Submarine Dynamic Modeling

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Abstract

This paper discusses the development of a dynamic model for a torpedo shaped submarine. Expressions for hydrostatic, added mass, hydrodynamic, control surface and propeller forces and moments are derived from first principles. Experimental data obtained from flume tests of the submarine are inserted into the model in order to provide computer simulations of the open loop behavior of the system.

Introduction

Autonomous underwater vehicles (AUV) are finding use in a variety of oceanographic survey applications (Foresti [2001] Dhanak [2001]). These vehicles present a comparatively low cost technology for underwater exploration with a freedom of motion superior to tethered-towed submersibles.

An experimental AUV (Figures 1 and 2) has been designed by Reid [2001] at Queensland University of Technology and the computer control system was installed by staff at the CSIRO, CMIT Automation Group. This AUV is torpedo shaped, approximately 1.5m long × 150mm diameter. Four, independently actuated, orthogonal, stern planes are used to control its attitude. Power to the DC motor, which drives the propellor, comes from a battery supply via a current amplifier, used to control the shaft speed.

The mathematical dynamic model, described in this paper, provides a useful tool for the understanding and tuning of the control system which automatically controls the attitude and depth of the submarine. This work tracks similar developments described by Nahon [1993, 1998]

Numerical values for the hydrodynamic coefficients which are contained within the mathematical model have been evaluated, where possible, from experimental data derived from the full size AUV or a half size model, inserted into an open flowing channel (flume),

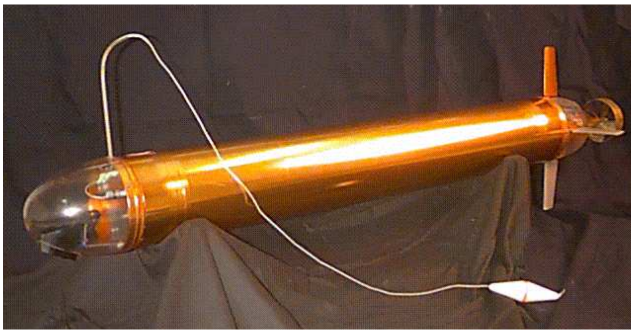

Figure 1: Torpedo shaped AUV: fully assembled

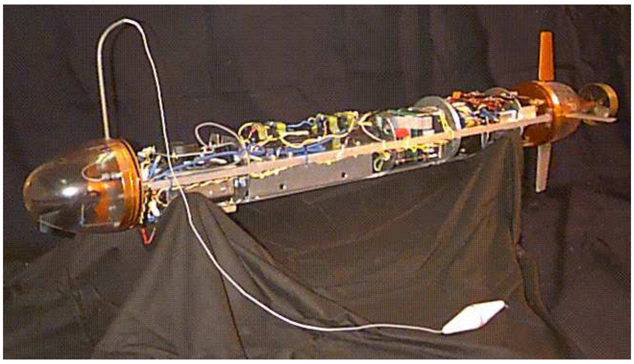

Figure 2: Torpedo shaped AUV: casing removed

585mm wide . These experimental procedures, described in detail in the Appendix, have provided data regarding relationships between angle of attack (α) and the coefficients of lift and drag (C_L, C_D), moment coefficient (C_M) and control surface effectiveness ($C_{L\delta f}$), plotted in this paper.

Equations of motion

Figure 3 shows the body frame of reference in which the equations of motion are written. Origin C_b is located at the centre of buoyancy and the centre of gravity lies at the point $\mathbf{r_G}=[x_G,y_G,z_G]^T$. The components of $\mathbf{r_G}$ are small since the submarine is deliberately designed to

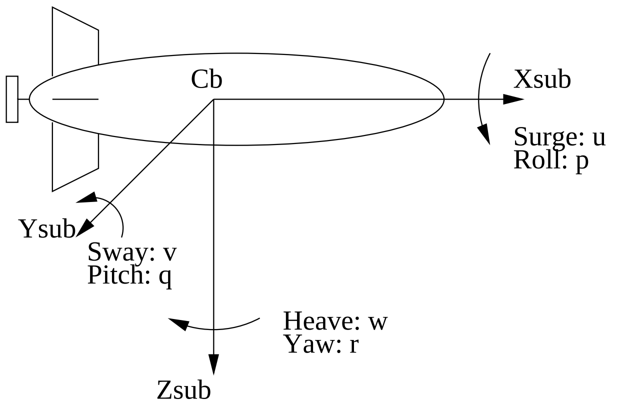

Figure 3: Body coordinate system

have the centre of gravity and the centre of bouyancy coincident. The following symbols are used for components in the X_{sub}-, Y_{sub}- and Z_{sub}- directions:

forces = $[X,Y,Z]^T$ moments = $[K,M,N]^T$
velocity $\mathbf{V} = [u,v,w]^T$ angular velocity $\omega = [p,q,r]^T$

Newton's equations of motion, for a rigid body with six degrees of freedom, relative to coordinates attached to the body at C_b, are $\Sigma F = m\mathbf{a_G}$ where

- m=mass of the submarine and
- $\mathbf{a_G}$=acceleration of the centre of mass.

Substituting $\mathbf{a_G} = \frac{\partial V}{\partial t} + \omega \times \mathbf{V} + \dot{\omega} \times \mathbf{r_G} + \omega \times \omega \times \mathbf{r_G}$ gives the following force equations in the X_{sub}-, Y_{sub}- and Z_{sub}-directions:

$$\begin{aligned} &m[\dot{u} - vr + wq - x_G(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pr + \dot{q})] \\ &= \sum X_{ext} \\ &m[\dot{v} - wp + ur - y_G(r^2 + p^2) + z_G(qr - \dot{p}) + x_G(qp + \dot{r})] \\ &= \sum Y_{ext} \\ &m[\dot{w} - uq + vp - z_G(p^2 + q^2) + x_G(rp - \dot{q}) + y_G(rq + \dot{p})] \\ &= \sum Z_{ext} \end{aligned} \tag{1}$$

Euler's equations of motion, for a rigid body with six degrees of freedom, relative to coordinates attached to the body at C_b, are $\Sigma M_B = \dot{H}_G + \mathbf{r_G} \times m\mathbf{a_G}$. The rate of change of angular momentum about the centre of gravity, $\dot{H}_G = [I]\dot{\omega} + \omega \times [I]\omega$, where [I] is the $[3 \times 3]$ diagonal inertia matrix $[I_{xx}, I_{yy}, I_{zz}]$ evaluated about principal axes located at the centre of gravity.

Substituting again for $\mathbf{a_G}$, neglecting small terms (eg x_G^2), gives the following moment equations in the X_{sub}-, Y_{sub}- and Z_{sub}-directions:

$$\begin{aligned} &I_{xx}\dot{p} + (I_{zz} - I_{yy})qr - m[y_G(\dot{w} - uq + vp) - z_G(\dot{v} - wp + ur)] \\ &= \sum K_{ext} \\ &I_{yy}\dot{q} + (I_{xx} - I_{zz})rp - m[z_G(\dot{u} - vr + wq) - x_G(\dot{w} - uq + vp)] \\ &= \sum M_{ext} \\ &I_{zz}\dot{r} + (I_{yy} - I_{xx})pq - m[x_G(\dot{v} - wp + ur) - y_G(\dot{u} - vr + wq)] \\ &= \sum N_{ext} \end{aligned} \tag{2}$$

Evaluation of Forces and moments

Five sets of forces/moments act on the hull:

(i)Hydrostatic forces

The orientation of the body frame relative to the world frame is described by Euler angles rotated in the order: roll ϕ, pitch θ, yaw ψ.

The static forces, weight (**W**) and buoyancy (**B**) act through the centre of gravity and centre of buoyancy respectively. When resolved onto the submarine body frame, these become:

$$\begin{aligned} X_{HS} &= -(W - B)\sin\theta \\ Y_{HS} &= (W - B)\cos\theta\sin\phi \\ Z_{HS} &= (W - B)\cos\theta\cos\phi \\ K_{HS} &= -y_G W\cos\theta\cos\phi - z_G W\cos\theta\sin\phi \\ M_{HS} &= -z_G W\sin\theta - x_G W\cos\theta\cos\phi \\ N_{HS} &= -y_G W\cos\theta\sin\phi - z_G W\sin\theta \end{aligned} \tag{3}$$

(ii)Added mass inertia forces

Added mass is a measure of the additional inertia created by water which accelerates with the submarine. The forces and moments created by added mass may be expressed :

$$\begin{aligned} X_A &= X_{\dot{u}}\dot{u} + X_{wq}wq + X_{qq}q^2 + X_{vr}vr + X_{rr}r^2 \\ Y_A &= Y_{\dot{v}}\dot{v} + Y_{\dot{r}}\dot{r} + Y_{ur_a}ur + Y_{wp}wp + Y_{pq}pq \\ Z_A &= Z_{\dot{w}}\dot{w} + Z_{\dot{q}}\dot{q} + Z_{uq_a}uq + Z_{vp}vp + Z_{rp}rp \\ K_A &= K_{\dot{p}}\dot{p} \\ M_A &= M_{\dot{w}}\dot{w} + M_{\dot{q}}\dot{q} + M_{uw_a}uw + M_{vp}vp + M_{rp}rp + M_{uq_a}uq \\ N_A &= N_{\dot{v}}\dot{v} + N_{\dot{r}}\dot{r} + N_{uv_a}uv + N_{wp}wp + M_{pq}pq + N_{ur_a}ur \end{aligned} \tag{4}$$

where eg. $X_{\dot{u}} = \frac{\partial X}{\partial \dot{u}}$ [kg] and $K_{\dot{p}} = \frac{\partial K}{\partial \dot{p}}$ [kg m^2] etc, are added masses and added mass moments of intertia. Axial $X_{\dot{u}}$ and rolling $K_{\dot{p}}$ added masses were estimated from an empirical relationships by Blevins[1984] and crossflow added masses ($Y_{\dot{v}}$, $Z_{\dot{w}}$=$Y_{\dot{v}}$, $M_{\dot{w}}$, $N_{\dot{v}}$=-$M_{\dot{w}}$, $Y_{\dot{r}}$=$N_{\dot{v}}$, $Z_{\dot{q}}$=$M_{\dot{w}}$, $M_{\dot{q}}$,$N_{\dot{r}}$=$M_{\dot{q}}$,) ,were evaluated numerically using the technique by Newmann [1980].

The remaining cross-terms result from added mass coupling and can be evaluated from the added mass terms already derived.

$$\begin{aligned} &X_{wq} = Z_{\dot{w}} && X_{qq} = Z_{\dot{q}} \\ &X_{vr} = -Y_{\dot{v}} && X_{rr} = -Y_{\dot{r}} \\ &Y_{ur_a} = X_{\dot{u}} && Y_{wp} = -Z_{\dot{w}} \\ &Y_{pq} = -Z_{\dot{q}} && \\ &Z_{uq_a} = -X_{\dot{u}} && Z_{vp} = Y_{\dot{v}} \\ &Z_{rp} = Y_{\dot{r}} && \\ &M_{uw_a} = -(Z_{\dot{w}} - X_{\dot{u}}) && M_{vp} = -Y_{\dot{r}} \\ &M_{rp} = (K_{\dot{p}} - N_{\dot{r}}) && M_{uq_a} = -Z_{\dot{q}} \\ &N_{uv_a} = -(X_{\dot{u}} - Y_{\dot{v}}) && N_{wp} = Z_{\dot{q}} \\ &N_{pq} = -(K_{\dot{p}} - M_{\dot{q}}) && N_{ur_a} = Y_{\dot{r}} \end{aligned} \tag{5}$$

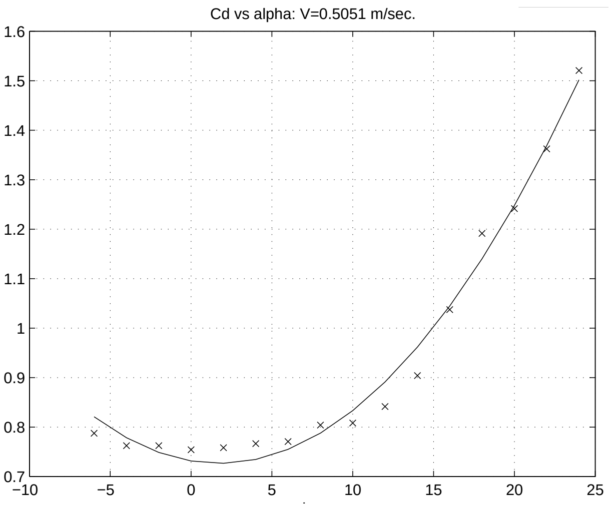

Figure 4: Hull Drag: C_D versus α, V=0.5m/sec. a=5.25 rad.$^{-2}$, b=-0.315 rad.$^{-1}$, c=0.732

(iii)Hydrodynamic forces and moments

Hull forces and moments

Drag is related to the fluid density ρ, submarine frontal area A_f and lies in the direction of the fluid velocity V.

$$D = \tfrac{1}{2}\rho C_D A_f V^2 \tag{6}$$

C_D is related to the angle of attack (α) through a parabolic relationship, plotted in Figure 4.

$$C_D = a\alpha^2 + b\alpha + c. \tag{7}$$

It is assumed that the sway (v) and heave velocity (w) are small compared with the surge (u).Angle of attack can be expressed: in the XZ-plane as $\tan\alpha \simeq \alpha = \frac{w}{u}$ [radians], or in the XY-plane as $\tan\beta \simeq \beta = \frac{v}{u}$ [radians].

Drag force, when viewed in the XZ-plane, may be resolved into the X_{sub}- and Z_{sub}- directions.

$$\begin{aligned} D_x &= -\tfrac{1}{2}\rho A_f C_D(u^2+w^2)cos\alpha \\ &\simeq -\tfrac{1}{2}\rho A_f C_D(u^2+w^2)(1-\tfrac{\alpha^2}{2}) \\ D_z &= -\tfrac{1}{2}\rho A_f C_D(u^2+w^2)sin\alpha \\ &\simeq -\tfrac{1}{2}\rho A_f C_D(u^2+w^2)\alpha \end{aligned} \tag{8}$$

Similarly, drag force, when viewed in the XY-plane, may be resolved into the X_{sub}- and Y_{sub}- directions.

$$\begin{aligned} D_x &= -\tfrac{1}{2}\rho A_f C_D(u^2+v^2)cos\beta \\ &\simeq -\tfrac{1}{2}\rho A_f C_D(u^2+v^2)(1-\tfrac{\beta^2}{2}) \\ D_y &= \tfrac{1}{2}\rho A_f C_D(u^2+v^2)sin\beta \\ &\simeq \tfrac{1}{2}\rho A_f C_D(u^2+v^2)\beta \end{aligned} \tag{9}$$

Expanding equations 8 and 9 , through C_D, α and β, and neglecting terms beyond second order, reveals that the components of total drag force in the X_{sub}-,Y_{sub}- and Z_{sub}- directions may be expressed:

$$\begin{aligned} X_d = &\; X_{u|u|}u\,|u| + X_{uv}uv + X_{uw}uw + \\ &\; X_{v|v|}v\,|v| + X_{w|w|}w\,|w| \\ Y_d = &\; Y_{uv_d}uv + Y_{v|v|}v\,|v| \\ Z_d = &\; Z_{uw_d}uw + Z_{w|w|}w\,|w| \end{aligned} \tag{10}$$

where:

$$\begin{aligned} &X_{u|u|} = -\tfrac{1}{2}\left(\rho A_f\right)c \\ &X_{uw} = X_{uv} = -\left(\tfrac{1}{2}\rho A_f\right)b \\ &X_{w|w|_d} = X_{v|v|} = -\left(\tfrac{1}{2}\rho A_f\right)\left(a+\tfrac{c}{2}\right) \\ &-Y_{v|v|} = Z_{w|w|} = -\left(\tfrac{1}{2}\rho A_f\right)b \\ &Z_{uw_d} = -Y_{uv_d} = -\left(\tfrac{1}{2}\rho A_f\right)c \end{aligned} \tag{11}$$

Lift L, acting at the centre of pressure, is generated perpendicular to the flow, as the submarine moves through the water. Relocating this force to act at the centre of buoyancy causes a pitching moment M to be created. Both lift and moment are directly proportional to the angle of attack and are plotted in Figures 5 and 6.

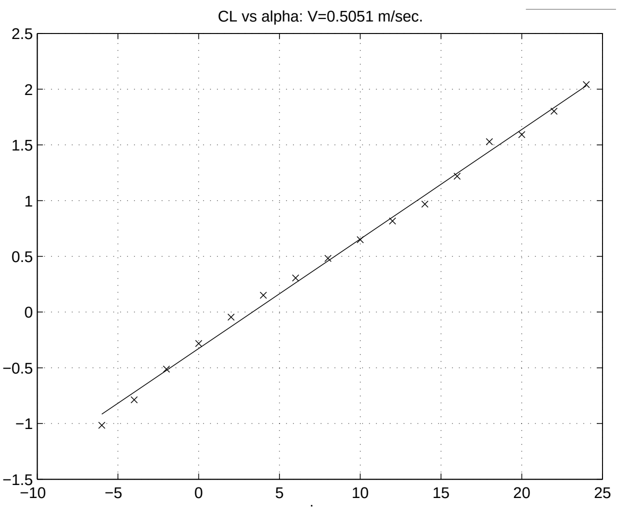

Figure 5: Hull Lift: C_L versus α, V=0.5m/sec. Slope: C_{L_α}=4.79 rad.$^{-1}$

$$\begin{aligned} L &= \tfrac{1}{2}\rho C_L A_f V^2 \quad C_{L_\alpha} = \tfrac{\partial C_L}{\partial \alpha} \\ M &= \tfrac{1}{2}\rho C_M A_f V^2 \quad C_{M_\alpha} = \tfrac{\partial C_M}{\partial \alpha} \end{aligned} \tag{12}$$

Lift force and pitching moment, when viewed in the XZ-plane are derived from equation 12:

$$\begin{aligned} Z_l &= -\tfrac{1}{2}\rho A_f C_{L_\alpha}(u^2+w^2)\alpha cos\alpha \\ M_l &= \tfrac{1}{2}\rho A_f C_{M_\alpha}(u^2+w^2)\alpha \end{aligned} \tag{13}$$

Similarly in the XY-plane,

$$\begin{aligned} Y_l &= \tfrac{1}{2}\rho A_f C_{L_\beta}(u^2+v^2)\beta cos\beta \\ N_l &= \tfrac{1}{2}\rho A_f C_{M_\beta}(u^2+v^2)\beta \end{aligned} \tag{14}$$

Using the expression of the angle of attack, under the assumption $u \gg w \; or \; v$ we have:

$$\begin{aligned} Y_{uv_l} &= -Z_{uw_l} = \tfrac{1}{2}\rho A_f C_{L_\alpha} \\ M_{uw_l} &= N_{uv_l} = \tfrac{1}{2}\rho A_f C_{M_\alpha} \end{aligned} \tag{15}$$

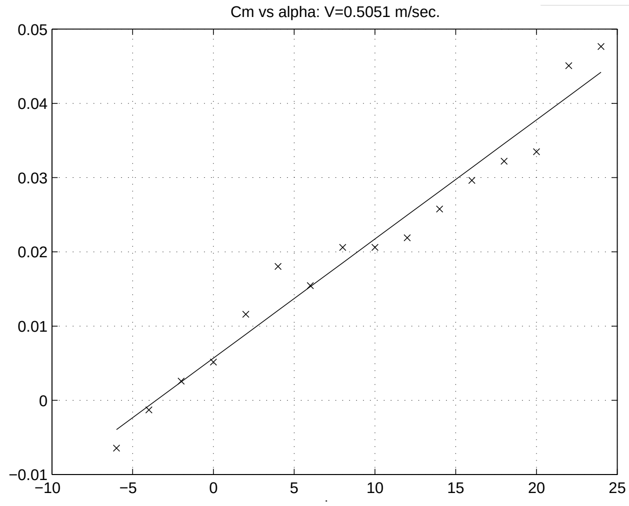

Figure 6: Hull Moment: C_m versus α, V=0.5m/sec. Slope: C_{m_α}=0.0974 rad.$^{-1}$

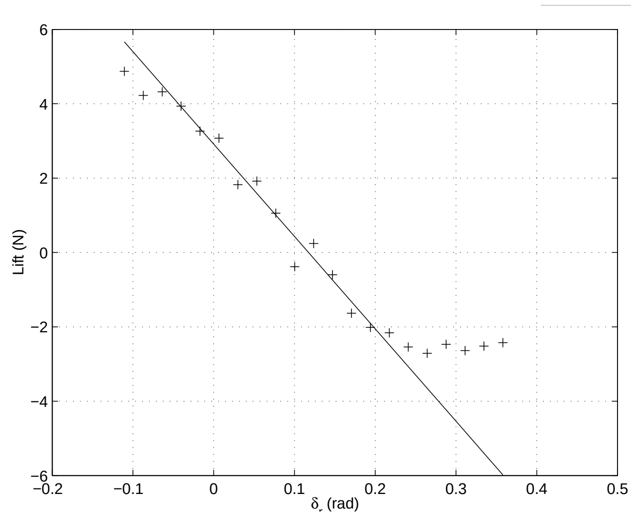

Figure 7: Fin Lift: Lift vs δ_r, V=0.81m/sec. NB: $|slope| = 2\,|Y_{uu\delta_r}|\,V^2$=-25 N.rad.$^{-1}$

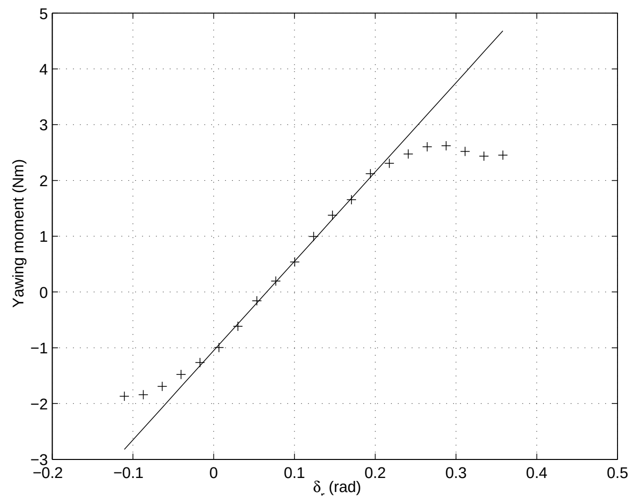

Figure 8: Fin Turning Moment: Moment vs δ_r, V=0.81m/sec. $|slope| = 2\,|N_{uu\delta_r}|\,V^2$=16 Nm.rad.$^{-1}$

(iv) Control surface forces and moments

Attitude of the vehicle is controlled by two horizontal stern planes, and two vertical rudders. Assuming diametrically opposite fins move together the empirical formula for fin lift is given as:

$$\begin{array}{l} L_{fin} = \frac{1}{2}\rho C_{L_{\delta f}} S_{fin} \delta_e v_e^2 \\ M_{fin} = x_{fin} L_{fin} \end{array} \tag{16}$$

where $C_{L_{\delta f}}$ is the rate of change of lift coefficient wrt fin effective angle of attack and S_{fin} is the fin planform area. δ_e is the effective fin angle in radians. For the rudder $\delta_e = \delta_r + \frac{v + x_{fin} r}{u}$ and stern plane $\delta_e = \delta_s - \frac{w - x_{fin} q}{u}$. Effective fin velocity, $v_e = u$ and x_{fin} is the axial position of the fin post in body-referenced coordinates.

These coefficients enable us to obtain the hydrodynamic coefficients from the equations for an individual fin lift and moment:

$$\begin{array}{l} Y_r = \frac{1}{2}\rho C_{L\delta f} S_{fin} \left[u^2\delta_r + uv + x_{fin} ur\right] \\ Z_s = -\frac{1}{2}\rho C_{L\delta f} S_{fin} \left[u^2\delta_s - uw + x_{fin} uq\right] \\ M_s = -\frac{1}{2}\rho C_{L\delta f} S_{fin} x_{fin} \left[u^2\delta_s - uw + x_{fin} uq\right] \\ N_r = -\frac{1}{2}\rho C_{L\delta f} S_{fin} x_{fin} \left[u^2\delta_r + uv + x_{fin} ur\right] \end{array} \tag{17}$$

Finally, we can separate the equation 17 into the following sets of fin lift coefficients:

$$\begin{array}{l} Y_{uu\delta_r} = Y_{uv_f} = \rho C_{L\delta f} S_{fin} \\ Z_{uu\delta_s} = -Z_{uw_f} = -\rho C_{L\delta f} S_{fin} \\ Y_{ur_f} = -Z_{uq_f} = \rho C_{L\delta f} S_{fin} x_{fin} \end{array} \tag{18}$$

and fin moment coefficients:

$$\begin{array}{l} M_{uu\delta_s} = -M_{uw_f} = -\rho C_{L\delta f} S_{fin} x_{fin} \\ N_{uu\delta_r} = N_{uv_f} = -\rho C_{L\delta f} S_{fin} x_{fin} \\ M_{uq_f} = N_{ur_f} = -\rho C_{L\delta f} S_{fin} x_{fin}^2 \end{array} \tag{19}$$

(v) Propeller forces and moments

The propeller provides forces $X_{prop}$and moments K_{prop} around the X axis of the body-fixed frame. Newman [1980] uses the non dimensional advance ratio $J = \frac{U}{nd}$ where U is the forward velocity, n the number of shaft revolution per unit time and d the propeller diameter. He assumes that, if there is no cavitation, the thrust and torque can be non dimensionalized so as to depend only on the advance ratio in the form:

$$\left.\begin{array}{l} \frac{T}{\rho n^2 d^4} = K_T(J) \\ \frac{Q}{\rho n^2 d^5} = K_Q(J) \end{array}\right\} \Rightarrow \left\{\begin{array}{l} X_{prop} = K_T(J)\rho n^2 d^4 \\ K_{prop} = K_Q(J)\rho n^2 d^5 \end{array}\right. \tag{20}$$

Propeller efficiency is the ratio of the work done by the propeller in developing the force UT divided by the work required to overcome the shaft torque $2\pi nQ$

It follows that:

$$\eta_p = \frac{UT}{2\pi nQ} = \frac{J}{2\pi}\frac{K_T}{K_Q} \tag{21}$$

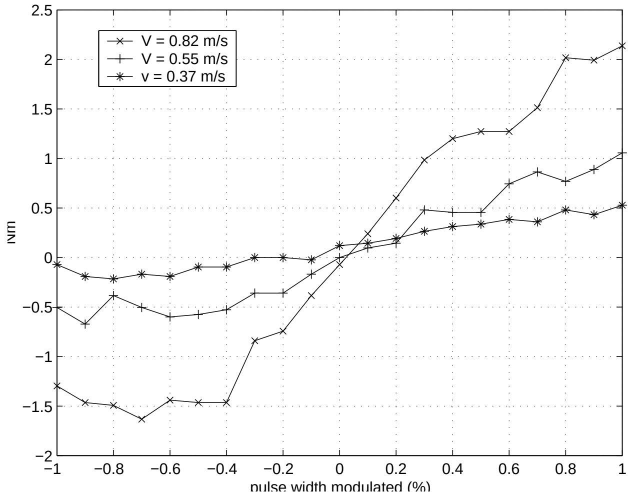

Figure 9: Fin Rolling Moment: Rolling Moment vs aileron angle, V=[0.37, 0.55, 0.82] m/sec. NB: $|K_{uu\delta_s}| = |K_{uu\delta_r}| = \frac{|slope|}{4V^2}$

Summation of forces and moments

Combining like cross-terms from equations 4, 5 10,15, 18 and 19 , we get the following hydrodynamic forces:

$$\begin{aligned}
Y_{uv} &= Y_{uv_l} + Y_{uv_f} + Y_{uv_d} \\
Y_{ur} &= Y_{ur_a} + Y_{ur_f} \\
Z_{uw} &= Z_{uw_l} + Z_{uw_f} + Z_{uw_d} \\
Z_{uq} &= Z_{uq_a} + Z_{uq_f} \\
M_{uw} &= M_{uw_a} + M_{uw_f} + M_{uw_l} \\
M_{uq} &= M_{uq_a} + M_{uq_f} \\
N_{uv} &= N_{uv_a} + N_{uv_f} + N_{uv_l} \\
N_{ur} &= N_{ur_a} + N_{ur_f}
\end{aligned} \tag{22}$$

Summing the forces and moments on the submarine

$$\begin{aligned}
\sum X_{ext} = \; & X_{HS} + X_{u|u|}u\,|u| + X_{\dot{u}}\dot{u} + X_{uv}uv + X_{uw}uw + \\
& X_{v|v|}v\,|v| + X_{vr}vr + X_{w|w|}w\,|w| + X_{wq}wq + \\
& X_{qq}qq + X_{rr}rr + X_{prop} \\
\sum Y_{ext} = \; & Y_{HS} + Y_{uu\delta_r}u^2\left(\delta_{r_{top}} + \delta_{r_{bottom}}\right) + Y_{ur}ur + \\
& Y_{uv}uv + Y_{v|v|}v\,|v| + Y_{\dot{v}}\dot{v} + Y_{wp}wp + Y_{pq}pq + Y_{\dot{r}}\dot{r} \\
\sum Z_{ext} = \; & Z_{HS} + Z_{uu\delta_s}u^2\left(\delta_{s_{right}} + \delta_{s_{left}}\right) + Z_{uw}uw + \\
& Z_{uq}uq + Z_{vp}vp + Z_{w|w|}w\,|w| + Z_{\dot{w}}\dot{w} + Z_{\dot{q}}\dot{q} + Z_{rp}rp \\
\sum K_{ext} = \; & K_{HS} + K_{\dot{p}}\dot{p} + K_{uu\delta_r}\left(-\delta_{r_{top}} + \delta_{r_{bottom}}\right) + \\
& K_{uu\delta_s}\left(-\delta_{s_{right}} + \delta_{s_{left}}\right) + K_{prop} \\
\sum M_{ext} = \; & M_{HS} + M_{uu\delta_s}u^2\delta_s + M_{uw}uw + M_{uq}uq + \\
& M_{vp}vp + M_{\dot{w}}\dot{w} + M_{\dot{q}}\dot{q} + M_{rp}rp \\
\sum N_{ext} = \; & N_{HS} + N_{uu\delta_r}u^2\delta_r + N_{ur}ur + N_{uv}uv + \\
& N_{\dot{v}}\dot{v} + N_{wp}wp + N_{pq}pq + N_{\dot{r}}\dot{r}
\end{aligned} \tag{23}$$

Simulation of Open Loop Behavior

The response of the submarine under the action of various actuator inputs has been plotted.Figure 10 shows the open loop response to inputs from the stern planes, causing the submarine to change depth. Figure 11 shows the yaw response of the submarine to various rudder inputs. Motor torque causes the submarine to turn more tightly to port.

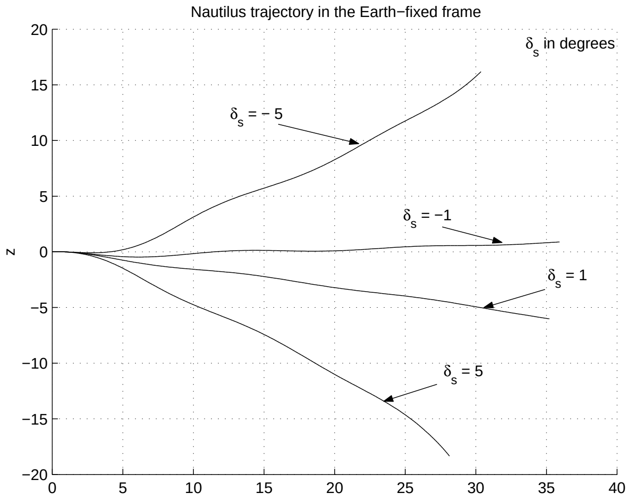

Figure 10: Simulated trajectory at various stern plane angles V=1.54 m/sec.

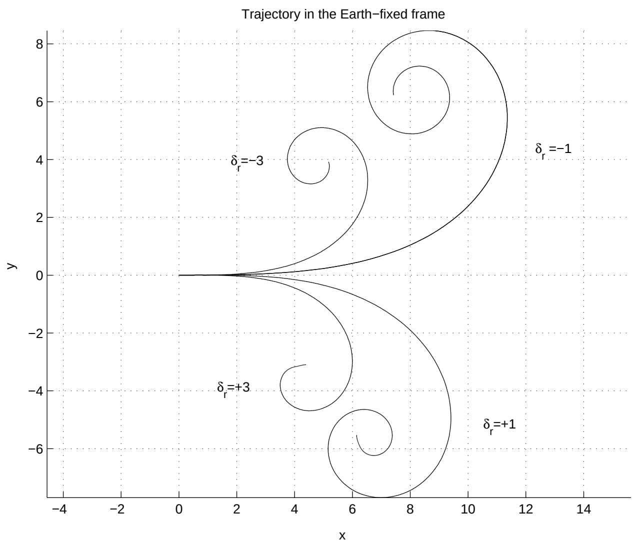

Figure 11: Simulated trajectory at various rudder angles V=1.54 m/sec., K_p=-0.543 Nm

Figure 12 predicts that the roll response is marginally stable. Natural frequency of the oscillation is determined by the offset of the centre of gravity which lies slightly below the centre of buoyancy. In reality this oscillation would tend to be damped out by viscous drag between the rolling submarine and the water. This effect is not included in the model. The offset in roll angle is due to the torque from the motor.

Figure 13 shows the response of the unpowered submarine as it surfaces under the action of its slightly positive buoyancy. It surfaces at a pitch angle of 5

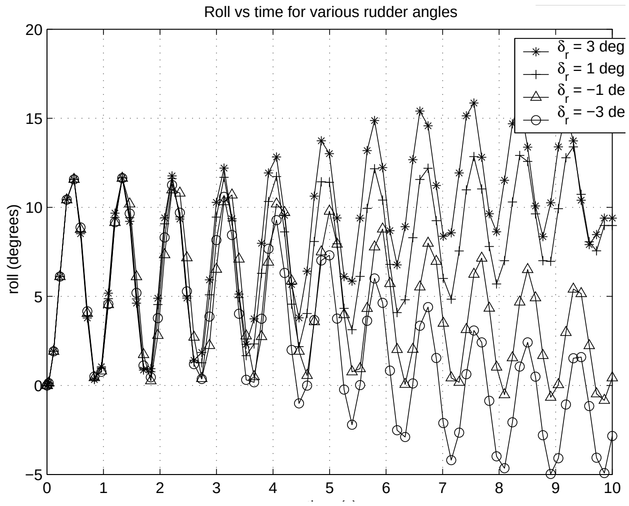

Figure 12: Simulated roll vs time at various rudder angles.

degrees.

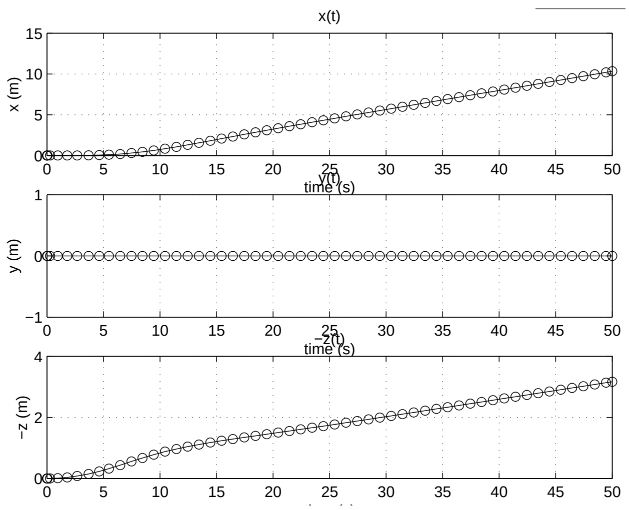

Figure 13: Simulated trajectory as the unpowered submarine surfaces.

Conclusions

A rigid body dynamic model discussed in this paper has produced a set of results for the open loop behavior of the submarine under the action of rudder, stern plane and motor inputs. The next phase of the project is to implement closed loop automatic control on depth, heading and roll angle of the submarine.This will be discussed in a forthcoming paper.

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Appendix: Experimental Data

Experimental data from which the submarine's hydrodynamic coefficients were derived came from tests carried out in an open flowing channel (flume). Figure 14 shows a half size scale model, suspended in the channel by a vertical shaft attached to a three axis dynamometer, capable of measuring forces in the x- (drag) and y- (lift) directions and a pitching moment about the z-axis.

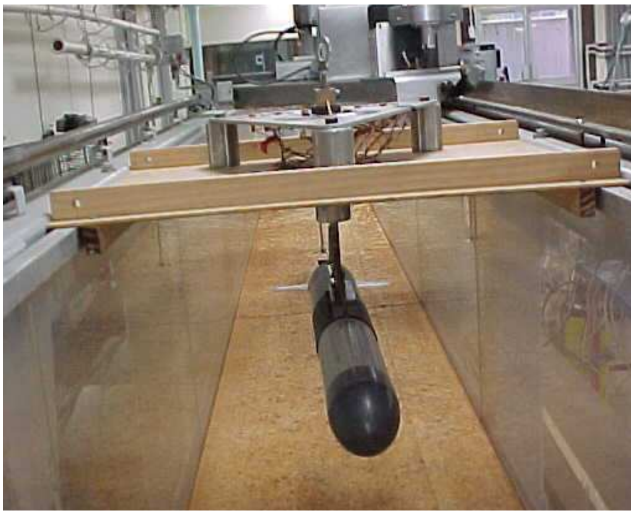

Figure 14:

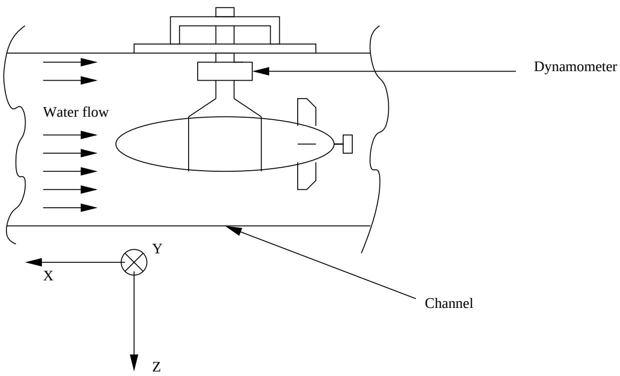

Figure 15:

The angle of attack α can be adjusted by rotating the shaft and setting the angle using a protractor, shown in Figure 15. Figures 4, 5 and 6 are the plots of drag, lift and moment coefficients versus angle of attack obtained using this experimental procedure.

$$C_D = \frac{F_x}{\frac{1}{2}\rho A_f V^2}, \quad C_L = \frac{F_y}{\frac{1}{2}\rho A_f V^2}, \quad C_M = \frac{M_z}{\frac{1}{2}\rho A_f V^2 L} \tag{24}$$

where: F_x and F_y are the measured forces in the x- and y-directions,M_z is the moment measured about the z-axis, A_f is the frontal cross sectional area, V is the free stream velocity, and L is the length of the submarine.

Data for figures 7 and 8 was obtained by suspending the full size submarine in the flume. Diametrically

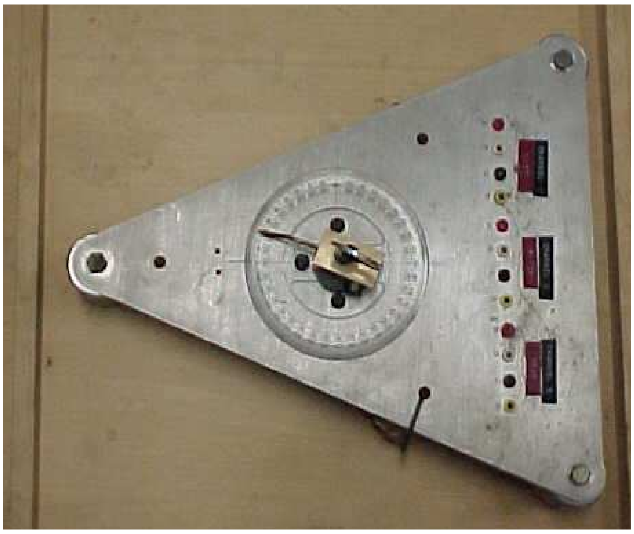

Figure 16:

opposite vertical fins were set at various angles (δ_f) and the lift and yawing moment measurements were recorded. Angle of attack α was held at zero degrees.

$$C_{L_{\delta_f}} = \frac{F_y}{\frac{1}{2}\rho A_f V^2}, \quad C_{M_{\delta_f}} = \frac{M_z}{\frac{1}{2}\rho A_f V^2 L} \tag{25}$$

Data for figure 9, again, was obtained by suspending the full size submarine in the flume. All four fins were offset equally to create a rolling moment, about the x-axis. Angle of attack α was held at zero degrees. The bending moment on the shaft was measured and recorded.

Parameter	Symbol	Value	Units
Mass	m	18.826	kg
Mass moment	Ixx	0.0727	kg.m^2
Mass moment	Iyy=Izz	1.77	kg.m^2
Length	L	1.391	m
Hull Radius	R	0.076	m
Fin distance from C_b	x_{fin}	0.537	m
Location of Centre of Mass	[X_g,Y_g,Z_g]	[-0.012, 0, 0.0048]	m

Non-linear Maneuvering Coefficients for Forces and Moments

Parameter	Value	Units	Description
X_{uu}	-3.11e+0	kg/m	Drag
$X_{\dot{u}}$	+4.21e-1	kg	Added Mass
X_{uw}	+3.01e+1	kg/m	Drag
X_{uv}	+3.01e-2	kg/m	Drag
X_{vv}	-5.19e+1	kg/m	Drag
X_{vr}	+2.72e+1	kg/rad	Added Mass Cross Term
X_{ww}	-5.19e+1	kg/m	Drag
X_{wq}	-2.72e+1	kg/rad	Added Mass Cross Term
X_{rr}	+1.83e+0	kg/rad	Added Mass Cross Term
X_{prop}	+7.38e+0 (variable)	N	Propeller Thrust
$Y_{uu\delta r}$	+1.19e+1	kg/(m.rad)	Fin Lift Force
Y_{uv}	-5.85e+1	kg/rad	Added Mass Cross-term, Fin Lift and Drag
Y_{ur}	+5.66e+0	kg/rad	Added Mass Cross-term and Fin Lift
Y_{vv}	+3.01e+0	kg/m	Drag
$Y_{\dot{v}}$	-2.72e+1	kg	Added Mass
Y_{wp}	+2.72e+1	kg/rad	Added Mass Cross Term
Y_{pq}	-1.83e+0	kg/rad	Added Mass Cross Term
$Y_{\dot{r}}$	-1.83e+0	kg	Added Mass
$Z_{uu\delta s}$	-1.19e+1	kg/(m.rad)	Fin Lift Force
Z_{uw}	-5.85e+1	kg/rad	Added Mass Cross-term, Fin Lift and Drag
Z_{uq}	-5.66e+0	kg/rad	Added Mass Cross-term and Fin Lift
Z_{vp}	-2.72e+1	kg/rad	Added Mass Cross Term
Z_{ww}	+3.01e+1	kg/m	Drag
$Z_{\dot{w}}$	-2.72e+1	kg	Added Mass
$Z_{\dot{q}}$	+1.83e+0	kg	Added Mass
Z_{rp}	-1.83e+0	kg/rad	Added Mass Cross Term

Non-linear Maneuvering Coefficients for Moments.

Parameter	Value	Units	Description
$K_{\dot{p}}$	-4.10e-2	kg.m^2 /rad^2	Added Mass
K_{prop}	-5.40e-1 (variable)	N.m	Propeller Torque
$K_{uu\delta r}$	+4.48e+0	kg/rad	Fin Rolling Moment
$K_{uu\delta s}$	+4.48e+0	kg/rad	Fin Rolling Moment
$M_{uu\delta s}$	-6.08e+0	kg/rad	Fin Lift Moment
M_{uw}	+2.40e+1	kg	Body and Fin and Munk Moment
M_{uq}	-4.93e+0	kg.m/rad	Added Mass Cross term and Fin Lift
M_{vp}	+1.83e+0	kg.m/rad	Added Mass Cross
$M_{\dot{w}}$	+1.83e+0	kg.m^2 /rad^2	Added Mass
$M_{\dot{q}}$	-4.34e+0	kg.m^2 /rad^2	Added Mass
M_{rp}	+4.30e+0	kg.m^2 /rad^2	Added Mass Cross Term
$N_{uu\delta r}$	-6.08e+0	kg/rad	Fin Lift Moment
N_{uv}	-2.40e+1	kg	Body and Fin and Munk Moment
N_{ur}	-4.93e+0	kg.m/rad	Added Mass Cross term and Fin Lift
$N_{\dot{v}}$	-1.83e+0	kg.m^2 /rad^2	Added Mass
N_{wp}	+1.83e+0	kg.m /rad	Added Mass Cross Term
N_{pq}	-4.30e+0	kg.m^2 /rad^2	Added Mass Cross Term
$N_{\dot{r}}$	-4.34e+0	kg.m^2 /rad^2	Added Mass