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Natural convection and heat transfer in attics subject to periodic thermal forcing

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Abstract

The heat transfer through the attics of buildings under realistic thermal forcing has been considered in this study. A periodic temperature boundary condition is applied on the sloping walls of the attic to show the basic flow features in the attic space over diurnal cycles. The numerical results reveal that, during the daytime heating stage, the flow in the attic space is stratified; whereas at the night-time cooling stage, the flow becomes unstable. A symmetrical solution is seen for relatively low Rayleigh numbers. However, as the $Ra$ gradually increases, a transition occurs at a critical value of $Ra$. Above this critical value, an asymmetrical solution exhibiting a pitchfork bifurcation arises at the night-time. It is also found that the calculated heat transfer rate at the night-time cooling stage is much higher than that during the daytime heating stage.

KEYWORDS: Natural convection; Periodic condition; Asymmetric flow; Nusselt number; Stream lines.
1. Introduction

Heat transfer through an attic space into or out of buildings is an important issue for attic shaped houses in both hot and cold climates. The heat transfer through attics is mainly governed by a natural convection process, and affected by a number of factors including the geometry, the interior structure and the insulation etc. One of the important objectives for design and construction of houses is to provide thermal comfort for occupants. In the present energy-conscious society, it is also a requirement for houses to be energy efficient, i.e. the energy consumption for heating or air-conditioning of houses must be minimized. A small number of publications are devoted to laminar natural convection in two dimensional isosceles triangular cavities in the vast literature on convection heat transfer.

The temperature and flow patterns, local wall heat fluxes and mean heat flux were measured experimentally by Flack [1, 2] in isosceles triangular cavities with three different aspect ratios. The cavities, filled with air, were heated/cooled from the base and
cooled/heated from the sloping walls covering a wide range of Rayleigh numbers. For the case of heated bottom surface it was found that the flow remained laminar for the low Rayleigh numbers. However, as the Rayleigh number increased, the flow eventually became turbulent. The author also reported the critical Rayleigh numbers of the transition from laminar to turbulent regimes. Kent [3] has also investigated the natural convection in an attic space for two different boundary conditions similar to Flack [1, 2]. The author observed that for top heating and bottom cooling case the flow is dominated by pure conduction and remains stable for higher Rayleigh numbers considered. However, the flow becomes unstable for sufficiently large Rayleigh number for the second case (top cooling and bottom heating).

A comparison study is performed by Ridoune et al [4] where the authors compare their numerical results produced for two different boundary conditions, (a) cold base and hot inclined walls (b) hot base and cold inclined walls with the experimental results obtained by Flack [1, 2]. A good agreement has been obtained between the numerical predictions and the experimental measurements of Nusselt number. A numerical study of above mentioned two boundary conditions has also performed by Ridoune et al [5]. However, the authors cut a significant portion of bottom tips and applied adiabatic boundary condition there. It is revealed from the analysis that the presence of insulated sidewalls, even of very small height, provides a huge gain of energy and helps keep the attic at the desired temperature with a minimum energy.

The attic problem under the night-time conditions was again investigated experimentally by Poulakakos and Bejan [6]. In their study, the authors modelled the enclosure as a right-angled triangle with an adiabatic vertical wall, which corresponded to
the half of the full attic domain. A fundamental study of the fluid dynamics inside an attic-shaped triangular enclosure subject to the night-time conditions was performed by Poulikakos and Bejan [7] with an assumption that the flow was symmetric about the centre plane. Del Campo et al. [8] examined the entire isosceles triangular cavities for seven possible combinations of hot wall, cold wall and insulated wall using the finite element method based on a stream function or vorticity formulation. A two dimensional right triangular cavity filled with air and water with various aspect ratios and Rayleigh numbers are also examined by Salmun [9].

The stability of the reported single-cell steady state solution was re-examined by Salmun [10] who applied the same procedures developed by Farrow and Patterson [11] for analysing the stability of a basic flow solution in a wedge-shaped geometry. Later Asan and Namli [12] carried out an investigation to examine the details of the transition from a single cell to multi cellular structures. Haese and Teubner [13] investigated the phenomenon for a large-scale triangular enclosure for night-time or winter day conditions with the effect of ventilation.

Holtzmann et al. [14] modelled the buoyant airflow in isosceles triangular cavities with a heated bottom base and symmetrically cooled top sides for the aspect ratios of 0.2, 0.5, and 1.0 with various Rayleigh numbers. They conducted flow visualization studies with smoke injected into the cavity. The main objective of their research was to validate the existence of the numerical prediction of the symmetry-breaking bifurcation of the heated air currents that arise with gradual increments in Rayleigh number. Ridoune and Campo [15] and Lei et al. [16] have also investigated the numerical prediction of the symmetry-breaking bifurcation. However, water has been used as the working fluid by Lei at al [16].
Ridoune and Campo [15] reported that as $Ra$ is gradually increased, the symmetric plume breaks down and fades away. Thereafter, a subcritical pitchfork bifurcation is created giving rise to an asymmetric plume occurring at a critical Rayleigh number, $Ra = 1.42 \times 10^5$. The steady state laminar natural convection in right triangular and quarter circular enclosures is investigated by Kent et al [17] for the case of winter-day temperature condition. A number of aspect ratios and Rayleigh numbers have been chosen to analyse the flow field and the heat transfer.

Unlike night-time conditions, the attic space problem under day-time (heating from above) conditions has received very limited attention. This may due to the fact that the flow structure in the attics subject to the daytime condition is relatively simple. The flow visualization experiments of Flack [1] showed that the daytime flow remained stable and laminar for all the tested Rayleigh numbers (up to about $5 \times 10^6$). Akinsete and Coleman [18] numerically simulated the attic space with hot upper sloping wall and cooled base. Their aim was to obtain previously unavailable heat transfer data relevant to air conditioning calculations. This study considered only half of the domain. For the purpose of air conditioning calculations, Asan and Namli [19] and Kent [20] have also reported numerical results for steady, laminar two-dimensional natural convection in a pitched roof of triangular cross-section under the summer day (day-time) boundary conditions.

Recently Saha [21] and Saha et al [22] have studied the natural convection inside the attic space subject to sudden and ramp heating/cooling boundary conditions. The authors have presented a detailed scaling analysis of the transient flow with different stages of flow development. A set of numerical simulations has also been performed to verify the scaling relations. In real situations, however, the attic space of buildings is subject to
alternative heating and cooling over a diurnal cycle. Therefore, the flow response and heat transfer in the attic space subject to a periodic thermal forcing are yet to be unveiled.

In this study, numerical simulations of natural convection in an attic space subject to diurnal temperature condition on the sloping wall have been carried out. The effects of the aspect ratio and Rayleigh number on the fluid flow and heat transfer have been discussed in details as well as the formation of a pitchfork bifurcation of the flow at the symmetric line of the enclosure.

2. Formulation of the problem

The physical system is sketched in Figure 1, which is an air-filled isosceles triangular cavity of variable aspect ratios. Here $2l$ is the length of the base or ceiling, $T_0$ is the temperature applied on the base, $T_A$ is the amplitude of temperature fluctuation on the inclined surfaces, $h$ is the height of the enclosure and $P$ is the period of the thermal forcing.

Under the Boussinesq approximations the governing continuity, momentum and energy equations take the following forms.

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  
(1)

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \nabla^2 u
\]  
(2)

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \nabla^2 v + g\beta (T - T_0)
\]  
(3)

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \kappa \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)
\]  
(4)
where $u$ and $v$ are the velocity components along $x$- and $y$-directions, $t$ is the time, $p$ is the pressure, $\nu$, $\rho$, $\beta$ and $\kappa$ are kinematic viscosity, density of the fluid, coefficient of thermal expansion and thermal diffusivity respectively, $g$ is the acceleration due to gravity and $T$ is the temperature.

The boundary conditions for the present numerical simulations are also shown in Figure 1. Here, the temperature of the bottom wall of the cavity is fixed at $T = T_0$. A periodic temperature boundary condition is applied to the two inclined walls. The Rayleigh number for the periodic boundary condition has been defined based on the maximum temperature difference between the inclined surface and the bottom over a cycle as

$$Ra \sim \frac{2g\beta T_0 h^3}{\kappa \nu}.$$  

Three aspect ratios 0.2, 0.5 and 1.0, four Rayleigh numbers, $1.5 \times 10^6$, $7.2 \times 10^5$, $1.5 \times 10^4$, and $1.5 \times 10^3$, and a fixed Prandtl number 0.72 are considered in the present investigation. Based on the experimental observations of Flack [1], which reported the critical Rayleigh number for the flow to become turbulent, we have chosen the maximum Rayleigh number, $Ra = 1.5 \times 10^6$ so that the flow stays in the laminar regime. It is understood that in real situations the Rayleigh number may be much higher than this and an appropriate turbulence model should be applied. This is beyond the scope of this study. In order to avoid the singularities at the tips in the numerical simulation, the tips are cut off by 5% and at the cutting points (refer to Figure 1) rigid non-slip and adiabatic vertical walls are assumed. We anticipate that this modification of the geometry will not alter the overall flow development significantly.

3. Numerical Scheme and Grid and time step dependence tests
Equations (1) - (4) are solved along with the initial and boundary conditions using the SIMPLE scheme. The Finite Volume method has been chosen to discretize the governing equations, with the QUICK scheme (see Leonard and Mokhtari [23]) approximating the advection term. The diffusion terms are discretized using central-differencing with second order accurate. A second order implicit time-marching scheme has also been used for the unsteady term.

Mesh and time step dependence tests have been carried out with three different meshes and three different time steps for each aspect ratio of $A = 0.2, 0.5$ and 1.0. The results of the mesh and time-step dependence tests are shown in Tables 1, 2 and 3, which show the temperature at three different positions in the cavity at specific times $t = 0.75P$ when the flow is the most unstable.

It is seen in Tables 1, 2 and 3 that the variation of the calculated temperature among the three meshes with respect to $T_a$ is very small (<1%). Based on these tests, any of the tested meshes would be sufficiently fine for resolving the flow. Therefore, mesh sizes of $360 \times 90, 320 \times 80$ and $270 \times 90$ for $A = 0.2, 0.5$ and 1.0 respectively have been selected for the numerical simulations. The time step for the aspect ratios 0.2 and 0.5 is adopted as 0.5s and for the aspect ratio 1.0 is 0.75s.

4. Flow response to the periodic boundary condition

In this section, the flow response to the periodic thermal forcing and the heat transfer through the sloping boundary are discussed for the case with $A = 0.5$, $Pr = 0.72$ and $Ra = 1.5 \times 10^6$. 
4.1 General flow response to diurnal heating and cooling

Since the initial flow is assumed to be isothermal and motionless, there is a start-up process of the flow response. In order to minimize the start-up effect, three full thermal forcing cycles are calculated in the numerical simulation before consideration of the flow. It is found that the start-up effect for the present case is almost negligible, and the flow response in the third cycle is identical to that in the previous cycle. In the following discussion, the results of the third cycle are presented.

Figure 2 shows snapshots of streamlines and the corresponding isotherms and vector field at different stages of the cycle. The flow and temperature structures, shown in Figure 2 at \( t = 2.00P \), represent those at the beginning of the daytime heating process in the third thermal forcing cycle. At this time, the inclined surfaces and the bottom surface of the enclosure have the same temperature, but the temperature inside the enclosure is lower than the temperature on the boundaries due to the cooling effect in the previous thermal cycle. The residual temperature structure, which is formed in the previous cooling phase, is still present at \( t = 2.00P \). The corresponding streamline contours at the same time show two circulating cells, and the temperature contours indicate stratification in the upper and lower section of the enclosure with two cold regions in the centre.

As the upper surface temperature increases further, a distinct temperature stratification is established throughout the enclosure by the time \( t = 2.05P \) (see Figure 2). The streamlines at this stage indicate that the centers of the two circulating cells have shifted closer to the inclined surfaces, indicating a strong conduction effect near those
boundaries. This phenomenon has been reported previously in Akinsete and Coleman [18] and Asan and Namli [19] for the daytime condition with constant heating at the upper surface or constant cooling at the bottom surface.

At $t = 2.25P$, the temperature on the inclined surfaces peaks. Subsequently, the temperature drops, representing a decreasing heating effect. Since the interior flow is stably stratified prior to $t = 2.25P$, the decrease of the temperature at the inclined surface results in a cooling event, appearing first at the top corner and expanding downwards as the surface temperature drops further. At $t = 2.45P$, two additional circulating cells have formed in the upper region of the enclosure, and the newly formed cells push the existing cells downwards. The corresponding temperature contours show two distinct regions, an expanding upper region responding to the cooling effect, and a shrinking lower region with stratification responding to the decreasing heating effect. By the time $t = 2.50P$, the daytime heating ceases; the lower stratified flow region has disappeared completely and the flow in the enclosure is dominated by the cooling effect. At this time, the top and the bottom surfaces again have the same temperature, but the interior temperature is higher than that on the boundaries.

As the upper inclined surface temperature drops below the bottom surface temperature ($t = 2.70P$, Figure 2), the cold-air layer under the inclined surfaces becomes unstable. At the same time, the hot-air layer above the bottom surface also becomes unstable. As a consequence, sinking cold-air plumes and rising hot-air plumes are visible in the isotherm contours and a cellular flow pattern is formed in the corresponding stream function contours. It is also noticeable that the flow is symmetric about the geometric symmetry plane at this time. However, as time increases the flow becomes asymmetric.
about the symmetric line (see isotherms at $t = 2.95P$). The large cell from the right hand side of the centreline, which is still growing, pushes the cell on the left of it towards the left tip. At the same time this large cell also changes its position and attempts to cross the centreline of the cavity and a small cell next to it moves into its position and grows.

At $t = 2.975P$, the large cell in the stream lines has crossed the centerline and the cell on the right of it grows and becomes as large as it is after a short time (for brevity figures not included). The flow is also asymmetric at this time. However, it returns to a symmetric flow at the time $t = 3.00P$ which is the same as that at $t = 2.00P$, and similar temperature and flow structures to those at the beginning of the forcing cycle are formed. The above described flow development is repeated in the next cycle.

The corresponding velocity vector (colored with velocity magnitude) of the isotherms and stream lines of Figure 2 is presented in Figure 3 except for the time $t = 2.975P$. At $t = 2.00P$ there are two rotating cells are seen in the vector filed as it is seen in the corresponding streamlines (see Figure 2). The left cell is rotating clockwise and the right one is anti-clockwise. The same scenario may be seen for $t = 2.05P$ and $t = 2.45P$. It is also observed that the velocity remains maximum in the vicinity of the inclined walls. At $t = 2.45P$ two more cells are visible on top of the existing cells whose direction of rotation is opposite to the existing cells. This newly formed cells eventually occupied the whole cavity with time (see at $t = 2.50P$). However, at night time cooling phase the flow is much more complicated. A number of counter-rotating cells are formed due to strong convection effect. The asymmetric behavior of the flow field can be seen at $t = 2.95P$ where a dominant large clockwise rotating cell moves to the centre line of the attic space.
The horizontal velocity profiles (velocity parallel to the bottom surface) and the corresponding temperature profiles evaluated along the line $DE$ shown in Figure 1 at different time instances of the third thermal forcing cycle are depicted in Figure 5. At the beginning of the cycle ($t = 2.00P$) the velocity is the highest near the roof of the attic (see Figure 5a), which is the surface driving the flow. At the same time, the body of fluid residing outside the top wall layer moves fast toward the bottom tips to fill up the gap. As time progresses the vertical velocity increases and the horizontal temperature decreases (see $t = 2.05P$). A three layer structure in the velocity field is found at $t = 2.45P$. At this time the top portion of the cavity is locally cooled and the bottom portion is still hot (see Figure 2). After that time the flow completely reverses at $t = 2.50P$. It is noted that at this time the horizontal velocity is lower than that at the beginning of the cycle despite that the temperatures on the sloping boundary and the ceiling are the same at both times (see Figure 5b). This is due to the fact that at the beginning of the cycle the flow is mainly dominated by convection as a result of the cooling effect in the second half of the previous thermal cycle. However, the flow is dominated by conduction at $t = 2.50P$ as a result of the heating effect in the first half of the current thermal cycle.

As mentioned above, at the beginning of the cycle ($t = 2.00P$) the temperatures on the horizontal and inclined surfaces are the same as shown in Figure 4(b). However the temperature near the mid point of the profile line is lower than that at the surfaces by approximately 0.5K, which is consistent with the previous discussion of the flow field. Subsequently the temperature of the top surface increases ($t = 2.05P$) while the bottom surface temperature remains the same. It is noteworthy that the top surface reaches its peak temperature at $t = 2.25P$ (for brevity the profile is not included). After this time the top
surface temperature starts to decrease which can be seen at time $t = 2.40P$. By comparing the temperature profiles at $t = 2.05P$ and $t = 2.45P$ shown in Figure 4(b), it is clear that the temperatures at both the top and bottom surfaces are the same for these two time instances. However, different temperature structures are seen in the interior region. The same phenomenon has been found at the times $t = 2.50P$ and $t = 2.00P$.

In Figure 4(c), the velocity profiles at the same location during the night-time cooling phase are displayed. In this phase the flow structure is more complicated. At $t = 2.55P$ the velocity near the bottom surface is slightly higher than that near the top. Again a three layer structure of the velocity field appeared which is seen at $t = 2.65P$, $2.75P$ and $2.85P$. The maximum velocity near the ceiling occurs at $t = 2.75P$ when the cooling is at its maximum. After that it decreases and the flow reverses completely at $t = 3.0P$. The corresponding temperature profiles for the night-time condition are shown in Figure 4(d). It is seen that the temperature lines are not as smooth as those observed for the daytime condition. At $t = 2.55P$, the temperature near the bottom surface decreases first and then increases slowly with the height and again decreases near the inclined surface. This behaviour near the bottom surface is due to the presence of a rising plume. Similar behaviour has been seen for $t = 2.75P$ and $2.85P$. However, at $t = 2.65P$ it decreases slowly after rapidly decreasing near the bottom surface. At $t = 3.00P$ again the bottom and top surface temperatures are the same with a lower temperature in the interior region.

4.2 Heat transfer through the attic

The Nusselt number, which has practical significance, is calculated as follows:
where the heat transfer coefficient $h_{\text{eff}}$ is defined by

$$h_{\text{eff}} = \frac{q}{T_A}.$$  \hspace{1cm} (6)

Here $q$ is the convective heat flux through a boundary. Since the bottom surface temperature is fixed at 295K and the sloping wall surface temperature cycles between 290K and 300K (refer to Figure 1), a zero temperature difference between the surfaces occurs twice in a cycle. Therefore, the amplitude of the temperature fluctuation ($T_A$) is chosen for calculating the heat transfer coefficient instead of a changing temperature difference, which would give an undefined value of the heat transfer coefficient at particular times.

Figure 5 shows the calculated average Nusselt number on the inclined and bottom surfaces of the cavity. The time histories of the calculated Nusselt number on the inclined surfaces exhibit certain significant features. Firstly, it shows a periodic behaviour in response to the periodic thermal forcing. Secondly within each cycle of the flow response, there is a time period with weak heat transfer and a period with intensive heat transfer. The weak heat transfer corresponds to the daytime condition when the flow is mainly dominated by conduction and the strong heat transfer corresponds to the night-time condition. At night, the boundary layers adjacent to the inclined walls and the bottom are unstable. Therefore, sinking and rising plumes are formed in the inclined and horizontal boundary layers. These plumes dominate the heat transfer through the sloping walls of the cavity.

Finally the calculated maximum Nusselt number on sloping surfaces is 8.72, occurring
during the night-time period, whereas the maximum value during the day time is only 3.48, for the selected Rayleigh number and aspect ratio.

The corresponding Nusselt number calculated on the bottom surface shows similar behavior as that of top surfaces. Note that the Nusselt number calculated using (5) is based on the total heat flux across the surfaces. Since the surface area of the top surface is larger (0.595m²) than the bottom surface (0.532m²), therefore the total surface heat flux on the top surfaces will be lower than that of the bottom surface. However, the integral of the heat transfer rate for a cycle on both surfaces has been calculated and it is found that both are the same.

4.3 Effect of the aspect ratio on the flow response

The flow responses to the periodic thermal forcing for the other two aspect ratios are shown in Figures 6 and 7, which are compared with the flow response for $A = 0.5$ shown in Figure 2. It is found that the aspect ratio of the enclosure has a great influence on the flow response as well as heat transfer. The residual effect of the previous cycle on the current cycle has been found similar for all aspect ratios (see at $t = 2.0P$ in Figures 2, 6 and 7) and the flow and temperature structures during the heating process is qualitatively the same for $A = 1.0$ and $A = 0.2$ as those for $A = 0.5$ for $Ra = 1.5 \times 10^6$. However, during the cooling phase there are significant changes of flow and heat transfer among these aspect ratios. For the night-time the high velocity area of these three aspect ratios exists between the two cells where the stream function gradient is higher. Therefore, the buoyancy drives the warm air upwards from the bottom of the geometry and at the same time the
gravitational force acts on the cold air downwards from the top. This upward and downward movement can be seen in the temperature contours as a form of rising and sinking plumes.

It has been revealed that the flow remains symmetric about the geometrical centreline throughout the cycle for aspect ratio $A = 0.2$, whereas, it is asymmetric during the cooling phase for the other two aspect ratios for $Ra = 1.5 \times 10^6$. It is also anticipated that the asymmetric solution is one of two possible mirror images of the solutions. Another noticeable variation with different aspect ratios is the formation of a circulation cell near the top of the enclosure. It is seen for $A = 1.0$ that there is an extra vortex (Figure 6 at $t = 2.95P$) on the top of the cavity, which is completely absent for $A = 0.5$ and $A = 0.2$. The flow and temperature fields for the smallest aspect ratio $A = 0.2$ are more complex, with several circulation cells on either side of the central line and many plumes alternately rising and falling throughout the domain, as seen in Figure 7. These cells and plumes are the result of flow instability described earlier.

Figure 8 illustrates the horizontal velocity and temperature profiles for aspect ratio $A = 1.0$ along the line $DE$ as shown in Figure 1 for $Ra = 1.50 \times 10^6$. Since the flow is stable and stratified during the day (the heating phase), the structures of the velocity and temperature profiles are qualitatively the same as those for other aspect ratios.

At the night-time the velocity and temperature profiles for $A = 1.0$ are more complicated than that for $A = 0.5$. As seen in Figure 8, at time $t = 2.55P$ when the upper surface temperature is lower than the bottom surface, the velocity near the bottom surface is slightly higher than that near the inclined surfaces. After that the velocity increases near both the surfaces until $t = 2.75P$. Since a plume-type instability dominates the flow during
the cooling phase and the flow has an asymmetric behaviour for a certain period of time, the horizontal velocity is in the same direction near both surfaces and is in an opposite direction in the middle (see $t = 2.8875P$). As the flow transits into the next thermal cycle, it becomes very weak. The corresponding temperature contours are plotted in Figure 8(b). It is seen that the temperature profiles near the bottom surface show a wave shaped for almost the whole cooling phase due to the rising plumes (see Figure 2). At the time $t = 2.8875P$, when three layers of the velocity structure is seen, the corresponding temperature profile also shows a wave structure.

Figure 9 shows the calculated average Nusselt number on the inclined surfaces of the cavity for three different aspect ratios. The time histories of the calculated Nusselt number exhibit certain common features. Within each cycle of the flow response, there is a time period with weak heat transfer and a period with intensive heat transfer for each aspect ratio. The weak heat transfer corresponds to the day time condition when the heat transfer is dominated by conduction and the strong heat transfer corresponds to the night-time condition when convection dominates the flows and the instabilities occur in the form of rising and sinking plumes. During the day time the heat transfer rate is almost the same for all three aspect ratios. However, at the night time the heat transfer rate for $A = 1.0$ is much smaller than that for the other two aspect ratios, and there is a fluctuation of the Nusselt number for a certain period of time. This fluctuation is absent in the other two aspect ratios. This may be due to the fact that less convective cells are present in the streamlines for $A = 1.0$ than those for the other two aspect ratios. Moreover, the movement of the dominating cell for $A = 1.0$ is faster than those for other two. In addition to this, an extra cell appears on the top of the cavity for the aspect ratio $A = 1.0$. It is also noticed that there is not much
difference in heat transfer for the aspect ratios $A = 0.5$ and 0.2. The highest average Nusselt numbers for $A = 1.0$, 0.5 and 0.2 are 6.55, 8.72 and 8.76 respectively.

4.4 Dependence of flow response and heat transfer on the Rayleigh number

Figure 10 shows snapshots of stream function and temperature contours for the aspect ratio 0.5 with three different Rayleigh numbers, $Ra = 1.5 \times 10^6$, $7.2 \times 10^4$ and $7.2 \times 10^3$. The contours for $Ra = 7.2 \times 10^4$ are qualitatively the same as for $Ra = 1.5 \times 10^6$. It is found that in the heating phase (i.e. when the upper wall temperature is higher than the temperature of the bottom) the flow structures are qualitatively similar for all Rayleigh numbers. However, in the cooling phase the flow behaviour is strongly dependent on the Rayleigh numbers. Stream function and temperature contours are presented at two different times, $t = 2.70P$ and $2.95P$ for each Rayleigh number in Figure 10. In the isotherms, rising and sinking plumes are visible for $Ra = 1.5 \times 10^6$ and $7.2 \times 10^4$ at both times. A cellular flow pattern is seen in the corresponding stream function contours for $Ra = 1.5 \times 10^6$. However, only two convective cells are present for $Ra = 7.2 \times 10^4$. If the Rayleigh number is decreased further ($Ra = 7.2 \times 10^3$), the flow becomes weaker. Only two cells are seen in the stream function contours and the temperature field is horizontally stratified (see the corresponding isotherms). At $t = 2.95P$, the flow seems to be asymmetric along the centre line for $Ra = 1.5 \times 10^6$. However, for the lower Rayleigh numbers the asymmetric behaviour is not visible.

Figure 11 shows the comparison of the Nusselt number among four Rayleigh numbers for a fixed aspect ratio 0.5. It is seen clearly that during the heating phase the heat transfer rate is weaker, whereas it is much stronger in the cooling phase. With the increase
of the Rayleigh number, the Nusselt number increases throughout the thermal cycle, but the rate of increase is much higher in the cooling phase compared to that in the heating phase. The maximum Nusselt number in the cooling phase for \( Ra = 1.5 \times 10^6 \) is about 2.5 times of the maximum Nusselt number during the heating phase. It is noticeable that for the lowest Rayleigh number \( Ra = 1.5 \times 10^3 \), the heat transfer rate during the heating and cooling phases are almost the same. The maximum Nusselt number for the four different Rayleigh numbers, \( Ra = 1.5 \times 10^6, 7.2 \times 10^5, 7.2 \times 10^4 \) and \( 7.2 \times 10^3 \) for the aspect ratio 0.5 are 8.65, 7.34, 4.26 and 3.11 respectively.

4.5 Transition between symmetric and asymmetric flows

The highest Rayleigh number considered in this study for the three aspect ratios is \( 1.5 \times 10^6 \). Except for \( A = 0.2 \), the flow in the cavity for the other two aspect ratios is observed to undergo a supercritical pitchfork bifurcation for this Rayleigh number, in which case one of two possible mirror image asymmetric solutions is obtained. This asymmetric behaviour was first reported numerically and experimentally by Holtzman et al. [14] in their study of the case of a sudden cooling boundary condition. If the flow is asymmetric, the horizontal velocity along the midplane of the isosceles triangle would be nonzero. Based on this hypothesis, Figure 12 illustrates the absolute value of maximum horizontal velocity along the geometric center line for \( A = 1.0 \) and 0.5. It is seen in this figure that, for both aspect ratios, the maximum horizontal velocity is zero up to approximately \( t = 0.70P \) in each cycle, suggesting that the flow is symmetric during this time. However, after this time the maximum horizontal velocity starts to increase, indicating that the flow becomes asymmetric. The asymmetry remains until shortly before
the end of each cycle when the flow returns to symmetric again. The same asymmetric behaviour of the flow is seen for the Rayleigh number $7.2 \times 10^5$ for the aspect ratios 0.5 and 1.0.

The same results have been found when the average Nusselt numbers obtained for both inclined surfaces are compared for the aspect ratios 1.0 and 0.5, which are shown in Figures 13 (a) and (b) respectively. It is seen that at about $t = 0.70P$, the calculated Nusselt numbers at the left and right inclined surfaces start to diverge, but later they meet again before the end of each cycle.

5. Conclusion

Natural convection in an attic space subject to periodic thermal forcing has been described in this study based on numerical simulations. Three aspect ratios of $A = 1.0, 0.5$ and 0.2 with four Rayleigh numbers of $Ra = 1.5 \times 10^6, 7.2 \times 10^5, 7.2 \times 10^4$ and $7.2 \times 10^3$ for each aspect ratio have been considered here. Many important features are revealed from the present numerical simulations. It is found that the flow response to the temperature variation on the external surface is fast, and thus the start-up effect is almost negligible. The occurrence of sinking cold-air plumes and rising hot-air plumes in the isotherm contours and the formation of cellular flow patterns in the stream function contours confirm the presence of the Rayleigh-Bénard type instability. It is also observed that the flow undergoes a transition between symmetry and asymmetry about the geometric symmetry plane over a diurnal cycle for the aspect ratios of $A = 1.0$ and 0.5 with the Rayleigh numbers $1.5 \times 10^6$ and $7.2 \times 10^5$. For all other cases the flow remains symmetric. A three-layer velocity
structure has been found along the line at $DE$ as shown in Figure 1 in both the daytime heating phase (due to local cooling effect in the upper sections of the inclined walls) and night-time cooling phase when the flow becomes asymmetric. Furthermore, the flow response in the daytime heating phase is weak, whereas the flow response in the night-time cooling phase, which is dominated by convection, is intensive. At lower Rayleigh numbers the flow becomes weaker for all aspect ratios, and no asymmetric flow behaviour has been noticed.

**Acknowledgements**

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**Reference**


[22] S. C. Saha, J. C. Patterson, C. Lei, Natural convection in attics subject to instantaneous and ramp cooling boundary conditions, Energy and Building, In Press:
doi:10.1016/j.enbuild.2010.02.010

Table 1 Parameters and results of mesh and time-step dependence test at $t = 0.75P$ for $A = 0.2$ and $Ra = 1.5 \times 10^6$.

<table>
<thead>
<tr>
<th>Mesh Size</th>
<th>Time Step(s)</th>
<th>Temperature at different points in the cavity (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$O$</td>
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<tr>
<td>180 × 45</td>
<td>1.00</td>
<td>292.4748</td>
</tr>
<tr>
<td>280 × 70</td>
<td>0.75</td>
<td>292.4953</td>
</tr>
<tr>
<td>360 × 90</td>
<td>0.50</td>
<td>292.4987</td>
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Table 2 Parameters and results of mesh and time-step dependence test at $t = 0.75P$ for $A = 0.5$ and $Ra = 1.5 \times 10^6$.

<table>
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<th>Mesh Size</th>
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<td>292.2819</td>
</tr>
<tr>
<td>240 × 60</td>
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</tr>
<tr>
<td>320 × 80</td>
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<td>292.3346</td>
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</table>

Table 3 Parameters and results of mesh and time-step dependence test at $t = 0.75P$ for $A = 1.0$ and $Ra = 1.5 \times 10^6$.

<table>
<thead>
<tr>
<th>Mesh Size</th>
<th>Time Step(s)</th>
<th>Temperature at different points in the cavity (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$O$</td>
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<tr>
<td>180 × 60</td>
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<tr>
<td>270 × 90</td>
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<td>292.2491</td>
</tr>
<tr>
<td>360 × 120</td>
<td>0.50</td>
<td>292.2521</td>
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</table>
Figure 1 A schematic of the geometry and boundary conditions of the enclosure.
Figure 2 A series of snapshots of stream function and temperature contours of the third cycle at different times for $A = 0.5$ and $Ra = 1.5 \times 10^6$. Left: streamlines; right: isotherms.
Figure 3 A series of snapshots of velocity vectors of the third cycle at different times for $A = 0.5$ and $Ra = 1.5 \times 10^6$. 
Figure 4 Horizontal velocity profile (left) and temperature profile (right) along $DE$ for $A = 0.5$ with $Ra = 1.5\times10^6$. 
Figure 5 Average Nusselt number on the top and bottom surfaces of the cavity for three full cycles where $Ra = 1.5 \times 10^6$ and $A = 0.5$, $Pr = 0.72$. 
Figure 6 A series of snapshots of stream function and temperature contours of the third cycle at different times for $A = 1.0$ and $Ra = 1.5 \times 10^6$. Left: streamlines; right: isotherms.
Figure 7 A series of snapshots of stream function and temperature contours of the third cycle at different times for $A = 0.2$ and $Ra = 1.5 \times 10^6$. Left: streamlines; right: isotherms.
Figure 8 Horizontal velocity profile (left) and temperature profile (right) along DE for $A = 1.0$ with $Ra = 1.50 \times 10^6$.

Figure 9 Comparison of the average Nusselt number of three aspect ratios for $Ra = 1.5 \times 10^6$ and $Pr = 0.72$. 
Figure 10 Snapshots of stream function contours (left) and isotherms (right) of the third cycle at different times and different Rayleigh numbers with fixed $A = 0.5$. 
Figure 11 Comparison of the average Nusselt number of four Rayleigh numbers for $A = 0.5$ and $Pr = 0.72$.

Figure 12 The maximum horizontal velocity along the symmetry line for (a) $A = 1.0$ and (b) $A = 0.5$ with $Ra = 1.5 \times 10^6$. 
Figure 13 Comparison of the average Nusselt number on two inclined surfaces of the enclosure for (a) $A = 1.0$ and (b) $A = 0.5$ with $Ra = 1.5 \times 10^6$