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Topological Map Induction using Neighbourhood Information of Places

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Abstract In topological mapping, perceptual aliasing can cause different places to appear indistinguishable to the robot. In case of severely corrupted or non-available odometry information, topological mapping is difficult as the robot is challenged with the loop-closing problem; that is to determine whether it has visited a particular place before.

In this article we propose to use neighbourhood information to disambiguate otherwise indistinguishable places. Using neighbourhood information for place disambiguation is an approach that neither depends on a specific choice of sensors nor requires geometric information such as odometry. Local neighbourhood information is extracted from a sequence of observations of visited places.

In experiments using either sonar or visual observations from an indoor environment the benefits of using neighbourhood clues for the disambiguation of otherwise identical vertices are demonstrated. Over 90% of the maps we obtain are isomorphic with the ground truth. The choice of the robot's sensors does not impact the results of the experiments much.

Keywords Autonomous mobile robots · topological mapping · colour histograms · sonar · neighbourhood information · perceptual aliasing · stochastic local search · particle filter

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1 Introduction

Successful navigation in its environment is an essential capability for an autonomous mobile robot to accomplish its mission. In robotics, *successful navigation* means reaching a destination from a starting location. In order to navigate sucessfully, robots commonly use a *map* as an internal representation of the spatial layout of its environment. Using a map enables a robot to plan and follow a trajectory into areas which are beyond its perception range at its current position. Autonomously building maps as spatial representations of the environment from sensor data is considered as one of the most important problems in the quest to build truly autonomous robots (Thrun, 2002).

Two major approaches to represent a robot's environment have emerged: *metric* and *topological* (Burgard and Hebert, 2008; Meyer and Filliat, 2003). The former models the surroundings using a metric map which preserves the relative spatial distances between the objects in the environment. The metric representation is particularly well suited for geometrically accurate continuous localisation and precise trajectory planning with respect to the map's coordinate system (Filliat and Meyer, 2003; Meyer and Filliat, 2003).

Topological maps are graphical models which represent the topology of an environment in an abstract and concise manner. According to Remolina and Kuipers (2004) there is no consensus about what exactly topological maps are and how they should be built. However, most of the topological map descriptions share common elements such as the representation of the environment through discrete *key places* and the *connectivity* relations among the places contained in the map (Marinakis and Dudek, 2010). A place is described with a feature vector derived from sensory data called a *fingerprint*. In this article, we define a place as a small connected region on the ground where the robot's perceptions are essentially similar.

One of the key challenges in inducing topological maps is to deal with the *perceptual aliasing* problem (Thrun, 2002) which is an instance of the data association problem (Ranganathan et al., 2006), also variously known as *closing the loop* (Hähnel et al., 2003) or *the revisiting* problem (Stewart et al., 2003). Perceptual aliasing arises if different places in the environment produce the same sensory perception and hence appear indistinguishable to the robot; hence it is uncertain whether sensor measurements taken at different points in time correspond to the same physical location.

Typically, perceptual aliasing is caused by measurement uncertainties inherent to robot perception, limited field of view (aperture problem) and repeated structures in the environment.

In this article we suggest to deal with the perceptual aliasing problem in topological mapping by using neighbourhood information. By the *neighbourhood information* we mean a window in sequence of visited places around a particular place. The novel idea here is inducing spatial adjacencies around places in a topological map that is represented as a connected graph by exploiting adjacency information from the sequence of visited places. Such neighbourhood information helps to distinguish places that appear identical to the robot. Using neighbourhood information for place disambiguation is a general approach which is neither bound to a specific choice of sensors nor requires geometric information.

We present two methods for topological mapping from a sequence of observations that integrate our method for place disambiguation. The first method employs a stochastic local search algorithm (Hoos and Stützle, 2004) for map induction and assumes a sequence of deterministic (noise free) observations of places. Using discrete, noise-free labels we assess the success rate in obtaining correct topological maps by using neighbourhood information. Second, a probabilistic map induction method is described that expresses neighbourhood information of places in terms of a likelihood. We show that integrating neighbourhood information into a probabilistic framework provides a powerful mechanism to induce purely topological maps from sequences of sonar measurements alone or sequences of visual perceptions alone in highly ambiguous environments.

The remainder of this article is organised as follows: Section 2 reviews previous work. Section 3 defines terms used in the rest of this article. In Section 4 we describe a stochastic local search algorithm for map induction and evaluate the proposed approach using artificially created random graphs. Section 5 describes a probabilistic map induction method from a sequence of noisy measurements and presents results from experiments using sonar data and visual data.

2 Related Work

Research in mapping has for the most part been concerned with the perceptual aliasing problem due to limited sensor capabilities. Many approaches to the mapping problem use Extended-Kalman-Filter (EKF) based mapping algorithms, often as part of the simultaneous mapping and localisation (SLAM) task (e.g. Leonard and Durrant-Whyte, 1991; Dissanayake et al., 2000, 2001; Milford and Wyeth, 2008; Thrun and Leonard, 2008). In an EKF the motion model and observation noise are assumed to be independent: the sensory noise is a function of the sensor physics, and is independent of the robot's motion noise. These techniques have mainly been implemented in conjunction with geometric maps, or topological maps that capture geometric components, as the EKF requires the motion model of the robot to be known.

Various approaches have been proposed to solve the correspondence problem using highly distinct fingerprints for easing place recognition (e.g. Tapus et al., 2004; Tapus and Siegwart, 2005; Valgren et al., 2007; Ramisa et al., 2009). The currently most successful approach for loop closure detection using visual data association is FAB-MAP which employs a probabilistic approach for image matching based on a *bag-of-words* model for performing localisation on trajectories up to 100km in length (Cummins and Newman, 2008, 2009; Posner et al., 2009). However, in many real world environments, such as office spaces or corridors, it is not always possible to generate distinctive fingerprints for different places because of the similarities inherent to the physical structure.

A common strategy in topological robotic mapping is to detect loop closures by combining place recognition with geometric trajectory information (e.g. Choset and Nagatani, 2001; Lamon et al., 2003; Angeli et al., 2008; Amigoni et al., 2009; Maddern et al., 2011; Ranganathan and Dellaert, 2011). Another strategy is to obtain topological maps by decomposing a previously generated geometric map into key areas (Choi et al., 2011) or extracting key places (Thrun, 1998; Mozos and Burgard, 2006). However, for both approaches the mapping process may be inaccurate or not successful without geometric trajectory information. In cases where no accurate model of the effect of robot motion is available, there is significant benefit not having to depend on it. In particular aerial robots, legged robots and under water robots cannot access odometry information and therefore require other mechanisms for coping with the loop-closure problem. The method for topological mapping we introduce in this article does not require geometric information or information about the robot's actions. However, such information could be integrated and exploited if available.

Approaches using behaviour-based control for exploration-based topological map induction have also been proposed. For generating a topological map Mataric (1990) combines boundary-following and goal-directed navigation behaviours with qualitative landmark identification. Pierce and Kuipers (1997) proposed a complete behaviour-based topological and metric map learning system from low-level sensorimotor control to topological environment representation using the spatial semantic hierarchy (Kuipers, 2000).

Ranganathan et al. introduced the idea of probabilistic topological maps, which is a sample-based representation that approximates the posterior distribution over topologies given the available sensor measurements. Mapping is performed through the use of Markov-Chain Monte Carlo based Bayesian inference over the space of all possible topologies (Ranganathan and Dellaert, 2011; Ranganathan et al., 2006; Ranganathan and Dellaert, 2005, 2004). While probabilistic topological maps are a general approach for mapping they are not capable of dealing with perceptual aliasing that occurs due to repeated structures in the environment unless geometric information is incorporated.

Other approaches incorporate a sequence of observations and actions to generate an automaton which corresponds to a topological map of the environment (Rivest and Schapire, 1990; Dean et al., 1993; Basye et al., 1995). Also, a strategy has been proposed to build a collection of candidate topologies and prune this collection to find the map which is most feasible according to a sequence of actions and observations (Remolina and Kuipers, 2004). These methods exploit the knowledge of the actions of the agent to disambiguate otherwise perceptually identical places. For example, two look-alike hotel rooms at each end of a corridor can be distinguished using the agent action history, as it needs to travel forward or backward towards one of the rooms. In contrast, our approach distinguishes the look-alike rooms using their neighbours as one room may be located next to a lobby whereas the other one is adjacent to a staircase.

Marinakis and Dudek (2010) recently proposed an approach for an extreme case of the loop-closing problem where the robot neither has the ability to obtain meaningful odometry measurements nor is it able to associate a unique label with any vertex or edge. In order to obtain useful topological maps, a robot is able to assign a relative ordering to the edges, leaving a vertex with reference to the edge by which it arrived. In order to obtain useful topological maps the relative ordering of edges is exploited through an exploration tree of plausible world models.

The problem we consider in this paper is similar, however, in our case, the robot neither is able to order nor to label travelled edges. Instead, the robot only exploits adjacency information from the sequence of visited places for building a topological map.

3 Notations and Definitions of Key Terms

In this section we formalise the problem of mapping with neighbourhood information for place disambiguation (Werner et al., 2008) and introduce notations and key terms.

A topological map is defined as a *labelled graph* $\mathbf{G} = (\mathbf{V}, \mathbf{E}, \mathbf{L})$ where the *vertices* $\mathbf{V}$ represent places and the *edges* $\mathbf{E}$ define the connectivity between places. The set $\mathbf{L}$ of vertex *labels* is a set of discrete symbols that abstract sensor readings through some processing like quantisation or clustering. Each vertex is mapped to a label $L(v)$.

Perceptual aliasing occurs when two or more vertices are mapped to the same label.

Definition 1 *Vertices x and $y \in \mathbf{V}$ are called* aliases *iff $L(x) = L(y)$.*

We denote the graph which represents the true topology of the environment by $\mathbf{G}_{env}$ (*environment graph*) and the map constructed by the robot by $\mathbf{G}_{map}$ (*map graph*). The environment graph is unknown to the agent and the only available information about the environment is a finite sequence of labels called *history.* The history is obtained by traversing the environment graph $\mathbf{G}_{env}$. Hence $\mathcal{H}^{1:T} = \ell^1, ..., \ell^T$ and $\ell^t \in \mathbf{L}_{env}$ for $t \in [1, T]^1$.

3.1 Adjacency Information: n-Grams

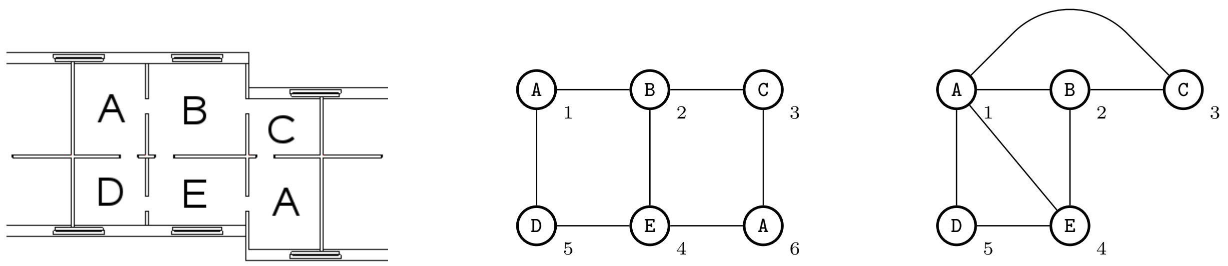
Our method exploits the neighbourhoods of vertices to disambiguate aliases. The history $\mathcal{H}$ contains some information about the neighbourhoods of $\mathbf{G}_{env}$. Consecutively visited vertices correspond to consecutive labels in the history and, reciprocally, consecutive labels in the history correspond to adjacent vertices in $\mathbf{G}_{env}$. In order to access the information about the neighbourhoods in $\mathbf{G}_{env}$ we use a set $\mathcal{H}$ of continuous slices of fixed length n ($n = 1, 3, 5, 7, ...$).

Definition 2 *An n-*gram *is a sequence of n labels.*

Using n-grams is very popular for statistical prediction and categorisation in speech and language processing (Brill, 2003) as well as in bioinformatics (Mansoori et al., 2009) but has not been used in the context robotic mapping. In the following we explain how n-grams can be exploited for topological mapping.

Definition 3 *The set of all n-grams which can be extracted from a history $\mathcal{H}$ is denoted* Grams($\mathcal{H}$,n).

[1] We use the notation *a:b* to denote the sequence of integers from *a* to *b*.



(a) Floor layout. (b) Environment graph. (c) Possible map graph.

Fig. 1 (a) Artificial environment example. The labels A,B,C,D and E denote the view the robot perceives in a particular room. (b) Topological map of this environment. The vertices are labelled with letters which denote the label (sensor view) of the vertex. The subscript number of each circle denotes the index of the corresponding vertex. Note the two aliases, labelled `A` (vertices 1 and 4).

The set Grams($\mathcal{H}$,n) is a *feature* derived from the history $\mathcal{H}$. A history of length m induces at most $m-n+1$ distinct n-grams.

Given a topological map $\mathbf{G}_{map}$, we can generate a history by traversing the map. The set of all possible n-grams that can be obtained by traversing $\mathbf{G}_{map}$ corresponds to a feature space on the map. The maximum number of distinct n-grams that are derivable from a graph is $|\mathbf{L}|^n$.

Definition 4 *The set* Grams$(\mathbf{G}, n)$ *denotes the set of all n-grams that can be obtained by traversing a topological map* $\mathbf{G}$.

To keep the notation simple, we will use $Grams(X,n)$ also when X is a history.

An example environment graph is shown in Figure 1(b). A possible history obtained from traversing this environment graph and, the extracted 3-grams from this history are shown in Table 1.

3.2 n-Consistency

Ideally, a topological map induced by an agent should be isomorphic to an environment graph $\mathbf{G}_{env}$. However, $\mathbf{G}_{env}$ is unknown and the only available information about the environment graph are local neighbourhood structures contained in the set of n-grams from the history. Consequently, we propose to measure the *consistency* of a map and a history by comparing the sets of n-grams that they generate.

history: $\mathcal{H} =<$ A, B, C, A, E, D, A, B, E, A, C, B, E, D, A, B, C $>$
$Grams(\mathcal{H}, 3) =$ {<A,B,A>,<A,B,C>,<E,B,A>, <A,D,A>,<E,D,A>,<B,A,B>, <D,A,B>,<B,C,B>,<A,C,B>, <B,E,B>,<D,E,B>,<B,E,A>, <C,B,C>,<C,B,E>,<C,A,C>, <C,A,E>,<D,A,D>,<D,E,D>, <A,E,D>,<E,B,E>,<E,D,E>, <E,A,E>,<A,C,A>,<A,E,A>}

Table 1 An example of a possible history $\mathcal{H}$ that could be obtained from the environment graph in Figure 1(b) and the set of 3-grams extracted from the history. Note, we allow the robot to perform U-turns.

Definition 5 *A map* $\mathbf{G}_{map}$ *and a history* $\mathcal{H}$ *are n-*consistent, *iff* $Grams(\mathbf{G}_{map}, n) = Grams(\mathcal{H}, n)$.

The parameter n denotes the length of the grams, that is considered for the consistency measure. For example, the graphs shown in Figure 1(b) and Figure 1(c) are 1-consistent as they induce the same set of 1-grams. The difference between these two graphs is not captured by 1-grams, as 1-grams do not contain any adjacency information. However, considering 3-consistency reveals the difference between the two graphs: the graph in Figure 1(c) induces 3-grams such as `C-A-D`, which are not contained in the 3-grams (shown in Table 1) of the graph in Figure 1(b).

4 Topological Mapping from a History of Deterministic Place Labels

This section demonstrates the discriminatory power of neighbourhood information for place disambiguation in topological mapping. For that purpose, we describe a method for inducing a topological map from a set of n-grams that are obtained by traversing an environment graph with discrete, noise free labels. We begin with introducing constraints that we wish the topological map graph to satisfy and then describe a stochastic local search method for topological map induction. The section is concluded with a discussion on the benefits and drawbacks of using neighbourhood information for place disambiguation on the basis of results from experiments on artificial environment graphs with deterministic labels.

4.1 Mapping Constraints

The aim of mapping in robotics is to generate a representation of the environment that is congruent with the real environment. Hence, in our case, the n-consistency requirement expresses a *hard constraint* that the map graph must satisfy.

A frequent problem in topological mapping is that the number of vertices is not known in advance. Even if we knew the number of vertices in advance, disambiguation would still be difficult unless every vertex is mapped to a distinct label (i.e. no aliasing).

A map graph, that is obtained by mapping each observation to a distinct vertex in a bijective manner, is n-consistent according to Definition 5, as the mapping method disregards multiple visits of the same place. However, if there are aliases, this map graph contains too many vertices and does not capture the connectivity of the environment appropriately and is therefore inappropriate for navigation. We propose to resolve this dilemma according to Occam's razor principle[2] by constructing a small map, minimising the number of vertices while maintaining consistency with the observations. Ideally, we would like to find the smallest map that explains the observed history. The objective of minimising the number of vertices is formulated as a *soft constraint*.

4.2 Map Induction using a Stochastic Local Search

The task of topologically mapping an environment is identical to the problem of partitioning the set of recorded labels such that each subset of the partition contains exactly those labels that were recorded at a particular place. That means, labels of places that were visited several times need to be assigned to the same subset of the partition. In general, this is a difficult task, as the number of different topologies over M observations is identical to the number of disjoint set partitions of the M-set. This number is called Bell number $b_M = \frac{1}{e}\sum_{i=0}^{\infty}\frac{i^M}{i!}$ and grows hyper exponentially with M (Nijenhuis and Will, 1978).

The combinatorial complexity motivates the use of a *stochastic local search* (SLS) which is a popular approach for solving difficult combinatorial optimisation problems (e.g. Richter et al., 2007). SLS algorithms have been successfully applied to *NP*-complete problems such as satisfiability and constraint satisfaction. Local search algorithms move from solution to solution in the space of candidate solutions (the search space) until a solution deemed good enough is found or a time limit has elapsed (Hoos and Stützle, 2004).

Algorithm 1 Algorithm for inducing a map graph $\mathbf{G}_{map}$ given a set of n-grams Grams$(\mathcal{H}, n)$. The algorithm is explained in detail in Section 4.2.

Require: $\mathbf{\Gamma} = \text{Grams}(\mathcal{H}, n)$
1: $G_{map} \leftarrow \emptyset$
2: **repeat**
3: select arbitrary $\gamma \in \mathbf{\Gamma}$
4: try to merge γ with $\mathbf{G}_{map}$ such that
  1. $\text{Grams}(\mathbf{G}_{map}, n) \subseteq \text{Grams}(\mathcal{H}, n)$
  2. Least number of new vertices are introduced
5: **if** merge successful **then**
6: $\mathbf{\Gamma} \leftarrow \mathbf{\Gamma} \setminus \text{Grams}(\mathbf{G}_{map}, n)$
7: **end if**
8: **until** $\mathbf{\Gamma} = \emptyset$ **or** maximum number of trials exceeded.
9: **if** $\mathbf{\Gamma} \neq \emptyset$ **then**
10: **return failure**
11: **else**
12: **return** $\mathbf{G}_{map}$
13: **end if**

4.2.1 Constraint Based Map Induction

In order to build a topological map $\mathbf{G}_{map}$ from a history $\mathcal{H}$, the set $\mathbf{\Gamma} = \text{Grams}(\mathcal{H}, n)$ of n-grams is extracted from the history. The extracted n-grams are then incrementally re-merged such that a map graph is obtained which satisfies the mapping constraints. Algorithm 1 lists the pseudo-code for the proposed algorithm.

The mapping process starts with an empty map graph $\mathbf{G}_{map} \leftarrow \emptyset$ and the set $\mathbf{\Gamma}$ initially contains all the n-grams extracted from the history. In the main loop, Algorithm 1 selects an arbitrary n-gram $\gamma \in \mathbf{\Gamma}$ and tries to merge (line 4) the n-gram with the current map graph (lines 2-8).

A merge of an n-gram γ with a map graph may introduce new edges and vertices such that γ can be derived from the extended map graph. Often, there are several possibilities to merge an n-gram with map graph. Hence, the merge option that satisfies the mapping constraints best is selected. According to the mapping constraints described in Section 4.1, the particular merge is selected that results in a map that is n-consistent with the history and requires fewest new vertices.

The n-grams that are accounted for by the newly created edges are removed from $\mathbf{\Gamma}$. That is, the n-grams that remain for merging are those which are not induced by the map graph yet (line 6). The mapping process is successfully finished when all n-grams are accounted for. That is, when $\mathbf{\Gamma} = \emptyset$ (lines 8 and 12). It may oc-

[2] "Entia non sunt multiplicanda praeter necessitatem" or "Entities should not be multiplied unnecessarily". William Occam (1285-1349).

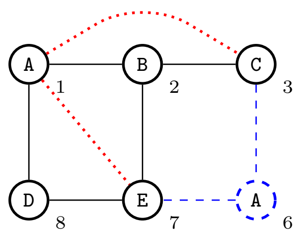


Fig. 2 A map graph is shown in black. Two possibilities to merge the 3-gram `C-A-E` are shown in dotted red and dashed blue. The dotted red option requires fewer vertices as only new connectivity is induced. However, the resulting map graph would violate the mapping constraints as it would be possible to observe the 3-gram `E-A-B` which does not appear in the extracted set of 3-grams from the history as shown in Table 1. Thus, the option which is displayed in dashed blue is selected.

cur that it is not possible to merge an n-gram with a map graph as only inconsistent mappings are obtained. Hence, after a maximum number of unsuccessful merge trials, the mapping process is terminated (lines 8) and a failure status is returned (line 10).

The merging relates the local adjacency information contained in the n-grams and the adjacencies of the vertices in the map graph. For example, Figure 2 shows two possibilities (shown in dotted red and dashed blue) to merge the 3-gram `C-A-E` to a map graph. Clearly, the possible edge addition shown in red dots requires fewer vertices. However, traversing the resulting map graph, we can obtain the 3-gram `E-A-B` which is not supported by the history thus the consistency constraint is violated. Hence, the merge shown in dashed blue is executed.

The proposed mapping method simultaneously determines the number of vertices, assigns appropriate labels to the vertices, and induces the connectivity of the vertices of the map graph.

4.2.2 Local Extrema and Search Restarts

Algorithm 1 aims to induce a map graph which is n-consistent with the set of n-grams extracted from a given history. However, an unfortunate order of selections of n-grams from $\mathbf{\Gamma}$ (line 3) can result in large graphs or make further valid merges impossible. We do not know how to find the optimal order efficiently in advance.

Our approach to solve the problem of finding the optimal order of merges is to restart the entire mapping process several times. Due to the arbitrary selections of n-grams different results may be obtained. We eventually keep the map graph which requires the smallest number of vertices.

4.3 Results from Experiments using Deterministic Labels

For the evaluating the stochastic local search algorithm for topological mapping we use artificial random graphs with discrete, noise free labels. Thus, we can consider the problem of different places that appear to be the same to the robot without additional complications such as measurement noise.

4.3.1 Artificial Random Graphs

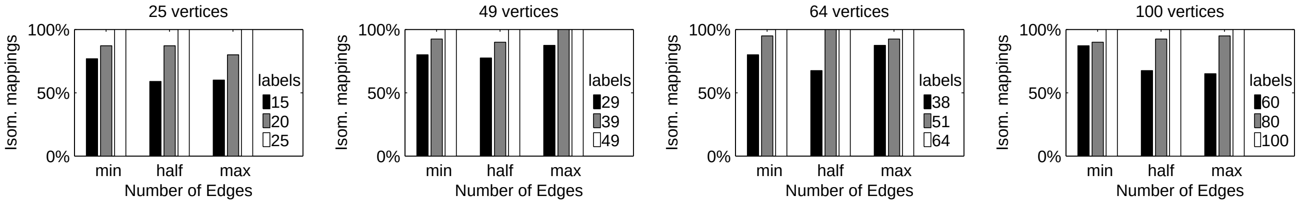
For the following evaluations, environment graphs are created with 25, 36, 49 and 100 vertices arranged and (possibly) connected in a rectangular grid. That means, the maximum degree of a vertex is four. We have chosen a grid layout to ensure planarity; most of the environments robots traverse are planar. On the grid, we simulate different edge densities: a fully connected grid, a grid with half of the edges of a fully connected grid and a minimally connected grid where each vertex has at most degree 2. The vertices of each graph are arbitrarily labelled with elements from a set of labels whose size was half, three quarters and the same size as the set of vertices. Reducing the size of the set of labels naturally increases the number of aliases in a graph. We assume, every environment graph is explored exhaustively such that the robot has collected all n-grams that are obtainable by traversing the environment graph.

4.3.2 Quality of the Topological Maps

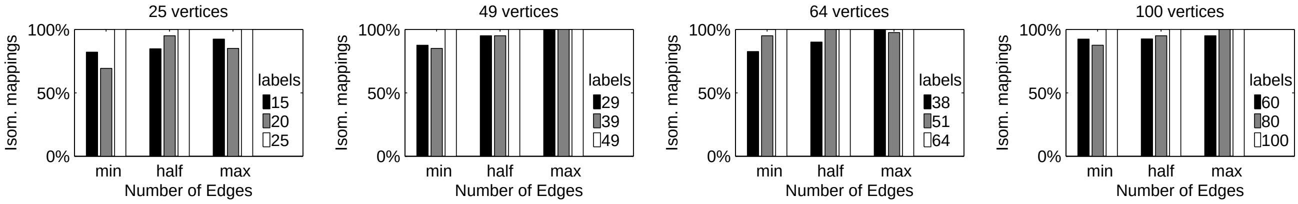
A good indication of the benefit of using neighbourhood information for place disambiguation in topological mapping is the quality of the obtained topological maps. Ideally, we would like a map that is isomorphic with the corresponding environment graph. Here, this can be decided because $\mathbf{G}_{env}$ is known.

Figure 3 shows a quality analysis of the proposed approach for inducing a topological map from a history. Clearly, for graphs with unambiguous labelling, all mappings are isomorphic (white bars). As expected, increasing the degree of ambiguity by reducing the size of the set of labels (grey and black bars), results in fewer isomorphic mappings. However, our method yields a high percentage of isomorphic mappings even for graphs with many ambiguous labels.

In comparison with the degree of aliasing, the edge density seems to have minor impact on the quality of the resulting maps. We can observe a slight tendency to fewer isomorphic mappings with decreasing edge density. This may be a consequence of fewer neighbours for

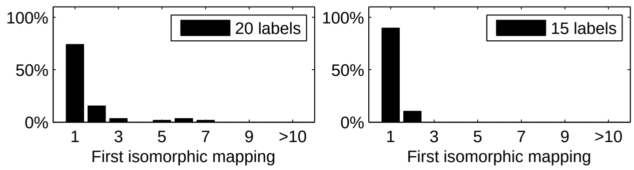


(a) Mapping with respect to 3-consistency.

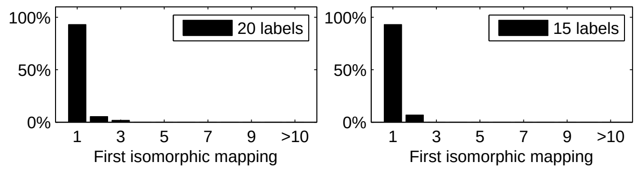


(b) Mapping with respect to 5-consistency.

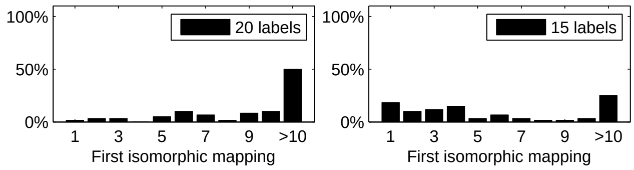
Fig. 3 Each chart shows the statistics of the resulting maps of 700 environment graphs using different edge densities (fully connected, half connected and minimally connected) and varying numbers of labels as described in Section 4.3.1. The top row shows mappings with respect to 3-consistency and the bottom row mappings with respect to 5-consistency. The white bars refer to mappings where each vertex is assigned a distinctive label (no ambiguity), the grey bars refer to mappings with an increased degree of ambiguity (size of the set of labels is 75% of the size of the set of vertices) and the black bars refer to mapping with high a high degree of ambiguity (the size of the set of labels is half the size of the set of vertices).



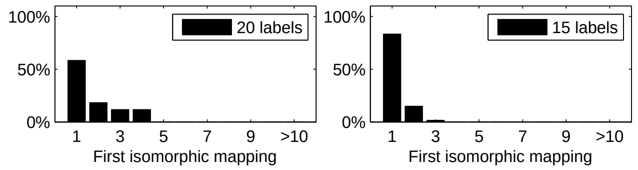
(a) Mapping of an environment graph with 25 vertices according to 3-consistency.



(b) Mapping of an environment graph with 25 vertices according to 5-consistency.



(c) Mapping of an environment graph with 100 vertices according to 3-consistency.



(d) Mapping of an environment graph with 25 vertices according to 5-consistency.

Fig. 4 Histograms of the first isomorphic map found out of 100 mapping trials for 2160 environment graphs with 25 vertices (a)-(b) and 100 vertices (c)-(d) with different numbers of edges and labels created according to the described schemata.

each vertex and hence less neighbourhood information for disambiguation.

The parameter that most impacts the quality of the mapping is the length n of the n-grams. Mapping with respect to 5-consistency yields more isomorphic mappings than mapping with respect to 3-consistency. This effect occurs as larger n-grams contain more information and hence are more discriminative than shorter n-grams.

The results demonstrate that using neighbourhood information is very beneficial for place disambiguation in topological mapping. The constraint of n-consistency between a history and the corresponding map graph governs the map building process in a manner that most ambiguities can be resolved and mostly isomorphic mappings are obtained. Note that our mapping method does not require any geometric information such as odometry. However, incorporating further information such the trajectory of the robot may increase the number of successful place disambiguations.

4.4 Mapping Performance

Because of the random selection of n-grams in the stochastic local search, we may obtain different maps in multiple runs of our mapping algorithm. Hence, it is

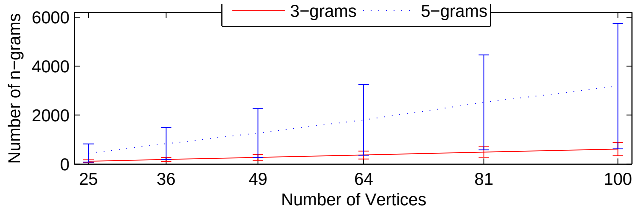


Fig. 5 The average size of the set of n-grams of the environment graphs used for the experiments are shown for $n \in \{3, 5, 7, 9, 11\}$. Note, the logarithmic scaling of the y-axis.

important to examine how many mapping trials our algorithm requires to execute until a map graph is found that is isomorphic with the environment graph. Thus, for each environment graph, we run the mapping process 100 times and analysed when the first isomorphic map graph was found.

The evaluations displayed in Figure 4 show, that an isomorphic mapping is usually found very quickly after a few trials. It seems, the mapping constraints tightly guide the mapping process so that mostly isomorphic mappings are obtained. Moreover, the results show that mapping larger graphs requires more trials on average. A possible reason for that occurrence may be that large environment graphs are more complicated to map as they contain more repeated structures than small environment graphs.

Mapping with respect to 5-consistency usually requires fewer trials to find an isomorphic map graph than mapping with 3-consistency as 5-grams contain more information for disambiguation than 3-grams. However, a trial in 5-consistency mapping requires more time than mapping with respect to 3-consistency.

The complexity of the proposed method mainly depends on the length n of the n-grams. As mentioned in Section 3.1 the size of the set of n-grams grows linearly with the number of labels of a graph but hyper-exponentially with n. Figure 5 supports that analysis on data from the experiments.

5 Induction of Topological Maps from an History of Noisy Measurements

In this section we present an approach for topological map induction from an history of noisy sensor readings. We describe a method for inducing a topological map using a Bayesian filter technique which integrates the disambiguation using neighbourhood information as proposed in Section 4.

As in Section 4 we assume the robot has collected all n-grams that are obtainable by traversing the environment graph. From a recorded history $\mathcal{H}^{1:T} = \ell^1, ..., \ell^T$ of noisy fingerprints of visited places, the set $\mathbf{\Gamma}_{env} = \text{Grams}(\mathcal{H}^{1:T}, n)$ of n-grams is computed after the exploration. The map is then inferred in a SLAM-like fashion by replaying the history with in addition $\mathbf{\Gamma}_{env}$ as background knowledge.

5.1 Bayesian Inference for Map Likelihood Estimation

Bayes filters recursively estimate a dynamic system's state from noisy observations (Arulampalam et al., 2002) and are widely used in robotic mapping and localisation (Fox et al., 2003). Here, the system state $\mathbf{s}^t = \{\mathbf{G}^t_{map}, p^t\}$ comprises the topological map $\mathbf{G}^t_{map}$ and the place p^t the robot is located at time t. We are interested in recursively estimating the posterior PDF (probability density function) $P(\mathbf{s}^t|\mathcal{H}^{1:t})$ over the states at time step t given the measurements $\mathcal{H}^{1:t}$ using Bayes' rule

$$P(\mathbf{s}^{t+1}|\mathcal{H}^{1:t+1}) = P(\mathbf{s}^{t+1}|\mathcal{H}^{t+1}, \mathcal{H}^{1:t}) \quad (1)$$

$$= \frac{P(\mathcal{H}^{t+1}|\mathbf{s}^{t+1}, \mathcal{H}^{1:t})P(\mathbf{s}^{t+1}|\mathcal{H}^{t:1})}{P(\mathcal{H}^{t+1}|\mathcal{H}^{t:1})} \quad (2)$$

$$\propto P(\mathcal{H}^{t+1}|\mathbf{s}^{t+1}, \mathcal{H}^{1:t})P(\mathbf{s}^{t+1}|\mathcal{H}^{t:1}). \quad (3)$$

The transition PDF from the t^{th} timestep to the $t+1^{\text{th}}$

$$P(\mathbf{s}^{t+1}|\mathcal{H}^{1:t}) = \sum_{\mathbf{v}} P(\mathbf{s}^{t+1}, \mathbf{s}^t = \mathbf{v}|\mathcal{H}^{1:t}) \quad (4)$$

$$= \sum_{\mathbf{v}} P(\mathbf{s}^{t+1}|\mathbf{s}^t = \mathbf{v}, \mathcal{H}^{1:t})P(\mathbf{s}^t = \mathbf{v}|\mathcal{H}^{1:t}) \quad (5)$$

depends only on the positions p^t and p^{t+1} the robot visits, hence

$$P(\mathbf{s}^{t+1}|\mathbf{s}^t, \mathcal{H}^{1:t}) = P(\mathbf{s}^{t+1}|\mathbf{s}^t = \mathbf{v}). \quad (6)$$

Putting everything together, we have

$$P(\mathbf{s}^{t+1}|\mathcal{H}^{1:t+1}) = P(\mathcal{H}^{t+1}|\mathbf{s}^{t+1}) \sum_{\mathbf{v}} P(\mathbf{s}^{t+1}|\mathbf{s}^t = \mathbf{v})P(\mathbf{s}^t = \mathbf{v}|\mathcal{H}^{1:t}) \quad (7)$$

where we exploit the conditional independence

$$P(\mathcal{H}^{t+1}|\mathbf{s}^{t+1}, \mathcal{H}^{1:t}) = P(\mathcal{H}^{t+1}|\mathbf{s}^{t+1}). \quad (8)$$

For map likelihood estimation, we recursively solve Equation 7 using a particle filter which is a sequential Monte-Carlo method used for Bayesian model inference (Arulampalam et al., 2002). In order to induce a topological map, we recursively compute the probability distribution $P(\mathbf{s}^t|\mathcal{H}^{1:t})$ at each time step. This distribution is obtained, recursively, in two stages: prediction and update.

5.1.1 Prediction Phase

Recall that $\mathbf{s}^t$ represents a pair made of a candidate map and the location of the robot within this map. To sample the set of pairs of (candidate map, robot location), we use a simplified motion model of the robot. Normally, the new location p_{t+1} of the robot is predicted using a conditional probability $P(\mathbf{s}^{t+1}|\mathbf{s}^t, \mathbf{u}^t)$ where $\mathbf{u}^t$ denotes the control command at time t. In the context of topological mapping with no information about the motion of the robot, the control command $\mathbf{u}^t$ is unknown. Therefore, we rely only on the probability $P(\mathbf{s}^{t+1}|\mathbf{s}^t)$ to predict the robot motion.

We estimate the PDF $P(\mathbf{s}^{t+1})$ with a particle system. For each particle in generation t , we create $K+1$ new particles. Each new particle represents a robot's predicted position in the map graph $\mathbf{G}^t_{map}$. The $K+1^{\text{st}}$ particle is special; it represents the robot at a new place (added vertex) in the map. Indeed, we have to accommodate the possibility that the robot could be entering a place that it has never visited so far. Therefore, the size of the population of particles grows to $N(K+1)$ before being resampled down to N particles. An edge is added between the vertex the robot currently visits and the vertex the robot is predicted to visit next if that particular link does not exist yet.

To introduce a preference bias for small maps that explain the history, we set the model transition probability

$$P(\mathbf{s}^{t+1}|\mathbf{s}^t) = \begin{cases} 1 \text{ if } |\mathbf{V}^{t+1}| = |\mathbf{V}^t| \\ \psi \text{ if } |\mathbf{V}^{t+1}| > |\mathbf{V}^t| \end{cases} \tag{9}$$

to favour small number of vertices ($\psi \in [0, 1]$).

5.1.2 Update Phase

In the update phase a measurement model is used to incorporate information from the sensors to obtain the posterior PDF $P(\mathbf{s}^{t+1}|\mathcal{H}^{1:t+1})$. The measurement model is given in terms of a measurement likelihood $P(\mathcal{H}^{t+1}|\mathbf{s}_i^{t+1})$. In particle filters, the measurement likelihood is computed by weighing the samples. In our algorithm a sample's weight comprises the similarity of the label ℓ_k of vertex $\mathbf{v}_k$ and the observation $\mathcal{H}^{t+1}$ (k denotes the vertex that represents the predicted location p_i^{t+1}) as well as the n-consistency of the current map graph estimate $\mathbf{G}_i^{t+1}$ and the history $\mathcal{H}^{1:t+1}$). Recall from Definition 5 that the n-consistency is measured by comparing the set of n-grams $\mathbf{\Gamma}_i^{t+1}$ that can be derived from the predicted map estimate and the set of n-grams $\mathbf{\Gamma}_{env}$ derived from the history. Thus, a particle's weight is computed with

$$w_i^{t+1} \propto P(\mathcal{H}^{t+1}|\mathbf{s}_i^{t+1}) \propto \exp\left(-\left(\frac{||\mathcal{H}^{t+1} - \ell_k||}{\sigma_l}\right)^2\right) \exp\left(-\left(\frac{d_H(\mathbf{\Gamma}_{env}, \mathbf{\Gamma}_i^{t+1})}{\sigma_c}\right)^2\right). \tag{10}$$

whereby the parameters σ_l and σ_c denote the standard deviations of the Gaussian distributions. The novel aspect here is performing a consistency check in every update step and including the consistency of history and map into the sample's weights.

However, it is not possible to perform a consistency check by testing whether $\text{Grams}(\mathbf{G}_{map}, n) = \text{Grams}(\mathcal{H}, n)$ as suggested in Definition 5 because the elements are not discrete and free of measurement noise. For noisy data we suggest to measure the degree of n-consistency of two sets of n-grams $\mathbf{\Gamma}_0$ and $\mathbf{\Gamma}_1$ using the Hausdorff distance

$$d_H(\mathbf{\Gamma}_0, \mathbf{\Gamma}_1) = \max(\max_{\alpha\in\mathbf{\Gamma}_0} \min_{\beta\in\mathbf{\Gamma}_1} d(\alpha, \beta), \max_{\beta\in\mathbf{\Gamma}_1} \min_{\alpha\in\mathbf{\Gamma}_0} d(\beta, \alpha)). \tag{11}$$

The smaller the Hausdorff distance the more n-consistent are $\mathbf{\Gamma}_0$ and $\mathbf{\Gamma}_1$. The distance of two n-grams α and β is computed using the maximum norm

$$d(\alpha, \beta) = ||\alpha - \beta||_\infty = \max_{k=0,\dots,n-1}(|\alpha_k - \beta_k|). \tag{12}$$

Thus, the distance between two sets of n-grams is computed by the largest distance of two fingerprints that are associated with the same vertex in a map.

The posterior distribution on topological maps is computed by drawing N samples from the proposal distribution.

5.1.3 Localisation

The place p^t that the robot occupies is implicitly estimated whenever the map graph is updated with a new observation. The vertex whose label is updated or added using the observation indicates the new location of the robot. If a new vertex is introduced the robot is by construction located at the place which corresponds to that vertex. The location, in turn, is used to guide the mapping process by introducing adjacencies between the current and the previous place occupied.

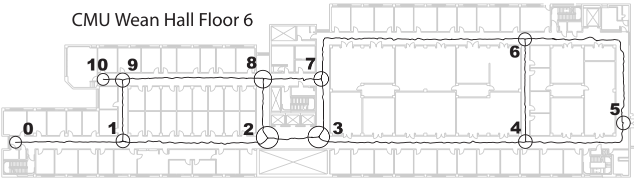


Fig. 6 The floor plan of Wean Hall Floor 6 at Carnegie Mellon University. Embedded is the topological graph (vertices and their connectivity) that reflects the ground truth of the topological map. The circles show the distance measure from the locus point to the closest obstacles which we use as fingerprints of the places.

5.2 Experiments

We demonstrate the generality of our approach for place disambiguation by presenting results from histories of solely sonar range measurements in Section 5.2.2 and histories of solely visual observations in Section 5.2.3.

5.2.1 Experiment Setup and Data Acquisition

Our experimental set up covers an indoor office environment area of about 20,000 square meters (Wean Hall at Carnegie Mellon University), see Figure 6. Our robot uses an ultrasonic sensor array and a panoramic camera to acquire information about the environment.

The robot traverses the environment using the Generalised Voronoi Graph (GVG) strategy as developed by Choset and Nagatani (2001). The GVG navigation method is based on the Voronoi diagram which is a skeleton graph of the environment. The points of the Voronoi diagram locally maximise the minimum distance to any obstacle. In indoor environments nodes of Voronoi diagram naturally to T-junctions or intersections of corridors as shown in Figure 6. We consider the Voronoi nodes as key places in the environment. Thus, once a locus point is identified, a sonar reading and a panoramic image are taken.

5.2.2 Map Induction from Sonar Readings

We have conducted several explorations and recorded a total of 105 sonar-based fingerprints of places (see Table 2). The sonar-based fingerprint of a place may be interpreted as the radius of the largest obstacle free disc that can be drawn around the locus points of Voronoi nodes. Clearly, such very simple fingerprints of places entail numerous topological ambiguities. Considering the means of each of the recorded fingerprints of particular places in Table 2, it is difficult to identify 11 distinguishable categories, resulting in a challenge for topological mapping and localisation algorithms. For example, places 2 and 3 appear similar to the robot as do places 3 and 4 or places 1, 4, 5, 6 and 9.

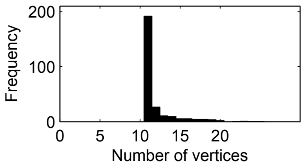
Given the database of recorded fingerprints of places and the ground truth environment graph, we can simulate random traversals of the environment. The robot starts at an initial vertex and selects an arbitrary adjacent vertex as next place. According to the vertex the robot occupies, a random observation from the database is sampled. For the following evaluations, 200 paths of length 100 were generated. Each path represents an exhaustive exploration of the environment. The posterior distribution on topological maps in the particle filter was approximated using 10 particles. Because of the severe level of ambiguities in our environment, we maintain 5-consistency between an history and the induced map as suggested from the results in Section 4.3.2.

Our approach aims at inducing a topological map which is consistent with the information given in the history. The consistency is measured through the Hausdorff distance (Equations 11 and 12) between the sets of n-grams which are derived from an history and the induced map graph. Figure 7(b) shows a histogram of the 5-consistencies of the obtained maps of the simulated random traversals. For most of the induced maps the 5-consistency error between the map and the underlying history is small and can be explained with the inherent uncertainties associated with sensory perception.

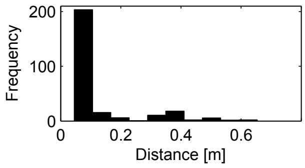
As mentioned earlier in this paper, the map that contains a vertex for every observation is a non-desirable topological map despite the fact that this map maximises the consistency between the induced map and the history. In order to escape such maps, we aim for small maps in terms of vertices. Figure 7(a) shows the number of vertices of the resulting topological maps. The clear peak of the histogram shows that most of the maps induced by our algorithm have eleven vertices, which is the same as the ground truth (see Figure 6). The algorithm does not induce smaller maps as they would violate the consistency criterion and hence have

id	Recordings	Mean [m]	Var
0	13	1.1713	0.0121
1	11	1.3600	0.0069
2	11	2.1641	0.0179
3	9	2.1624	0.0077
4	9	1.3350	0.0107
5	7	1.2695	0.0255
6	8	1.3104	0.0008
7	7	1.4983	0.0024
8	10	1.6768	0.0134
9	10	1.3537	0.0009
10	10	1.1606	0.0110

Table 2 Database of sonar readings recorded from several traversals of the experimental environment. The indices of the places correspond to Figure 6.



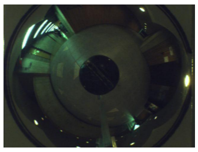
(a) Number of vertices of the induced map graphs



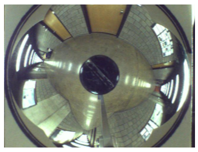
(b) The Hausdorff distance of the induced map graphs.

Fig. 7 The number of vertices (a) and the consistency (b) are shown for the simulated histories of sonar readings. The consistency is denoted by the Hausdorff distance between the sets of n-grams from the histories and the the induced induced maps. The clear peak at 11 vertices indicates that most obtained maps contain as many vertices as the environment graph (see Figure 6). Moreover, most of the induced maps have only small consistency errors caused by measurement noise.

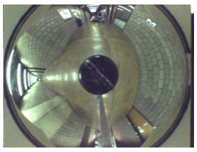
(a) Place 2



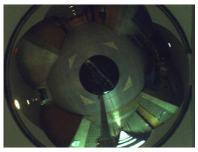
(b) Place 4



(c) Place 5



(d) Place 7



(e) Place 9

(f) Place 10

Fig. 8 Example image recordings from the identified places using the GVG method in Wean Hall at Carnegie Mellon University. It is apparent that due to the structure of the building several physically different places appear visually similar to the robot.

low probability. Larger maps may occur if an observation is mapped to an inappropriate vertex because measurement noise unbalances the competing goals of maintaining consistency while minimising the number of vertices in the map.

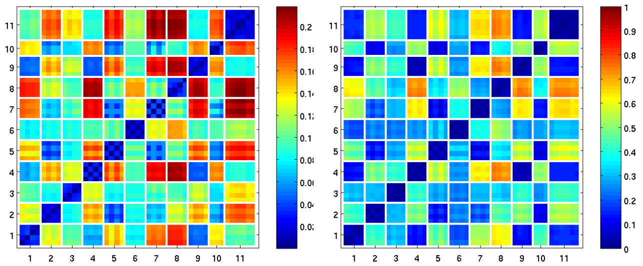
Moreover, the sampling of new map candidates from a posterior distribution can sample map candidates which are not optimal and hence create larger maps. In general, reducing the strength of the minimisation constraint (by increasing ψ in Equation 9) results in larger maps whereas strengthening minimisation decreases the map size but may result in inconsistencies. Also, by reducing the penalty for adding vertices, the resulting map usually contains more vertices as the method is less robust against measurement noise. Reciprocally, using a strong penalty for adding vertices may result in distorted connectivities as places with different views may be mapped to the same vertex.

In experiments using shorter grams, that means maintaining 3-consistency or 1-consistency between the map and the history, we have obtained maps with small consistency error, however, fewer vertices than the number of places contained in the environment. That occurs as 3-grams contain less adjacency information than 5-grams and therefore less information for place disambiguation is available during the mapping process. Using 1-grams does not consider any adjacency information between places thus the mapping process does nothing else than clustering history similar to the work of Ranganathan et al. (2006).

5.2.3 Visual Appearance based Map Induction

We have conducted similar experiments as described in the previous section using panoramic images in order to demonstrate that using neighbourhood information for place disambiguation is a general method and not bound to a specific choice of perception. Using the GVG exploration method, our robot has recorded a database of 45 panoramic images of places. In some areas of our experimental environment bright ceiling lights are installed whereas some other areas have wall lights. As a result, the images the robot takes at each place suffer from loss of clarity of visual information within shadows or near strong lights (over and under exposed regions). We use a method proposed by Vonikakis and Andreadis (2007) to enhance the acquired images by lightening under exposed regions and darkening over exposed regions without affecting the correctly exposed ones.

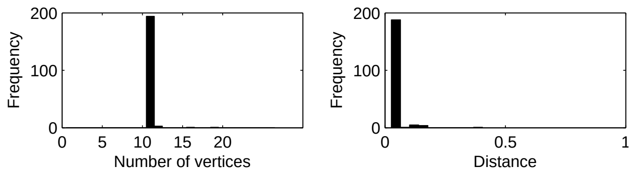
We use colour histograms as fingerprints of places as they represent the visual appearance of places in a compact manner, are easy and fast to compute and are rotation invariant for panoramic images. Usually, a colour histogram is computed by calculating an N-bin histogram for each of the R, G and B colour bands. Unfortunately, this type of colour histogram loses the 3D spatial information of the RGB tuples in colour space. In order to retain the 3D spatial information, histograms in 3D RGB space, where the histogram consists of N^3 equally sized bins (Werner et al., 2007). In order to retain the 3D spatial information, we use histograms in 3D RGB space, where the histogram consists of N^3 bins. Using $N = 5$ keeps the system fast despite potentially large sets of n-grams. In comparison, a single standard SIFT feature as suggested by Lowe (2004) is represented through a 128 dimensional vector, whereby thousands of such features may be identified in a single image. Figure 9 shows a distance matrix of the fingerprints from the enhanced images. It is apparent that the environment contains numerous topological ambiguities when using colour histograms as fingerprints of the places.



(a) Distance matrix of the data set using standard colour histograms with 3×256 bins.

(b) Distance matrix of the data set using colour histograms in 3D colour space with 125 bins.

Fig. 9 Distance matrix of the colour histogram-based fingerprints of places (see Figure 6). It is apparent that using 3D information increases the discrimination of colour histograms compared to standard colour histograms. However, fingerprints of some different places remain indistinguishable. According to the colour bar, blue denotes small distance of fingerprints. The histograms are normalized so the maximum distance of two histograms is 1.



(a) Number of vertices of the induced map graphs.

(b) Hausdorff distance of the induced map graphs.

Fig. 10 The number of vertices (a) and the consistency (b) of the induced maps are shown for the simulated histories of colour histograms. The consistency is denoted by the Hausdorff distance between the sets of n-grams from the histories and the the induced induced maps. Similarly to the results of sonar-based fingerprints of places, most of the induced maps have small consistency error and 11 vertices which corresponds to real environment.

We have simulated random traversals using the database of colour histograms and the ground truth. For the following evaluations, 200 paths of length 100 were generated. The results shown in Figure 10 are similar to the experiments using sonar-based fingerprints of places. The consistency error of most histories and corresponding topological maps is small and explainable through measurement noise. Moreover, we found all map graphs (193, or 97%) with the same number of vertices as the environment graph to be isomorphic to the environment graph. Rarely, larger maps may occur if an observation is not mapped to the proper vertex because of measurement noise.

6 Conclusion

In this paper we have addressed the problem of topological mapping in the extreme case where a single robot neither has the capability to obtain odometry measurements nor it is able to acquire unique fingerprints of places. Thus, several places in an environment may appear indistinguishable to the robot.

Many recent approaches to the topological mapping problem in robotics do not fully address the problem of indistinguishable places of real environments. On the contrary, most methods try to avoid these ambiguities by using highly discriminative fingerprints of places or incorporating geometric information for place disambiguation. The method we introduced in this article complements existing approaches in that it still works when some places cannot be distinguished based only on sensor data. These places can be indistinguishable because they are inherently similar or because the discriminative power of the sensors is too weak. Our method achieves this distinction between places without the need for additional geometric information. To demonstrate this capability, we have integrated our approach for place disambiguation into a method for map induction by place recognition and show the complementary benefits.

We have shown that even in the case of highly ambiguous fingerprints of places it is possible to induce spatial adjacencies around places in a topological map that is represented as a connected graph by exploiting sequential neighbourhood information (n-grams) from the sequence of visited places. Our approach infers topological maps that are consistent with the adjacency information obtained from the history and aims for small maps based on the principle of Occams's razor. We have demonstrated in experiments using sonar readings or visual information that using neighbourhood information for place disambiguation is a viable and general approach which is neither bound to a specific choice of sensors nor requires geometric information.

Our method requires the selection of the length n of the n-grams. The parameter n captures the relevant size of the neighbourhood of the places. If the adjacent places (3-grams) are not sufficient to distinguish two or more ambiguous places, one has to consider larger neighbourhoods (i.e. the neighbours of a place's neighbours (5-grams), or even bigger neighbourhoods). However, increasing the size of the neighbourhood entails an increase of the computational effort of the mapping process. We do not have a solution for deciding on the size of the neighbourhood. Our experiments have shown that even in environments with severe ambiguities, we obtain topological maps that are isomorphic with the environment.

Using neighbourhood information only for place disambiguation it is not possible to truly map symmetric environments. However, including further information such as odometry, the degree of vertices, order of travelled edges or even clues on whether the robot has conducted a U-turn during exploration may help to properly map even symmetric environments.

It was not the purpose of this paper to discuss the properties of geometric information in robotic mapping but to demonstrate the great benefits of using neighbourhood information for place disambiguation for inducing a topological map from a sequence of observations. Indeed, including further sensory information it may be possible to relax the strong consistency constraint so one can adapt our method to map only partially explored environments.

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