Dwell-time and run-time control for DC mass rapid transit railways

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Abstract

Dwell-times at stations and inter-station run-times are the two of major operational parameters to maintain train schedule in railway service. The current practices on dwell-time and run-time control, however, are only optimal with respect to certain nominal traffic conditions, but not necessarily the current service demand. The advantages of dwell-time and run-time control on trains are therefore not fully taken. This paper presents the application of a dynamic programming approach, with the aid of an event-based model, to devise an optimal set of dwell-times and run-times for trains under given operational constraints over a regional level. Since train operation is interactive and of multi-attributes, dwell-time and run-time coordination among trains is a multi-dimensional problem. The computational demand on devising trains' instructions, which is the prime concern for the operators in real-time applications, is excessively high. To properly reduce the computational demand in the provision of appropriate dwell-times and run-times for trains, this paper proposes to divide a DC railway line into a number of regions and each region is overseen by one Dwell-time and Run-time Controller (*DRC*). To demonstrate the performance and feasibility of the controller in formulating the dwell-time and run-time solutions for real-time applications, the results of the three studies are discussed.

List of symbols

AT_x^i arrival-time of train *i* at station *x*

NDW_x^i nominal dwell-time of train *i* at station *x*

$NRT_{x-1,x}^i$ nominal run-time of train *i* between station *x*-1 and *x*

$[AT]_k$ arrival-times set of trains at successive stations at stage *k*

$[NDW]_{k-1,k}$ a set of nominal dwell-times for trains at successive stations between stages *k*-1 and *k*

$[NRT]_{k-1,k}$ a set of nominal run-times for trains at successive inter-station runs between stages *k*-1 and *k*

$[RDW]_{k-1,k}$ a set of dwell-time extensions or reductions of trains at successive stations with respect to the nominal dwell-time schedule between stages *k*-1 and *k*

$[RRT]_{k-1,k}$ a set of run-time extensions or reductions of trains at successive inter-station runs with respect to the nominal run-time schedule between stages *k*-1 and *k*

τ number of control actions

T a set of possible stage transformations

$x^g(j)$ g^{th} state at stage *j*

$x(j)$ a set of possible states at stage *j*

$\hat{x}(j)$ a set of possible states at stage *j* after state grouping

$F(x^g(j))$ a set of stage-to-stage cost(s) of reaching the given state '$x^g(j)$' in stage *j*

M a set of the sets of the minimum costs of reaching all possible states in successive stages

X a set of all the possible ordered sets of states in successive stage transformations

$x^{\bullet}$ one element in X

x^{*} optimal path

$c^{g}(j)$ a set of individual dwell-time and/or run-time adjustments to the trains with respect to the corresponding nominal schedule at $x^{g}(j)$

$c(j)$ a set of control actions leading to each element in $x(j)$

C a set of all the possible ordered sets of control actions made to attain the final stage in successive stage transformations

$c^{\bullet}$ one element in C

c^{*} a set of the optimal control actions to attain each element in x^{*} in successive stages

1 Introduction

Owing to the cost effectiveness and environmental friendliness, the number of metro systems has been growing rapidly around the world for a few decades. To meet the population and social activities throughout the day, a reliable train service is inevitable. However, when taking all the track-related constraints, control variables and operational requirements into account, regulation of train operation to match the time-varying passenger demand becomes a very complicated problem because of its non-linear and multi-dimensional properties. Since a large amount of operational parameters are involved, a quick solution for train control is not always possible. Further, the common practice on train control is usually based on a set of specified operational criteria. Adjustment on train operation either through service headway, dwell-times at stations or inter-station run-times is thus confined to a certain extent.

To enhance the flexibility and capability of train control, a dynamic train controller to maintain train schedule [1] according to the current traffic scenarios is desired.

Dwell-time control [2] is the commonly adopted means for train scheduling in practice because of its simplicity. To reduce energy consumption of trains and maintain service at the same time, inter-station run-time control is more preferable to achieve train coordination, particularly at off-peak hours. A trade-off between service quality and energy consumption of a train movement in an inter-station run can be easily accomplished with coast control [3-5], except for certain track geometry and speed restrictions, as run-time decreases and energy consumption increases monotonically when the coasting point shifts from the starting station to the next.

This paper describes regulation and coordination of multi-train operation with mixed dwell-time and run-time control by dynamic programming approach (DP) [6,7]. As computation time is critical for real-time applications, an event-based model [8,9] on train movement, which does not require calculation on every detail, is the tool to evaluate the possible control actions during the optimisation process.

Trains are running in a sequential order on a line with the separation governed by the headway and each train carries its own corresponding dwell-time and run-time schedule. With a large number of trains running at the same time, the problem of dwell-time and run-time coordination among trains is extensively complex. Size of solution space for train operation inevitably inflates with numbers of trains and stations and hence the computational demand becomes excessive. In order to take full advantage of dwell-time and run-time control in railway applications, dividing a line into a number of control regions is proposed and each region is controlled by one Dwell-time and Run-time Controller (*DRC*). With a number of *DRC*s along a line, the solution space of train control is relatively smaller and hence computational demand is kept manageable.

In DC metro systems, the traction power to trains is mainly supplied by the two nearest substations [10]. In this study, the section of track between two adjacent substations is defined as a control region, which usually covers a few passenger stations. As a result, the number of substations in the metro system determines the number of *DRC*s required. Given the system operational requirements, energy demand and headway are 'allocated' to *DRC*s from a central level of control. Each *DRC* then calculates the sets of dwell-times and run-times for trains in a region. An on-board train-based controller (*TBC*) [11], which is integrated into the Automatic Train Operation (*ATO*), may be employed in each train to determine the necessary control measures based on the given operational constraints by a *DRC*. Hence, a hierarchical train control is possible and the decision-making process of train operation is vertically divided into three layers, with the *DRC*s located in the middle.

This paper focuses on the design of the *DRC* and the regulation of train service within a region. The study explores the feasibility of an online traffic flow optimisation technique. With dynamic programming, the advantage of guaranteed optimality of the solution is exploited. Three studies will be conducted to demonstrate the controller's flexibility with various operational requirements through simulation.

2 Problem formulation

In order to explain the problem of train regulation in a region, a simple track with 5 stations between two adjacent substations (i.e. within one control region) is given here. As shown in Fig. 1, a train is assumed to be at station '0' initially and it will reach station '4' through three intermediate stops. The total travelling time (i.e. dwell-times and run-times through three intermediate stops) and energy consumption of the whole journey from station '0' to '4' may vary, depending on the corresponding dwell-times at stations and run-times in successive inter-station runs. There are a large number of dwell-time and run-time combinations for the train to reach station '4' and each produces different overall run-time and energy consumption.

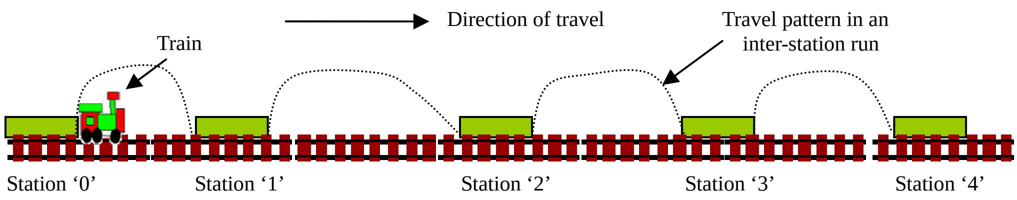


Fig. 1 4 inter-station runs

To represent the train movement through the three intermediate stations, a traffic flow model is to be established, as shown in Fig. 2, where the numbered states correspond to the train's possible arrival-times at the stations. The lines connecting the states represent the travelling time $T_{x,x+1}$ between stations 'x' and 'x+1' (i.e. the dwell-time at stations 'x' and the possible run-times between the two stations) as well as the corresponding energy consumptions $E_{x,x+1}$ of the train.

For easy illustration in Fig. 2, the possible number of states evolved at each station is limited at 3, excluding the one at the initial station. The control actions taken by the operator on each state at stations evolves into the same set of states at the subsequent stations. For instance, states 1, 2 and 3 lead to the same states 4, 5 and 6 at station '2'. There are a total of twelve states required in representing the traffic conditions of a train through station '0' to '4' in this example; whilst the number of paths to reach the station '4' through different combinations of successive states is $3^3 = 27$.

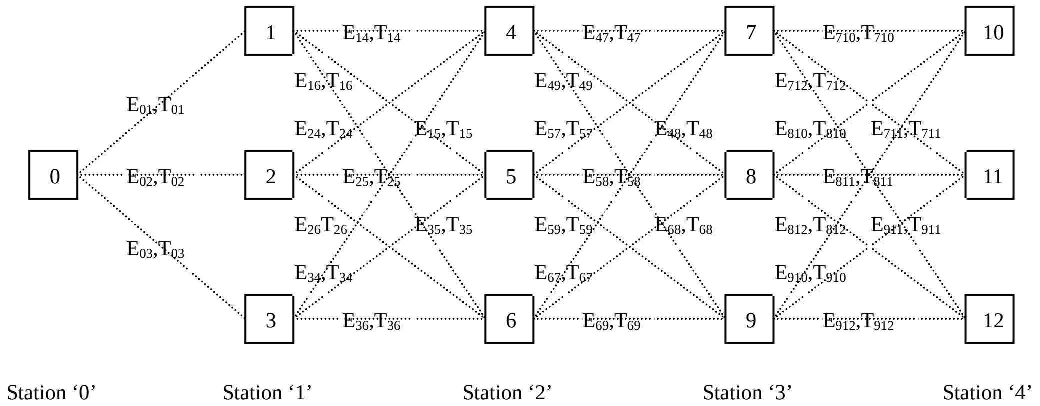


Fig. 2 State diagram of a single train operation

However, the number of possible states at the next stations is more likely to increase in practice. The size of the state diagram expands as the train moves along the line and hence the problem of train control becomes more complex. Given the train's three possible control actions at a state, the maximum number of states required (excluding the initial state) to represent the flow of traffic and paths to reach station '4' through different combinations of states are determined as $3^1 + 3^2 + 3^3 + 3^4 = 120$.

To further represent multiple train movement on a line (each train carries its own unique dwell-time and run-time schedule), a state corresponds to the arrival times of trains at stations. In other words, number of stations over a region implies the number of trains to be controlled at a state. The size of state diagram inflates with the number of trains and control actions over the region. To take one set of control actions on trains based on the given operational constraints, the number of inter-station runs over a region is the boundary of train control per cycle. The ultimate aim of *DRC* introduced here is to find the set of dwell-times at stations and inter-station run-times for trains in a region, to meet a particular operational criterion (i.e. run-time or minimum energy consumption).

3 Traffic flow model

As dwell-time and run-time is calculated in successive stages, formulation of the traffic flow model to link the control actions and their corresponding operational performance is possible. The train's operational performance between links, which can be determined by train simulator, depends on its operation mode (i.e. acceleration, coasting and braking).

Under given system constraints and operational requirements within a region, regulation of train operation can be attained by either heuristic or classical approaches. Heuristic methods usually consume less memory but they do not guarantee optimal solution. A better solution is attained in a longer simulation time. Classical methods ensure the optimality of solution but an analytical model to relate various system parameters is needed to be formulated. Dynamic programming is one of the classical optimisation techniques and it divides the multi-stage problem into a series of single-stage problem. Dynamic programming is adopted to solve the problem of train control in this study because this traffic regulation problem can be formulated into a multi-stage problem.

To establish an event-based traffic flow model with DP, an event represents a state and the links between events are the transformation between states in the state-space model. In *DRC*, an event denotes arrival-time set of trains at stations, while the links between events are inter-station runs for trains.

3.1 Schedule and control

3.1.1 Nominal train schedule

Inter-station run-time depends on the exact train movement between stations, and a particular run can be described by the difference between the arrival times at two successive stations:

$$AT_x{}^i - AT_{x-1}{}^i = NDW_{x-1}{}^i + NRT_{x-1,x}{}^i \tag{1}$$

Trains are running in sequential order on the same railway line, as shown in Fig. 3, with their separations governed by the headway. Each train has its own set of arrival-times at stations. To represent multi-train operation in the traffic flow model, a state at a particular stage *k* is defined as the set of arrival-times of trains at successive stations.

$$State : [AT]_k = \left[AT_x{}^{i+k},\ AT_{x-1}{}^{i+k+1},\ AT_{x-2}{}^{i+k+2},\ \ldots\ ,\ AT_{x-n}{}^{i+k+n} \right]_k \tag{2}$$

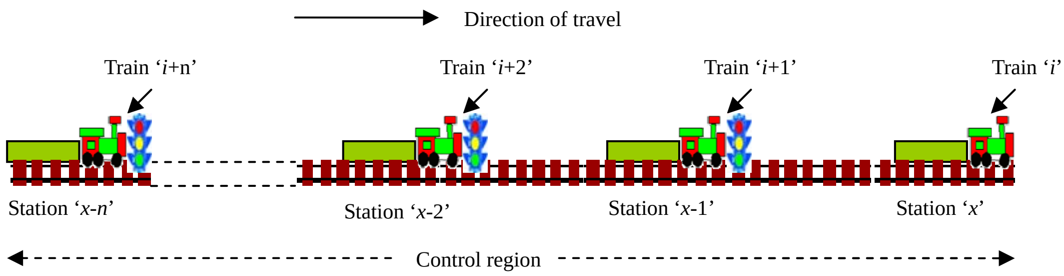


Fig. 3 Multi-train operation

The dynamic behaviour of multi-train operation with respect to the nominal schedule between two states is therefore expressed as:

$$[AT]_k - [AT]_{k-1} = [NDW]_{k-1,k} + [NRT]_{k-1,k} \tag{3}$$

where,

$$[NDW]_{k-1,k} = \left[NDW_x{}^{i+k-1}, NDW_{x-1}{}^{i+k}, NDW_{x-2}{}^{i+k+1}, \quad \ldots, \quad NDW_{x-n}{}^{i+k+n-1}\right]_{k-1,k} \tag{4}$$

$$[NRT]_{k-1,k} = \left[NRT_{x-1,x}{}^{i+k}, NRT_{x-2,x-1}{}^{i+k+1}, \quad \ldots, \quad NRT_{x-n,x-n+1}{}^{i+k+n-1}\right]_{k-1,k} \tag{5}$$

In order to maintain the train service, regulation of dwell-times at stations, and run-times in successive inter-station runs with respect to the nominal schedule, are the two viable control actions.

3.1.2 Dwell-time and run-time control

Dwell-time of trains at stations can be either extended or reduced with respect to the nominal schedule. Eqn (3) then becomes,

$$[AT]_k - [AT]_{k-1} = [NDW]_{k-1,k} + [RDW]_{k-1,k} + [NRT]_{k-1,k} \tag{6}$$

where,

$$[RDW]_{k-1,k} = \left[RDW_x{}^{i+k-1}, RDW_{x-1}{}^{i+k}, RDW_{x-2}{}^{i+k+1}, \quad \ldots, \quad RDW_{x-n}{}^{i+k+n-1}\right]_{k-1,k} \tag{7}$$

Since the nominal train schedule (i.e. $[NDW]_{k-1,k}$ and $[NRT]_{k-1,k}$) is constant, $[RDW]_{k-1,k}$ is the possible control variable to maintain the train schedule. Similarly, when run-time control is adopted, the dynamic behaviour of train operation is expressed by:

$$[AT]_k - [AT]_{k-1} = [NDW]_{k-1,k} + [NRT]_{k-1,k} + [RRT]_{k-1,k} \tag{8}$$

where,

$$[RRT]_{k-1,k} = \left[RRT_{x-1,x}{}^{i+k}, RRT_{x-2,x-1}{}^{i+k+1}, \quad \ldots, \quad RRT_{x-n,x-n+1}{}^{i+k+n-1}\right]_{k-1,k} \tag{9}$$

To further enhance the flexibility of train control, both $[RDW]_{k-1,k}$ and $[RRT]_{k-1,k}$

are introduced into Eqn (3) and the mixed control is described by:

$$[AT]_k - [AT]_{k-1} = [NDW]_{k-1,k} + [RDW]_{k-1,k} + [NRT]_{k-1,k} + [RRT]_{k-1,k} \qquad (10)$$

3.2 State formulation

To demonstrate how the event-based model is applied to represent traffic flow, an example, in which there are 4 stations, is given. The traffic condition is shown in Table 1.

3.2.1 Initialisation

Given the traffic condition, the state at the initial stage '0' is calculated by:

Assume train 1 departs at station '0' when time = 0 sec;

AT_3^1 (i.e. Arrival-time of train 1 at station '3') = (135+25+121+25+90) sec

= 396 sec

With 120 sec headway, AT_2^2 = (120+135+25+121) = 401 sec

AT_1^3 = (120×2 +135) = 375 sec

AT_0^4 = (120×3-25) = 335 sec

An initial state $[396, 401, 375, 335]_0$ is obtained.

Table 1 – Traffic conditions

Headway	120 sec		
Nominal dwell-time	25 sec		
Time extension of train service	5 %		
Nominal run-times	Inter-station run (sec)		
	0-1	1-2	2-3
	135	121	90
Number of control steps in run-time	0 or 7 sec	0 or 6 sec	0 or 5 sec

* Dwell-time control is not introduced in this application.

3.2.2 State evolution

Given the corresponding run-time extensions (i.e. 7, 6 and 5 sec) at the stations with respect to the nominal schedule in successive inter-station runs, and no dwell-time control is introduced, a new state at the next stage is calculated by Eqn. (8),

$$\begin{bmatrix} AT_3^{\ 2} \\ AT_2^{\ 3} \\ AT_1^{\ 4} \end{bmatrix}_1 = \begin{bmatrix} NDW_2^{\ 2} \\ NDW_1^{\ 3} \\ NDW_0^{\ 4} \end{bmatrix}_{0,1} + \begin{bmatrix} NRT_{2,3}^{\ 2} \\ NRT_{1,2}^{\ 3} \\ NRT_{0,1}^{\ 4} \end{bmatrix}_{0,1} + \begin{bmatrix} RRT_{2,3}^{\ 2} \\ RRT_{1,2}^{\ 3} \\ RRT_{0,1}^{\ 4} \end{bmatrix}_{0,1} + \begin{bmatrix} AT_2^{\ 2} \\ AT_1^{\ 3} \\ AT_0^{\ 4} \end{bmatrix}_0$$

$$= \begin{bmatrix} 25 \\ 25 \\ 25 \end{bmatrix}_{0,1} + \begin{bmatrix} 90 \\ 121 \\ 135 \end{bmatrix}_{0,1} + \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}_{0,1} + \begin{bmatrix} 401 \\ 375 \\ 335 \end{bmatrix}_0 = \begin{bmatrix} 521 \\ 527 \\ 502 \end{bmatrix}_1 \text{ sec}$$

With 120 sec headway, AT_0^5 = 120×4-25 = 455 sec. A possible state [521, 527, 502, 455]$_1$ at stage "1" is attained. A similar approach of calculation on the arrival-times set of trains is carried out at the later stages.

Excluding the initial stage, the number of possible states at a particular stage in the state-space traffic model depends on: (1) the number of possible states in the previous stage, and (2) the number of possible control actions, τ , taken by the operator on each state at stations.

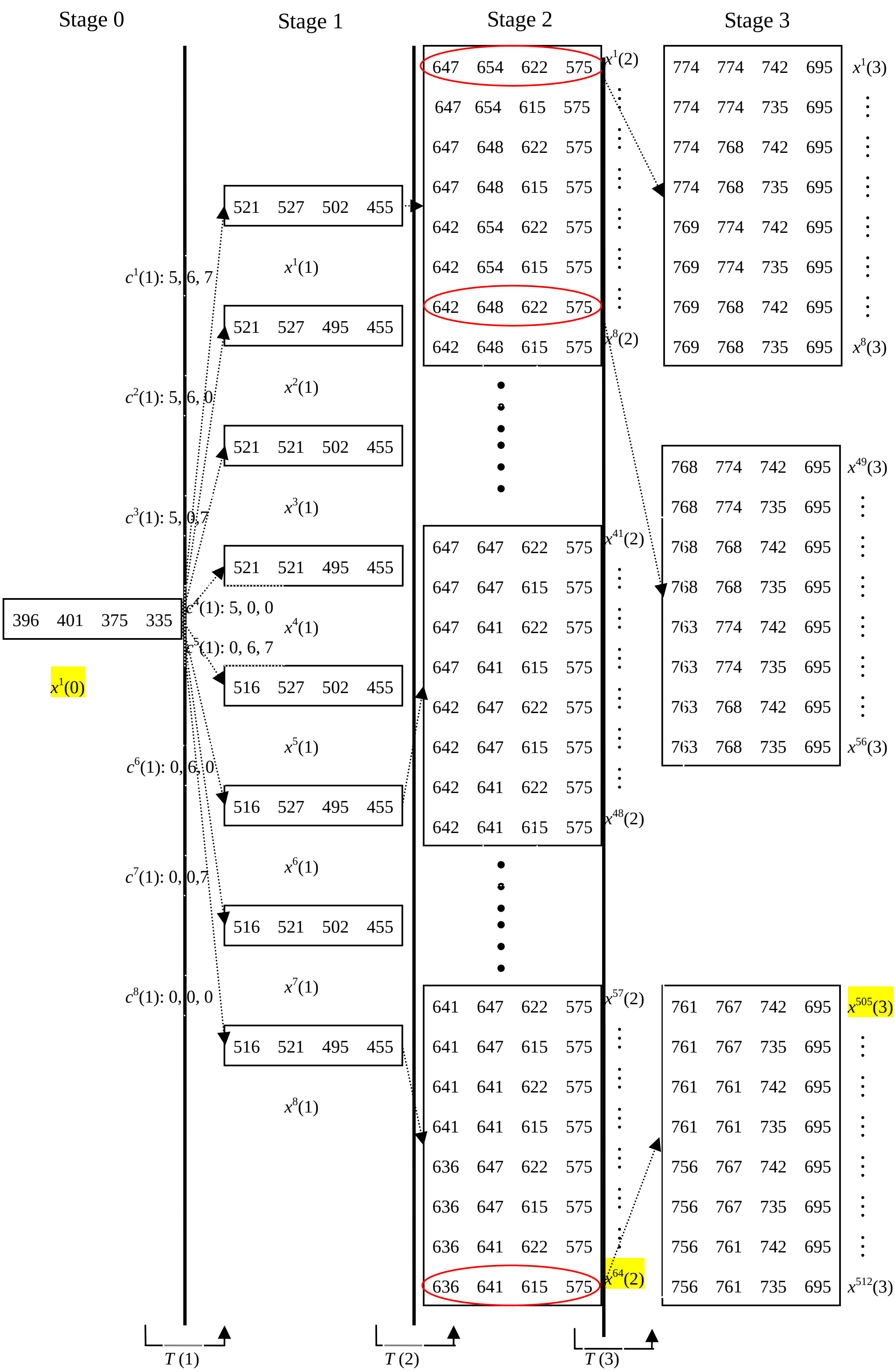
$$Number\ of\ states\ at\ stage\ (k+1) = Number\ of\ states\ at\ stage\ k \times \tau \qquad (11)$$

$$\tau = (Number\ of\ steps\ in\ control\ \mathrm{var}iable)^{Number\ of\ stations - 1} \qquad (12)$$

A larger number of states is therefore obtained in later stages and the number of stages increases with the number of stations in the region.

3.3 Operation with state diagram

Fig. 4 shows the event-based traffic flow model representing train movement with all combinations of states in successive stages. It is only a simple case of four trains travelling through four successive stations with different arrival-times. ‘7 or (/) 0’, ‘6 / 0’ and ‘5 / 0’ sec(s) are the three sets of two run-time extension levels in the three inter-station runs ‘0-1’, ‘1-2’ and ‘2-3’ respectively, and a new train is fed into the line at station 1 with 120 sec headway. The initial state is the same as derived in Section 3.2.1. The traffic conditions summarised in Table 1 are employed. Each stage has a number of states representing the possible arrival-times of trains at successive stations. A stage transformation corresponds to one inter-station run for the trains and the number of stage transformations increases with the number of stations. There are a total of 512 (i.e. $2^3 \times 2^3 \times 2^3 = 8^3$) combinations of the arrival-times of trains at the final stage, even with only two possible run-time extension levels in each of the three inter-station runs.



$c^m(k)$: x, y, z are the corresponding run-time extensions with respect to the nominal schedule in successive inter-station runs from stage k-1 to k

AT_0	AT_1	AT_2	AT_3	indicates the corresponding arrival-times of trains at successive stations

Fig. 4 Complete state-space diagram

With the given traffic conditions, the set of stage transformation T required in successive stages and all the possible sets of control actions in C to reach the final stage are expressed in Eqns (13) and (14) respectively.

$$T\{T(1),\ldots,T(j)\} = \{T(1),T(2),T(3)\} \tag{13}$$

$$C = \{\{c^{p}(1),\ldots,c^{q}(n)\};\quad \ldots\ldots;\quad \{c^{w}(1),\ldots,c^{v}(n)\}\} \tag{14}$$

$$\text{where, } c^{p}(1), c^{w}(1) \in c(1) \text{ and } c^{q}(n),\ c^{v}(n) \in c(n)$$

$T(j)$ is the transformation from stage *j*-1 to stage *j*; whilst $c(j)$ is the set of control actions leading to each element in $x(j)$. j is the number of inter-station runs within the specified control region and $x(j)$ is a set of possible states in stage *j*. Hence,

$$\begin{aligned}
C &= \{\{c^{1}(1), c^{1}(2), c^{1}(3)\}; \{c^{1}(1), c^{1}(2), c^{2}(3)\};\quad \ldots\ldots\ldots;\quad \{c^{8}(1), c^{64}(2), c^{512}(3)\}\} \\
&= \{\{(5,6,7),(5,6,7),(5,6,7)\}; \{(5,6,7),(5,6,7),(5,6,0)\};\quad \ldots;\quad \{(5,6,7),(5,6,7),(0,0,0)\}; \\
&\quad \{(5,6,7),(5,6,0),(5,6,7)\}; \{(5,6,7),(5,6,0),(5,6,0)\};\quad \ldots;\quad \{(5,6,7),(5,6,0),(0,0,0)\}; \\
&\quad \vdots \\
&\quad \{(5,6,0),(5,6,7),(5,6,7)\}; \{(5,6,0),(5,6,7),(5,6,0)\};\quad \ldots;\quad \{(5,6,7),(5,6,7),(0,0,0)\}; \\
&\quad \vdots \\
&\quad \{(0,0,0),(5,6,7),(5,6,7)\}; \{(0,0,0),(5,6,7),(5,6,0)\};\quad \ldots;\quad \{(0,0,0),(5,6,7),(0,0,0)\}; \\
&\quad \vdots \\
&\quad \{(0,0,0),(0,0,0),(5,6,7)\}; \{(0,0,0),(0,0,0),(5,6,0)\};\quad \ldots;\quad \{(0,0,0),(0,0,0),(0,0,0)\}\}
\end{aligned}$$

For each sequence $c^{\bullet}$ control actions in C, there is a corresponding sequence of states $x^{\bullet}$ in successive stages.

$$\begin{aligned}
c^{\bullet} &= \{(5,6,7),(5,6,7),(5,6,7)\} \quad\rightarrow\quad x^{\bullet} = \{x^{1}(0), x^{1}(1), x^{1}(2), x^{1}(3)\} \\
c^{\bullet} &= \{(5,6,7),(5,6,7),(5,6,0)\} \quad\rightarrow\quad x^{\bullet} = \{x^{1}(0), x^{1}(1), x^{1}(2), x^{2}(3)\} \\
&\vdots \\
c^{\bullet} &= \{(5,6,7),(5,6,7),(0,0,0)\} \quad\rightarrow\quad x^{\bullet} = \{x^{1}(0), x^{1}(1), x^{1}(2), x^{8}(3)\} \\
&\vdots
\end{aligned}$$

$$c^{\bullet} = \{(5,6,7),(5,6,0),(0,0,0)\} \quad \rightarrow \quad x^{\bullet} = \{x^1(0), x^1(1), x^2(2), x^{16}(3)\}$$

$$\vdots$$

$$c^{\bullet} = \{(5,6,0),(5,6,7),(5,6,7)\} \quad \rightarrow \quad x^{\bullet} = \{x^1(0), x^2(1), x^9(2), x^{65}(3)\}$$

$$\vdots$$

$$c^{\bullet} = \{(0,0,0),(0,0,0),(5,6,7)\} \quad \rightarrow \quad x^{\bullet} = \{x^1(0), x^8(1), x^{64}(2), x^{505}(3)\}$$

$$\vdots$$

$$c^{\bullet} = \{(0,0,0),(0,0,0),(0,0,0)\} \quad \rightarrow \quad x^{\bullet} = \{x^1(0), x^8(1), x^{64}(2), x^{512}(3)\}$$

For example, when the control actions $c^{\bullet} = \{(0,0,0),(0,0,0),(5,6,7)\}$ is applied in successive stage transformations and train 1 departs at station '0' when time = 0 sec. The corresponding sequence of states in successive stages is:

$$x^{\bullet} = \{x^1(0),\ x^8(1),\ x^{64}(2),\ x^{505}(3)\}$$

and the states are highlighted in Fig. 4.

Fig. 4 also shows the total number of possible states at stages 1, 2 and 3 are 8, 64 and 512 respectively. State $x^1(1)$ and $x^5(1)$, representing two different sets of arrival-times of trains at stations (i.e. $[521, 527, 502, 455]$and $[516, 527, 502, 455]$), produce two sets of states $x^1(2)\ldots x^8(2)$ and $x^{33}(2)\ldots x^{40}(2)$ at stage 2 with the corresponding control actions. Similarly, the other ordered states in $x(1)$, in which their arrival-time elements are not the same with each other, provide the different set of states at stage 2 as follows:

$$x^2(1) \ = \ [521,527,495,455] \ \rightarrow x^9(2)\ldots x^{16}(2) \ \text{and}$$

$$x^6(1) \ = \ [516,527,495,455] \rightarrow x^{41}(2)\ldots x^{48}(2) \ ;$$

$$x^3(1) \ = \ [521,521,502,455] \ \rightarrow x^{17}(2)\ldots x^{24}(2) \ \text{and}$$

$$x^7(1) \ = \ [516,521,502,455] \rightarrow x^{49}(2)\ldots x^{56}(2) \ ;$$

$$x^4(1) \;=\; [521, 521, 495, 455] \;\rightarrow x^{25}(2)\ldots x^{32}(2) \text{ and}$$

$$x^8(1) \;=\; [516, 521, 495, 455] \rightarrow x^{57}(2)\ldots x^{64}(2) \;;$$

To further elaborate on state evolution at stage 3, an example is shown below:

$$x^1(2) = [647, 654, 622, 575] \;\rightarrow x^1(3)\ldots x^8(3) \text{ and}$$

$$x^{33}(2) \;=\; [647, 654, 622, 575] \rightarrow x^{257}(3)\ldots x^{263}(3);$$

$$x^5(2) = [642, 654, 622, 575] \;\rightarrow x^{33}(3)\ldots x^{40}(3) \text{ and}$$

$$x^{37}(2) \;=\; [642, 654, 622, 575] \;\rightarrow x^{289}(3)\ldots x^{296}(3);$$

$$x^{17}(2) = [641, 654, 622, 575] \;\rightarrow x^{129}(3)\ldots x^{136}(3) \text{ and}$$

$$x^{49}(2) \;=\; [641, 654, 622, 575] \;\rightarrow x^{385}(3)\ldots x^{392}(3);$$

$$x^{21}(2) \;=\; [636, 654, 622, 575] \;\rightarrow x^{161}(3)\ldots x^{168}(3) \text{ and}$$

$$x^{53}(2) \;=\; [636, 654, 622, 575] \rightarrow x^{417}(3)\ldots x^{424}(3)$$

3.4 Grouping

Even though the optimal solution on train operation can be attained with the complete state-space diagram, computational demand is inevitably heavy and memory storage requirement becomes huge as the number of inter-station runs increases. To minimise the extensive state increment in the state-space traffic flow model in stages, grouping of states in a stage is a viable means. The notion of state grouping is to combine some of the states within a stage, if they have the same arrival-times of trains at successive stations, prior to the stage optimisation. The optimal solution remains with state grouping.

Fig. 5 shows that state $x^1(1)$ and $x^5(1)$ at stage 1 produce the same set of states at stage 2 (i.e. $x^1(2) \ldots x^8(2)$ and $x^{33}(2) \ldots x^{40}(2)$) through different control actions. The pairs of states $x^1(2)$ and $x^{33}(2)$, $x^2(2)$ and $x^{34}(2)$, $x^3(2)$ and $x^{35}(2)$, ………, $x^8(2)$ and $x^{40}(2)$, have the same arrival-time elements. These 8 pairs of states can be combined to form new states $\hat{x}^1(2) \ldots \hat{x}^8(2)$ at stage 2 through state grouping, based on the comparison of $M(x^1(2))$ and $M(x^{33}(2))$, ……, $M(x^8(2))$ and $M(x^{40}(2))$, where $M(x^k(j))$ is the minimum cost to reach $x^k(j)$ from the initial stage, and the cost $M(x^g(2))$ of reaching state $x^g(2)$ is determined by:

$$M(x^g(2)) = \{\min(F(x^g(2))_i + M(x^k(1)))\}$$

such that $x^k(1)$ reaches $x^g(2)$ with cost $F(x^g(2)_i)$. $F(x^g(2)_i)$ is the minimum cost to reach $x^g(2)$ from $x^k(1)$. i is the number of possible states in stage 1 to reach $x^g(2)$.

With state grouping, for each new state, $x^g(2)$ (i.e. each of $\hat{x}^1(2) \ldots \hat{x}^{32}(2)$), the corresponding set of $F(x^g(2)) = \{F(x^g(2))_1, \ldots, F(x^g(2))_{t_g}\}$, is formed. t_g is the number of possible states in stage '*j*-1' to reach '$x^g(j)$'. Similarly, the sets of cost reaching each new state, $\hat{x}^1(2) \ldots \hat{x}^{64}(2)$, at stage 3 are attained. State grouping is performed in each stage and hence the number of states at each stage is significantly reduced. To illustrate the advantage of state grouping, Table 2 shows the total number of possible states before and after grouping with different number of control variable steps and inter-station runs.

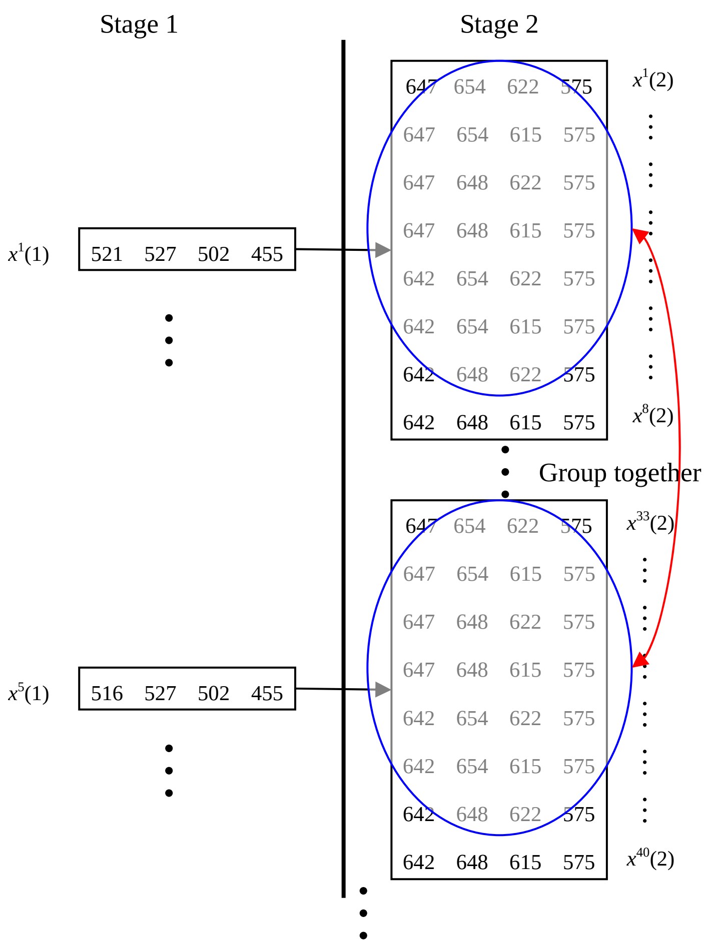


Fig. 5 State reduction through grouping

Table 2 – Number of states at the final stage without/with grouping

Number of inter-station runs	Number of steps in control variable*		
	2	3	4
	Without grouping		
1	2	3	4
2	16	81	256
3	512	19683	262144
4	65536	43046721	4294967296
5	33554432	$8.472886094 \times 10^{11}$	$1.125899907 \times 10^{15}$
	With grouping		
1	2	3	4
2	8	27	64
3	64	729	4096
4	1024	59049	1048576
5	32768	14348907	1073741824

* Run-time adjustment is the control variable in successive inter-station runs in this application.

3.5 Cost function

An objective function on service quality and another on energy consumption, as well as a combined one, are introduced here to evaluate how well the chosen control actions lead to the desired operational requirements.

I. *Service quality*

Service regularity is the major concern to passengers and operators when accessing quality of train service. Suppose all trains carrying the same traction characteristics and the separations of trains are scheduled to be the same, headway is the indicator of the service regularity. The following cost function penalises deviation from the nominal headway, and a low cost implies the chosen solution leading to the desired service.

$$Cost_{Ser} = \frac{\sum_{i=0}^{k} \left| \frac{(H_d \times (1+\varepsilon)) - (T_{k-i}{}^{i+2} - T_{k-i}{}^{i+1})}{H_d \times (1+\varepsilon)} \right|}{k} \qquad (15)$$

where k is the total number of inter-station runs in a region; H_d is the nominal headway between the trains; ε is the maximum allowable percentage of headway deviation from the nominal schedule. ε can be either positive or negative. Headway of trains is lengthened when ε is positive; $T_{k-i}{}^{i+2}$ and $T_{k-i}{}^{i+1}$ are the two arrival-time of trains '*i*+2' and '*i*+1' at the station '*k*-*i*' respectively. Headway of trains is therefore the arrival-time difference between the two trains at station '*k*-*i*'. The range of cost is between $\left(\frac{\varepsilon}{1+\varepsilon}\right)$ and 0.

II. Energy consumption

Energy consumption is the other concern of the operators as higher energy consumption implies higher operation cost. The following cost function of energy consumption is given to encourage energy reduction in the inter-station runs. Energy consumption of specific train movement between two stations is determined by a single train simulator [12].

$$Cost_{Energy} = Sgn\left(\frac{1}{k} \frac{\sum_{r=1}^{k} E_A{}^r - (1+\vartheta)\sum_{r=1}^{k} E_S{}^r}{(1+\vartheta)\sum_{r=1}^{k} E_S{}^r} \right) \quad (16)$$

where k is the total number of inter-station runs in a region; $E_S{}^r$ is the energy consumption of a train with the nominal run-time in an inter-station run 'r'; $E_A{}^r$ is the actual energy consumption of a train in an inter-station run 'r'; ϑ is the percentage of energy consumption deviation with respect to that in the nominal run-time. ϑ can be set as either positive or negative by the operators. Energy reduction is attained with the corresponding run-time extension of train when ϑ is negative. The energy cost function is only applicable when run-time control is employed and the range of return cost is between $\frac{\vartheta}{1+\vartheta}$ and 0.

III. Overall cost function

To reflect the relative importance of service quality and energy consumption on the overall cost function, the following expression is adopted.

$$Cost_{Overall} = W_{Ser} \cdot Cost_{Ser} + W_{Energy} \cdot Cost_{Energy} \quad (17)$$

Subject to: $W_{Ser} + W_{Energy} = 1$; $0 \le W_{Ser} \le 1$; $0 \le W_{Energy} \le 1$

where W_{Ser} *and* W_{Energy} are the weightings assigned to service quality and energy consumption respectively.

3.6 Optimal path

An example is given here to illustrate the formulation of the optimal path by DP. Referring to Table 1 and Fig. 4, two run-time extension levels (i.e. control steps) are adopted in each of the three inter-station runs, i.e. either 0 or 5 sec in inter-station 1; 0 or 6 sec in inter-station run 2; and 0 or 7 sec in the inter-station run 3. The run-times are allowed to extend by 5% with respect to the nominal train service.

To demonstrate the approach to obtain the optimal solution with the given operational requirements in DP, $\left\{F\left(x^g(1)\right), \; for \; 1 \le g \le 8\right\}$, $\left\{F\left(x^g(2)\right), \; for \; 1 \le g \le 32\right\}$ and $\left\{F\left(x^g(3)\right), \; for \; 1 \le g \le 64\right\}$ denote the three sets of costs of reaching all the possible states $\left\{x^1(1), ..., x^8(1)\right\}$, $\left\{x^1(2), ..., x^{32}(2)\right\}$ and $\left\{x^1(3), ..., x^{64}(3)\right\}$ in the successive stages respectively. The three sets of costs to reach the corresponding elements in $x(1), x(2)$ and $x(3)$ are calculated by Eqns (15), (16) and (17).

At the beginning, $\left\{x^1(1), ..., x^8(1)\right\}$ are evolved from stage 0 with the corresponding run-time extensions of (5,6,7), (5,6,0), (5,0,7), (5,0,0), (0,6,7), (0,0,7) and (0,0,0) sec (i.e. $c(2)$) in successive inter-station runs, from the initial stage. The minimum cost $M\left(x(1)\right) = \left\{\min F\left(x^g(1)\right), \; for \; 1 \le g \le 8\right\}$ of reaching states $\left\{x^1(1), ..., x^8(1)\right\}$ in stage 1 are then computed. Determination of the minimum cost, $M\left(x(j)\right)$, to reach the given states $x^g(j)$ is illustrated in Fig. 6. Since there is only one initial state in stage 0, no optimisation is required. The control action, $c^*(1)$, which minimizes $\{F(x^g(1)),\ for\ 1 \le g \le 8\}$, is recorded.

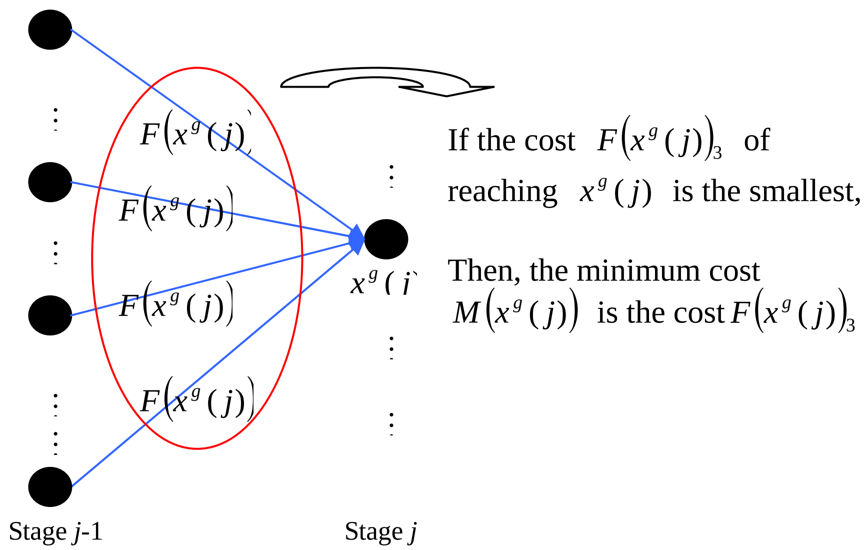


Fig. 6 Minimum cost $M(x^g(j))$ of reaching state $x^g(j)$ in stage j

With the minimum cost $M(x(1))$ for each element in $x(1)$, the set $M(x(2))$ are deduced as shown in Table 3. The states in stage 1, from which the elements in $x(2)$ are reached, are available and the corresponding costs are denoted by $F(x^g(2))$. Table 3 also reveals the minimum cost, $M(x(3))$, to reach all the possible states in $x(3)$ in stage 3. When the control action $c^*(3)$ is deduced, the overall optimal path with the corresponding control decisions and states in successive stages are obtained.

$$Optimal\ \ path: \qquad x^*(3) \xrightarrow{c^*(3)} x^*(2) \xrightarrow{c^*(2)} x^*(1) \xrightarrow{c^*(1)} x^*(0) \qquad (18)$$

Suppose '$x^1(0)$ → $x^8(1)$ → $x^{32}(2)$ → $x^{57}(3)$' is the optimal path under the given traffic conditions and operational requirements shown in Table 1, the corresponding control actions of run-time extensions made in the three inter-station runs are (0,0,0) sec in the stage transformations '0 to 1' and '1 to 2'; and (5,6,7) sec in stage transformation '2 to 3'.

Table 3 – Optimisation at stage 2 and 3

States in stage 2	States in $x(1)$ to reach each element in $x(2)$	Minimum cost $M(x(2))$ in stage 2	Control action
$x^1(2)$	$x^1(1)$ $x^5(1)$	$\underset{c(2)}{Min}\left\{\begin{matrix} F(x^1(2))_1 + M(x^1(1)) \\ F(x^1(2))_2 + M(x^5(1)) \end{matrix}\right\}$	(5,6,7)
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$x^{10}(2)$	$x^2(1)$ $x^6(1)$	$\underset{c(2)}{Min}\left\{\begin{matrix} F(x^{10}(2))_1 + M(x^2(1)) \\ F(x^{10}(2))_2 + M(x^6(1)) \end{matrix}\right\}$	(5,6,0)
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$x^{19}(2)$	$x^3(1)$ $x^7(1)$	$\underset{c(2)}{Min}\left\{\begin{matrix} F(x^{19}(2))_1 + M(x^3(1)) \\ F(x^{19}(2))_2 + M(x^7(1)) \end{matrix}\right\}$	(5,0,7)
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$x^{28}(2)$	$x^4(1)$ $x^8(1)$	$\underset{c(2)}{Min}\left\{\begin{matrix} F(x^{28}(2))_1 + M(x^4(1)) \\ F(x^{28}(2))_2 + M(x^8(1)) \end{matrix}\right\}$	(5,0,0)
States in stage 3	States in $x(2)$ to reach each element in $x(3)$	Minimum cost $M(x(3))$ in stage 3	Control action
$x^1(3)$	$x^1(2)$ $x^5(2)$ $x^{17}(2)$ $x^{21}(2)$	$\underset{c(3)}{Min}\left\{\begin{matrix} F(x^1(3))_1 + M(x^1(2)) \\ F(x^1(3))_2 + M(x^5(2)) \\ F(x^1(3))_3 + M(x^{17}(2)) \\ F(x^1(3))_4 + M(x^{21}(2)) \end{matrix}\right\}$	(5,6,7)
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$x^{42}(3)$	$x^{10}(2)$ $x^{14}(2)$ $x^{26}(2)$ $x^{30}(2)$	$\underset{c(3)}{Min}\left\{\begin{matrix} F(x^{42}(3))_1 + M(x^{10}(2)) \\ F(x^{42}(3))_2 + M(x^{14}(2)) \\ F(x^{42}(3))_3 + M(x^{26}(2)) \\ F(x^{42}(3))_4 + M(x^{30}(2)) \end{matrix}\right\}$	(5,6,0)
$\vdots$ $\vdots$	$\vdots$ $\vdots$	$\vdots$ $\vdots$	$\vdots$ $\vdots$
$x^{64}(3)$	$x^{12}(2)$ $x^{16}(2)$ $x^{28}(2)$ $x^{32}(2)$	$\underset{c(3)}{Min}\left\{\begin{matrix} F(x^{64}(3))_1 + M(x^{12}(2)) \\ F(x^{64}(3))_2 + M(x^{16}(2)) \\ F(x^{64}(3))_3 + M(x^{28}(2)) \\ F(x^{64}(3))_4 + M(x^{32}(2)) \end{matrix}\right\}$	(0,0,0)

4 Results and discussions

This section demonstrates the functions and versatility of the proposed *DRC* under various traffic conditions and operation requirements. The simulation is conducted on an IBM-compatible PC with PIII-866 MHz CPU with 256MB memory. A number of studies have been carried out to evaluate the controller's performance in terms of capability, optimality and flexibility.

4.1 State simplification

This study illustrates the possible state simplification in the event-based traffic flow model by state-grouping. The traffic conditions and operational constraints are given in Table 4.

Table 4 – Traffic conditions and operational requirements

Traffic conditions			
Number of inter-station runs	3		
Nominal dwell-times at stations	30 sec		
Nominal inter-station run-times	Run 0-1	Run 1-2	Run 2-3
	135 sec	121 sec	90 sec
Operational constraints			
Control method	Mixed (i.e. Dwell-time and run-time)		
Control space*	Run-time: +10%	Dwell-time: +10%	
Steps in control variable	Dwell-times at stations: 3, 2, 1, 0 sec		
	Inter-station run 0-1	Inter-station run 1-2	Inter-station run 2-3
	12, 8, 4, 0 sec	12, 8, 4, 0 sec	9, 6, 3, 0 sec
Operational requirements	Headway is extended by 10% (i.e. ε is 0.1 in $Cost_{Ser}$)		

*Nominal schedule is used as a reference to regulate the service.

Results:

Table 5 – Number of states at stages

Case I: Operation with complete state-space diagram (i.e. without grouping)				
Stage	0	1	2	3
Number of states	1	64	4096	262144
Case II: Operation with dynamic programming (i.e. state grouping)				
Stage	0	1	2	3
Number of states	1	64	1024	4096

Table 6 – Optimal cost and computational demand

Case	I	II
Cost	0.1269	0.1269
Computation time (sec)	35.6	0.73
Physical memory storage (MB)	26	1.27

Discussions:

Table 5 shows that the number of possible states at the intermediate stages (i.e. stage 2 and 3) increases substantially in Case I. Likewise, in Case II, where grouping is introduced, the number of states also increases, but to a smaller extent, especially at stage 3. With grouping, a number of states in the proceeding stage can be reduced and a significant reduction on the scale of states expansion is then accomplished.

Simulation results also show that the controller delivers the same cost at the final state in both Cases I and II. It has therefore verified that the controller provides the same solution (i.e. same optimal path and cost) with identical traffic conditions and operational requirements with and without state grouping. The computation time and memory requirement are significantly reduced as a result of the state grouping.

4.2 Control steps and inter-station runs

This study investigates the impact of the number of control steps and inter-station runs on computational demand under given operational conditions. Four cases are undertaken and shown in Cases I to IV respectively. Cases I to III with 3 inter-station runs are set up to investigate the effect of control steps on computational demand; while a further test (i.e. Case IV) with 4 inter-station runs is carried out to evaluate the increase on the demand when compared to Case I, for they have the same number of control steps. The traffic conditions and operational constraints are given in Table 7.

Table 7 – Traffic conditions and operational constraints

Traffic conditions				
Nominal inter-station run-times	Run 0-1	Run 1-2	Run 2-3	Run 3-4
	135sec	121 sec	90 sec	76 sec
Operational constraints				
	Case I	Case II	Case III	Case IV
Inter-station runs involved	Station 0 to 3	Station 0 to 3	Station 0 to 3	Station 0 to 4
Steps in control variable at stations	(Station 0 to 1) 14, 7, 0 sec (Stations 1 to 2) 12, 6, 0 sec (Stations 2 to 3) 8, 4, 0 sec	(Station 0 to 1) 12, 8, 4, 0 sec (Stations 1 to 2) 12, 8, 4, 0 sec (Stations 2 to 3) 9, 6, 3, 0 sec	(Station 0 to 1) 12, 9, 6, 3, 0 sec (Stations 1 to 2) 12, 9, 6, 3, 0 sec (Stations 2 to 3) 8, 6, 4, 2, 0 sec	(Station 0 to 1) 14, 7, 0 sec (Stations 1 to 2) 12, 6, 0 sec (Stations 2 to 3) 8, 4, 0 sec (Stations 3 to 4) 8, 4, 0 sec
Control space*	Run-time: +10%			
Operational requirements	Headway is extended by 6% (i.e. ε is 0.06 in $Cost_{Ser}$)			

*Nominal schedule is used as a reference to regulate the service.

Results:

Table 8 – Optimal cost and computational demand

Case	I	II	III	IV
Cost	0.073	0.071	0.069	0.0704
Computation time (sec)	0.14	0.841	4.035	17.445
Physical memory storage (MB)	0.168 (i.e.172 KB)	1.28	6.98	28.8

Discussions:

Simulation results show that the computational time and memory storage requirements increase drastically with the number of inter-stations runs and control steps. The computational demand is the highest in Case IV. Given the same number of control steps at each inter-station run in Cases I and IV (i.e. 3 steps), the computational time and physical storage with 4 inter-stations runs (i.e. Case IV) increases by more than a hundred times, when compared to those with 3 inter-station runs (i.e. Case I). With Cases I, II and III, the increase on computational demand is much lower since the number of inter-station runs determines the number of possible stage transformations in the traffic flow model. The number of possible states at a particular stage depends on: (1) the number of possible states from the previous stage; and (2) the number of possible solutions each state can generate (i.e. a solution composes of a set of extended run-times in successive inter-station runs). Therefore, a larger number of states are usually required with more inter-station runs, and hence the computation time and memory storage.

Further, more steps in control variables imply a finer resolution of run-time extensions in the control space. When comparing the optimal solution attained in Cases I, II and III, Case III's optimal solution stands out with the lowest cost as stated in Table 8, since more precise train control is attained with a finer resolution of control step. The computation time and memory storage requirement, however, increases with more steps in control variables.

4.3 Dwell-time and run-time

This study examines the train performance with different control methods. Dwell-time, run-time and the combined control are adopted in Cases I, II and III respectively, which are subject to three operational requirements shown in tests A, B and C. Service extension (i.e. *S*) is the main interest in test A (i.e. $W_S = 1$ and $W_E = 0$), while energy reduction (i.e *E*) is the focus in test B (i.e. $W_S = 0$ and $W_E = 1$) and their combination (i.e. *S&E*) is introduced in test C. The weighting on service, W_S, and energy, W_E, are of the same importance in test C (i.e. $W_S = W_E = 0.5$).

Table 9 – Traffic conditions and operational constraints

<table>
<tr><th colspan="10">Traffic conditions</th></tr>
<tr><td>Number of inter-station runs</td><td colspan="9">4</td></tr>
<tr><td>Nominal dwell-times at stations</td><td colspan="9">20 sec</td></tr>
<tr><td rowspan="2">Nominal inter-station run-times</td><td colspan="2">Run 0-1</td><td colspan="2">Run 1-2</td><td colspan="2">Run 2-3</td><td colspan="3">Run 3-4</td></tr>
<tr><td colspan="2">135sec</td><td colspan="2">121 sec</td><td colspan="2">90 sec</td><td colspan="3">76 sec</td></tr>
<tr><th colspan="10">Operational constraints</th></tr>
<tr><td rowspan="2">Control method</td><td colspan="3">Case I</td><td colspan="3">Case II</td><td colspan="3">Case III</td></tr>
<tr><td colspan="3">Dwell-time</td><td colspan="3">Run-time</td><td colspan="3">Mixed</td></tr>
<tr><td>Control space*</td><td colspan="4">Run-time: +10%</td><td colspan="5">Dwell-time: +10%</td></tr>
<tr><td rowspan="3">Steps in control variable</td><td colspan="9">Dwell-times at stations: 2, 1, 0 sec</td></tr>
<tr><td colspan="2">Inter-station run 0-1</td><td colspan="2">Inter-station run 1-2</td><td colspan="2">Inter-station run 2-3</td><td colspan="3">Inter-station run 3-4</td></tr>
<tr><td colspan="2">14, 7, 0 sec</td><td colspan="2">12, 6, 0 sec</td><td colspan="2">8, 4, 0 sec</td><td colspan="3">8, 4, 0 sec</td></tr>
<tr><td>Operational requirements</td><td colspan="9">Headway is extended by 4% (i.e. ε is 0.04 in $Cost_{Ser}$)
Energy is reduced by 8% (i.e. ϑ is 0.08 in $Cost_{Energy}$)</td></tr>
<tr><td rowspan="2">Cost function**</td><td colspan="3">Case I</td><td colspan="3">Case II</td><td colspan="3">Case III</td></tr>
<tr><td>A</td><td>B</td><td>C</td><td>A</td><td>B</td><td>C</td><td>A</td><td>B</td><td>C</td></tr>
</table>

	S	*E*	*S&E*	*S*	*E*	*S&E*	*S*	*E*	*S&E*
Weighting factor	$W_S = 0.5$, $W_E = 0.5$ (For all case C only)								

*Nominal schedule is used as a reference to regulate the service.

***S*: service plays dominant (i.e. $W_S = 1$); *E*: energy plays dominant (i.e. $W_E = 1$); *S&E*: Both service and energy are taken into account with their corresponding weights.

Results:

Table 10 – Optimal costs

Case	Cost		
	A	B*	C
I	0.128	-	0.107
II	0.075	0	0.042
III	0.086	0	0.046

*Adjustments of dwell-time of trains at stations is not applicable to achieve energy saving

Discussions:

Table 10 shows that run-time control (i.e. Case II) is more likely to meet the train operational requirement either in terms of service, energy and their combination, since the controller delivers the optimal path with the lowest cost as shown in tests A and C of Case II, when compared with the corresponding tests of Case I and III respectively. The optimal cost with dwell-time control is the highest under the same operational requirements – the cost given in test A (i.e. 0.128) of Case I is higher than that of Cases II (i.e. 0.075) and III (i.e. 0.086). Dwell-time at stations is always shorter than the inter-station run-time, a smaller control space is therefore available to maintain the train service and hence a relatively less flexible train control is attained. Further, when energy saving is taken into account, dwell-time control is not preferred as it does not change energy demand.

With mixed control, both dwell-time and run-time control are adjusted to meet the operational requirements and a flexible train control is to be expected. The optimal cost attained in tests A and C of Case III is, however, slightly higher than the one in the corresponding tests of Case II even though mixed control is applied. It is because dwell-time extensions at stations inevitably affect the train service quality. Moreover, the optimisation problem of train scheduling gets more complicated and the solution space becomes larger with the introduction of mixed control. More computational time and higher memory storage requirement are required. Run-time adjustment in successive inter-station runs thus provides a relatively better performance on train regulation.

5 Conclusions

With the aid of an event-based traffic flow model, adjustment of train operation in a region through dwell-time and run-time control by dynamic programming has been presented. From the viewpoint of energy saving, run-time control is superior to dwell-time control because a longer run-time between stations implies lower energy consumption. However, energy reduction cannot be achieved by lengthening the waiting time of train at station unless regenerative braking is taken into account with coordination of trains approaching and leaving stations. Further, a high flexibility of train regulation can be achieved with run-time control when compared to dwell-time control because run-time between stations is usually much longer than the dwell-time at stations and hence a larger control space is available for the operators to maintain the train service.

The complexity and size of the event-based traffic flow model depends on the number of inter-stations runs and the number of possible sets of control actions. To reduce the computation demand of decision-making process of trains' instruction, a line is divided into a number of control regions. Size of region depends on the configuration of the traction supply system and each region usually covers few passenger stations. With a number of *DRC*s in work, the computation demand on each *DRC* is relatively low. *DRC* is thus able to deliver the optimal control actions for the trains for real-time applications.

6 Acknowledgements

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