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## DELAY DISTRIBUTION BASED ROBUST $H_\infty$ CONTROL OF NETWORKED CONTROL SYSTEMS WITH UNCERTAINTIES

Chen Peng, Dong Yue, and Yu-Chu Tian

### ABSTRACT

Network induced delay in networked control systems (NCS) is inherently non-uniformly distributed and behaves with multifractal nature. However, such network characteristics have not been well considered in NCS analysis and synthesis. Making use of the information of the statistical distribution of NCS network induced delay, a delay distribution based stochastic model is proposed to link Quality-of-Control and network Quality-of-Service for NCS with uncertainties. From this model together with a tighter bounding technology for cross terms,  $H_\infty$  NCS analysis is carried out with significantly improved stability results. Furthermore, a memoryless  $H_\infty$  controller is designed to stabilize the NCS and to achieve the prescribed disturbance attenuation level. Numerical examples are given to demonstrate the effectiveness of the proposed method.

**Key Words:** Networked control systems,  $H_\infty$  control, stochastic distribution, linear matrix inequalities, uncertainties.

### I. INTRODUCTION

Networked control systems (NCS) use data networks to close both information and control loops. Significant effort is being made in the networked control community to develop general NCS networking

technologies, e.g. those based on Ethernet or wireless Ethernet, for control applications requiring remote and distributed operations. Typical examples include remote medical treatment, urban heating systems, and wide area plant automation over Internet Protocol (IP) networks. Due to its well-developed infrastructure, wide acceptance, simplicity and affordability [1], IP networking has been promoted for networked control. Although IP networking is convenient for communications in large-scale and remotely distributed systems, there are still major difficulties in design and implementation of NCS over IP networks [2]. Fundamentals of NCS over data communication networks can be found in [1, 3] and references therein for related works.

An NCS integrates information, communications, and control with control loops being closed through the network. Among many problems in NCS, how to deal with time-varying network induced delay is challenging, and has been one of our recent research focuses, e.g. [2, 4–6]. Network induced delay degrades control performance or even causes system instability. There have been some reports in the open literature on disturbance attenuation and robust  $H_\infty$  control of systems without transmission delay of control signals [7].

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C. Peng is with the School of Electrical and Automation Engineering, Nanjing Normal University, 78 Bancang Street, Nanjing 210042, China (e-mail: 63050@njnu.edu.cn).

D. Yue is with the School of Electrical and Automation Engineering, Nanjing Normal University, 78 Bancang Street, Nanjing 210042, China (e-mail: medongy@njnu.edu.cn).

Y.-C. Tian (corresponding author) is with the Faculty of Information Technology, Queensland University of Technology, GPO Box 2434, Brisbane 4001, Australia; (e-mail: y.tian@qut.edu.au).

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Much effort has also been made on this topic for NCS [3, 8, 9]. By introducing some slack matrix parameters and considering the lower bound of network-induced delay, Yue *et al.* have studied the robust  $H_\infty$  control of uncertain NCS and obtained less conservative results than previous work [8]; but the computational demand is high for the controller feedback gain due to the introduced slack matrices. Under the assumption that the network-induced delay is less than the sampling period and satisfies Bernoulli random binary distribution, Yang *et al.* have considered the observer-based  $H_\infty$  controller problem for NCS with random communication delays [3]; however, they have neglected the synchronization of the control signals from the controller to actuator. Gao, Chen and Lam have studied a special type of delay system, which contains multiple successive delay components in the state [10, 11]. Less conservative results can be expected from [10, 11] because more information on the inner delay distribution properties has been utilized.

Although much research has been conducted in NCS analysis and synthesis, the statistical features of NCS communication networks have not been fully captured. For example, many published papers of theoretical investigations have inherently assumed an uniform distribution of the NCS network-induced delay between its lower and upper bounds. Recent advances, have revealed that IP however, based NCS network-induced delay is non-uniformly distributed [1] and behaves with a multifractal nature [6]. It is our belief that the more the statistical features of the NCS networks are utilized in NCS analysis and synthesis, the better the control performance obtained. The problem of  $H_\infty$  robust analysis and synthesis for uncertain NCS with consideration of the stochastic distribution of the network delay remains open. Furthermore, simultaneous consideration of networks and control is crucial for system co-design, which links NCS quality-of-control (QoC) and NCS network quality-of-service (QoS).

The overall aim of this paper is to develop a delay distribution-based robust  $H_\infty$  control method for a class of uncertain NCS over IP networks. To make use of the delay characteristics of the NCS networks, we propose a new stochastic NCS model to capture the non-uniform distribution of the network delays. From this model together with a tighter bounding technology, delay dependent criteria for NCS stability and control synthesis are derived using a linear matrix inequality (LMI) approach. Because of the use of the delay distribution information, much less conservative results can be obtained than those from existing references. Since the obtained conditions for the existence of admissible controllers are not expressed as strict LMIs, the cone

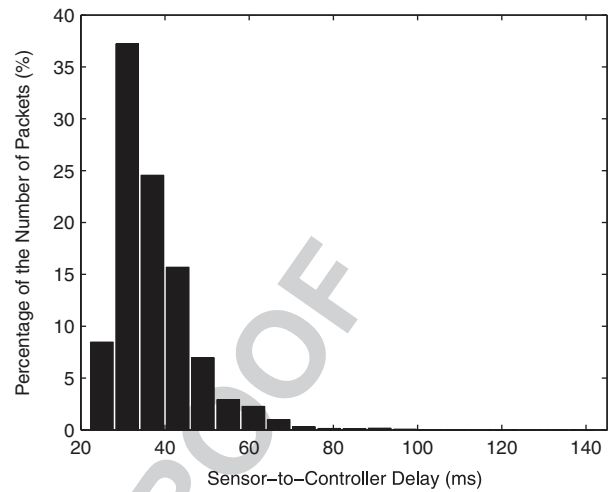


Fig. 1. Histograms of the network delays reported in [2].

complementary linearisation procedure is employed to solve the non-convex feasibility problem.

**Notation.**  $\mathbb{N}$  stands for positive integers,  $\mathbb{R}^n$  denotes the  $n$ -dimensional Euclidean space,  $\mathbb{R}^{n \times m}$  is the set of  $n \times m$  real matrices,  $I$  is the identity matrix of appropriate dimensions. Matrix  $X > 0$  (respectively,  $X \geq 0$ ) for  $X \in \mathbb{R}^{n \times n}$  means that  $X$  is a real symmetric positive definite (respectively, positive semi-definite).  $\mathbb{E}\{x\}$  is the expectation of  $x$ .  $\Pr\{\cdot\}$  means probability. For an arbitrary matrix  $B$  and two symmetric matrices  $A$  and  $C$ ,  $\begin{bmatrix} A & B \\ * & C \end{bmatrix}$  denotes a symmetric matrix, where  $*$  denotes the entries implied by symmetry.  $\text{Ones}(n, m)$  is an  $m$ -by- $n$  matrix of ones.

## II. NON-UNIFORM DISTRIBUTION OF NETWORK DELAY IN IP BASED NCS

IP-based network delays display irregular behaviour. To illustrate the characteristics of such delays, Tipsuwan *et al.* measured Round-Trip Time delays from different Ethernet network nodes for 24 hours [1]. Recently, Tian *et al.* have used the open source package ns2 to model and simulate typical scenarios of NCS over IP networks. For the scenarios described in [2] for an NCS over 10 Mbps IP networks, the corresponding histograms of the delays are shown in Fig. 1 [4].

Figure 1 shows that the histograms skew to the left, indicating the higher probability to have delays shorter than the median and mean delays. Actual delays can be much longer than the median and mean, but with

1 much lower probability. Generally speaking, although  
 2 communication delay  $\tau(t)$  is time-varying, it is non-  
 3 uniform. In most cases, small delays are dominant while  
 4 large delays are exiguous, implying that the probability  
 5 of small delays is bigger than that of large delays. This  
 phenomenon can be mathematically described as

$$\begin{cases} 1) & \tau_0 \leq \tau(t) \leq \tau_2 < \infty, \forall t \geq 0 \\ 2) & \Pr[\tau(t) \in [\tau_0, \tau_1]] = \delta(t), \\ & \Pr[\tau(t) \in [\tau_1, \tau_2]] = 1 - \delta(t) \end{cases} \quad (1)$$

7 where  $\delta(t) \in [0, 1]$  is a stochastic variable,  $\tau_1$  is the  
 8 bound which means  $\Pr\{\tau(t) \in [\tau_0, \tau_1]\} = \delta(t)$ .

9 It is worth mentioning that the general statistical  
 10 distribution of the network induced delay described in  
 11 Eq. (1) can be regarded as one of the key characteristics  
 12 of NCS network QoS. The fundamental difficulty from  
 13 this new description of network QoS is how to improve  
 14 the QoS under this new framework. In the following  
 15 section, we devote to linking the QoS and QoC.

### III. DELAY DISTRIBUTION BASED NCS CONTROL

19 Consider a class of control systems governed by

$$\dot{x}(t) = (A + \Delta A)x(t) + Bu(t) + B_1w(t), \quad (2)$$

$$z(t) = Cx(t), \quad (3)$$

$$x(t) = \varphi(t), t \in [-\eta_2, -\eta_1] \quad (4)$$

21 where  $x(t) \in \mathbb{R}^n$ ,  $u(t) \in \mathbb{R}^m$  and  $z(t) \in \mathbb{R}^p$  are the state  
 22 vector, control input vector and controlled output vec-  
 23 tor, respectively.  $A$ ,  $B$ ,  $B_1$  and  $C$  are constant ma-  
 24 trices with appropriate dimensions.  $\varphi(t)$  is the initial  
 25 condition of the system.  $\eta_2$  and  $\eta_1$  are the lower and  
 26 upper delay bounds, respectively.  $\Delta A$  denotes the pa-  
 27 rameter uncertainties satisfying  $\Delta A = HF(t)E_1$ , where  
 28  $H$  and  $E_1$  are constant matrices with appropriate di-  
 29 mensions, and  $F(t)$  is an unknown time-varying ma-  
 30 trix, which is Lebesgue measurable in  $t$  and satisfies  
 31  $F^T(t)F(t) \leq I$ .  $w(t) \in L_2[t_0, \infty)$  denotes the external  
 32 perturbation. Throughout this paper, it is assumed that  
 33 system (2)–(4) is controlled over a network.

The following assumptions commonly used in  
 NCS research in the literature are made in this work.

- 35 1. The sensors are clock-driven; and the controllers  
and actuators are event-driven;

2. All state variables are available for measurement;  
and the data is transmitted with a single-packet;  
and
3. Signal quantization and communication errors  
over the network are not considered.

Using a similar modeling technique in [5, 8, 9], we  
 model the closed-loop control system for (2)–(4) as

$$\begin{aligned} \dot{x}(t) &= (A + \Delta A)x(t) + Bu(t) + B_1w(t), \\ t &\in [i_k h + \tau_{i_k}, i_{k+1} h + \tau_{i_{k+1}}) \end{aligned} \quad (5)$$

$$\begin{aligned} u(t^+) &= Kx(t - \tau_{i_k}), \\ t &\in \{i_k h + \tau_{i_k}, k = 1, 2, \dots\} \end{aligned} \quad (6)$$

where  $u(t^+) = \lim_{t \rightarrow t+0} u(t)$ ,  $h$  is the sampling pe-  
 riod,  $i_k (k = 1, 2, 3, \dots)$  are some integers such that  
 $\{i_1, i_2, i_3, \dots\} \subset \{0, 1, 2, 3, \dots\}$ ,  $K$  is the state feedback  
 gain, the network-induced delay  $\tau_{i_k}$  is the time from  
 the instant  $i_k h$  when sensors sample from the plant to  
 the instant when actuators send control actions to the  
 plant. Here, it is assumed that the control computation  
 and other overhead delays are included in  $\tau_{i_k}$  [9].

In Equation (5), it is not required to have  $i_{k+1} > i_k$ .  
 If  $i_{k+1} - i_k = 1$ , it means that there is no data packet  
 dropout in the transmission. If  $i_{k+1} > i_k + 1$ , there are  
 dropped packets but the received packets are in or-  
 dered sequence. If  $i_{k+1} < i_k$ , it means out-of-order  
 packet arrival sequences occur; a typical scenario  
 is that  $u(i_{k+1}) = Kx(i_{k+1}h)$  is implemented after  
 $u(i_k h) = Kx(i_k h)$ . Discarding the old data packet will  
 help reduce network-induced delay  $\tau_{i_k}$ , which in turn  
 makes the system tolerate a larger amount of data  
 packet loss. Therefore, the time stamping technique  
 is employed in this paper to implement the message  
 rejection, implying that the latest message is kept and  
 old messages are discarded.

Define  $\eta(t) = t - i_k h$ ,  $t \in [i_k h + \tau_{i_k}, i_{k+1} h + \tau_{i_{k+1}})$ ,  
 $k = 1, 2, 3, \dots$ , in every interval  $[i_k h + \tau_{i_k}, i_{k+1} h + \tau_{i_{k+1}})$ .  
 It follows that

$$\tau_{i_k} \leq \eta(t) \leq (i_{k+1} - i_k)h + \tau_{i_{k+1}}. \quad (7)$$

Since  $x(i_k h) = x(t - (t - i_k h))$ , from eqs. (6) and (7),  
 we have

$$u(t^+) = Kx(t - \eta(t)), \quad \eta_1 \leq \eta(t) \leq \eta_2 \quad (8)$$

where  $\eta_1 = \inf_k \{\tau_{i_k}\}$ ,  $\eta_2 = \sup_k [(i_{k+1} - i_k)h + \tau_{i_{k+1}}]$ .

Similar to [12], define two sets  $\Omega_1 = \{t: \eta(t) \in [\eta_0, \eta_1]\}$   
 and  $\Omega_2 = \{t: \eta(t) \in [\eta_1, \eta_2]\}$ . It is clear that  
 $\Omega_1 \cup \Omega_2 = \mathbb{R}^+$  and  $\Omega_1 \cap \Omega_2 = \mathbf{0}$ . In terms of the defi-  
 nitions of  $\Omega_1$  and  $\Omega_2$ , it is seen that  $t \in \Omega_1$  means the

1 event  $\eta(t) \in [\eta_0, \eta_1)$  occurs and  $t \in \Omega_2$  means the event  
 2  $\eta(t) \in [\eta_1, \eta_2)$  occurs. Furthermore, we can define a  
 3 stochastic variable  $\delta(t)$  as

$$\delta(t) = \begin{cases} 1, & t \in \Omega_1 \\ 0, & t \in \Omega_2. \end{cases} \quad (9)$$

5 To make use of the distribution information of the  
 6 NCS network described in (1), a new feedback control  
 7 law is proposed below to replace control law (8):

$$\begin{cases} u(t) = \delta(t)Kx(t - \eta_1(t)) \\ \quad + (1 - \delta(t))Kx(t - \eta_2(t)), \\ \eta_1(t) \in [\eta_0, \eta_1), \eta_2(t) \in [\eta_1, \eta_2) \end{cases} \quad (10)$$

9 where

$$\begin{cases} \eta_1(t) = \delta(t)\tau(t), \\ \quad \text{and } \delta(t) = 1 \text{ if } \eta(t) \in [\eta_0, \eta_1) \\ \eta_2(t) = (1 - \delta(t))\eta(t), \\ \quad \text{and } \delta(t) = 0 \text{ if } \eta(t) \in [\eta_1, \eta_2). \end{cases} \quad (11)$$

11 Substituting (10) into (5), we obtain the following  
 12 model for the closed-loop control systems with the con-  
 13 trolled output vector and initial conditions being copied  
 to eqs. (3) and (4), respectively.

$$\begin{aligned} \dot{x}(t) = & (A + \Delta A)x(t) + \delta(t)BKx(t - \eta_1(t)) \\ & + (1 - \delta(t))BKx(t - \eta_2(t)) + B_1w(t) \end{aligned} \quad (12)$$

15 **Remark 1.** Inspired by the work [3, 12–14], assume  
 16 that  $\delta(t)$  in (12) is a Bernoulli Distributed Sequence  
 17 (BDS) with

$$\begin{cases} \Pr\{\delta(t) = 1\} = \mathbb{E}\{\delta(t)\} := \bar{\delta} \\ \Pr\{\delta(t) = 0\} = 1 - \mathbb{E}\{\delta(t)\} := 1 - \bar{\delta} \end{cases} \quad (13)$$

19 where  $0 \leq \bar{\delta} \leq 1$  is a constant. Different from the work  
 20 [3, 13, 14], where BDS is employed to deal with the  
 21 systems with missing measurements, this paper utilises  
 22 BDS to describe the probability distribution of the NCS  
 23 network delay.

25 **Remark 2.** It is seen that system (12) makes use of  
 26 more information about the distribution characteristics  
 27 of the NCS network induced delay. Less conservative  
 28 results can be expected from this formulation than those  
 29 from the models without considering the non-uniform  
 delay distribution. This will be verified through numer-  
 ical examples in Section VI.

**Remark 3.** When  $\delta(t) \triangleq 0$  or 1, model (12) is reduced to

$$\dot{x}(t) = (A + \Delta A)x(t) + BKx(t - \eta(t)) \quad (14)$$

$$x(t) = \varphi(t), \quad t \in [-\eta_2, 0] \quad (15)$$

This special case implies that the statistical distribution  
 characteristics of input  $u(t)$  is not considered in system  
 analysis and synthesis. This case has been investigated  
 in [5, 15, 16]. We will show in Section VI that their  
 results are the special cases of our results.

To facilitate further development, Eq. (12) can be  
 rewritten as

$$\dot{x}(t) = \varphi(t) + (\delta(t) - \bar{\delta})\psi(t) \quad (16)$$

$$\begin{cases} \varphi(t) \triangleq Ax(t) + \bar{\delta}BKx(t - \eta_1(t)) \\ \quad + (1 - \bar{\delta})BKx(t - \eta_2(t)) \\ \quad + B_1w(t) + HF(t)E_a \\ \psi(t) \triangleq BKx(t - \eta_1(t)) \\ \quad - BKx(t - \eta_2(t)) \end{cases} \quad (17)$$

Now, let us formulate some practically computable  
 criteria for the asymptotic stability of the NCS described  
 by (12). The following definition is useful in deriving  
 the criteria.

**Definition 1.** For all admissible uncertainties, system  
 (12) is said to be robustly asymptotically stable in the  
 mean-square with an  $H_\infty$  norm bound  $\gamma$  if the following  
 conditions hold:

1. For system (12) with  $w(t) \triangleq 0$ , and  $\delta(t)$  described  
 in (1) and (13), the trivial solution (equilibrium  
 point) is globally and asymptotically stable in the  
 mean square if the following holds:

$$\lim_{t \rightarrow \infty} \mathbb{E}|x(t, \delta(t))|^2 = 0 \quad (18)$$

2. Under the assumption of zero initial condition, the  
 controlled output  $z(t)$  satisfies

$$\mathbb{E}\{\|z(t)\|_2\} \leq \gamma \mathbb{E}\{\|w(t)\|_2\} \quad (19)$$

for any nonzero  $w(t) \in L_2[0, \infty)$ , where  $\gamma$  is a  
 prescribed scalar.

#### IV. $H_\infty$ PERFORMANCE ANALYSIS

Existing results of asymptotical stability of time-  
 varying delay systems are delay distribution independ-  
 ent. This section will develop stability criteria that are

1 delay distribution dependent. An innovative approach  
 2 will be proposed to construct a Lyapunov functional that  
 3 utilizes the information of the delay distribution and  
 4 lower bound of the delay. The main results of our NCS  
 5 stability analysis are summarized below in Theorem 1.

6 **Theorem 1.** For given constants  $\eta_i (i=0, 1, 2)$ ,  $\gamma > 0$   
 7 and matrix  $K$ , if there exist matrices  $P$ ,  $Q$ ,  $R_j$   
 8 ( $j=1, \dots, 4$ )  $> 0$  with appropriate dimensions, scalar  
 9  $\varepsilon > 0$ , such that the following matrix inequality holds,  
 10 then system (12) is asymptotically stable in the mean-  
 11 square with an  $H_\infty$  norm bound  $\gamma$ .

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & \Pi_{14} \\ * & \Pi_{22} & 0 & 0 \\ * & * & \Pi_{33} & 0 \\ * & * & * & -\varepsilon I \end{bmatrix} < 0 \quad (20)$$

12 where

$$\Pi_{11} = \begin{bmatrix} \Psi_{11} & R_1 + \bar{\delta}PBK & R_4 & \Psi_{14} & PB_1 & PH \\ * & -R_1 - R_3 & R_3 & 0 & 0 & 0 \\ * & * & \Psi_{33} & 0 & 0 & 0 \\ * & * & * & -R_2 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & -\varepsilon I \end{bmatrix},$$

$$\Psi_{11} = -R_1 - R_2 - R_4 + PA + A^T P + Q + C^T C,$$

$$\Psi_{33} = -Q - R_3 - R_4, \Psi_{14} = R_2 + (1 - \bar{\delta})PBK,$$

$$\Pi_{12} = \text{diag}\{A, \bar{\delta}BK, 0, (1 - \bar{\delta})BK, B_1, H\}^T \\ \times \text{Ones}(6, 4),$$

$$\Pi_{13} = \text{diag}\{0, (BK)^T, 0, -(BK)^T, 0, 0\} \text{Ones}(6, 4),$$

$$\Pi_{22} = \text{diag} \left\{ -\eta_1^{-2} R_1^{-1}, -\left(\frac{\eta_1 - \eta_0}{2}\right)^{-2} R_3^{-1}, \right. \\ \left. -\left(\frac{\eta_0 + \eta_1}{2}\right)^{-2} R_4^{-1}, -\eta_2^{-2} R_2^{-1} \right\},$$

$$\Pi_{14} = [\varepsilon E_a \ 0 \ 0 \ 0 \ 0 \ 0]^T,$$

$$\Pi_{33} = [\bar{\delta}(1 - \bar{\delta})]^{-1} \Pi_{22}.$$

13 **Proof.** Define  $\mu = \frac{\eta_0 + \eta_1}{2}$ . Construct a Lyapunov-  
 14 Krasovskii functional candidate as

$$V(x_t) = \sum_{i=1}^5 V_i(x_t) \quad (21)$$

where

$$V_1(x_t) = x^T(t)Px(t),$$

$$V_2(x_t) = \int_{t-\mu}^t x^T(s)Qx(s)ds,$$

$$V_3(x_t) = \sum_{i=1}^2 \int_{-\eta_i}^0 \int_{t+s}^t \dot{x}^T(v)\eta_i R_i \dot{x}(v) dv ds$$

$$V_4(x_t) = \int_{-\mu}^{-\eta_1} \int_{t+s}^t \dot{x}^T(v)(\mu - \eta_1)R_3 \dot{x}(v) dv ds$$

$$V_5(x_t) = \int_{-\mu}^0 \int_{t+s}^t \dot{x}^T(v)\mu R_4 \dot{x}(v) dv ds,$$

$R_j > 0, j=1, \dots, 4$ . It follows from Eq. (13) that  $\mathbb{E}\{\delta - \bar{\delta}\} = 0$ ,  $\mathbb{E}\{(\delta - \bar{\delta})^2\} = \bar{\delta}(1 - \bar{\delta})$ , where  $\delta$  means  $\delta(t)$ . Then, the mathematical expectation of the generator  $\mathcal{L}V(x_t)$  for the evolution of  $V(x_t)$  along the solutions of system (12) is given by

$$\begin{aligned} \mathbb{E}\{\mathcal{L}V_1(x_t)\} &= 2\mathbb{E}\{x^T(t)P\dot{x}(t)\} \\ &= 2\mathbb{E}\{x^T(t)P\varphi(t) \\ &\quad + (\delta - \bar{\delta})x^T(t)P\psi(t)\} \\ &= 2x^T(t)P\varphi(t) \end{aligned} \quad (22)$$

$$\begin{aligned} \mathbb{E}\{\mathcal{L}V_2(x_t)\} &= x^T(t)Qx(t) - x^T(t - \mu)Qx(t - \mu) \end{aligned} \quad (23)$$

According to (16), we have

$$\begin{aligned} &\mathbb{E} \left\{ \sum_{i=1}^2 \dot{x}^T(t)\eta_i^2 R_i \dot{x}(t) \right\} \\ &= \mathbb{E} \left\{ \sum_{i=1}^2 [\varphi(t) + (\delta - \bar{\delta})\psi(t)]^T \eta_i^2 R_i \right. \\ &\quad \left. \times [\varphi(t) + (\delta - \bar{\delta})\psi(t)] \right\} \\ &= \sum_{i=1}^2 \varphi^T(t)\eta_i^2 R_i \varphi(t) \\ &\quad + 2\mathbb{E}(\delta - \bar{\delta}) \sum_{i=1}^2 \psi^T(t)\eta_i^2 R_i \varphi(t) \\ &\quad + \mathbb{E}[(\delta - \bar{\delta})^2] \sum_{i=1}^2 \psi^T(t)\eta_i^2 R_i \psi(t) \\ &= \sum_{i=1}^2 \varphi^T(t)\eta_i^2 R_i \varphi(t) \\ &\quad + \bar{\delta}(1 - \bar{\delta}) \sum_{i=1}^2 \psi^T(t)\eta_i^2 R_i \psi(t) \end{aligned} \quad (24)$$

1 Applying Jensen's inequality [17], when  $R_i > 0$ , we have

$$\begin{aligned}
 & -\sum_{i=1}^2 \int_{t-\eta_i(t)}^t \dot{x}^T(v) \eta_i R_i \dot{x}(v) dv \\
 & \leq \sum_{i=1}^2 \begin{bmatrix} x(t) \\ x(t-\eta_i(t)) \end{bmatrix}^T \\
 & \quad \times \begin{bmatrix} -R_i & R_i \\ * & -R_i \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\eta_i(t)) \end{bmatrix} \quad (25)
 \end{aligned}$$

3 From (24) and (25), the following  $\mathbb{E}\{\mathcal{L}V_3(x_t)\}$  can be more evolved.

$$\begin{aligned}
 & \mathbb{E}\{\mathcal{L}V_3(x_t)\} \\
 & = \sum_{i=1}^2 \mathbb{E} \left\{ \dot{x}^T(t) \eta_i^2 R_i \dot{x}(t) \right. \\
 & \quad \left. - \int_{t-\eta_i}^t \dot{x}^T(v) \eta_i R_i \dot{x}(v) dv \right\} \\
 & \leq \sum_{i=1}^2 \mathbb{E} \left\{ \dot{x}^T(t) \eta_i^2 R_i \dot{x}(t) \right. \\
 & \quad \left. - \int_{t-\eta_i(t)}^t \dot{x}^T(v) \eta_i R_i \dot{x}(v) dv \right\}. \quad (26)
 \end{aligned}$$

5 The same operations can be applied to  $\mathbb{E}\{\mathcal{L}V_4(x_t)\}$   
 7 and  $\mathbb{E}\{\mathcal{L}V_5(x_t)\}$ . Then, considering (22)-(26) together,  
 and similar to the method in [18] to deal with uncertainties, we have

$$\begin{aligned}
 \mathbb{E}\{\mathcal{L}V(x_t)\} & \leq \mathbb{E}\{\xi^T(t) [\Pi_{11} - \Pi_{12} \Pi_{22}^{-1} \Pi_{12}^T \\
 & \quad - \Pi_{13} \Pi_{33}^{-1} \Pi_{13}^T] \xi(t) - z(t)^T z(t) \\
 & \quad + \gamma^2 w(t)^T w(t) \\
 & \quad + \varepsilon [F(t) E_a]^T [F(t) E_a]\} \quad (27)
 \end{aligned}$$

9 where  $\xi^T(t) = [x^T(t), x^T(t-\eta_1(t)), x^T(t-\mu), x^T(t-\eta_2(t)), \varpi^T(t), [F(t) E_a]^T]$ ,  $\Pi_{ij}$  ( $i, j = 1, 2, 3, 4$ ) is defined in Theorem 1.

11 By Schur complement, (20) implies that

$$\begin{aligned}
 & \Pi_{11} - \Pi_{12} \Pi_{22}^{-1} \Pi_{12}^T - \Pi_{13} \Pi_{33}^{-1} \Pi_{13}^T \\
 & \quad + \Pi_{14} \varepsilon^{-1} \Pi_{14}^T < 0. \quad (28)
 \end{aligned}$$

It is clear that there exists a scalar  $\varepsilon > 0$  such that

$$\begin{aligned}
 & \xi^T(t) \Pi_{14} \varepsilon^{-1} \Pi_{14}^T \xi(t) \\
 & \quad - \varepsilon [F(t) E_a]^T [F(t) E_a] \geq 0 \quad (29)
 \end{aligned}$$

Then, combining (28) and (29), we have

$$\begin{aligned}
 & \xi^T(t) [\Pi_{11} - \Pi_{12} \Pi_{22}^{-1} \Pi_{12}^T - \Pi_{13} \Pi_{33}^{-1} \Pi_{13}^T] \xi(t) \\
 & \quad + \varepsilon [F(t) E_a]^T [F(t) E_a] \leq 0. \quad (30)
 \end{aligned}$$

With (30), (27) implies that

$$\mathbb{E}\{\mathcal{L}V(x_t)\} \leq \mathbb{E}\{\gamma^2 w(t)^T w(t) - z(t)^T z(t)\} \quad (31)$$

Under zero initial condition, integrating both sides of (31) from  $t_0$  to  $t$  and letting  $t \rightarrow \infty$ , we have

$$\mathbb{E} \left\{ \int_{t_0}^{\infty} z(s)^T z(s) ds \right\} \leq \mathbb{E} \left\{ \int_{t_0}^{\infty} \gamma^2 w(s)^T w(s) ds \right\} \quad (19)$$

thus  $\mathbb{E}\{\|z(t)\|_2\} \leq \gamma \mathbb{E}\{\|w(t)\|_2\}$ .

Next, we can prove the asymptotic stability of system (12) in mean-square. From (20) and (27), we have  $\mathbb{E}\{\mathcal{L}V(x_t)\} < 0$ . It can now be concluded from Lyapunov stability theory that the dynamics of system (12) is robustly asymptotically stable in the mean square. Then, by Definition 1, the result is established. This completes the proof.

**Remark 4.** Matrix inequality (20) is not an LMI due to nonlinear terms in  $\Pi_{22}$ . But it is easy to transformed into an LMI. Analysis of the transformation is omitted.

**Remark 5.** Model transformation is one of the sources of conservativeness [17]. Free weighting matrices also introduce conservativeness [19]. Peng *et al.* derived less conservative results based on an augmented matrix method, in which neither free weighting matrices nor any model transformation have been employed [19]. Using the convexity of the matrix function and employing fewer unknown variables, Park *et al.* obtained improved results [20]. Compared with Moon's inequality [21] and the slack variable matrix method [22] to deal with cross terms in [8, 15, 23], our proof of Theorem 1 uses neither model transformation nor free variable matrices. It is also worth mentioning that all variable matrices  $P$  and  $R_i$  in Theorem 1 appear in the Lyapunov-Krasovskii functional (21); and their physical interpretations are clear. Nevertheless, how to get less conservative results with reduced computational power demand is still challenging.

1 **Remark 6.**  $\bar{\delta}$  reflects network QoS. The larger  $\bar{\delta}$  is,  
 2 the more network induced delays fall into the interval  
 3  $[\eta_0, \eta_1)$ . It will be seen later that a larger  $\bar{\delta}$  and smaller  
 4 interval  $[\eta_0, \eta_1)$  correspond to a larger allowable  $\eta_2$ ,  
 5 which is the upper delay bound. This links the NCS  
 6 QoC with the network QoS, and will help achieve an  
 7 optimized co-design of NCS networks and control to  
 maximize the overall performance of the NCS.

### V. ROBUST $H_\infty$ CONTROLLER DESIGN

11 With Theorem 1, we are now ready to design the  
 12 feedback gain  $K$  to make system (12) robustly asymp-  
 13 totically stable in the mean square with an  $H_\infty$  norm  
 bound  $\gamma$ . The main results of the control design are sum-  
 marized below in Theorem 2.

15 **Theorem 2.** Given scalars  $\eta_i (i = 0, 1, 2), \gamma > 0$ . System  
 16 (12) is asymptotically stable in mean-square with feed-  
 17 back gain  $K = VX^{-T}$  and the  $H_\infty$ -norm constraint (19)  
 18 is achieved for all nonzero  $\omega(t)$ , if there exist matri-  
 19 ces  $\tilde{Q}, \tilde{R}_j > 0 (j = 1, \dots, 4)$ , a non-singular matrix  $X$ ,  
 20 matrix  $V$  with appropriate dimension, scalar  $\varepsilon > 0$ , such  
 21 that the following matrix inequality holds

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} & XC^T \\ * & \Sigma_{22} & 0 & 0 & 0 \\ * & * & \Sigma_{33} & 0 & 0 \\ * & * & * & -\varepsilon I & 0 \\ * & * & * & * & -I \end{bmatrix} < 0, \quad (32)$$

23 where

$$\Sigma_{11} = \begin{bmatrix} \tilde{\Psi}_{11} & \tilde{R}_1 + \bar{\delta}BV & \tilde{R}_4 & \tilde{\Psi}_{14} & B_1 & H \\ * & -\tilde{R}_1 - \tilde{R}_3 & \tilde{R}_3 & 0 & 0 & 0 \\ * & * & \tilde{\Psi}_{33} & 0 & 0 & 0 \\ * & * & * & -\tilde{R}_2 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & -\varepsilon I \end{bmatrix},$$

$$\tilde{\Psi}_{11} = -\tilde{R}_1 - \tilde{R}_2 - \tilde{R}_4 + AX^T + XA^T + \tilde{Q},$$

$$\tilde{\Psi}_{14} = \tilde{R}_2 + (1 - \bar{\delta})BV, \tilde{\Psi}_{33} = -\tilde{Q} - \tilde{R}_3 - \tilde{R}_4,$$

$$\Sigma_{22} = \text{diag} \left\{ -\eta_1^{-2} X \tilde{R}_1^{-1} X, -\left(\frac{\eta_1 - \eta_0}{2}\right)^{-2} X \tilde{R}_3^{-1} X, \right.$$

$$\left. -(\mu)^{-2} X \tilde{R}_4^{-1} X, -\eta_2^{-2} X \tilde{R}_2^{-1} X \right\}$$

$$\Sigma_{12} = \text{diag}\{XA^T, \bar{\delta}V^T B^T, 0, (1 - \bar{\delta})V^T B^T,$$

$$B_1^T, H^T\} \text{Ones}(6, 4),$$

$$\Sigma_{13} = \text{diag}\{0, V^T B^T, 0, -V^T B^T, 0, 0\} \text{Ones}(6, 4),$$

$$\Sigma_{14} = [\varepsilon E_a X \ 0 \ 0 \ 0 \ 0 \ 0]^T,$$

$$\Sigma_{33} = [\bar{\delta}(1 - \bar{\delta})]^{-1} \Sigma_{22}.$$

**Proof.** Pre- and post-multiply both sides of (20) with  
 25  $\text{diag}(X, X, X, X, I, I, I, I, I, I, I, I, I, I, I)$  and its  
 26 transpose. Defining  $V = KX^T, X = P^{-1}, \tilde{Q} = XQX,$   
 27  $\tilde{R}_i = XR_i X (i = 1, \dots, 4)$ , and applying Schur comple-  
 ment, we have (32). This completes the proof.  $\square$

29 At this stage, if the above problem of (32) has  
 a solution, we say that there exists a feedback gain  
 30  $K = VX^{-T}$  that guarantees the asymptotical stability of  
 system (12) in the mean square. However, the nonlinear  
 31 equality matrix conditions  $X \tilde{R}_i^{-1} X$  in (32) result in  
 32 difficulties for obtaining such a solution. The following  
 33 method is developed to overcome the difficulties.

34 Define  $M_i, i = 1, \dots, 4$ , such that  $X \tilde{R}_i^{-1} X \geq M_i$ ,  
 35 and define  $S_i = M_i^{-1}$  and  $U_i = \tilde{R}_i^{-1}$ . Replace condition  
 36 (32) with the following (33) and (34)

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} \\ * & \Gamma_{22} & 0 & 0 \\ * & * & \Gamma_{33} & 0 \\ * & * & * & -\varepsilon I \end{bmatrix} < 0, \quad (33)$$

$$\begin{bmatrix} S_i & X \\ X & U_i \end{bmatrix} \geq 0, i = 1, \dots, 4 \quad (34)$$

39 where  $\Gamma_{22} = \text{diag}\{-\eta_1^{-2} M_1, -(\frac{\eta_1 - \eta_0}{2})^{-2} M_3, -\mu^{-2} M_4,$   
 $-\eta_2^{-2} M_2\}, \Gamma_{33} = [\bar{\delta}(1 - \bar{\delta})]^{-1} \Gamma_{22}.$

41 Using the cone complementarity approach [24],  
 42 we formulate the following nonlinear minimization  
 43 problem with considerations of LMI conditions instead  
 of the original non-convex feasibility problem

$$\text{Min: Trace} \left\{ XP + \sum_{i=1}^4 (S_i M_i + U_i \tilde{R}_i) \right\} \quad (35)$$

$$\text{Subject to : (33), (34) and } \begin{bmatrix} P & I \\ * & X \end{bmatrix} > 0,$$

$$\begin{bmatrix} M_i & I \\ * & S_i \end{bmatrix} > 0, \begin{bmatrix} \tilde{R}_i & I \\ * & U_i \end{bmatrix} > 0 \quad (36)$$



1 For this optimization problem, the following algorithm  
 2 is employed to obtain a suboptimal maximum of  $\eta_2$  for  
 3 given  $\eta_0, \eta_1, \gamma$  and  $\bar{\delta}$ . The convergence of the algorithm  
 4 is guaranteed in terms of the similar result of [24].

5 **Algorithm 1.** Finding a suboptimal maximum  $\eta_2$  with  
 6 respect to  $\gamma$

Q1

- 7 1.
- 8 2. Choose a sufficiently little initial  $\eta_2 > 0$ , such that  
 9 there exists a feasible solution to (33) and (34).  
 10 Set  $\eta_{02} = \eta_2$ .
- 11 3. Search a feasible set  $\{X_0, P_0, M_{0i}, U_{0i}, \tilde{R}_{0i}, S_{0i}$   
 12  $(i = 1, \dots, 4)\}$  satisfying LMI in (33) and (35).  
 13 Set  $k = 0$ .
- 14 4. Solve the following LMI problem for the variables  
 15  $\{X, P, M_i, U_i, \tilde{R}_i, S_i (i = 1, \dots, 4)\}$

$$\text{Min : } \text{tr} \left( X_k P + P_k X + \sum_{i=1}^4 [S_{ki} M_i + M_{ki} S_i + U_{ki} \tilde{R}_i + \tilde{R}_{ki} U_i] \right) \quad (36)$$

Subject to: (33) and (34).

17 Set  $X_{k+1} = X, P_{k+1} = P, \tilde{R}_{(k+1)i} = \tilde{R}_{ki}, S_{(k+1)i} = S_{ki},$   
 18  $M_{(k+1)i} = M_{ik}, U_{(k+1)i} = U_{ki}.$

- 19 5. If (32) is satisfied, return to Step 2 after increasing  
 20  $\eta_2$  to some extent; if (32) is not satisfied within  
 21 a specified number of iterations, then exit. Other-  
 22 wise, set  $k = k + 1$  and go to Step 3.

23 **Remark 7.** It is worth mentioning that there are al-  
 24 ternative algorithms to solve out the non-LMIs in  
 25 Theorem 3. One of such algorithms is to linearise the  
 26 nonlinear item  $X \tilde{R}_i^{-1} X$  by setting  $X = \varepsilon_i \tilde{R}_i$  in (32),  
 27 where  $\varepsilon_i$  is a prescribed constant. Another algorithm  
 28 is to replace the nonlinear item  $X \tilde{R}_i^{-1} X$  with  $\tilde{R}_i - 2X$   
 29 [25]. Although the cone complementarity-based Al-  
 30 gorithm 1 will increase computational complexity in  
 31 comparison with the aforementioned methods, it gives  
 32 less conservative results for the examples to be dis-  
 33 cussed later in this paper.

34 So far, the controller has been designed to sat-  
 35 isfy Definition 1. The results in Theorem 2 also suggest  
 36 some other types of optimization problems for control  
 37 design, e.g. for the optimal  $H_\infty$  control problem

$$\begin{cases} \text{Minimize: } \gamma \\ \text{Subject to: } (33) \text{ and } (34) \end{cases} \quad (37)$$

## VI. NUMERICAL EXAMPLES

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This section aims to demonstrate the effectiveness  
 and applications of the methodology developed in this  
 paper.

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### 6.1 Performance analysis

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Consider the following system investigated in [16]

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} u(t) \quad (38)$$

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When the effect of external perturbations on the system  
 is considered, (38) can be expressed as

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$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} u(t) + \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} w(t) \quad (39)$$

$$z(t) = [0 \quad 1]x(t) \quad (40)$$

Two cases are considered below for stability anal-  
 ysis of system (39)-(40).

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**Case 1.1.** For (38), the state feedback controller  
 designed in [16] is  $u(t) = [-3.75 - 11.5]x(t)$ . It is de-  
 signed without considering the presence of NCS net-  
 works.

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1. Without consideration of the distribution of the  
 network induced delay, the control law of [16] is  
 employed, i.e.  $u(t) = -[3.75, 11.5]x(t)$ . Using  
 the criteria from [5, 15, 16, 23, 26] and two special  
 cases of Theorem 1 of this paper ( $\eta_0 = 0, \bar{\delta} = 0$   
 or 1), we obtain the respective maximum allow-  
 able transfer intervals (MATI), also called MADB  
 [23], which guarantees the asymptotic stability of  
 system (38) controlled over a network. The re-  
 sults are shown in Table I, which indicates that our  
 Theorem gives the same result as the existing best.
2. When the delay distribution is considered, the  
 control law of [16] is modified to  $u(t) = -$   
 $\delta(t)[3.75, -11.5]x(t - \eta_1(t)) - (1 - \delta(t))[3.75,$   
 $11.5]x(t - \eta_2(t))$ . Our MATI results are listed in  
 Table II. Existing methods in the open literature  
 cannot handle this case.

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**Case 1.2.** Consider the  $H_\infty$  performance of sys-  
 tem (39)-(40) under the given controller and the  $H_\infty$   
 disturbance attenuation level  $\gamma$ .

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Table I. MATI results without consideration of network characteristics.

Criterion	[16]	[26]	[23]	[5]
MATI	0.00045	0.0538	0.7805	0.8695
Criterion	Th.1 <sup>†</sup>	[15]	Th.1 <sup>‡</sup>	
MATI	0.8695	0.9410	0.9410	

<sup>†</sup> $\eta_0 = 0, \bar{\delta} = 0$ , finding the maximum  $\eta_2$ .

<sup>‡</sup> $\eta_0 = 0$  and  $\bar{\delta} = 1$ , finding the maximum  $\eta_1$ .

Table II. MATI  $\eta_2$  with consideration of network characteristics.

$\bar{\delta}$	$\eta_0 = 0.00$	$\eta_0 = 0.02$	$\eta_0 = 0.04$
	$\eta_1 = 0.02$	$\eta_1 = 0.04$	$\eta_1 = 0.06$
0	0.8695	0.8695	0.8695
0.2	0.9721	0.9720	0.9719
0.4	1.1224	1.1222	1.1217
0.6	1.3746	1.3741	1.3732
0.8	1.9440	1.9429	1.9411
1	$\infty$	$\infty$	$\infty$

Table III. MATI  $\eta_2$  under  $\gamma = 1$  and with consideration of network characteristics.

$\bar{\delta}$	$\eta_0 = 0.00$	$\eta_0 = 0.02$	$\eta_0 = 0.04$
	$\eta_1 = 0.02$	$\eta_1 = 0.04$	$\eta_1 = 0.06$
0	0.8695	0.8695	0.8695
0.2	0.9710	0.9677	0.9622
0.4	1.1197	1.1114	1.0976
0.6	1.3696	1.3539	1.3273
0.8	1.9340	1.9042	1.8539
1	$\infty$	$\infty$	$\infty$

1 1. When the distribution of the network induced delay is not considered, i.e.  $\bar{\delta} = 0$ , we obtain the minimum allowable  $\gamma = 1$  for  $\eta_1 = 0.8695$ .

3 2. When the distribution of the network induced delay is taken into account, we obtain the upper bound  $\eta_2$  under different lower bounds  $\eta_0$  and  $\eta_1$ , as shown in Table III. Again, existing methods in the open literature cannot handle this case.

5 From Tables I, II and III, we can draw the following general conclusions.

- 7 • When the distribution of the network induced delay is not considered, the result in this paper is as good as the existing best in the open literature.
- 9 • It is seen from Tables II and III that when  $\bar{\delta} \neq 0$ , i.e. when the distribution of the network induced delay is considered, less conservative results can

be obtained.

- 17 • For fixed  $\bar{\delta} (\bar{\delta} \rightarrow 0$  or  $1)$ ,  $\eta_2$  decreases as  $\eta_0$  or  $\eta_1$  increases. When  $\eta_0$  and  $\eta_1$  are fixed,  $\eta_2$  increases with the increase of  $\bar{\delta}$ . These results reflect the relationship between QoS and QoC. For example, the larger the probability of the network induced delay in the lower interval  $[\eta_0, \eta_1)$  (QoS), the larger the allowable network induced delay upper bound  $\eta_2$  (QoC).
- 19 • In Tables II and III, for any  $\eta_0 \leq \eta_1 \leq 0.8695$ , when  $\bar{\delta} \rightarrow 0$  we have  $\eta_2 = 0.8695$ . When  $\bar{\delta} \rightarrow 1$ , we have  $\eta_2 \rightarrow \infty$ , implying that most of the network induced delays  $\eta(t)$  fall into  $[\eta_0, \eta_1)$  and the probability of  $\eta(t) \in [\eta_1, \eta_2)$  trends to zero. Therefore, when  $\bar{\delta} \rightarrow 1$ , the effect of  $\eta_2$  on system stability is negligible.

## 6.2 Controller design

For controller design, we study the networked control problem for an uninterruptible power system (UPS). The objective is to control the PWM inverter over an IP based NCS network such that the output AC voltage is kept at the desired value while maintaining robustness against load disturbances. A UPS with 1 KVA is considered in this example. The continuous-time model can be obtained from the discrete model described in [3] at half-load operating point.

$$\dot{x} = \begin{bmatrix} 0.0449 & -0.093 & 0 \\ 0.1469 & -0.0906 & 0 \\ -6.6297 & 6.2635 & -4.2872 \end{bmatrix} x$$

$$+ \Delta A(t)x + \begin{bmatrix} 0.0238 \\ -0.0678 \\ -3.8693 \end{bmatrix} u(t)$$

$$+ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} w(t) \tag{41}$$

$$z(t) = [0.1 \ 0 \ 0]x(t) \tag{42}$$

where  $\|\Delta A\| \leq 0.01$ .

Two cases are considered below for controller design of the system.

**Case 2.1.** Let us design an  $H_\infty$  controller with or without consideration of the IP network characteristics. Assume that the disturbance attenuation level  $\gamma = 0.5$ .

Without consideration of the delay distribution of the NCS network ( $\bar{\delta} = 0$ ), using Theorem 2 and

Algorithm 1, we have the maximum allowable network delay  $\eta_2 = 1.0$ .

When the network characteristics are considered ( $\bar{\delta} = 0.5, \eta_0 = 0, \eta_1 = 0.2$ ), we have the maximum allowable delay  $\eta_2 = 1.7$ . The corresponding

$$X = \begin{bmatrix} 12.9 & 18.2 & 8.45 \\ 18.2 & 20.5 & 42.27 \\ 8.45 & 42.27 & 930.03 \end{bmatrix}$$

and  $V = [-137.6118, 273.698, -463.8711]$ , respectively.

To practically simulate the time-varying network delay, a stochastic delay table is established, in which the number of delay values  $\eta(t) \in [0, \eta_1]$  is  $N_1$ , and the number of delay values  $\eta(t) \in [\eta_1, \eta_2]$  is  $N_2$ , respectively. For given  $\bar{\delta}$ , it is required to have  $N_1:N_2 = \bar{\delta} : (1 - \bar{\delta})$  in order to model the distribution of the network induced delay.

With the delay bounds obtained above, the feedback controller gain  $K$  can be computed as  $K = [-14.1371, 2.6782, -0.0432]$ . Furthermore, different distributions of the network induced delay will give different control results.

**Case 2.2.** Let us design the  $H_\infty$  controller over an IP network described in [1] with  $\bar{\delta} = 0.5, \eta_1 = 0.0629$ , and  $\eta_2 = 0.7562$ . The purpose is to minimize the  $H_\infty$  index, i.e. to solve the problem (37).

Solving the optimization problem (37) using the LMI Toolbox yields the the minimum allowable value of  $\gamma = 0.23$  and  $K = [-35.2031, 1.9184, 0.0472]$ .

However, if the network characteristics are not considered, i.e.  $\bar{\delta} = 0$ , we have  $\gamma = 0.36$  and  $K = [-22.1321, 2.0866, 0.0223]$  for  $\eta_2 = 0.7562$ . The results are more conservative.

In summary, it is seen from the examples that the more information of the network characteristics is considered in NCS analysis and design, the less conservative results can be obtained. Therefore, the delay distribution based approach proposed in this paper is effective to improve the NCS control performance.

## VII. CONCLUSION

The disturbance attenuation problem for NCS over IP based networks has been investigated based on a Lyapunov–Krasovskii functional method. A delay distribution based stochastic control approach that links QoC and QoC is developed. Improved criteria for  $H_\infty$  performance analysis and  $H_\infty$  control synthesis have been derived using this approach together with a tighter

bounding technology to deal with cross terms. Simulation results have been given to demonstrate the effectiveness of the proposed approach.

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**Chen Peng** was born in China in 1972. He received his B.Sc., M.Sc. and Ph.D. from the Chinese University of Mining Technology in 1996, 1999 and 2002 respectively. He was a Post-doctoral Research Fellow at Nanjing Normal University.

From November 11 2004 to January 2005, he was a Research Associate at Hong Kong University. From July 2006 to August 2007, he was a Research Associate at Queensland University of Technology. He is currently a full associate Professor at Nanjing Normal University. His research interests include networked control systems, and fuzzy systems and delay systems.



**Dong Yue** was born in 1964 in Henan, China. He received the B.S. degree in mechanical engineering from the Guilin Institute of Electrical Engineering, Guilin, China, in 1985, the M.S. degree in applied mathematics from the Anhui University, Huainan, China, in 1991, and the Ph.D. degree in control engineering from the

South China University of Technology, Guangzhou, China, in 1995.

From June 1999 to September 1999, he was a research associate at City University of Hong Kong. From 2000 to 2001, he worked at Pohang University of Science and Technology as a senior scientist. From June to September, 2002, he was a research associate at Hong Kong University. From March 2004 to March 2005, he was a visiting scholar at Central Queensland University, Australia. From January, 2007 to March, 2007, he was a visiting scholar at Brunel University, USA. He is currently a professor of control theory and control engineering at Nanjing Normal University and director of Research Center for Information and Control Engineering Technology. His research interests include