The Anti-Competitive Effects of Brand-Controlled "Pseudo-Generics" in the Canadian Pharmaceutical Market," *Canadian Public Policy*, 24, 21-32.

H. Hotelling (1929): "Stability in Competition," *Economic Journal*, 39, 41-57.

A. Irmen and J.-F. Thisse (1998): "Competition in Multi-Characteristic Spaces: Hotelling Was Almost Right," *Journal of Economic Theory*, 78, 76-102.

J.P. Johnson and D.P. Myatt (2003): "Multiproduct Quality Competition: Fighting Brands and Product Line Pruning," *American Economic Review*, 93, 748-774.

K. Judd (1985): "Credible Spatial Preemption," *RAND Journal of Economics*, 16, 153-166.

N. Kaldor (1935): "Market Imperfections and Excess Capacity," *Economica*, 2, 33-50.

M.L. Katz and C. Shapiro (1994): "Systems Competition and Network Effects," *Journal of Economic Perspectives*, 8, 93-115.

G. Keller and S. Rady (2003): Price Dispersion and Learning in a Dynamic Differentiated Goods-Duopoly," *RAND Journal of Economics*, 34, 139-166.

H.J. Kind, T. Nilssen, and L. Sorgard (2003): "Advertising on TV: Under- or Overprovision?," Working Paper, Norwegian School of Economics and Business Administration, Bergen.

K.C. Lancaster (1966): "A New Approach to Consumer Theory," *Journal of Political Economy*, 74, 132-157.

K.S. Lee, G.H. Lim, and S.J. Tan (2001): "Gray Marketing - A Low Cost Market Penetration Strategy for Entrepreneurs," in: *Research at the Marketing/ Entrepreneur Interface*, G.E. Hills, W.-S. Siu & D. Malewicki (ed.), Institute for Entrepreneural Studies, Chicago.

R. Levitan and M. Shubik (1972): "Price Duopoly and Capacity Constraints," *International Economic Review*, 13, 111-122.

E. Maskin and J. Tirole (1987): "A Theory of Dynamic Oligopoly III: Cournot Competition," *European Economic Review*, 31, 947-968.

E. Maskin and J. Tirole (1988): "A Theory of Dynamic Oligopoly I: Overview and Quantity Competition with Large Fixed Costs," *Econometrica*, 56, 540-570.

E. Maskin and J. Tirole (1988): "A Theory of Dynamic Oligopoly II: Price Competition, Kinked Demand Curves, and Edgeworth Cycles," *Econometrica*, 56, 541-600.

F. Mathewson and R. Winter (1997): "Tying as a Response to Demand Uncertainty," *RAND Journal of Economics*, 28, 566-583.

C. Matutes and P. Regibeau (1992): "Compatibility and Bundling of Complementary Goods in Duopoly," *Journal of Industrial Economics*, 40, 37-54.

R.P. McAfee, J. McMillan, and M.D. Whinston (1989): "Multiproduct Monopoly, Commodity Bundling, and Correlation of Values," *Quarterly Journal of Economics*, 104, 371-383.

M. Mussa and S. Rosen (1978): "Monopoly and Product Quality," *Journal of Economic Theory*, 18, 301-317.

B. Nalebuff (2004): "Bundling as an Entry Barrier," *Quarterly Journal of Economics*, 119, 159-187.

J.F. Nash (1950): "Equilibrium Points in N-Person Games," *Proceedings of the National Academy of Sciences*, 36, 48-49.

D. Neven (1986): "On Hotelling's Competition with Non-Uniform Consumer Distribution," *Economics Letters*, 21, 121-126.

R. Porter (1983): "A Study of Cartel Stability, the Joint Executive Committee, 1880-1886," *Bell Journal of Economics*, 14, 301-314.

R. Rees (1993): "Collusive Equlibrium in the Great Salt Duopoly," *Economic Journal*, 103, 833-848.

J.-C. Rochet and J. Tirole (2003): "Platform Competition in Two-Sided-Markets," *Journal of the European Economic Association*, 1, 990-1029.

U. Ronnen (1991): "Minimum Quality Standards, Fixed Costs, and Competition," *RAND Journal of Economics*, 22, 490-504.

J.J. Rotemberg and G. Saloner (1986): "A Supergame-Theoretic Model of Price Wars During Booms," *American Economic Review*, 76, 390-407.

S.C. Salop (1979): "Monopolistic Competition with Outside Goods," *Bell Journal of Economics*, 10, 141-156.

F.M. Scherer (1996): *Industry Structure, Strategy, and Public Policy*, New York: Harper-Collins.

F.M. Scherer (2000): "The Parmaceutical Industry," in: *Handbook of Health Economics*, A.J. Culyer & J.P. Newhouse (ed.), Amsterdam, North-Holland: Elsevier Science Publishers, B.V.

R. Schmalensee (1984): "Gaussian Demand and Commodity Bundling," *Journal of Business*, 57, 211-230.

K.M. Schmidt (1995): "Contract Theory," Lecture Notes, University of Munich.

J.A. Schumpeter (1954): *History of Economic Analysis*, New York: Oxford University Press.

D.J. Seidmann (1991): "Bundling as a Facilitating Device: A Reinterpretation of Leverage Theory," *Economica*, 58, 491-499.

A. Shaked and J. Sutton (1982): "Relaxing Price Competition through Product Differentiation," *Review of Economic Studies*, 49, 3-13.

C. Shapiro (1989): "Theories of Oligopoly Behavior", in: *The Handbook of Industrial Organization*, I, R. Schmalensee & R.D. Willig (ed.), Amsterdam, North-Holland: Elsevier Science Publishers, B.V.

D.F. Spulber (1989): "Product Variety and Competitive Discounts," *Journal of Economic Theory*, 48, 510-525.

L.A. Stole (1995): "Nonlinear Pricing and Oligopoly," *Journal of Economics & Management Strategy*, 4, 529-562.

L.A. Stole (2003): "Price Discrimination and Imperfect Competition," Working Paper, University of Chicago, forthcoming in: *The Handbook of Industrial Organization*, III.

T. Tabuchi and J.-F. Thisse (1995): "Asymmetric Equilibria in Spatial Competition," *International Journal of Industrial Organization*, 13, 213-227.

J.-F. Thisse and X. Vives (1988): "On the Strategic Choice of Spatial Price Policy," *American Economic Review*, 78, 122-137.

US Congressional Budget Office (1998): *How Increased Competition from Generic Drugs has Affected Prices and Returns in the Pharmaceutical Industry*, Washington.

US Federal Trade Commission (1985): *Statistical report: Annual Line of Business Report: 1977*, Washington.

H.R. Varian (1989): "Price Discrimination," in: *The Handbook of Industrial Organization*, I, ed.: R. Schmalensee / R.D. Willig, Amsterdam, North-Holland: Elsevier Science Publishers, B.V.

M.D. Whinston (1990): "Tying, Foreclosure, and Exclusion," *American Economic Review*, 80, 837-859.

J. Wright (2003): "Optimal Card Payment Systems," *European Economic Review*, 47, 587-612.

J. Wright (2004): "The Determinants of Optimal Interchange Fees in Payment Systems," *Journal of Industrial Economics*, forthcoming.

Lebenslauf von Markus Reisinger

Persönliche Daten

Geburtsdatum:	18.10.1974
Geburtsort:	München

Ausbildung

09/1981 - 07/1985:	Grundschule an der Hans-Kerle-Straße, Gröbenzell
09/1985 - 07/1994:	Gymnasium Gröbenzell; Abschluß: Abitur; Notendurchschnitt: 1.9
11/1995 - 07/2000:	Grund- und Hauptstudium der Volkswirtschaftslehre, Ludwig-Maximilians-Universität München; Abschluß: Diplom; Note: 1.23
06/2000 - 07/2004:	Doktorand und wissenschaftlicher Mitarbeiter am Seminar für dynamische Modellierung, Ludwig-Maximilians-Universität München; Abschluß: Dissertation; Gesamtnote: summa cum laude (Alumni-Preis für die beste Dissertation im Jahr 2004 an der Volkswirtschaftlichen Fakultät)

München, den 20. Dezember 2004