Teaching Mental Computation Strategies in Early Mathematics

Ann Margaret Heirdsfield

[*Insert introductory figure here*]

Research into Practice

A primary aim of research in mathematics education is to improve the quality of mathematics education. Some would argue, however, that it has little impact on teaching (Steen 1999), as it remains at a distance from schools and it is difficult for teachers to keep abreast of research developments.

Teachers need access to the research literature so that they can design the most effective research-based mathematics lessons to engage children in significant learning. Given the magnitude of research, teachers require a great deal of support accessing and interpreting relevant research to assist them in developing curriculum and practices.

Mental computation—that is, calculating in the head—is a relatively new topic in mathematics curricula for primary-age children (e.g., National Council of Teachers of Mathematics 2000; Queensland Studies Authority 2004). It is an important skill because it enables children to learn more deeply how numbers work, make decisions about procedures, and create strategies for calculating (Varol & Farran, 2007; Vershaffel, Luwel, Torbeyns, & Van Dooren 2007)), thus promoting number sense—a well developed understanding of number **(**Maclellan 2001). The importance of enhancing children's math reasoning processes and ability to represent, communicate, and connect ideas cannot be underestimated. NAEYC and NCTM (2002) suggest that these are components of high-quality mathematics education for young children.

When teaching mental computation, the emphasis should be on children developing their own strategies by exploring, discussing, and justifying their thinking and solutions. For some teachers, this requires a shift in beliefs and attitudes about what and how to teach mathematics in primary grades. Teachers need support to make these shifts.

With this in mind, I worked with Pam and Sue, two primary-grade teachers, to help them improve their practice and introduce this new topic—mental computation—in their classrooms. Pam and Sue taught 6-, 7-, and 8-year olds at a Brisbane (Australia) elementary school. This article describes how these teachers engaged children in developing strategic thinking skills in math, applying research to their classroom practice in order to do so.

What we did

In the first 10-week term (of a four-term year), I gave Pam and Sue research literature about mental computation (learn more in the research section that follows). The three of us interviewed the children individually to identify the mental computation strategies (if any) they already used—for example, calculating 26 + 19 as 26 + 20 – 1.

Second term, we developed a curriculum based on research and the children's current knowledge about math. In the classrooms, Pam and Sue taught the children and I assisted by observing the lessons, providing feedback, suggesting content for future lessons. Finally, in the third term, Pam, Sue, and I re-interviewed all the children to identify whether the children had developed further mental computation strategies.

The research

The relevant research focuses on three themes: (1) mental computation strategies, (2) models to support children's development of efficient mental computation strategies, and (3) a concept map for mental computation.

Mental computation strategies. Teachers need a language—commonly understood words and terms—to talk about children's strategies and to understand children's explanations of their strategies. However, there is little common terminology for identifying mental computation strategies in the literature. For instance, among three ways to compute 37 + 28,

- 30 + 20 = 50, 7 + 8 = 15, 50 + 15 = 65 is known as the *split* method in England, as the *1010* (ten-ten) procedure in The Netherlands, and also as *partitioning*
- 37 + 20 = 57, 57 + 8 = 65 is known as *jumping*, *bridging through 10*, *sequencing*, or *cumulative* (*N10*)
- 37 + 30 = 67, 67 – 2 = 65 is called *compensation* or *N10C*

And children may use additional strategies.

To give Pam and Sue an idea of the types of strategies the children might develop—or already use—I summarized the strategies mentioned in international literature from The Netherlands, the United States, the United Kingdom, and Australia. I did not suggest using any particular terminology, as there is little consensus in the literature. The teachers described strategies in their own way, and they encouraged the children to describe their own strategies.

Models. The literature recommends three particular models or representations children can use to support mental computation strategies (Blöte, Klein, & Beishuizen 2000): the 99 chart, the 100 chart, and the empty number line (ENL) (see below). I suggested ways the teachers could introduce the models and how children might use them.

Some manipulatives are not effective in supporting children's mental computation strategies. For example, Multibase Arithmetic Blocks (MAB) represent numbers according to their place values. When using MAB, children tend to "split" numbers into their place values, for example tens and ones, and therefore, they do not develop efficient mental computation strategies such as *jumping* or *compensation*. (Beishuizen 1993).

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

52–28: 52–2=50, 50–6=44; 44-20=24

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11	12	13	14	15	16	17	18	19	20
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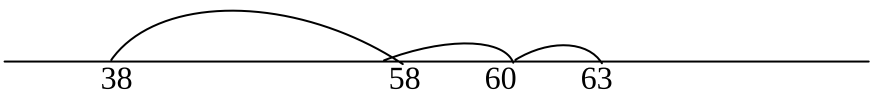
38+25: 38+20=58, 58+2=60, 60+3=63



38+25: 38+20=58, 58+2=60, 60+3=63

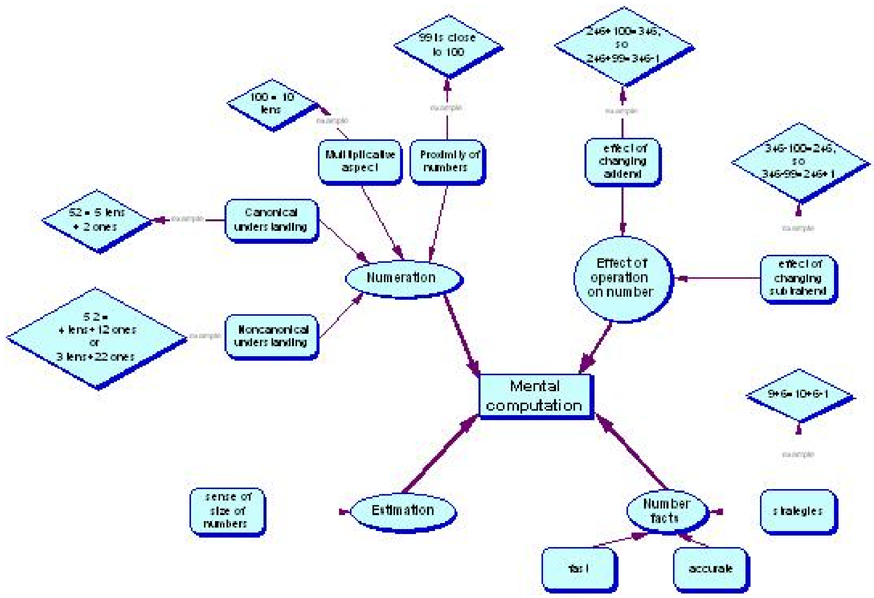
99 chart and 100 chart and ENL

Concept map. In my own research, I had developed a map of math concepts helpful in mental computation. The map is for teacher use. It identifies linked understandings for proficiency in mental computation (see "Concept Map for Mental Computation"). For instance, a child might solve 246 + 199 by strategizing that 246 +

200 = 446, and 446 – 1 = 445. To use that strategy, the child needs a grasp of the following concepts or understandings:

- *numeration*—understanding size and value of number (in particular, *proximity of number*—that is, 199 is close to 200, and one too many was added during calculation—not 10 too many or 100 too many);
- *the effect of an operation on a number* (that is, the effect of changing the addend 199 to 200—of adding too much, so having to reduce the final sum);
- *number facts*—“knowing” 9 + 6 = 15. Alternatively, using a number facts strategy such as *bridging through 10* —for example, 9 + 6 = 10 + 6 – 1); and
- *estimation*—to check the reasonableness of the solution

Looking at the concept map, Pam and Sue could see that mental computation is part of a bigger picture, and thus children must understand associated concepts and make connections across them.



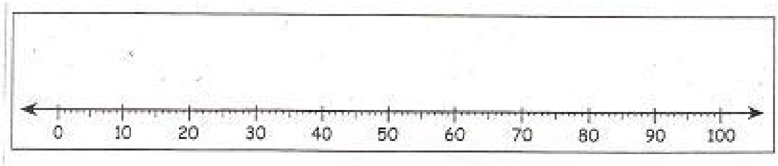
Concept Map for Mental Computation

What the teachers did

Pam and Sue planned their teaching based on their review of research and our interviews with each child. Some children had already developed strategies, while others used simple counting-by-one strategies, and still others were unable to even attempt to solve the problems.

In their planning, the teachers focused on asking children questions, such as**,** "How did you solve this?", "Who solved the problems in a similar way?", "How is your method similar?", "How is yours different?" Pam and Sue offered the children problems they could solve and models to assist them to calculate. In addition, they helped children establish connections among mathematical concepts (identified in the concept map) through careful sequencing of their lessons. For instance, examples of the types 46 + 20 were presented before 46 + 24, which were presented before 46-29. Their teaching practices focused on encouraging the children to think strategically.

Empty number line. The children had no previous experience with this model, so Pam and Sue introduced a number line with graduations, where 10s were labeled and divisions between 10s marked (see "Graduated Number Line"). They encouraged the children to suggest where numbers would sit on their number lines and to explain how they knew where to place the numbers. Then, to encourage the children to use the *jump* method for calculations, the teachers had the children jump forward or backwards from these numbers by 10s. The teachers drew the children's attention to the connection between jumping by 10s and adding and subtracting multiples of 10s (for example, 73 – 40 is the same as starting at 73 and jumping back four 10s).



Graduated Number Line

The teachers then introduced the *empty* number line. They drew a straight line (with no markings) on the chalkboard and asked the children to solve the problem, 95 + 30.

Sue: How can we use this number line to solve this problem?

Natasha: Ninety-five should go at the right because that's where 95 goes.

James: But we'll have to draw a longer line to solve it.

Tanya: We could put 95 to the left so we can jump [*by 10s*] to the right.

Pam and Sue encouraged the children to share their strategies with each other so that the children could learn other strategies from one another. Everyone used their number line to calculate and to demonstrate their strategies.

The 99 chart and 100 chart. Pam and Sue introduced these tools by asking the children to find various numbers in them.

Sue: How did you find 26, Lachlan?

Lachlan: I put my finger down the line *[the first column in the 99 chart].* I found the 20s first, and then I looked along 6.

Sue: Did anyone find 26 a different way?

Helen: Yes, I looked for 6 *[in the top row]* and went down to 26.

Some children discovered number patterns when counting in 10s and were eager to share their discoveries with the class:

Melissa (running her finger down the 9s column)**:** Nineteen, 29, 39, 49, . . . 99.

Pam: What is happening here?

Mark: They are all in the same column.

Mary: They all end in 9.

Jane: They are all counting in 10s.

Pam: How will we add 19 to 17?

Nick: I got 36. I went down *[10—from 17]* and then down *[by another 10]* and then back one—adding on two 10s, which is 20, then you go back one [*for 19]*.

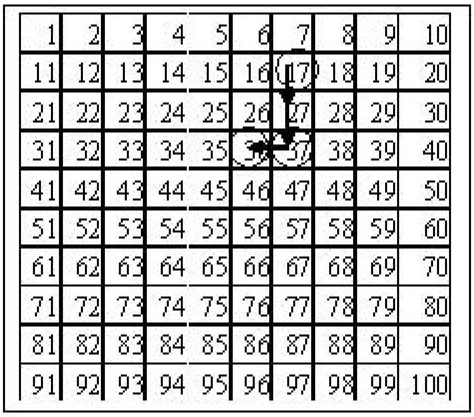


Figure 4. *Nick's solution strategy for 17 + 19*

Nick's solution strategy for 17 + 19

Although the children used three models (100 chart, 99 chart, and empty number line) to help them calculate, the focus was not on the models. Rather, the teachers focused on the children's strategic thinking. The children were free to choose any model (or no model) to aid in solving the examples. Pam and Sue constantly encouraged the children to explain their reasoning, compare their strategies with others' strategies, and critique the strategies.

I joined Pam and Sue's classes for mental computation activities, but I was not present for other math classes. In my absence, Pam and Sue designed lessons to address related topics, such as numeration and number facts strategies—topics identified in the concept map.

What the teachers learned

Pam and Sue said that the research focusing on the strategies, models, and linked knowledge (concept map) helped them in developing an approach to teaching math.

Researcher: What other things helped you design your lessons?

Pam: Concept maps ... have become part of my planning. I used your concept map to identify where the children were at and what each child needed. . . . I now construct a concept map before I teach anything new. It helps me see the links.… I also think about the links between lessons in my planning. [*The concept map*] changed my way of thinking. It made me more aware of why we teach certain things. [*Concepts and skills*] don't come up in isolation.

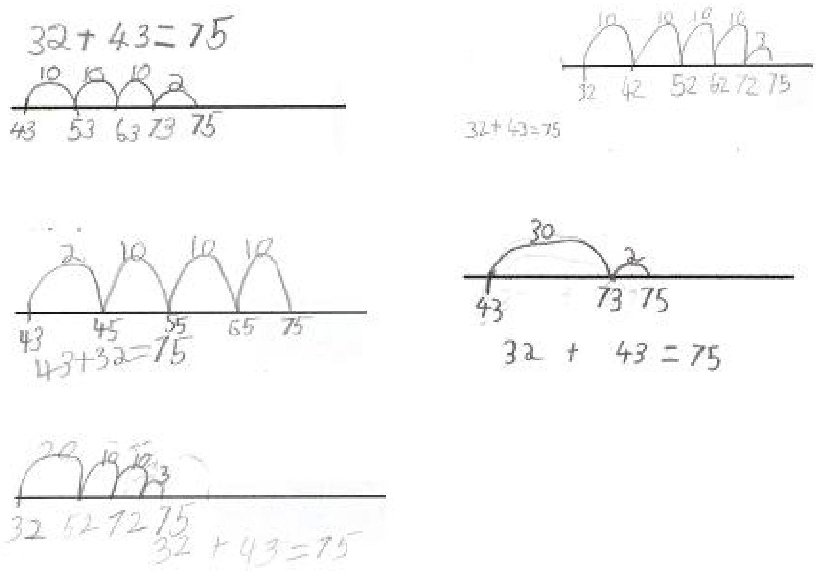
Having been introduced to concept maps, Pam and Sue developed their own concept maps before teaching new topics in any curriculum area, as well as in mathematics. They said that in-class assistance from a researcher and background readings helped them offer the children more interesting and engaging lessons.

The children were enthusiastic about participating. They saw more of a purpose in mathematics; for instance, the children could see why learning number facts strategies, place-value concepts, counting in 10s, and so on, helped them calculate.

With the emphasis on process rather than product, Pam and Sue noted that the children were developing more confidence in their ability to "do" mathematics. They were also becoming more competent in using a variety of appropriate strategies. The teachers were pleased that the children were becoming “more mathematically inclined” and more engaged in mathematics.

What the children learned

Pam and Sue used the models of the 99 and 100 charts and the empty number line to support the children’s mental computation, but they did not show them specific strategies to use. Instead, they encouraged the children to devise their own. As a result, the children developed a variety of strategies. For instance, using the empty number line to calculate 43 + 32, some children jumped in 1s first and then 10s, while others jumped in 10s first and then 1s (see “Five Strategies for Computing 43 + 32”).

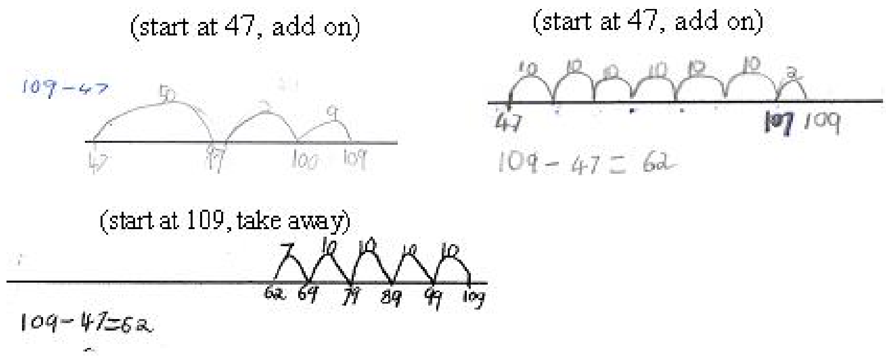


Five strategies for 43 + 32.

The children eagerly shared their ideas and listened to others. Here is part of a classroom discussion about using the ENL to solve the problem, 109 – 47:

Pam:	What’s different about this example?
Mandy:	It’s a take away.
Ben:	We’ll have to start at the other end.
Pam:	Ben said that we’ll have to start at the other end.
Tim:	No we don’t. We can start at 47 and add on.

After this initial discussion, the children used both “add on” and “take away” strategies to do subtraction (see “Children’s Strategies for 109 – 47”).



Children’s strategies for 109 – 47.

Over the school term, many children developed appropriate strategies. In the initial and closing interviews, we did not allow the children to use models to help them calculate. They had to solve the questions mentally. In the first interview, several children had been unable to solve many problems or had used their fingers for counting.

When we interviewed the children for the second time, they all had developed strategies, and some children had developed very powerful strategies. Many children, like Molly (6 years old), had used finger counting initially to solve 5 + 9. In the second interview, Molly said, “Ten plus 5 equals 15, less 1 equals 14.”

Most children solved examples like this using efficient strategies similar to Molly’s. They also used efficient strategies for operations with larger numbers. For instance, Mitchell said, “Take 1 off 6 [*in 26*], add [*it*] to 9; 10 plus 25 equals 35.” Jackson solved 26 + 9 using this strategy: “I got 1 from the 6 [*in 26*], and there was 5 left; [*25 plus 10 equals*] 35.”

Subtraction is often more difficult than addition for children, and although most children could not solve many subtraction examples before the instruction, many children developed efficient subtraction strategies. For instance, Lachlan (7 years old) solved 30 – 19 by strategizing, "Nineteen is close to 20. Thirty minus 20 is 10. The answer's 11." Jackson used a different but equally efficient strategy: "Ten less is 20. But it's two 10s. So 19 less is 11." Finally, Molly solved 134 – 99 using a sophisticated strategy: "Turn 99 into 100 [*by adding 1*], and then put 1 on [*134 to make*] 135. 135 take 100 is 35."

Concluding comments

Through involvement in this project, the teachers came to understand and incorporate into their teaching some principles to advance children's learning (not only in mental computation):

1. Determine the existing knowledge of the children (in this project, the teachers interviewed the children one-on-one).
2. Identify associated concepts that are necessary for connected understanding (I gave the teachers a concept map for mental computation, but they formulated their own when introducing other topics).
3. Teach the associated concepts and support the children in "seeing" the connections.
4. Maintain a classroom environment in which children feel safe to explore, share, critique, and justify their strategies and solutions; where the process is just as important as the product.

The focus in these two classrooms was not only helping children develop mental computation strategies, but also helping them develop higher order thinking—reasoning, critiquing, and making sense of numbers and operations—in both what they did and what they said. This teaching experiment has given fruitful insight into young children's potential to develop and efficiently use a range of mental computation strategies. The success of this teaching experiment depended on the teachers being informed by research and being assisted to put this research into practice.

References

Beishuizen, M. 1993. Mental strategies and materials or models for addition and subtraction up to 100 in Dutch second grades. *Journal for Research in Mathematics Education* 24 (4): 294–323.

Blöte, A.W., A.S. Klein, & M. Beishuizen. 2000. Mental computation and conceptual understanding. *Learning and Instruction* 10: 221–47.

Maclellan, E. 2001. Mental calculation: Its place in the development of numeracy. *Westminster Studies in Education* 24 (2): 145–54.

NAEYC & NCTM. 2002. *Early childhood mathematics: Promoting good beginnings.* Joint position paper. Washington, DC: NAEYC. www.naeyc.org/positionstatements/mathematics

NCTM (National Council of Teachers of Mathematics). 2000. *Principles and standards for school mathematics.* Reston, VA: Author. nctm.org

Queensland Studies Authority. 2004. *Mathematics Years 1–10 syllabus.* www.qsa.qld.edu.au/downloads/syllabus/kla_maths_syll.pdf

Steen, L. A. 1999. Theories that gyre and gimble in the wabe. *Journal for Research in Mathematics Education* 30 (2): 235–41.

Varol, F., & D. Farran. 2007. Elementary school students' mental computation proficiencies. *Early Childhood Education Journal*, 35 (1): 89-94.

Verschaffel, L., Luwel, K., Torbeyns, J., & W. Van Dooren. 2007. Developing adaptive expertise: A feasible and valuable goal for (elementary) mathematics education? *Ciencias Psicologicas,* 1 (1): 27-35.

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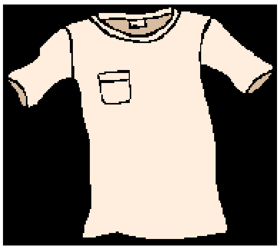

$99 $36

Interviewer: How much do the CD player and the CD cost altogether?

Interview 1
Mitchell (age 7): It’s too big. I can’t do it.

Interview 2 (eight weeks later)
Mitchell: Take 1 off the 6; that makes 100. One hundred plus 35 equals 135.
[Introductory figure]

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
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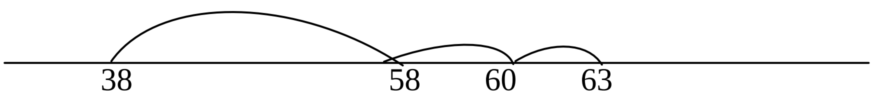

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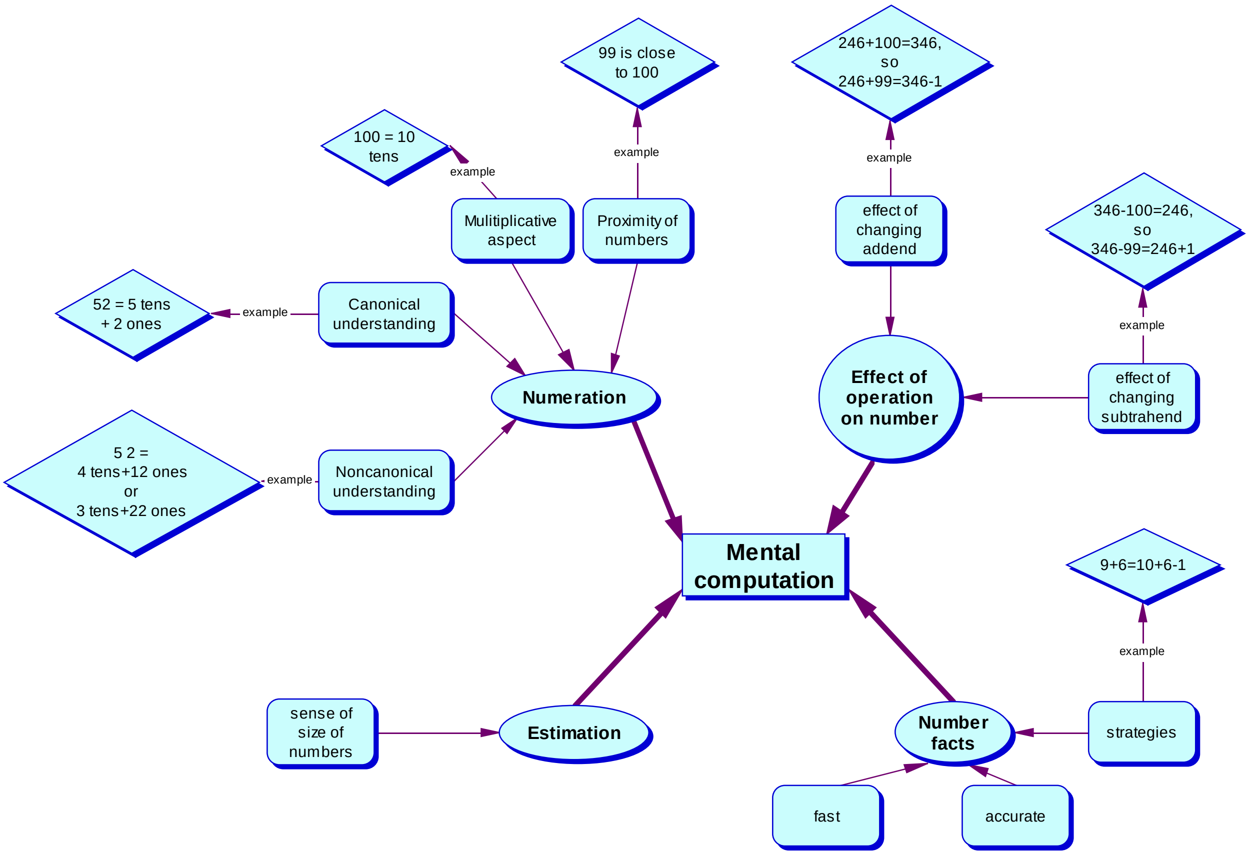
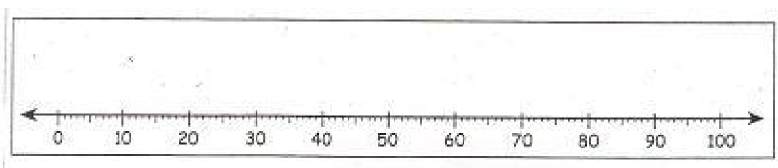
[Figure 1]
99 chart, 100 chart and ENL



[Figure 2]
Concept Map for Mental Computation



[Figure 3] Graduated Number Line

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
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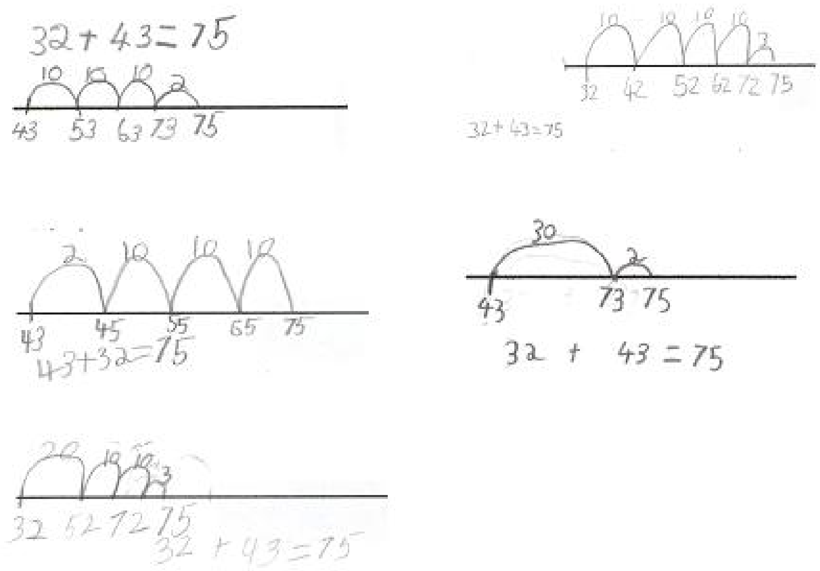
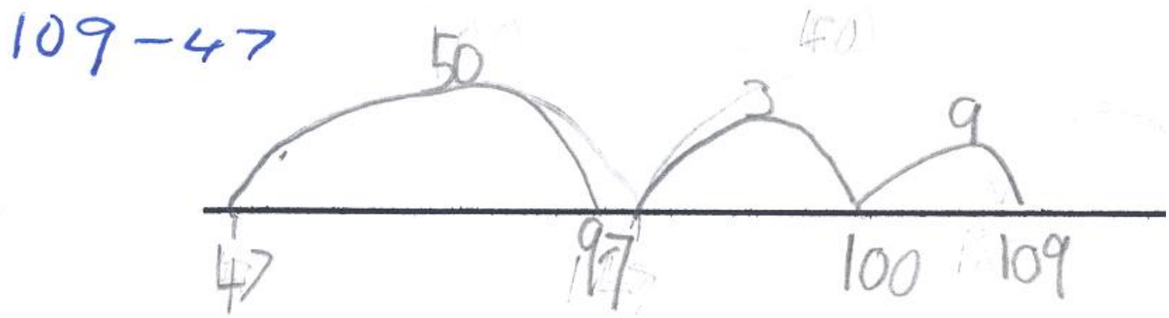
[Figure 4] Nick’s Strategy for 17 + 19

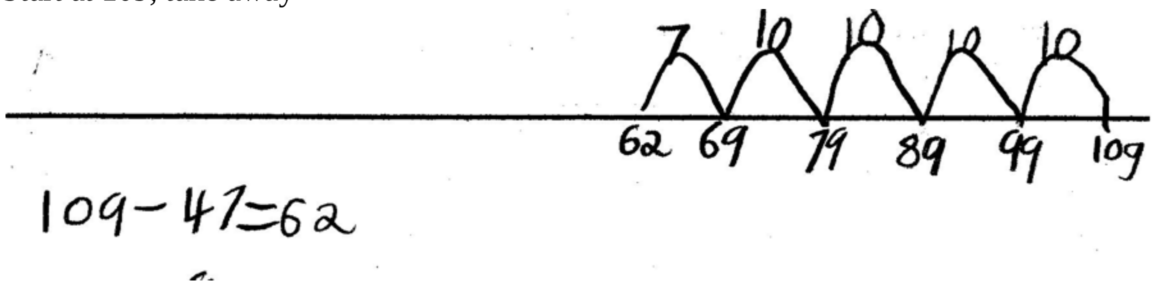


[Figure 5] Five Strategies for 43 + 32

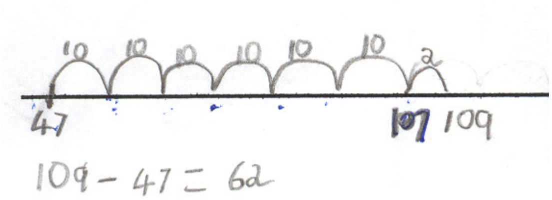
Start at 47, add on



Start at 109, take away



Start at 47, add on



[Figure 6] Children’s Strategies for 109 – 47