A review of natural convection and heat transfer in the attic-shaped space

Suvash C. Saha[1*], M. M. K. Khan[2]

[1]School of Engineering and Physical Sciences, James Cook University

Townsville, QLD - 4811, Australia

[2]School of Engineering & Built Environment, Central Queensland University,

Rockhampton Campus, Qld-4702, Australia

[*]Corresponding author: Dr. Suvash C Saha, Email: s_c_saha@yahoo.com

Tel: +61-7-4779-7523, Fax: +61-7-4781-6788

Abstract

This paper presents an extensive review of studies regarding natural convection heat transfer in the triangular enclosure with primary focus on those with a direct application to heat transfer in attic-shaped space. To provide thermal comfort to the occupants in attic-shaped buildings and to minimise the energy costs associated with heating and air-conditioning, increasing research activities have been devoted to topics relevant to heat transfer in attics over the last three decades. There are two basic thermal boundary conditions of attic are considered to represent hot and cold climate or day and night time. A significant number of studies on other topics related to the attic space have also been performed recently such as attics subject to localized heating and attics filled with porous media.

KEYWORDS: Attic; natural convection; heat transfer; asymmetric flow; porous media.

Contents

1. Introduction

Buoyancy-induced fluid motions in cavities have been discussed widely because of the applications in nature and engineering. A large body of literature exists on the forms of internal and external forcing, various geometry shapes and temporal conditions (steady or unsteady) of the resulting flows. Especially for the classic cases of rectangular, cylindrical or other regular geometries, many authors have investigated imposed temperature or boundary heat fluxes. Reviews of these research can be found in [1-2]. However, the rectangular cavity is not an adequate model for many geophysical situations where a variable (or sloping) geometry has a significant effect on the system. That is why, the convective flows in triangular shaped enclosures have received recent attention in parallel with the rectangular or square enclosure.

In the previous studies of triangular geometries, researchers mainly focused on two applications. Firstly, the fluid dynamics and heat transfer in an attic space with imposed temperature differences between the horizontal base and the sloping upper boundaries [3-9]. Secondly, the fluid dynamics and heat transfer in reservoir sidearms, seashores, lakes and other shallow water bodies with a sloping bottom surface [10-13].

Heat transfer through the attic space into or out of buildings is an important issue for attic-shaped houses in both hot and cold climates. One of the most important objectives for design and construction of houses is to provide thermal comfort for occupants. In the present energy-conscious society, it is also a requirement for houses to be energy efficient, i.e. the energy consumption for heating or air-conditioning must be minimized. Because of the relevance to these objectives, research related to the heat transfer in attics has been conducted for about three decades.

From the available literature it is observed that two basic sets of temperature boundary conditions in the context of an attic have been considered previously: night-time or winter-time cooling (cold top and hot bottom) and day-time or summer-time heating (hot top and cold bottom). Other topics related to the attic space have also been researched recently such as attics subject to localized heating, and attics filled with porous media. Some of the important studies are presented as a tabular form in Table 1.

2. Night-time boundary conditions

2.1 Symmetric flow assumption

The night-time or winter-time boundary conditions mean the sloping walls of the attic are isothermally cooled and the bottom is heated or adiabatic. For this type of boundary condition the flow is potentially unstable due to Rayleigh Bénard instability. The instability occurs as a form of sinking and rising plumes as it is seen in Fig.1. The temperature contours and the stream lines are shown in Fig.1 for $Ra = 10^4$, Pr = 0.72 and $A = 1.0$. The top inclined walls are cooled and the bottom horizontal wall is heated isothermally. Note that the flow seems to be symmetric along the geometric centerline. It is found in the literature that Flack [3] was the first who adopted an isosceles triangle for his experimental model and conducted flow visualizations and heat transfer measurements for the night-time conditions. From the temperature difference between the boundaries the author obtained heat transfer data and observed the motions of suspended particles qualitatively. Also, the velocities at a few selected locations were measured using a laser velocimeter. The velocity measurements were made primarily to aid in the general understanding of the structure and direction of the flow. The author also showed the temperature profiles along the apparatus centerline. It was also found that initially, at low Rayleigh numbers, the flow remained laminar. However, as the Rayleigh number was increased, the flow eventually became turbulent. The author mentioned that four Bénard type convective cells were present in the laminar flow regime, but there was no mention of the development of the cellular flow pattern, or the relative positions of the cells in the stream lines.

The night-time attic problem was again investigated experimentally by Poulikakos and Bejan [4]. However, the author considered an right-angled triangle instead of full isosceles triangular domain where an adiabatic thermal condition applied to the vertical wall. The focus of their investigation was on the flow regime with a Rayleigh number range 10^6 to 10^9, higher than considered previously. It was observed that the flow inside the enclosure was turbulent which was consistent with Flack's [3] observation. Moreover, the authors showed a plot of the Nusselt number versus the Rayleigh number as a demonstration of heat transfer. They also visualize the flow using the thymol blue pH indicator technique.

A fundamental study of the fluid dynamics inside an attic-shaped triangular enclosure at night-time was performed by Poulikakos and Bejan [5] with the assumption that the flow was symmetric about the center plane. The study was undertaken in three distinct parts. Firstly the flow and temperature fields in the cavity are determined theoretically on the basis of an asymptotic analysis which is valid for shallow spaces i.e. $A \rightarrow 0$. They were able to show that in the $A \rightarrow 0$ limit the circulation consists of a single elongated cell driven by the cold upper wall. The net heat transfer in this limit is dominated by pure conduction. Secondly, scaling analysis examined the transient behaviour of the attic for fluids $Pr > 1$. The transient phenomenon begins with the sudden cooling of the upper sloping wall. The authors mention that by properly identifying the timescales of various features that develop inside the enclosures, it is possible to predict theoretically the basic flow features that will endure in the steady state. Finally, they focused on a complete sequence of transient numerical simulations covering the ranges of parameters such as the Rayleigh number, aspect ratio and Prandtl number. The authors reported that the resulting flow patterns in the enclosure corresponded to a single convective cell, for the range of conditions considered.

Salmun [6] studied the problem of natural convection inside a two dimensional triangular geometry filled with air or water, with various aspect ratios and Rayleigh numbers ranging between 10^2 and 10^5. Following the earlier simulations, only half of the domain was examined. The center plane was insulated and this was thought to adequately model the entire domain.

Numerical solutions of the time dependent problem were obtained using two different numerical techniques. The general flow structure corresponded to a single convective cell for low values of the *Ra* and to a multi-cellular regime for the high values of this parameter. The author disagreed with some results obtained by Poulikakos and Bejan [4] regarding the heat transfer mechanism. The problem regarding the stability of the single-cell steady state solution was examined by Salmun [7]. It is noted that the author attempted to present the results obtained from a linear stability analysis of the steady state asymptotic solution in a shallow triangular enclosure and to show that it is not stable to the type of instabilities expected in fluid layers heated differentially along horizontal boundaries.

Latter, an investigation to examine the details of the transition from single cell to multi cell flow was carried out by Asan and Namli [8]. The results of their study showed that both the height-base ratio and the Rayleigh number had a profound influence on the temperature and flow field. In addition, it was shown that as the aspect ratio decreased, the transition to multi cell flow took place at higher Rayleigh numbers. The results also showed that once a secondary cell began to form, it caused the primary cell to shift towards the plane of symmetry. However, since it was assumed that the plane of symmetry could be modeled as an adiabatic surface, the flow patterns obtained are comparable with those from works mentioned above.

Haese and Teubner [14] investigated the phenomenon for a large-scale triangular enclosure for night-time condition. The authors point out that for a realistic attic space, Rayleigh numbers as high as 10^{10} or 10^{11} would be encountered. This study focused on the existing building structure. It is interesting to note that, contrary to the previously mentioned work of [8], the authors of this study reported that the shift of multi cellular flow is accelerated by a decrease in the aspect ratio, for the same Rayleigh number. This is consistent with the results presented by

Salmun [5], which indicated that the number of cells that developed, increased as the aspect ratio decreased.

2.2 Asymmetric flow behavior

The studies mentioned above assumed that the flow was symmetric about the geometric centre plane. However, Holtzman *et al.* [15] for the first time examined the validity of this assumption. The authors pointed out that at low *Ra*, symmetric solutions are obtained, indicating that a symmetry assumption is valid in agreement with the single cell solutions found in previous studies. However, as the Rayleigh number increases, a pitchfork bifurcation is observed in which two steady asymptotic mirror image solutions can be found. However, it was reported that only asymmetric solutions were stable beyond a critical Rayleigh number, if a finite perturbation was applied. To confirm the numerical predictions of the flow patterns and the existence of a symmetry-breaking bifurcation, a flow visualization study was conducted. The flow patterns were observed by slowly injecting smoke into the enclosure. The isotherms and the stream lines are plotted in Fig.2 for $Ra = 10^6$, Pr = 0.72 and $A = 0.5$ (top) and $A = 0.2$. The top inclined walls are cooled and bottom wall is heated. It is found that unlike Fig.1 the flow seems to be asymmetric along the symmetric line of the geometry. A large dominating convecting cell can be seen in the stream lines along the mid plane in both aspect ratio.

Transient thermal convection in an air-filled isosceles triangular cavity heated from below and cooled from above has also been studied numerically by Ridouane and Campo [16] for a fixed aspect ratio $A = 0.5$ over an extensive range of Rayleigh numbers. The influence of *Ra* on the flow and temperature patterns is analyzed and discussed for two contrasting scenarios, which correspond to increasing and decreasing *Ra*. Two steady-state solutions were obtained numerically using appropriate initial perturbations. For increasing *Ra*, it is confirmed that the symmetric flow is achievable at relatively low *Ra* numbers. However, as *Ra* is continually increased, the symmetric plume breaks down and fades away. Thereafter, a subcritical pitchfork bifurcation is created, giving rise to an anti-symmetric plume occurring at a critical Rayleigh number, $Ra=1.5\times10^5$. The evolution of the flow structure with time is studied in detail to illustrate how this physical transition manifests. The multiplicity of solutions is not a simple theoretical finding, because experimental evidence taken from the specialized literature supports it in a convincing manner. The existing ranges of these solutions are reported for both scenarios, i.e., increasing and decreasing *Ra*. The emergence of a hysteresis phenomenon is observed. This implies that the critical *Ra* characterizing the transition from a steady-state solution to another steady-state solution depends markedly on the scenario considered. In fact, when increasing *Ra*, this value is equivalent to $Ra = 1.5\times10^5$, but when decreasing *Ra* this value decreases to $Ra = 1.0\times10^4$. In the subinterval between these two critical values of *Ra*, both solutions are possible depending solely on the initial conditions. Quantitatively, for a fixed *Ra* in this subinterval, the difference in the mean Nusselt number between the symmetrical and the asymmetrical solutions is found to be about 1% – 3%.

Lei *et al.* [17] studied both experimentally and numerically the natural convection flow in an isosceles triangular enclosure subject to abrupt heating from the base and simultaneous cooling from the inclined surfaces. The authors used water as the working fluid for flow visualization using a shadowgraph technique. The numerical simulation has been carried out for a fixed aspect ratio $A = 0.5$, and a range of Rayleigh numbers. They classified the transient flow development in the enclosure into three distinct stages, an initial stage, a transitional stage, and a steady or quasi-steady stage in both the experiments and numerical simulations. They claim that the initial stage flow is characterized by the growth of thermal boundary layers adjacent to all the interior surfaces and the initiation of primary circulations. The transitional stage flow is characterized by the appearance of convective instabilities in the form of rising and sinking thermals and the formation of cellular flow structures. The steady-state flow at low Rayleigh numbers is characterized by symmetric flows about the geometric symmetry plane, and the quasi steady flow at relatively higher Rayleigh numbers is characterized by the pitchfork bifurcation, which results in alternative occurrence of convective instabilities from the two sides of the enclosure and the oscillation of the upwelling flow near the centre. They also established two important time scales from the numerical simulation based of the curved fitting of the data; time scale for the steady or quasi-steady flow as

$$t_s = 8257.9 Gr^{-0.532} \tag{1}$$

and the time scale for the transition of the flow from symmetry to asymmetry due to the pitchfork bifurcation as follows

$$t_c = 9.46 \times 10^4 Gr^{-0.564} \tag{2}$$

The authors also established a co-relation between the Nusselt number and the Grashof number as

$$\left|Nu\right|_{base} = 0.826 Gr^{0.207} \text{ and } \left|Nu\right|_{top} = 0.743 Gr^{0.207} \tag{3}$$

All studies reported for the attic space problem are for the two dimensional domain. The authors claimed that two dimensional simulations are good enough to characterize the flow.

Saha et al. [18-20] have demonstrated the heat transfer through the attics of buildings under realistic thermal forcing condition. The authors applied a periodic temperature boundary condition on the sloping walls of the attic to show the basic flow features in the attic space over diurnal cycles. Their numerical results revealed that, the flow in the attic space is stratified during the daytime heating stage; whereas the flow becomes unstable at the night-time cooling stage. When the Rayleigh number is lower the flow is symmetric. However, as it is seen in [15-16] for the sudden cooling case, a transition occurs at a critical value of *Ra*. Above this critical value, an asymmetrical solution exhibiting a pitchfork bifurcation arises at the night-time. They also claimed that the calculated heat transfer rate at the night-time cooling stage is much higher than that during the daytime heating stage. The same authors [21] again studied the fluid dynamics in the attic space by focusing on its transient response to sudden and linear changes of temperature along the two inclined walls. The authors felt that the transient behaviour of an attic space is relevant to our daily life. They applied the sudden and the ramp cooling boundary condition on the sloping walls of the attic space. A theoretical understanding of the transient behaviour of the flow in the enclosure is performed using the scaling analysis. The authors established that a proper identification of the time scales, the velocity and the thickness relevant to the flow that develops inside the cavity makes it possible to predict theoretically the basic flow features. These features will survive once the thermal flow in the enclosure reaches a steady state. They also predict a time scale for the cooling down of the whole cavity together with the heat transfer scales through the inclined walls through scaling analysis. The verification of those scalings by the numerical simulations has also been performed in their studies.

In addition to the above studies, night-time boundary condition is considered by many other researchers [43-48]. Omri et al [43] considered the steady natural convection in isosceles triangular cavities heated from bellow by a uniform heat flux. The authors identified three important flow mechanisms that drive a circulation in the enclosure. The first mechanism is the symmetrical structure of the flow as describe before. This structure is seen when the values of *Ra* is smaller or the isosceles cavity is less wider for all values of *Ra*. The second mechanism is the asymmetrical flow structure. If the Rayleigh number exceeds a critical value an asymmetric bifurcation sets in. A dominating large vortex develops inside the cavity. The third mechanism is the multi-cellular flow structure in wider cavities (see Fig. 2d.). They also identified a boundary layer regime as an effect of the vent area on the thermal exchange. The authors found that as the number of cells is increased, the quantity of heat transferred from the bottom to the upper layers by convection is increased. The numerical simulation of 3D behaviour of the flow has been considered by Ridouane and Campo [45] in a pentahedral spaces. Roy et al. [44] presented a finite element simulation for uniform and sinusoidal distribution of the temperature on the bottom surface.

3. Day-time heating conditions

Unlike night-time conditions, the attic space problem under daytime conditions has received very limited attention. This is might be the fact that the flow structure for this type of boundary condition is simple. The boundary conditions for day-time or summer-time are that the sloping walls of the attic space are isothermally heated and the bottom surface is cooled or adiabatic. Flack [3] first investigated the flow dynamics and heat transfer for the day-time boundary condition on the triangular enclosure. The author found that under this conditions, the flow inside the enclosure remained laminar for Rayleigh numbers up to 4.9×10^7. It was found that the resulting heat transfer data could be correlated with heat fluxes calculated for one-dimensional conduction, suggesting that the heat transfer through the enclosure was dominated by conduction. Under the daytime conditions, the heat transfer rates and flow velocities were significantly lower than those under the night-time conditions. The authors established a co-relation between the Nusselt number and the Grashof number (*Ra*/Pr) as

$$Nu = c_1 Gr^{c2} \tag{4}$$

where c_1 and c_2 were calculated based on the aspect ratios, e.g. for $A = 1$, $c_1 = 0.19$ and $c_2 = 0.30$.

With the continuation of the previous work for air conditioning calculation, Asan and Namli [9] reported results for steady, laminar, two-dimensional natural convection in a pitched roof of triangular cross-section under the summer-day boundary condition. The results showed that the aspect ratio has a profound influence on the temperature and flow field. On the other hand, the effect of Rayleigh number is not significant for $A < 1$ and $Ra < 10^5$. For small Rayleigh numbers, two counter rotating vortices were present in the enclosure and the eye of the vortices was located at the center of the half of the cross-section. With the increase of the Rayleigh number, a secondary vortex developed and the newly developed secondary vortex pushed the eye of the primary vortex further towards the inclined wall. The transition from a two-vortex solution to a multiple vortex solution was dependent on the Rayleigh number and the aspect ratio. They noticed that near the place of intersection of the cold horizontal wall and hot inclined wall a considerable proportion of the heat transfer across the base wall of the region takes place. The authors also showed the relationship between the mean Nusselt number, the Rayleigh number, and the aspect ratio.

Latter, Akinsete and Coleman [22] numerically simulated the attic space with a hot upper sloping wall and cooled base. Their aim was to obtain previously unavailable heat transfer data, relevant to air conditioning calculations. This study considered only half of the domain. The authors considered two forms of heating on the hot wall including isothermal heating and constant heat flux heating. They found that the calculated flow remained laminar in this study, which agrees with Flack's [3] experiment for day-time heating conditions.

Recently, Saha et al. [23] and Saha [24-25] investigated the fluid flow and heat transfer inside a triangular enclosure due to sudden and ramp heating on the inclined walls and adiabatic bottom wall scaling analysis and direct numerical simulations for both $Pr < 1$ and $Pr > 1$. The authors showed that the development of the unsteady natural convection boundary layer under the inclined walls might be classified into three distinct stages including a start-up stage, a transitional stage and a steady state stage. These three stages can be clearly identified in the analytical as well as numerical results. The authors considered an improved new triple-layer integral approach of scaling analysis for $Pr > 1$ [24-25] to obtain major scaling relations of the velocity, thicknesses, Nusselt number and the flow development time of the natural convection boundary layer and verified by direct numerical simulations over a wide range of flow parameters. The new scaling results can properly demonstrate the Pr dependency on the fluid flow and heat transfer.

Saha and Khan [26] numerically investigated the coupled thermal boundary layers which develop on both sides of a partition placed in an isosceles triangular enclosure along its middle symmetric line. The authors imposed a sudden temperature difference between two zones of the enclosure to trigger the natural convection. The observed from the numerical simulations that the coupled thermal boundary layers development adjacent to the partition undergoes three distinct stages; namely an initial stage, a transitional stage and a steady state stage. Time dependent features of the coupled thermal boundary layers as well as the overall natural convection flow in the partitioned enclosure have also been discussed and compared with the non-partitioned enclosure. Moreover, The examined the heat transfer as a form of local and overall average Nusselt number through the coupled thermal boundary layers and the inclined walls.

There are more studies for the case of day-time heating is available in the literature [27-42]. Most studies are conducted using numerical simulations and steady state case. However, some dealt with experimental observation [30-31, 33, 39]. Anderson et al. [30] confirmed the correlation established by Ridouane and Campo [28] given by

$$Nu = 0.286A - 0.286Gr^{1/4} \tag{5}$$

and suggested that this relation might be suitable for predicting the natural convection heat transfer coefficients in full scale attic enclosures. Latter, same authors [31] experimentally investigated the suppression of natural convection by attaching a single baffle at different places along the symmetry line in the attic space. The authors found that the heat transfer coefficient changes with increasing baffle length and can be predicted using a modified form of the above correlation (5) as,

$$Nu = 0.286A - 0.286Gr^{1/4}\left(1 - \frac{L}{H}\right)^{0.2} \tag{6}$$

where, L is the baffle length and H is the height of the attic space. Karyakin and Sokovishin [32] investigated natural convection in the full domain of the attic space by considering two cases of thermal boundary conditions: lateral heating of the enclosure with an insulated base and heating from above. The results of calculations agree with the available experimental data given in [33]. The lateral heating is also investigated for wide range of aspect ratio and Grashof number both experimentally and numerically by Flack et al [39].

4. Porous triangular enclosure

Natural convection of a triangular cavity filled with porous media was first investigated by Bejan and Poulikakos [49]. The authors analytically studied the natural convection in a wedge-shaped porous layer cooled from above and showed that the flow pattern can differ fundamentally from Bénard circulation encountered in constant-thickness horizontal layers. Latter, Varol and co-authors [50-59] comprehensively investigated the steady state natural convection in porous triangular enclosures. They applied different types of boundary condition to show the suppression and enhancement of the heat transfer. They have added thin fin on the surfaces [51-53] and found that the heat transfer significantly reduced in presence of the fin. The authors also considered non-uniform heating on the surfaces [55-56] and a square body inside the porous cavity.

The finite element simulations for the porous triangular enclosure has been performed by Basak and co-authors [60-61]. Basak et al. [60] investigated heating effects for two cases involving uniform and non-uniform heating of bottom wall in a triangular cavity with the aspect ratio 1. They established that the local heat transfer rates at specific locations along the top and bottom walls may be enhanced either with uniform or non-uniform heating of bottom wall for various Prandtl number regimes corresponding to various materials. The authors also claimed that the heating scenarios would be further useful to obtain guideline for efficient material processing within triangular cavities.

Basak et al. [61] numerically studied the phenomena of natural convection in an isosceles triangular enclosure filled with a porous media. They carried out the study in two cases depending on various thermal boundary conditions; firstly, two inclined walls are uniformly heated and the bottom wall is isothermally cooled and secondly, two inclined walls are non-uniformly heated and the bottom wall is isothermally cooled. The authors found that at low Darcy numbers the heat transfer is primarily due to conduction irrespective of the *Ra* and Pr. They also found that as Rayleigh number increases, there is a change from conduction dominant region to convection dominant region for a specific Darcy number and obtained the critical Rayleigh number. For low and high Prandtl number limits they established some interesting features of stream function and isotherm.

5. Other studies

The study of natural convection in the triangular enclosures due to the localized heating and in presence of flush mounted heater or protruding isothermal heater is seen in the literature [62-67]. All studies considered the steady state simulations. The heating effects are shown as a form of isotherm and streamlines and the heat transfer are shown as the Nusselt number. Studies of entropy generation and heat flow using the Bejan's heatline concept are conducted by Basak and co-authors [68-70]. They used finite element analysis for the numerical simulations in their studies. Natural convection of heated vertical wall and cooled hypotenuse of a right-angled triangular cavity is studied by Ridouane et al. [71]. A double diffusive natural convection is also studied by Hari et al. [72] and Chamkha et al. [73].

The turbulent natural convection inside the attic space has also been studied [74-76]. Ridouane et al. [74] dealt with turbulent natural convection inside an air-filled isosceles triangular enclosure which represents conventional attic spaces of houses and buildings with pitched roofs and horizontally suspended ceilings. Their dimensional model corresponds the nondimensional values of Rayleigh numbers equal to 1.58×10^9 and 5×10^{10}. The authors modelled the turbulent flow by a low-Reynolds-number $k - \varepsilon$ model. They found that the average Nusselt number associated to the right inclined wall is found to be 25% higher than the corresponding value connected to the left inclined wall. Another interesting study of Brownian motion of nanoparticles in a triangular enclosure has been investigated by Ghasemi and Aminossadati [77]. The authors found an optimum value for the solid volume fraction at high Rayleigh numbers, which results in the maximum heat transfer rate. They claimed that this is in contradiction to the results of the analysis in which Brownian motion is neglected.

6. Concluding remarks

In this study literature related to attic shaped enclosure has been reviewed in details. The study on this geometry started from 1979 by Flack [33]. However, recently an increasing interests have been developed because of its application of our daily life. Until the year 2000, the flow inside the attic space for night-time boundary conditions considered as symmetric. Hotlzman et al. [15] first showed the asymmetric behaviour of the flow by both experimentally and numerically. This is an interesting phenomena for the fluid flow which is at the regime before reaching turbulent. It is found from the above review that most works performed in the laminar and transition regimes. However, in reality the air flow inside the real attic seems to be turbulent for the higher Rayleigh numbers ($Ra > 10^{10}$). Therefore, more study should be undertaken in this regime. Moreover, for higher Rayleigh numbers, even in the transitional regime, the flow might have 3D behaviour. More study on 3D geometry of the attic space is required. Above all, Scaling analysis for the sudden/ramp heating and cooling have been performed well with numerical verification. However, scaling for diurnal heating and cooling is yet to be unrevealed.

References

[1] S. Ostrach, Natural convection in enclosures, J. Heat Transfer 110 (1988) 1175-1190.

[2] J. M. Hyun, Unsteady buoyant convection in an enclosure, Adv. Heat Transfer 24 (1994) 277-320.

[3] R. D. Flack, The experimental measurement of natural convection heat transfer in triangular enclosures heated or cooled from bellow, J. Heat Transfer 102 (1980) 770-772.

[4] D. Poulikakos and A. Bejan, Natural convection experiments in a triangular enclosure, J. Heat Transfer 105 (1983) 652-655.

[5] D. Poulikakos and A. Bejan, The fluid dynamics of an attic space, J. Fluid Mech. 131 (1983) 251-269.

[6] H. Salmun, Convection patterns in a triangular domain, Int. J. Heat Mass Transfer 38 (1995) 351-362.

[7] H. Salmun, The stability of a single-cell steady-state solution in a triangular enclosure, Int. J. Heat Mass Transfer 18 (1995) 363-369.

[8] H. Asan and L. Namli, Numerical simulation of buoyant flow in a roof of triangular cross-section under winter day boundary conditions, Energy Buildings 33 (2001) 753-757.

[9] H. Asan and L. Namli, Laminar natural convection in a pitched roof of triangular cross-section: summer day boundary conditions, Energy Buildings 33 (2000) 69-73.

[10] C. Lei, and J. C. Patterson, Unsteady natural convection in a triangular enclosure induced by absorption of radiation. J. Fluid Mech. 460 (2002) 181-209.

[11] G. M. Horsch and H. G. Stefan, Convective circulation in littoral water due to surface cooling. Limnol. Oceanogr. 33 (1988) 1068-1083.

[12] Y. Mao,C. Lei, and J. C. Patterson, Unsteady near-shore natural convection induced by surface cooling, J. Fluid Mech. 642 (2010) 213–233.

[13] Y. Mao,C. Lei, and J. C. Patterson, Unsteady natural convection in a triangular enclosure induced by absorption of radiation, J. Fluid Mech. 622 (2009) 75-102.

[14] P. M. Haese and M. D. Teubner, Heat exchange in an attic space, Int. J. Heat Mass Transfer 45 (2002) 4925-4936.

[15] G. A. Holtzman, R. W. Hill and K. S. Ball, Laminar natural convection in isosceles triangular enclosures heated from bellow and symmetrically cooled from above, J. Heat Transfer 122 (2000) 485-491.

[16] E. H. Ridouane and A. Campo, Formation of a pitchfork bifurcation in thermal convection flow inside an isosceles triangular cavity, Physics Fluids 18 (2006) 074102.

[17] C. Lei, S. Armfield and J. C. Patterson, Unsteady natural convection in a water-filled isosceles triangular enclosure heated from below, Int. J. Heat Mass Transfer 51 (2008) 2637-2650.

[18] S. C. Saha, C. Lei, J. C. Patterson, Natural convection in a triangular enclosure subject to periodic thermal forcing, In Proceedings of the 13th International Heat Transfer Conference, Sydney, Australia, 13-18 August 2006. Paper No. NCV-43. (ISBN 1-56700-226-9)

[19] S. C. Saha, C. Lei and J. C. Patterson, Effect of aspect ratio on natural convection in

attics subject to periodic thermal forcing, ANZIAM J., 48 (2007) C677-C691,

[20] S. C. Saha, J. C. Patterson and C. Lei, Natural convection and heat transfer in attics subject to periodic thermal forcing, Int. J. Thermal Sci. 49 (2010) 1899-1910.

[21] S. C. Saha, J. C. Patterson, C. Lei, Natural convection in attics subject to instantaneous and ramp cooling boundary conditions, Energy Buildings 42 (2010) 1192-1204.

[22] V. A. Akinsete, and T. A. Coleman, Heat transfer by steady laminar free convection in triangular enclosures, Int. J. Heat Mass Transfer 25 (1982) 991-998.

[23] S. C. Saha, J. C. Patterson and C. Lei, Natural convection in attic-shaped spaces subject to sudden and ramp heating boundary conditions, Heat Mass Transfer 46 (2010) 621-638.

[24] S. C. Saha, Unsteady natural convection in a triangular enclosure under isothermal heating. Energy Buildings, 43 (2011) 701-709.

[25] S. C. Saha, Scaling of free convection heat transfer in a triangular cavity for Pr > 1. Energy Buildings, under review.

[26] S. C. Saha, M. M. K. Khan, Unsteady buoyancy driven flows and heat transfer through coupled thermal boundary layers in a triangular enclosure, Energy Buildings, under review.

[27] H. F. Oztop, Y. Varol and A. Koca, Laminar natural convection heat transfer in a shed roof with or without eave for summer season, Applied Thermal Engineering, 27 (2007) 2252–2265.

[28] E. H. Ridouane and A. Campo, Experimental-based correlations for the characterization of free convection of air inside isosceles triangular cavities with variable apex angles, Exp. Heat Transfer 18 (2005) 81–86.

[29] S. C. Tzeng, J. H. Liou, and R. Y. Jou, Numerical simulation-aided parametric analysis of natural convection in a roof of triangular enclosures, Heat Trans. Eng. 26 (2005) 69–79.

[30] T. N. Anderson, M. Duke and J.K. Carson, Experimental determination of natural convection heat transfer coefficients in an attic shaped enclosure, Int. Comm. Heat Mass Transfer 37 (2010) 360–363.

[31] T. N. Anderson, M. Duke and J.K. Carson, Suppression of natural convection heat transfer coefficients in an attic shaped enclosure, Int. Comm. Heat Mass Transfer 37 (2010) 984–986.

[32] Y. E. Karyakin and Y. A. Sokovishin, Transient natural convection in triangular enclosures, Int. J. Hear Mass Transfer 31 (1988) 1759-1766.

[33] R. D. Flack, The measurement of natural convective heat transfer in triangular enclosures. J. heat transfer 101 (1979) 648-654.

[34] E. H. Ridouane, A. Campo, M. McGarry, Numerical computation of buoyant airflows confined to attic spaces under opposing hot and cold wall conditions, Int. J. Thermal Sci. 44 (2005) 944–952.

[35] E. F. Kent, Numerical analysis of laminar natural convection in isosceles triangular enclosures, Proc. IMechE, Part C: J. Mechanical Engineering Science, 223 (2009) 1157-1169.

[36] E. F. Kent, Numerical analysis of laminar natural convection in isosceles triangular enclosures for cold base and hot inclined walls, Mechanics Research Communications 36 (2009) 497–508.

[37] E. H. Ridouane, A. Campo and M. Hasnaoui, Benefits derivable from connecting the bottom and top walls of attic enclosures with insulated vertical side walls, Numer. Heat Transfer, Part A 49 (2006) 175–193.

[38] T. Basak, S. Roy, Ch. Thirumalesha, Finite element analysis of natural convection in a triangular enclosure: Effects of various thermal boundary conditions, Chemical Engineering Science 62 (2007) 2623 – 2640.

[39] R. D. Flack, K. Brun and F. J. Schnipke, Measurement and prediction of natural convection velocities in triangular enclosures, Int. J. Heat Fluid Flow 16 (1995) 106-113.

[40] H. Küçük and H. Gedikli, Natural convection in a triangular cross section roof under daylight conditions, J. Thermal Science Technology, 30 (2010) 99-110.

[41] T. Anderson, M. Duke and J. Carson, Convection suppression in a triangular-shaped enclosure, Computational Thermal Sciences, 1 (2009) 309–321.

[42] T. Basak, S. Roy, S. K. Babu and A.R. Balakrishnan, Finite element analysis of natural convection flow in a isosceles triangular enclosure due to uniform and non-uniform heating at the side walls, Int. J. Heat Mass Transfer 51 (2008) 4496–4505.

[43] A. Omri, M. Najjari and S. B. Nasrallah, Numerical analysis of natural buoyancy-induced regimes in isosceles triangular cavities, Numer. Heat Transfer, Part A 52 (2007) 661–678.

[44] S. Roy, T. Basak, Ch. Thirumalesha, Ch. M. Krishna, Finite element simulation on natural convection flow in a triangular enclosure due to uniform and non-uniform bottom heating, J. Heat Transfer 130 (2008) 032501.

[45] E. H. Ridouane and A. Campo, Numerical simulation of the 3D behavior of thermal buoyant airflows in pentahedral spaces, Int. J. Heat Fluid Flow 29 (2008) 1360–1368.

[46] E. H. Ridouane and A. Campo, Relationship between thermal convection intensity and aspect ratio of two triangular cavities inscribed in horizontal rectangular cavities, Int. J. Num. Meth. Heat Fluid Flow 16 (2006) 338-355.

[47] E. F. Kent, Laminar Natural Convection in Isosceles Triangular Roofs in Wintertime Conditions, Heat Trans. Eng. 31 (2010) 1068–1081.

[48] E. F. Kent, E. Asmaz and S. Ozerbay, Laminar natural convection in right triangular enclosures, Heat Mass Transfer 44 (2007) 187–200.

[49] A. Bejan and D. Poulikakos, Natural convection in an attic shaped space filled with porous material, J. Heat Transfer 104 (1982) 241-247.

[50] Y. Varol, H. F. Oztop and A. Varol, Free convection in porous media filled right-angle triangular enclosures. Int. Comm. Heat Mass Transfer 33 (2006) 1190-1197.

[51] Y. Varol, H. F. Oztop and A. Varol, Effects of thin fin on natural convection in porous triangular enclosures. Int. J. Thermal Sci. 46 (2007) 1033-1045.

[52] Y. Varol, H. F. Oztop and A. Varol, Natural convection in porous triangular enclosures with a solid adiabatic fin attached to the horizontal wall. Int. Comm. Heat Mass Transfer 34 (2007) 19-27.

[53] Y. Varol, H. F. Oztop and I. Pop, Natural convection in porous media-filled triangular enclosure with a conducting thin fin on the hot vertical wall, Proc. IMechE. Part C: J. Mechanical Engineering Science, 222 (2008) 1735-1743

[54] Y. Varol, H. F. Oztop and I. Pop, Conjugate heat transfer in porous triangular enclosures with thick bottom wall, Int. J. Num. Meth. Heat Fluid Flow 19 (2009) 650-664.

[55] Y. Varol, H. F. Oztop and I. Pop Entropy generation due to natural convection in non-uniformly heated porous isosceles triangular enclosures at different positions, Int. J. Heat Mass Transfer 52 (2009) 1193–1205.

[56] Y. Varol, H. F. Oztop, M. Mobedi and I. Pop, Visualization of natural convection heat transport using heatline method in porous non-isothermally heated triangular cavity, Int. J. Heat Mass Transfer 51 (2008) 5040–5051.

[57] Y. Varol, H. F. Oztop and T. Yilmaz, Two-dimensional natural convection in a porous triangular enclosure with a square body, Int. Comm. Heat Mass Transfer 34 (2007) 238–247.

[58] Y. Varol, H. F. Oztop and I. Pop Influence of inclination angle on buoyancy-driven convection in triangular enclosure filled with a fluid-saturated porous medium, Heat Mass Transfer 44 (2008) 617–624.

[59] H. F. Oztop, Y. Varol, I. Pop, Investigation of natural convection in triangular enclosure filled with porous media saturated with water near 4°C, Energy Conversion Management 50 (2009) 1473–1480.

[60] T. Basak, S. Roy, S. K. Babu, Natural convection and flow simulation in differentially heated isosceles triangular enclosures filled with porous medium, Chemical Engineering Science 63 (2008) 3328 - 3340.

[61] T. Basak, S. Roy, S. K. Babu and I. Pop, Finite element simulations of natural convection flow in an isosceles triangular enclosure filled with a porous medium: Effects of various thermal boundary conditions, Int. J. Heat Mass Transfer 51 (2008) 2733–2741.

[62] A. Koca, H. F. Oztop and Y. Varol, The effects of Prandtl number on natural convection in triangular enclosures with localized heating from below, Int. Comm. Heat Mass Transfer 34 (2007) 511-519.

[63] Y. Varol, A. Koca and H. F. Oztop, Natural convection in a triangular enclosure with flush mounted heater on the wall, Int. Comm. Heat Mass Transfer 33 (2006) 951-958.

[64] Y. Varol, H. F. Oztop and T. Yilmaz, Natural convection in triangular enclosures with protruding isothermal heater, Int. J. Heat Mass Transfer 50 (2007) 2451-2462.

[65] A. Koca, H. F. Oztop and Y. Varol, Natural convection analysis for both protruding and flush-mounted heaters located in triangular enclosure, Proc. IMechE, Part C: J. Mechanical Engineering Science, 222 (2008) 1203-1214.

[66] Y. Varol, H. F. Oztop and A. Koca, Entropy production due to free convection in partially heated isosceles triangular enclosures, Applied Thermal Engineering 28 (2008) 1502–1513.

[67] E. H. Ridouane and A. Campo, Effects of attaching baffles onto the inclined walls of attic frames for purposes of energy conservation, Heat Trans. Eng. 28 (2007) 103–111.

[68] T. Basak, G. Aravind, S. Roy and A. R. Balakrishnan, Heatline analysis of heat recovery and thermal transport in materials confined within triangular cavities, Int. J. Heat Mass Transfer 53 (2010) 3615–3628.

[69] R. S. Kaluri, R. Anandalakshmi, T. Basak, Bejan’s heatline analysis of natural convection in right-angled triangular enclosures: Effects of aspect-ratio and thermal boundary conditions, Int. J. Thermal Sci. 49 (2010) 1576-1592

[70] T. Basak, R. S. Kaluri and A. R. Balakrishnan, Effects of thermal boundary conditions on entropy generation during natural convection, Numer. Heat Transfer, Part A 59 (2011) 372–402.

[71] E. H. Ridouane, A. Campo and J. Y. Chang, Natural Convection Patterns in Right-Angled Triangular Cavities With Heated Vertical Sides and Cooled Hypotenuses, J. Heat Transfer 127 (2005) 1181.

[72] I. Hajri, A. Omri and S. B. Nasrallah, A numerical model for the simulation of double-diffusive natural convection in a triangular cavity using equal order and control volume based on the finite element method, Desalination 206 (2007) 579-588.

[73] A. J. Chamkha, M. A. Mansour, S. E. Ahmed, Double-diffusive natural convection in inclined finned triangular porous enclosures in the presence of heat generation/absorption effects, Heat Mass Transfer 46 (2010) 757–768.

[74] E. H. Ridouane, A. Campo, M. Hasnaoui, Turbulent natural convection in an air-filled isosceles triangular enclosure, Int. J. Heat Fluid Flow 27 (2006) 476–489.

[75] M. Ghasemi, M. Fathabadi and A. Shadaram, Numerical analysis of turbulent natural convection heat transfer inside a triangular-shaped enclosure utilizing computational fluid dynamic code, Int. J. Num. Meth. Heat Fluid Flow 18 (2008) 14-23.

[76] S.O. Talabi, V.O.S. Olunloyo, O.M. Kamiyo, M.W. Collins and T.G. Karayiannis, Flow field and Reynolds stress distribution in low turbulence natural convection in a triangular cavity, International Centre for Heat and Mass Transfer, ICHMT Digital Library Online, 10 (2006) 511-514.

[77] B. Ghasemi, S.M. Aminossadati, Brownian motion of nanoparticles in a triangular enclosure with natural convection, Int. J. Thermal Sci., 49 (2010) 931-940.

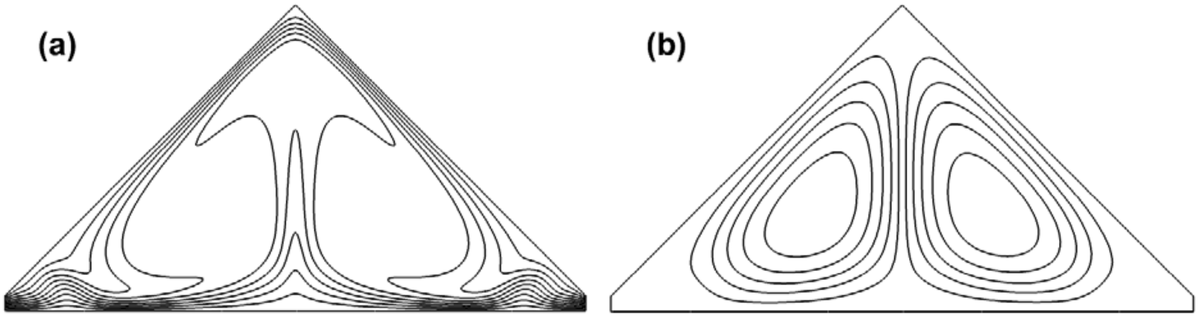

Fig. 1. Steady state (a) Temperature contours and (b) stream lines for Ra = 10^4, Pr = 0.72 and A = 1.0.

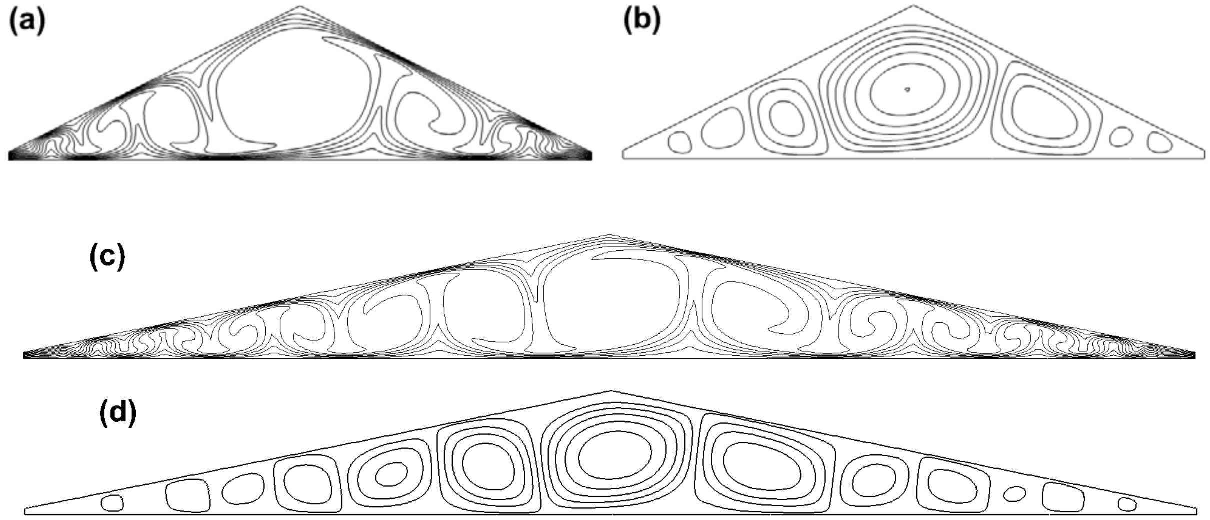

Fig. 2. Steady state (a-c) Temperature contours and (b-d) stream lines for *Ra* = 10^6, Pr = 0.72 and *A* = 0.5 (top) and *A* = 0.2 (bottom).

Table 1

Selected studies of two dimensional attic space

References	Types of study	Aspect ratio	Fluid Pr values	*Ra* range	Steady/ unsteady	Temperature difference	Configuration
[24]	Scaling & Numer.	0.2, 0.5, 1.0	5 - 10^2	10^6 - 10^8	Unsteady	1 (non-dimension)	Two inclined walls are heated & bottom is adiabatic
[23]	Scaling & Numer.	0.1, 0.2, 0.5, 1.0	0.72	1.5×10^6 - 3.0×10^7	Unsteady	1 (sudden), $\min(\tau/\tau_p,1)$ (ramp)	Two inclined walls are heated (both sudden and ramp) & bottom is adiabatic
[21]	Scaling & Numer.	0.2, 0.5, 1.0	0.72	10^4 - 7.2×10^6	Unsteady	10*K* (sudden), 0*K* - 10*K* (ramp)	Two inclined walls are cold (both sudden and ramp) & bottom is adiabatic
[18-20]	Numer.	0.2, 0.5, 1.0	0.72	7.2×10^3 - 1.5×10^6	Unsteady	-5.0 - 5.0*K*	$T = (295+5\sin(2\pi t/P))K$ on two inclined walls & 295*K* on bottom wall
[17]	Exp. & Numer.	0.5	7.06	4.17×10^4 - 3.34×10^6	Unsteady	0.2*K* - 16*K*	Bottom is heated and top walls are cold
[5]	Numer.	0.1-1.0	0.72, 7.1	10^2 - 10^5	Unsteady	1 (Non-dimension)	Half domain: bottom wall is heated, top cooled and the vertical wall is insulated.
[6]	Analyt. & Numer.	0.2	0.72	72 - 72000	Unsteady	-	Half domain: Bottom heated, top cooled and insulated vertical.
[15]	Exp. & Numer.	0.2, 0.5, 1.0	0.71	10^3 - 10^5	Steady	-	Bottom heated and top inclined walls cooled
[16]	Numer.	0.5	0.71	10^3 - 10^6	Unsteady	-	Heated bottom, cooled upper walls
[4]	Asympt., Scaling & Numer.	0.2, 0.4, 1.0	0.72, 6.0	1.4×10^1 - 1.4×10^5	Unsteady	-	Half domain: Bottom cold, insulated vertical & hot upper wall
[7-8]	Numer.	0.125-1.0	0.7	10^3 - 10^6	Steady	1 (Non-dimension)	Cold (or hot) bottom wall and hot (or cold) top walls
[13]	Exp.	0.58-1.73	0.7	1.23×10^5 - 3.56×10^6	-	3°C - 4°C	Cold (or hot) bottom wall and hot (or cold) top walls
[34]	Numer.	0.577, 1.0, 1.73	0.70	7.0×10^2 - 2.0×10^6	Steady	-	Two inclined walls are heated (or cooled) & bottom is cooled (or heated)
[33]	Exp.	0.58-1.73	0.7	2.03×10^6 - 6.30×10^6	-	-	One inclined wall is heated and another one is cooled while bottom is insulated

Table 1 (*continued*)

References	Types of study	Aspect ratio	Fluid Pr values	*Ra* range	Steady/unsteady	Temperature difference	Configuration
[44]	Numer.	0.5	0.026-10^2	10^3 - 10^5	-	1 (non-dimension)	Top inclined walls are cold and bottom is heated (isothermal or sinusoidal)
[14]	Numer.	0.2, 0.5, 1.0	0.7	7.1×10^2 - 7.1×10^{10}	Steady	-	Half domain: cold inclined wall, insulated vertical wall and warm bottom wall without and with inflow, out flow at different locations
[75]	Numer.	0.58, 1.0	0.7	5.34×10^3 - 5.34×10^6	Turbulent	-	Half domain: cold inclined wall, insulated vertical wall and warm bottom wall
[46]	Numer.	1.0, 2.0	0.71	7.1×10^2 - 7.1×10^5	Unsteady	-	Top walls are cold, bottom wall is hot.
[36]	Numer.	0.27-3.73	0.71	10^3 - 10^5	Steady	1 (non-dimension)	Inclined walls are heated and base is cooled.
[40]	Numer.	0.25-1.0	0.7	10^3 - 10^6	Steady	-	Inclined walls are heated and base is cooled.
[43]	Numer.	0.18-5.67	0.7	10^2 -10^6	Steady	-	Cold top and heated bottom (heat flux)
[32]	Numer.	0.5-4.0	0.71, 1.0	7.1×10^2 - 7.1×10^7	Unsteady	-	(a) bottom adiabatic, inclined walls are cold and hot (b) hot inclined surfaces and cold bottom.